1. There exist an integer solution for $\frac{10x^2-4}{11}$.

   False. This is because it implies a $x \in \mathbb{N}$ where $10x^2 - 4 = 0$ in $\mathbb{F}_{11}$. In other words, $-x^2 = 4$ or $x^2 = -4 = 7$. However, $(\frac{7}{11}) = -(\frac{11}{7}) = -(\frac{2}{7})(\frac{2}{7}) = -1$. This contradicts 7 being a non-quadratic residue in $\mathbb{F}_{11}$.

2. Assume that $8n+3$ is prime and also assume there exist an $n \in \mathbb{Z}$ where $8n+3 \mid 10x^2 - 4$. This implies that $8n+3 \mid 5n-2$.

   True. We note that $10x^2 - 4 = (5x+2)(5x-2)$. Meanwhile, $(8n+3)\cdot(-5) + (5n+2)\cdot 8 = -40n - 15 + 40n + 16 = 1$. Hence, for any $n \in \mathbb{Z}$, $\gcd(8n+3, 5n+2) = 1$. By Euclid Lemma, $8n+3 \mid 5x-2$.

3. For all primes p, if $\nu_p(n) = 1$, $\nu_p(L_n) = \nu_p(n!)$ where $L_n = lcm(1, \cdots, n)$.

   False. If $n = 14$, $\nu_7(14) = 1$ but $\nu_p(L_{14}) = 1$ and $\nu_p(14!) = 2$.

4. Let p be a prime. $\Phi_p(a)$ is odd if and only if a is even.

   True. We note that $\Phi_p(x) = \frac{x^p-1}{x-1} = \sum_{i=1}^{p-1} x^i + 1$. Since $\Phi_p(a)$ is even, $\Phi_p(a) - 1 = \sum_{i=1}^{p-1} a^i$ is even. By contradiction, if we assume a is odd, then a^i is odd, and a sum of odd is odd, a contradiction. Hence, a must be even. For the converse, if a is even, all a^i are even and the sum of evens are even. Hence, $\sum_{i=1}^{p-1} a^i$ is even and $\sum_{i=1}^{p-1} a^i + 1 = \Phi_p(a)$ is odd.

5. There does not exist a ring R where for all $n \in \mathbb{N}$, for $x \in R$, $f(x) = x^n$ is an isomorphism.

   False. Consider $\mathbb{F}_2$. For any $n \in \mathbb{N}$, $f(0) = 0$ and $f(1) = 1$. Hence, f is an identity map thus is isomorphic in $\mathbb{F}_2$.

6. The ideal $(x, x+1) = \{xa + (x+1)b : a, b \in \mathbb{Z}[x]\}$ of $\mathbb{Z}[x]$ is a prime ideal.

   False. Note that $x(-1) + (x+1)(1) = 1$. Hence, since $1 \in (x, x+1)$, $(x, x+1) = \mathbb{Z}[x]$. The ideal is not proper, so it cannot be a prime ideal.

7. Let $\mathbb{F}_p, \mathbb{F}_{p^d}$ be fields. If there exist an $f(x) \in \mathbb{F}_p[x]$ where $f(x)$ has a root α in $\mathbb{F}_{p^d}$, then $\deg(f) \mid d$.

   True. By Proposition 9.21, there exist a ring homomorphim of $\mathbb{F}_p[x]/f(x) \to \mathbb{F}_p^d[x]$. We note that $|\mathbb{F}_p[x]/f(x)| = p^{\deg(f)}$ by Lemma 9.18. By Theorem 10.1 (d), this homomorphim implies that $\deg(f) \mid d$.

8. Let $m \geq 3$. For all primes $m \mid p-1$, there exist a root for $\Phi_m(x)$ in $\mathbb{F}_p$

   True. Since $p \geq m$, $m \mid p-1$ and there exist an element a in $\mathbb{F}_p$ where $o(a) = m$ by Proof of Theorem 10.7. From HW7 3 (b), this implies that $\Phi_m(a) = 0$ in $\mathbb{F}_p$.

9. Consider a set with two elements $A = \{a, b, c\}$. Let g be the identity map for A. Let be f where $a \mapsto b$, $b \mapsto a$, $c \mapsto c$. If $\circ$ denotes function composite, then $(\{g, f\}, \circ)$ is a cyclic group.

   True We take notice that $f \circ f = f^2 = g$, so $\{g, f\} = \{f^2, f\} = \langle f \rangle$.

10. Let $n \in \mathbb{Z}$ be odd and $n - 1 = u \cdot 2^k$ where u is odd and $k = \nu_2(n-1)$. If $a \in (\mathbb{Z}/m\mathbb{Z}^\times)$ yields $a^{u \cdot 2^d} = -1$ in $\mathbb{Z}/m\mathbb{Z}$ for $d \leq k - 1$, then m is not prime.

    False. Since $d \leq k - 1$, we can square $a^{u \cdot 2^d}$ until we get $a^{u \cdot 2^k} = a^{n-1} = 1$. This is simply Fermat's Little Theorem, and does not imply anything about whether n is prime or not. Instead, what Miller Rabbin tests is if $a^{u \cdot 2^d} \neq -1$ and -1 implies n is probably prime instead.