1. We define the function

$$h(x) = (f(b) - f(a))g(x) - (g(b) - g(a))f(x)$$

Since f, g are continuous and differentiable on $[a, b]$, we not that h must be continuous and differentiable on $[a, b]$ too. We then note that:

$$\begin{aligned} xh(a) &= (f(b) - f(a))g(a) - (g(b) - g(a))f(a) \\ &= f(b)g(a) - f(a)g(a) - g(b)f(a) + g(a)f(a) \\ &= g(a)f(b) - g(b)f(a) \end{aligned}$$

$$\begin{aligned} h(b) &= (f(b) - f(a))g(b) - (g(b) - g(a))f(b) \\ &= f(b)g(b) - f(a)g(b) - g(b)f(b) + g(a)f(b) \\ &= g(a)f(b) - g(b)f(a) \end{aligned}$$

It should be clear that $h(a) = h(b)$. We then take note that

$$h'(x) = (f(b) - f(a))g'(x) - (g(b) - g(a))f'(x)$$

Since $h(a) = h(b)$, by Rolle's Theorem, there exist a $c \in (a, b)$ where $h'(c) = 0$, which gives $0 = (f(b) - f(a))g'(c) - (g(b) - g(a))f'(c)$ or $(f(b) - f(a))g'(c) = (g(b) - g(a))f'(c)$ as desired.