Probability and statistics

Assignment - 1

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① We can break this question as :
No digit repeating, one digit repeating and Two digit repeating.

Case I :- Pick any 5 and permute them in
$^9P_5 = 9 \times 8 \times 7 \times 6 \times 5 = 15120$ ways.

Case II :- choose 4 from 9 and then choose one number from 4 ~~an~~ choose numbers and arrange them, can be done in :

$$^9P_4 \times {}^4C_1 \times \frac{5!}{2!} = 7560 \times 4 = 30240 \text{ ways.}$$

Case III :- choose 3 from 9 and then choose two from that 3 chousen numbers, can be done in $^9C_3 \times {}^3C_2 \times \frac{5!}{2! \times 2!} = 7560$

Total no. of ways $= 15120 + 30240 + 7560$
$= 52920.$

② Let the names of the committee be committee 1, committee 2, committee 3,
Committee 1: choose 3 from 12 in $^{12}C_3$ ways
Committee 2: choose 4 from remaining 9 in $^9C_4$ ways
Committee 3: choose 5 from remaining 5 in $^5C_5$ ways

⇒ Total no. of ways $= {}^{12}C_3 \times {}^9C_4 \times {}^5C_5$
$= 27720$ ways.

③ Considering french and English delegates are to sit together as a single entity, so there are 9 entities which can be arranged in 9! ways & 2! ways for arranging french & English delegates.

for the second cond^{n}, we calculate the no. of ways in which Russia & US delegates can sit together & subtract it from the total, can be done in $8! \times 2! \times 2!$ ways.

$\Rightarrow$ Total $= 9! \times 2! - 8! \times 2! \times 2! = 564480$.

(4) Let 'R' denotes Red balls & 'G' denotes Green balls & 'B' denotes Blue balls.

With Replacement :-

$S_1 = \{RR, RG, RB, GR, GG, GB, BR, BG, BB\}$

$|S_1| = 9$.

$S_2 \Rightarrow$ Without Replacement :-

$S_2 = \{RG, RB, GR, GB, BR, BG\}$

$|S_2| = 6$.

(5) E: Event that sum of the dice is odd.

$E = \{(1,2)(1,4)(1,6)(2,1)(2,3)(2,5)(3,2)(3,4)(3,6)(4,1)(4,3)(4,5)(5,2)(5,4)(5,6)(6,1)(6,3)(6,5)\}$.

$\Rightarrow |E| = 18$

F: Event that atleast one of the dice lands on 1.

$F = \{(1,1)(1,2)(1,3)(1,4)(1,5)(1,6)(2,1)(3,1)(4,1)(5,1)(6,1)\}$

$\Rightarrow |F| = 11$

G: Event that the sum is 5.

$G = \{(1,4)(2,3)(3,2)(4,1)\}$

$\Rightarrow |G| = 4$

EF: Event ~~[illegible]~~ of $(E \cap F)$.

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$EF = E \cap F = \{(1,2)(1,4)(1,6)(2,1)(4,1)(6,1)\}$

$\Rightarrow |EF| = 6.$

$E \cup F = \{(1,1)(1,2)(1,3)(1,4)(1,5)(1,6)(2,1)(2,3)$
$(2,5)(3,2)(3,4)(3,6)(3,1)(4,1)(4,3)(4,5)$
$(5,1)(5,2)(5,4)(6,1)(6,3).(6,5)\}$

$\Rightarrow |E \cup F| = 23.$

$F \cap G = \{(1,4)(4,1)\} \Rightarrow |F \cap G| = 2$

$E \cap \bar{F} = \{(2,3)(2,5)(3,2)(3,4)(3,6)(4,3)(4,5)$
$(5,2)(5,4)(5,6)(6,3)(6,5)\}$

$\Rightarrow |E \cap \bar{F}| = 12$

$E \cap F \cap G = EFG = \{(1,4)(4,1)\}$

$\Rightarrow |EFG| = 2.$

6. (i) As, we have 5 components and each of these 5 components have 2 choices; either working or failed.

$\Rightarrow$ No. of outcomes $= 2 \times 2 \times 2 \times 2 \times 2 = 2^5 = 32$ outcomes

(ii) To simplify assuming five components as 5 digit numbers in which '1' used for stating working & 0 $\rightarrow$ Not working.

A: 1 & 2 both are working

so, $A = \{$ (11000, 11001, 11010, 11011, 11100, 11101, 11110, 11111 $\}$ $\Rightarrow |A| = 8.$

B: 3 & 4 both are working

so, $B = \{$ 00110, 00111, 01110, 01111, 10110, 10111, 11110, 11111 $\}$ $\Rightarrow |B| = 8.$

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C : 1, 3 & 5 all are working :
$C = \{ 10101, 10111, 11101, 11111 \}$
$\Rightarrow |C| = 4.$

(iii) Since, two components are failed, we are left with 3 components each having 2 choices (working or failed)
$\Rightarrow 2 \times 2 \times 2 = 2^3 = 8$ outcomes.

7 (i) Sample space (S) = {(1,g) (0,g) (1,f) (0,f) (1,s) (0,s)}
(ii) A : Serious condition.
So, $A = \{ (1,s) (0,s) \} \Rightarrow |A| = 2$
(iii) B : Uninsured patient
So, $B = \{ (0,g) (0,f) (0,s) \} \Rightarrow |B| = 3.$
(iv) $(\overline{B \cup A}) = \{ (g,1) (f,1) (s,1) (s,0) \}$
$\Rightarrow |\overline{B \cup A}| = 4$

8 $P(A) = 0.3, \quad P(B) = 0.5.$
(i) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$= 0.3 + 0.5 - 0$ (as mutually exclusive)
$= 0.8.$

(ii) $P(A \cap \overline{B}) = P(A) = 0.3$

(iii) $P(A \cap B) = 0.$

9 Let 'S' be the Sample space
'N' be the no. of student's who wear Necklace
'R' be the no of students who wear Rings.
So,
~~$P(\bar{S}) = 0.60$~~, $P(S) = 0.40, P(R) = 0.20, P(N) = 0.30$
$P(\overline{R \cup N}) = 0.60$

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(i) A ring or a Necklace

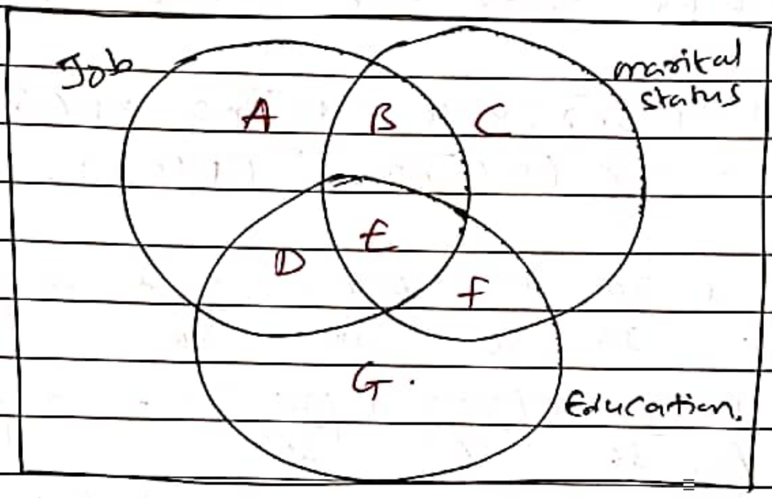
$P(R \cup N) = 1 - P(\overline{R \cup N}) = 1 - 0.60 = 0.40$

(ii) A ring and a Necklace.

$P(R \cap N) = P(R) + P(N) - P(R \cup N)$

$= 0.20 + 0.30 - 0.40 = 0.10$

(10)



Let us name each part in Venn Diagram.

So, Given 1000 Subscribers:

So, $A+B+C+D+E+F+G = 1000$. ——— (i)

Also, we can say that

$A+B+D+E = 312$	$D+E = 42$
$B+C+E+F = 470$	$E+F = 147$
$D+E+F+G = 525$	$B+E = 86$, $E = 25$.

By putting $E=25$ in all other equations we got, $B=61$, $F=122$, $D=17$, $G=361$, $C=262$, $A=209$.

On adding all these derived values i.e, $A+B+C+D+E+F+G = 1057$ which is not equal to eq^2 (i). So we can say that the numbers reported in the study must be INCORRECT.

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(11) Probability of 5 or 7 occuring as a sum.

$P(5) = \{(1,4)\ (2,3)\ (3,2)\ (4,1)\}$

$P(7) = \{(1,6)\ (2,5)\ (3,4)\ (4,3)\ (5,2)\ (6,1)\}$

P(sum is neither 5 nor 7).

$\Rightarrow P(5) = \frac{4}{36},\ P(7) = \frac{6}{36},\ P(\overline{5 \cap 7}) = \frac{26}{36}$

Probability that the sum of dies is 5 first.

$\Rightarrow P(5) + P(\overline{5\cap 7}) \times P(5) + P(\overline{5\cap 7}) \times P(\overline{5 \cap 7}) \times P(5) \times \cdots \times (P(\overline{5\cap 7}))^n (P(5))$.

$= \frac{4}{36} + \frac{26}{36} \times \frac{4}{36} + \left(\frac{26}{36}\right)^2 \times \frac{4}{36} + \cdots + \left(\frac{26}{36}\right)^n \left(\frac{4}{36}\right)^1$

$= \frac{4}{36}\left[1 + \frac{26}{36} + \left(\frac{26}{36}\right)^2 + \left(\frac{26}{36}\right)^3 + \cdots + \left(\frac{26}{36}\right)^n\right]$

$\cancel{= \frac{4}{36}\left[\left(\frac{26}{36}\right)\right]}$ This is an infinite GP with $r = 26/36$.

$\Rightarrow S_n = \frac{1}{1-r}$ when $n \to \infty$.

$= \frac{4}{36}\left[\frac{1}{1 - 26/36}\right] = \frac{4}{36} \times \frac{36}{10} = \boxed{\frac{2}{5}}$.

(12) ~~Let E: Event that atleast one dice lands on 6.~~

~~$E = \{(1,6)(2,6)(3,6)(4,6)(5,6)(6,6)(6,1)(6,2)(6,3)(6,4)(6,5)\}$~~

$\Rightarrow |E| = 11$.

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(12) F: Dice lands on different numbers.

So, $P(F) = \frac{30}{36}$.

E: Atleast one dice lands on '6'.

$P(E) = \{(1,6)\ (2,6)\ (3,6)\ (4,6)\ (5,6)\ (6,6)\ (6,1)$
$(6,2)\ (6,3)\ (6,4)\ (6,5)\}$.

~~P(E) =~~

So, $P\left(\frac{E}{F}\right) = \frac{P(E \cap F)}{P(F)}$

$= \frac{10/36}{30/36} = \frac{1}{3}$.

(13) Let's assume that the seat assigned to the 5th person is free when the last person enters the train. Now, if the seat assigned to the 5th person is free when the last person enters then it was also free when the 5th person boarded, so 5th person would have taken it then.

Hence, the seat assigned to the 5th person can't be free when the last person enters. Simillarly, the seat for ith person to board can't be free when the last person boards, bcz if it would have been free then the ~~2nd person~~ ith person would have taken it.

So,

when the last person boards, the only possibilities for empty seats are the correct seat or the seat assigned to the first person. Hence, the probability that the last person gets to sit in his assigned seat is $\frac{1}{2}$.

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(14) Case I : 'X' doesn't ask about 'Y'.
Probability of 'X' passing the exam is 2/3
⇒ X is Happy in 2/3 of the cases.

Case II : 'X' asks about 'Y'.
→ Y has failed the exam.
P(Y failed) = 1/3.
⇒ X is Happy in 1/3 of the cases.
→ Y has passed the exam.
Now, probability of 'X' passing the exam given that 'Y' has already passed is 1/2.

So, we can conclude that 'X' has NOT mistaken in his calculations and he will be more happy when he doesn't ask about Y.
Relation with Monty Hall :—
P(failure) is higher than P(success) in Monty Hall and we know that we should always switch so we can go ~~from~~ either from success to failure or failure to success. As, our chances of failure are higher on the initial choose, so after switching the chances of failure will become our chances of ~~failure~~ success.

(15) Let 'r' denotes the no. of red ~~balls~~ pen and 'b' ~~denotes~~ the no. of blue

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(15) Let 'r' denotes the no. of red pens.

(i) 'b' denotes the no. of blue pens.

So, Probability of both red $= \frac{{}^{r}C_2}{{}^{r+b}C_2} = \frac{1}{2}$

So, $\frac{r(r-1)}{(r+b)(r+b-1)} = \frac{1}{2}$

$\Rightarrow 2r^2 - 2r = r^2 + rb - r + rb + b^2 - b.$

$\Rightarrow 2r^2 - r^2 - 2r + r \mp 2rb \mp b^2 + b = 0.$

$\Rightarrow r^2 - r \mp 2rb \mp b^2 + b = 0.$

$\Rightarrow r^2 - r(1+2b) + (b^2 \mp b) = 0$

Solving for 'r':

$$r = \frac{-(-(1+2b)) \pm \sqrt{(1+2b)^2 - 4(-b^2 \mp b)}}{2}$$

$$= \frac{1+2b \pm \sqrt{1+4b+4b^2+4b^2 \mp 4b}}{2}$$

$$= \frac{1+2b \pm \sqrt{1+8b^2}}{2}$$

So, for minimum no. of pen we should choose as minimum no. of blue pen as possible to get a Natural no. red pen. As pen cannot be ~~Float~~ fraction or negative.

Case 1: $b = 1$, $r = \frac{3 \pm 3}{2} \rightarrow r = 0$ (not possible as $r \geq 2$)

$\rightarrow r = 3$

$\Rightarrow$ No. of red pens $= 3$, No. of blue pens $= 1$.

So, Total no. of minimum pens $= 3 + 1 = 4$

(ii) Blue pens are even then $b = 2x$ where $x \in 1, 2, 3 \ldots$

So, $r = \frac{1 + 4x \pm \sqrt{1 + 32x^2}}{2}$

~~$x = 1$~~

~~$r = \frac{5 \pm \sqrt{33}}{2}$~~

~~r is fraction~~

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when $n = 1$, since r is fractional

$r = \frac{5 \pm \sqrt{33}}{2}$, number $\Rightarrow$ ~~n~~ $\neq$ Incorrect

when $n = 2$, since r is fractional

$r = \frac{9 \pm \sqrt{129}}{2}$, number $\Rightarrow$ Incorrect

when $n = 3$,

$r = \frac{13 \pm \sqrt{289}}{2} = \frac{13 \pm 17}{2}$

$\therefore r = 15$ ~~or~~ $r = -2$ (Not possible).

Thus,

minimum no. of red pens when blue pens are even is 15.

No. of blue pens $= 2 \times 3 = 6$.

Therefore,

Minimum no. of pens when blue pens are ~~not~~ even is $\boxed{21}$.