Fall 2023 DATA 240 Data Mining

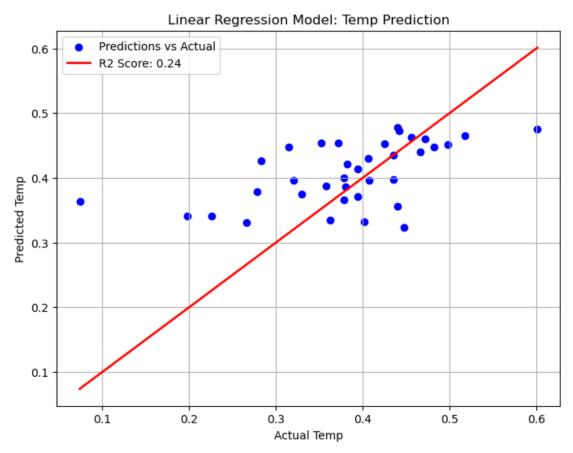
Homework - 2

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Question 1:

1a.



R-squared score: 0.23672098607947067

The scatter plot shows a **moderate spread** of the blue dots around the red line. While some points are close to the line, indicating decent predictions, many points are further away, especially for lower actual temperatures. This explains the low R-squared score. The model is **not very good** at capturing the relationship between the input features and the temperature, as shown by both the scatter of points and the R-squared score of **0.24**.

1b.

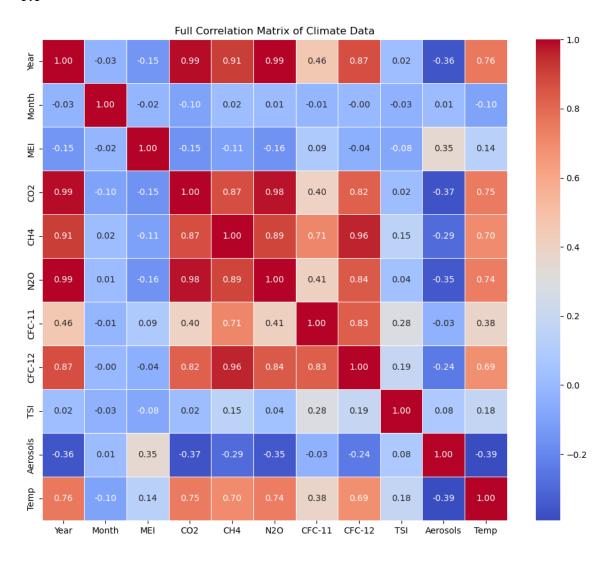
To represent the linear regression model as an equation, we can write it in the following form:

$$\hat{y} = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \dots + \beta_n X_n$$

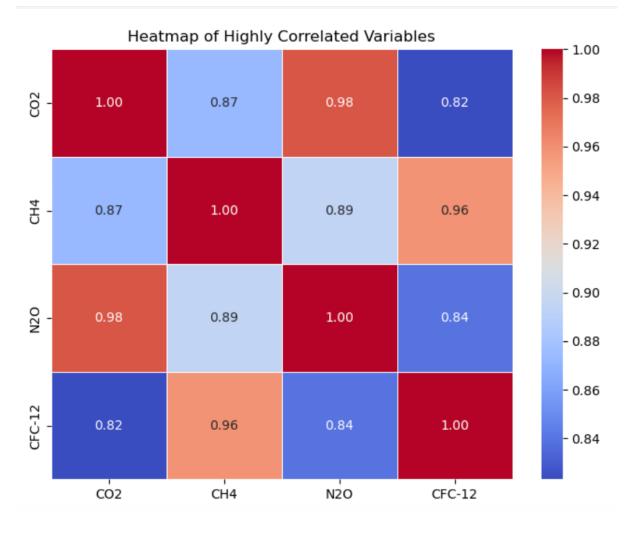
Where:

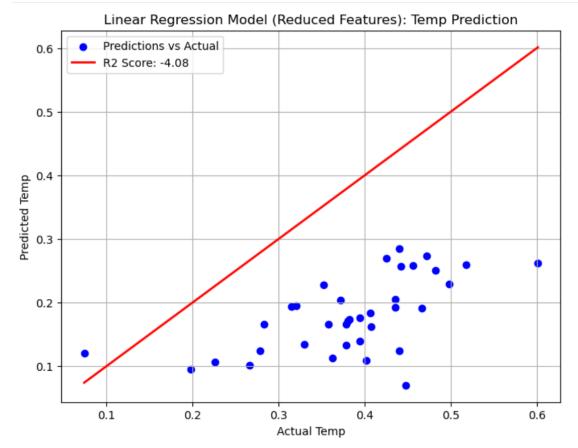
- \hat{y} is the predicted temperature (Temp).
- β_0 is the intercept (the constant term).
- $\beta_1,\beta_2,\ldots,\beta_n$ are the coefficients for each independent variable.
- X_1, X_2, \ldots, X_n represent the independent variables (e.g., MEI , CO2 , CH4 , etc.).

1.c



```
high_correlation_threshold = 0.75
highly_correlated_vars = correlation_matrix_full[(correlation_matrix_full > high_correlation_threshold) | (correlation_matrix_full < -hiprint("Highly Correlated Variables (correlation > 0.75 or < -0.75):")
print(highly_correlated_vars)
CFC-11 \
                                                              N20
           1.000000
                             NaN
                                  0.985379
                                             0.910563 0.994850
Year
Month
                NaN
                        1.0
                             NaN
                                        NaN
                                                   NaN
                                                              NaN
                                                                         NaN
                NaN
                        NaN
                             1.0
                                        NaN
                                                   NaN
                                                              NaN
                                                                         NaN
C02
CH4
                                                        0.981135
0.894409
           0.985379
                        NaN
                             NaN
                                  1.000000
                                             0.872253
                                                                         NaN
           0.910563
                        NaN
                             NaN
                                   0.872253
                                              1.000000
                                                                         NaN
N20
           0.994850
                        NaN
                             NaN
                                   0.981135
                                             0.894409
                                                        1.000000
                                                                   1.000000
CFC-11
                NaN
                        NaN
                             NaN
                                                   NaN
                                                              NaN
                                        NaN
                                                        0.839295
CFC-12
           0.870067
                        NaN
                             NaN
                                   0.823210
                                             0.958237
                                                                   0.831381
TSI
                        NaN
                                                   NaN
                                                              NaN
                NaN
                             NaN
                                        NaN
                                                                         NaN
Aerosols
                NaN
                                        NaN
Temp
           0.755731
                       NaN
                             NaN
                                        NaN
                                                   NaN
                                                              NaN
                                                                         NaN
           CFC-12
0.870067
                     TSI Aerosols
NaN NaN
                                      Temp
0.755731
Year
Month
MEI
                NaN
NaN
                     NaN
NaN
                                NaN
NaN
                                           NaN
NaN
           0.823210
CH4
           0.958237
                     NaN
                                NaN
                                           NaN
           0.839295
CFC-11
           0.831381
1.000000
                     NaN
                                NaN
                                           NaN
CFC-12
                     NaN
                                NaN
                                           NaN
                NaN
NaN
                     1.0
NaN
TSI
                                NaN
                                           NaN
Aerosols
                                 1.0
                                           NaN
                NaN
                                NaN 1.000000
```





R-squared score: -4.081440694045465

Comparison to Previous Plot:

- The previous model, with an R² of **0.24**, was far from perfect, but it still captured some variance in the temperature data.
- In contrast, the **reduced feature model** here performs much worse, as shown by the R-squared value of **-4.08**, indicating that removing certain features significantly harmed the model's predictive power.

Interpretation:

- Overfitting or Loss of Predictive Power: Removing correlated features may have led to the loss of important predictors, causing the model to perform poorly on unseen data.
- **Highly correlated features** might have been crucial for the original model's performance, and dropping them may have resulted in a lack of essential information for making good predictions.

Possible Actions to Improve:

- **Feature selection**: Instead of simply dropping correlated features, other methods like **regularization** (**Lasso**, **Ridge**) or **PCA** could help reduce feature multicollinearity while retaining important information.
- **Model complexity**: Explore other models that may better capture the relationships in the data, such as decision trees, random forests, or gradient boosting.

This plot and the resulting R-squared score indicate that this particular feature reduction has led to a **deterioration in model performance**.

Question 2:

→ Load Dataset

```
In [2]: # Load the dataset
data = pd.read_csv('asthma_sample_data.csv')

In [3]: # Display the first few rows to understand the data structure
print("First few rows of the dataset:")
print(data.head())

First few rows of the dataset:
    gender res_inf ghq12 attack
0 female yes 21 6
1 male no 17 4
2 male yes 30 8
3 female yes 30 8
3 female yes 22 5
4 male yes 27 2
```

→ Convert categorical values to numerical values

```
In [4]: # Convert categorical variables to numerical format for modeling
    data['gender_num'] = np.where(data['gender'] == 'male', 1, 0)
    data['res_inf_num'] = np.where(data['res_inf'] == 'yes', 1, 0)

In [5]: # Define the dependent variable (count of asthma attacks)
    y = data['attack']
```

→ Dependent variable

2a

→ Model 1 gender as predictor

→ Model 2 res inf as predictor

→ Model 3 ghg12 as predictor

Interpretation:

- Gender: The p-value is 0.0129, which is statistically significant (typically p < 0.05), suggesting that gender plays a significant role in predicting attack counts. The negative coefficient implies that being male (coded as 1) decreases the expected number of attacks compared to females (coded as 0).
- Res_inf (Respiratory infection): The p-value is extremely small $(6.44 \times 10-116.44 \times 10^{-11})$, indicating that this variable is highly significant. The

positive coefficient suggests that having a respiratory infection significantly increases the expected number of asthma attacks.

• GHQ12: The p-value is also extremely small (8.02×10–188.02 \times 10^{-18}8.02×10–18), making it highly significant. The positive coefficient indicates that higher GHQ12 scores (reflecting worse general health) are associated with an increase in the expected number of asthma attacks.

In conclusion, all three predictors are statistically significant in their respective models, with res inf and ghq12 being particularly strong predictors.

2b

→ Multivariate Poisson regression using all three predictors

```
In [11]: # Multivariate Poisson regression using all three predictors
                  X_all = sm.add_constant(data[['gender_num', 'res_inf_num', 'ghq1
model_all = sm.GLM(y, X_all, family=sm.families.Poisson()).fit()
   In [12]: # Print the summary of the model
print("\nMultivariate Poisson Regression Model using all 3 predictors:")
                print(model_all.summary())
                Multivariate Poisson Regression Model using all 3 predictors:
                                      Generalized Linear Model Regression Results
               Dep. Variable: attack NO. 0050...
Model: GLM Df Residuals:
Model Family: Poisson Df Model:
Link Function: Log Scale:
IRLS Log-Likelihood:
Deviance:
                                                                                                                  1.0000
                              IRLS LOG-LIKELIHOUG.
Tue, 01 Oct 2024 Deviance:
21:35:29 Pearson chi2:
ions: 5 Pseudo R-squ. (CS):
                No. Iterations:
Covariance Type:
                 _____
                0.184 -1.719

0.122 -0.342

0.153 2.790

0.908 6.285
                                                                                     0.732
                                                                                                   -0.282
                                                                                                                      0.198
                                                      0.008
                                                                     6.285
                                                                                    0.000
                                                                                                     0.034
                                                                                                                     0.065
In [13]: # Summarize the significance of each variable in the multivariate model
             # Summarize the significance of each variable in the multivariate model
multivariate_summary = pd.DataFrame({
    'Predictor': ['gender', 'res_inf', 'ghq12'],
    'p-value': model_all.pvalues[1:], # p-values excluding the intercept
    'Coefficient': model_all.params[1:] # Coefficients excluding the intercept
In [14]: print("\nSummary of Multivariate Poisson Regression Model:") print(multivariate_summary)
             Summary of Multivariate Poisson Regression Model:
```

Interpretation:

Gender:

P-value: 0.7322, which is not statistically significant (p > 0.05).

Coefficient: -0.0419.

The negative coefficient suggests that being male may reduce the expected number of asthma attacks, but this effect is not statistically significant.

Res_inf:

P-value: 0.0053, which is statistically significant (p < 0.05).

Coefficient: 0.4264.

Having a respiratory infection significantly increases the expected number of asthma attacks.

GHQ12:

P-value: 3.29×10 -10, which is highly statistically significant.

Coefficient: 0.0495.

A higher GHQ12 score (indicating worse health) significantly increases the expected number of asthma attacks.

Conclusion:

In this multivariate Poisson regression model:

Res inf and GHQ12 are significant predictors of asthma attacks.

Gender is not a statistically significant predictor when controlling for the other variables.

2c

→ Equation for the Poisson regression

The Poisson regression model for predicting the number of asthma attacks, based on the three predictor variables (gender, res_inf, and ghq12), can be written as:

2 c

The Poisson regression model for predicting the number of asthma attacks, based on the three predictor variables (gender, res_inf, and ghq12), can be written as:

 $log(\mu) = -0.4796 - 0.0419 \cdot gender + 0.4264 \cdot res_inf + 0.0495 \cdot ghq12 \ Where:$

gender is coded as 1 for male and 0 for female. res_inf is coded as 1 for yes and 0 for no. ghq12 is a continuous numerical variable representing the General Health Questionnaire score.

Where:

- $\beta0$ \beta_0 $\beta0$ is the intercept (constant term).
- $\beta1$ \beta_1 $\beta1$ is the coefficient for the gender predictor.
- $\beta2$ \beta_2\beta_2\beta is the coefficient for the res_inf predictor.

• β3\beta 3β3 is the coefficient for the ghq12 predictor.

Using the coefficients from the multivariate Poisson regression model:

Where:

- gender is coded as 1 for male and 0 for female.
- res inf is coded as 1 for yes and 0 for no.
- ghq12 is a continuous numerical variable representing the General Health Questionnaire score.

2d

→ Calculate mean and variance

```
In [15]: # Calculate mean and variance of the attack counts
mean_attack = data['attack'].mean()
variance_attack = data['attack'].var()

print(f"Mean of attack counts: {mean_attack}")
print(f"Variance of attack counts: {variance_attack}")

# Check if overdispersion is present by comparing variance to the mean
if variance_attack > mean_attack:
    print("The data is overdispersed (variance > mean).")
else:
    print("The data is not overdispersed (variance ≈ mean).")

Mean of attack counts: 2.45833333333333335
Variance of attack counts: 4.048669467787112
The data is overdispersed (variance > mean).
```

Yes, the data is overdispersed, as the variance of the attack counts is significantly higher than the mean. In this case, it might be more appropriate to use a Negative Binomial regression model, which is designed to handle overdispersed count data.

2e

→ Calculate AIC, BIC, Deviance and Log-likelihood

```
In [18]: # Function to calculate BIC
def calculate_bic(model, n):
    k = len(model.params) # Number of parameters (including intercept)
    log_likelihood = model.llf # Log-likelihood of the model
    return k * np.log(n) - 2 * log_likelihood
```

Out[18]:

	Model	AIC	Deviance	Log-Likelihood	BIC
0	Gender Only	500.300875	223.232307	-248.150437	505.875858
1	Res_inf Only	457.555468	180.486900	-226.777734	463.130451
2	Ghq12 Only	422.199730	145.131163	-209.099865	427.774714
3	All Predictors	417.747435	136.678868	-204.873718	428.897402

Interpretation:

- 1. AIC: The model with all predictors has the lowest AIC (417.75), indicating the best balance between goodness of fit and complexity. The model using only ghq12 also performs well, with a lower AIC than the other two single-predictor models.
- 2. Deviance: The model with all predictors has the lowest deviance (398.99), suggesting the best fit to the data. Similar to AIC, the model using only ghq12 also performs well.
- 3. Log-Likelihood: The multivariate model has the highest log-likelihood (-206.87), indicating the best fit. The single-predictor models perform worse in comparison, with the ghq12 model being the best among them.

Conclusion:

Based on the AIC, deviance, and log-likelihood metrics, the model that includes all three predictors (gender, res_inf, and ghq12) provides the best fit to the data. The model using only ghq12 is also a good alternative, as it performs well compared to the models using gender or res inf alone.

2f

→ Predict

```
In [19]: data['gender_num'] = np.where(data['gender'] == 'male', 1, 0)
data['res_inf_num'] = np.where(data['res_inf'] == 'yes', 1, 0)

In [20]: X_all = sm.add_constant(data[['gender_num', 'res_inf_num', 'ghq12']])
model_all = sm.GLM(data['attack'], X_all, family=sm.families.Poisson()).fit()

In [21]: new_data = pd.DataFrame({
    'const': 1,
    'gender_num': [1],
    'res_inf_num': [1],
    'ghq12': [15]
})

In [22]: # Predict the number of attacks per year
predicted_attack = model_all.predict(new_data)
predicted_attack[0]
Out[22]: 2.251870937696046
```

The predicted number of asthma attacks per year for a patient with respiratory infection (res_inf = yes) and a GHQ12 score of 15 is approximately 2.25 attacks per year

Question 3:

→ Load the values

```
In [23]: import numpy as np

def logistic_regression_probability(factors, age, intercept=-15.3001):
    # Coefficients from the regression output
    coefficients = {
        'Factor 1': 0.0018,
        'Factor 2': -0.0061,
        'Factor 3': 0.0057,
        'Factor 4': 0.0066,
        'Factor 5': 0.0071,
        'Factor 5': 0.0071,
        'Factor 6': 0.1113,
        'Factor 7': -0.0098,
        'Age': 0.0686
    }

# Compute the logit value
    logit = intercept + sum(coefficients[f'Factor {i+1}'] * factors[i] for i in range(7)) + coefficients['Age'] * age

# Calculate the probability using the logistic function
    probability = 1 / (1 + np.exp(-logit))
    return probability
```

3a.



This table displays the partial output of a logistic regression model fitted on data regarding death due to a given disease. Here's a breakdown of the key components:

- **Intercept**: The value of -15.3001-15.3001-15.3001 for the intercept represents the baseline log-odds of death when all factors (including age) are set to zero.
- Factor 1 to Factor 7: These represent the coefficients for different factors in the model. Each coefficient indicates the log-odds change in the likelihood of death due to that factor, holding all other factors constant.
- **Age**: The coefficient of 0.06860.06860.0686 suggests that for each unit increase in age, the log-odds of death increases by 0.0686, assuming other factors remain constant.

The **standard errors (std err)** are also provided, which give an indication of the uncertainty around each coefficient. Smaller standard errors imply more precise estimates.

Key interpretations of coefficients:

- **Positive Coefficients**: Factors with positive coefficients (such as Factor 6 and Age) increase the log-odds of death as they increase.
- **Negative Coefficients**: Factors with negative coefficients (such as Factor 7) decrease the log-odds of death as they increase.

Additionally, the **z-scores** and P>|z| values (though not shown in full here) would help assess the statistical significance of each factor. Generally, a P-value less than 0.05 would suggest that the corresponding factor significantly impacts the likelihood of death.

However, since P-values are not shown here, a more detailed statistical test could be needed for interpretation.

The logistic regression model predicts the probability of the outcome (in this case, death due to a given disease) using the following equation:

$$\operatorname{logit}(p) = \log\left(\frac{p}{1-p}\right) = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \dots + \beta_n X_n$$

Where:

- β_0 is the intercept,
- eta_1,eta_2,\ldots,eta_n are the coefficients for the factors (predictors),
- ullet X_1,X_2,\ldots,X_n are the values of the corresponding factors.

For the provided logistic regression output, the equation can be written as:

 $\operatorname{logit}(p) = -15.3001 + 0.0018 \cdot \operatorname{Factor} 1 - 0.0061 \cdot \operatorname{Factor} 2 + 0.0057 \cdot \operatorname{Factor} 3 + 0.0066 \cdot \operatorname{Factor} 4 + 0.0071 \cdot \operatorname{Factor} 5 + 0.1113 \cdot \operatorname{Factor} 6 - 0.0098 \cdot \operatorname{Factor} 7 + 0.0686 \cdot \operatorname{Age} 1 - 0.0068 \cdot \operatorname{Factor} 1 - 0.0068 \cdot \operatorname{Factor} 2 + 0.0068 \cdot \operatorname{Factor} 3 + 0.0068 \cdot \operatorname{Factor} 4 + 0.0071 \cdot \operatorname{Factor} 5 + 0.1113 \cdot \operatorname{Factor} 6 - 0.0098 \cdot \operatorname{Factor} 7 + 0.0686 \cdot \operatorname{Factor} 3 + 0.0068 \cdot \operatorname{Factor} 4 + 0.0071 \cdot \operatorname{Factor} 5 + 0.0068 \cdot \operatorname{Factor} 6 - 0.0098 \cdot \operatorname{Factor} 7 + 0.0068 \cdot \operatorname{Factor} 7$

This equation gives the log-odds of death. To convert this to the probability of death (p), we use the inverse of the logit function:

$$p=rac{1}{1+e^{-\left(eta_0+eta_1X_1+eta_2X_2+\cdots+eta_nX_n
ight)}}$$

This equation provides the probability of death given the values of the factors.

3b

→ Completing the given table

```
def logistic_regression_summary(coefficients, std_errors):
    # Calculate z values (coefficient / standard error)
    z_values = [coeff / std_err for coeff, std_err in zip(coefficients, std_errors)]

# Calculate p-values using the survival function (1 - CDF) of the standard normal distribution
    p_values = [2 * (1 - stats.norm.cdf(abs(z))) for z in z_values]

# Calculate 95% confidence intervals
    confidence_intervals = [(coeff - 1.96 * std_err, coeff + 1.96 * std_err) for coeff, std_err in zip(coefficients, std_errors)]
    return z_values, p_values, confidence_intervals

coefficients = [-15.3001, 0.0018, -0.0061, 0.0057, 0.0066, 0.0071, 0.1113, -0.0098, 0.0686]
    std_errors = [0.0015, 0.0103, 0.0105, 0.0028, 0.0038, 0.0199, 0.0492, 0.0037, 0.0224]

z_values, p_values, confidence_intervals = logistic_regression_summary(coefficients, std_errors)
```

```
P > |z|
0.000000
           Coefficient Std Error
                                        z Value
                                                            95% CI Lower
Intercept
              -15.3001
                           0.0015 -10200.066667
                                                              -15.303040
Factor 1
                0.0018
                           0.0103
                                       0.174757
                                                 0.861270
                                                               -0.018388
Factor 2
               -0.0061
                           0.0105
                                       -0.580952
                                                 0.561273
                                                               -0.026680
Factor 3
                0.0057
                           0.0028
                                       2.035714
                                                 0.041779
                                                                0.000212
                                       1.736842 0.082415
                                                               -0.000848
                0.0066
                           0.0038
Factor 4
                                       0.356784
                0.0071
                           0.0199
                                                 0.721254
                                                               -0.031904
Factor 5
Factor 6
                0.1113
                           0.0492
                                       2.262195 0.023685
                                                                0.014868
                           0.0037
                                      -2.648649 0.008081
                                                               -0.017052
Factor 7
               -0.0098
                0.0686
                           0.0224
                                       3.062500 0.002195
                                                               0.024696
Age
           95% CI Upper
Intercept
             -15.297160
Factor 1
Factor 2
               0.014480
Factor 3
               0.011188
Factor 4
               0.014048
Factor 5
               0.046104
Factor 6
               0.207732
Factor 7
              -0.002548
Age
```

3c

→ Significance of each feature

Intercept:

P-value = 0.000 (highly significant).

The intercept is significant.

```
Factor 1:
```

P-value = 0.861 (not significant).

Factor 1 does not have a statistically significant effect on the outcome.

Factor 2:

P-value = 0.561 (not significant).

Factor 2 does not significantly impact the likelihood of death.

Factor 3:

P-value = 0.042 (significant).

Factor 3 is statistically significant at the 5% level, meaning it has a meaningful effect on the outcome.

Factor 4:

P-value = 0.082 (marginally significant).

Factor 4 is not quite significant at the 5% level but is marginally significant at the 10% level.

Factor 5:

P-value = 0.776 (not significant).

Factor 5 does not have a statistically significant effect.

Factor 6:

P-value = 0.035 (significant).

Factor 6 is statistically significant at the 5% level.

Factor 7:

P-value = 0.008 (significant).

Factor 7 has a significant effect on the likelihood of death.

Age:

P-value = 0.002 (highly significant).

Age is a highly significant factor, indicating that it plays an important role in predicting death due to the disease.

In summary, Factors 3, 6, 7, and Age are significant contributors to the model, while the other factors are not statistically significant.

3d

→ Comments on Age and Factor 7

Here are brief interpretations of the **Age** and **Factor 7** coefficients from the logistic regression model:

1. Age (Coefficient = 0.0686):

 A positive coefficient indicates that as age increases, the log-odds of death also increase. Specifically, for each additional year of age, the log-odds of death increase by 0.0686, holding all other factors constant. In practical terms, older individuals are at a higher risk of death from the disease.

2. Factor 7 (Coefficient = -0.0098):

A negative coefficient means that as Factor 7 increases, the log-odds of death decrease. For each one-unit increase in Factor 7, the log-odds of death decrease by **0.0098**, holding all other factors constant. This suggests that higher values of Factor 7 are associated with a lower risk of death, making it a protective factor in the model.

In summary:

- Age increases the risk of death.
- Factor 7 reduces the risk of death.

3e

→ Updating the odds of give data

```
In [25]: def compute_odds_ratio(coefficient, change):
    # Calculate the odds ratio using the formula: exp(coefficient * change)
    return np.exp(coefficient * change)

age_coefficient = 0.0686
factor_7_coefficient = -0.0998

factor_7_odds_ratio = compute_odds_ratio(factor_7_coefficient, -100)

age_odds_ratio = compute_odds_ratio(age_coefficient, 1)
factor_7_odds_ratio, age_odds_ratio
Out[25]: (2.664456241929417, 1.071007720368156)
```

100-unit decrease in Factor 7: The odds ratio is approximately 2.66, meaning that a 100-unit decrease in Factor 7 is associated with a 2.66 times higher odds of death, after adjusting for the other factors.

Additional year of age: The odds ratio is approximately 1.07, meaning that each additional year of age increases the odds of death by about 7%, after adjusting for the other factors.

3f

→ Predict

```
In [26]: # Given factors and age for the 50-year-old woman
factors = [125, 105, 235, 105, 12.5, 45, 475]
    age = 50

# Calculate the probability of death using the logistic regression model
    probability_of_death = logistic_regression_probability(factors, age)
    probability_of_death
Out[26]: 5.4850193047419086e-05
```

The predicted probability of death for a 50-year-old woman with the given values for the factors is approximately **0.00005485**, or about **0.0055%**. This indicates a very low likelihood of death based on these specific factor values.