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Sequence Data -

Sequential Pattern Mining

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Agenda

- What is Sequence Data?
- Sequential Pattern Mining
  - GSP
  - Prefixspan

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Data Mining / Machine Learning Methods

	Vector Data	Set Data	Sequence Data / Time Series Data	Graph Data
Classification	Logistic Regression			
Clustering	K-means Hierarchical Clustering Gaussian Mixture Models			
Prediction / Regression	Linear Regression Generalized Linear Models			
Frequent Pattern Mining		Apriori	GSP; PrefixSpan	
Similarity Search				

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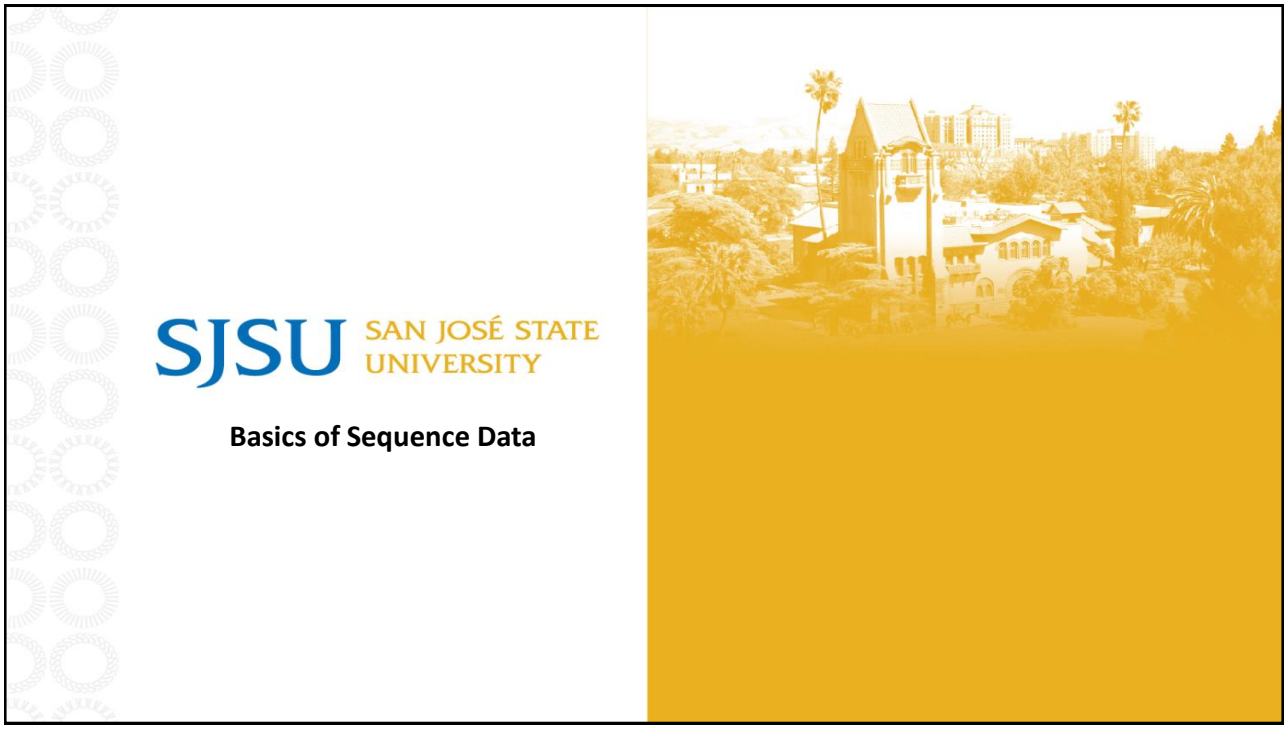

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What is Sequence Data?

- A sequence dataset consists of ordered elements or events
- Recorded with or without notion of time
- Transaction dataset vs. sequence dataset

Transaction Dataset

TID	itemsets
10	a, b, d
20	a, c, d
30	a, d, e
40	b, e, f

Sequence Dataset

SID	sequences
10	<a(abc)(ac)d(cf)>
20	<(ad)c(bc)(ae)>
30	<(ef)(ab)(df)cb>
40	<eg(af)cbc>

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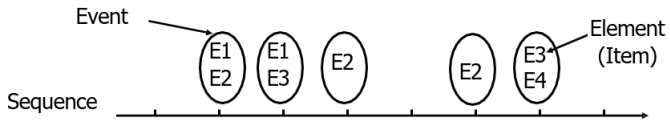
Sequence Data

Sequence Database	Sequence	Event	Element
Customer	Purchase history of a given customer	A set of items bought by a customer at time t	Books, diary products, CDs, etc
Web Data	Browsing activity of a particular Web visitor	A collection of files viewed by a Web visitor after a single mouse click	Home page, index page, contact info, etc
Event data	History of events generated by a given sensor	Events triggered by a sensor at time t	Types of alarms generated by sensors
Genome sequences	DNA sequence of a particular species	An element of the DNA sequence	Bases A,T,G,C



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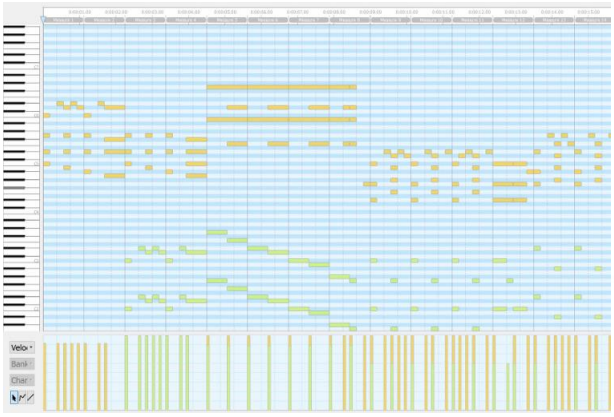
Examples of Sequence Data

Music Sequence



DNA Sequence

```
MD106  ATGCTTAGTAATCCCTACTTTAAGTCCGTTTTGTGGCTGATTGGCTTCGGAGGAATGGG
NEWC   ATGCTTAGTAATCCTTACTTTAAATCCGTTTTGTGGCTGATTGGCTTCGGAGGAATGGG
W501   ATGCTTAGTAATCCCTACTTTAAGTCCGTTTTGTGGCTGATTGGCTTCGGAGGAATGGG
MD199  ATGCTTAGTAATCCCTACTTTAAGTCCGTTTTGTGGCTGATTGGCTTCGGAGGAATGGG
C1674  ATGCTTAGTAATCCCTACTTTAAGTCCGTTTTGTGGCTGATTGGCTTCGGAGGAATGGG
SIM4   ATGCTTAGTAATCCCTACTTTAAGTCCGTTTTGTGGCTGATTGGCTTCGGAGGAATGGG

MD106  CTACGGCCTAATGGTGCTAACAGAGCCGAACGTCGACAAAATAGAGCGCATCAAAGCCT
NEWC   CTACGGCCTAATGGTGCTAACCGAGCCGAACGTCGACAAAATAGAGCGCATCAAAGCCT
W501   CTACGGCCTAATGGTGCTAACCGAGCCGAACGTCGACAAAATAGAGCGCATCAAAGCCT
MD199  CTACGGCCTAATGGTGCTAACCGAGCCGAACGTCGACAAAATAGAGCGCATCAAAGCCT
C1674  CTACGGCCTAATGGTGCTAACCGAGCCGAACGTCGACAAAATAGAGCGCATCAAAGCCT
SIM4   CTACGGCCTAATGGTGCTAACCGAGCCGAACGTCGACAAAATAGAGCGCATCAAAGCCT

MD106  CCGTTTCAAGTACCAAACTGAGTGCGGATGAGCAGCGAAAGGCTCTGTTTATGAAGAAG
NEWC   CCGTTTCAAGTACCAAACTGAGTGCGGATGAGCAGCGAAAGGCTCTGTTTATGAAGAAG
W501   CCGTTTCAAGTACCAAACTGAGTGCGGATGAGCAGCGAAAGGCTCTGTTTATGAAGAAG
MD199  CCGTTTCAAGTACCAAACTGAGTGCGGATGAGCAGCGAAAGGCTCTGTTTATGAAGAAG
C1674  CCGTTTCAAGTACCAAACTGAGTGCGGATGAGCAGCGAAAGGCTCTGTTTATGAAGAAG
SIM4   CCGTTTCAAGTACCAAACTGAGTGCGGATGAGCAGCGAAAGGCTCTGTTTATGAAGAAG

MD106  CTGCAGGAGGCGTCCACCACCAGTGCCCCAATCTACAGGTCAGCGGCCGAGAAATAG
NEWC   CTGCAGGAGGCGTCCACCACCAGTGCCCCAATCTACAGGTCATCGGCCGAGAAATAG
W501   CTGCAGGAGGCGTCCACCACCACTGCCCCAATCTACAGGTCATCGGCCGAGAAATAG
MD199  CTGCAGGAGGCGTCCACCACCAGTGCCCCAATCTACAGGTCAGCGGCCGAGAAATAG
C1674  CTGCAGGAGGCGTCCACCACCAGTGCCCCAATCTACAGGTCAGCGGCCGAGAAATAG
SIM4   CTGCAGGAGGCGTCCACCACCAGTGCCCCAATCTACAGGTCAGCGGCCGAGAAATAG
```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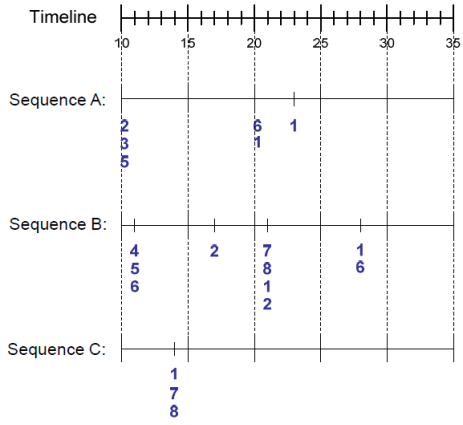
Sequence Data with Timestamp

Sequence Database

Sequence ID	Timestamp	Events
A	10	2, 3, 5
A	20	6, 1
A	23	1
B	11	4, 5, 6
B	17	2
B	21	7, 8, 1, 2
B	28	1, 6
C	14	1, 8, 7



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Definitions of Sequences, Events…

- A sequence is an ordered list of events, denoted $< e_1\ e_2\ \dots\ e_l >$
- An event is a *non-empty* collection of items/elements: $e_1 =$ (abc)
- Given two sequences $\alpha = < a_1\ a_2\ \dots\ a_n >$ and $\beta = < b_1\ b_2\ \dots\ b_m >$
- α is called a subsequence of β, denoted as $\alpha \subseteq \beta$, if there exist integers $1 \le j_1 < j_2 < \dots < j_n \le m$ such that $a_1 \subseteq b_{j1}$, $a_2 \subseteq b_{j2}$,…, $a_n \subseteq b_{jn}$
- β is a super sequence of α
- Examples
  - α = <(ab)d> and β = < (abc)(de)>
  - α = <a(bc)dc> and β = <a(abc)(ac)d(cf)>

SID	Sequence
10	<a(abc)(ac)d(cf)>
20	<(ad)c(bc)(ae)>
30	<(ef)(ab)(df)cb>
40	<eg(af)cbc>

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Subsequence Exercises

Data Sequence	Subsequence	Contain?
< (2 4)(3 5 6)(8) >	< (2)(8) >	
< (1 2)(3 4) >	< (1)(2) >	
< (2 4)(2 4)(2 5) >	< (2)(4) >	
<(2 4)(2 5)(4 5)>	< (2)(4)(5) >	
<(2 4)(2 5)(4 5)>	< (2)(5)(5) >	
<(2 4)(2 5)(4 5)>	< (2 4 5) >	

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Definitions of Sequences, Events…

- Length of a sequence, |s|, is given by # of elements in the sequence

  e.g. What is the length of < (ef)(ab)(df)cb > ?

- A k-sequence is a sequence that contains k events:

  e.g. <(ab)(a)> is a 2-sequence and has a length of 3

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Support of a Sequence

- Support of a Sequence α: # of sequences in a sequence dataset D that contains a sequence α on and is denoted as sup(α):

$$\sup(\alpha) = |\{S \mid S \in D \text{ and } \alpha \sqsubseteq S\}|$$

- α is frequent if sup(α) ≥ σ (*minsup* threshold)
- A frequent sequence is called sequential pattern

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Example: Support of a Sequence

Consider the sequence dataset as shown:

- Example 1: sup(<(a)>) =?
- Example 2: sup(<(b)>) =?
- Example 3: sup(<(a)(b)>) =?
- Example 4: sup(<(a b)>) =?

SID	Sequence
10	<(ab)(c)(a)>
20	<(a)(b)(c)>
30	<(b)(c)(d)>
40	<(b)(ab)(c)>

sup(<(a)>) = 3 (75%)
sup(<(b)>) = 4 (100%)
sup(<(c)>) = 4 (100%)
sup(<(a)(c)>) = 3 (75%)
sup(<(ab)>) = 2 (50%)
sup(<(b)(c)>) = 4 (100%)
sup(<(ab)(c)>) = 2 (50%)

SID	Sequence
10	<(ab)(c)(a)>
20	<(a)(b)(c)>
30	<(b)(c)(d)>
40	<(b)(ab)(c)>

SID	Sequence
10	<(ab)(c)(a)>
20	<(a)(b)(c)>
30	<(b)(c)(d)>
40	<(b)(ab)(c)>

SID	Sequence
10	<(ab)(c)(a)>
20	<(a)(b)(c)>
30	<(b)(c)(d)>
40	<(b)(ab)(c)>

SID	Sequence
10	<(ab)(c)(a)>
20	<(a)(b)(c)>
30	<(b)(c)(d)>
40	<(b)(ab)(c)>

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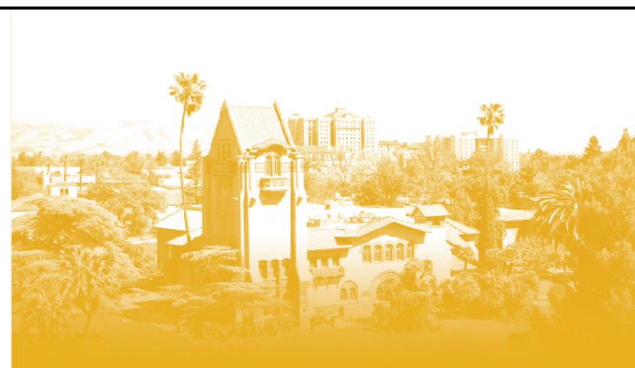

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Sequential Pattern Mining

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What Is Sequential Pattern Mining?

- Given a set of sequences and support threshold, find the complete set of **frequent subsequences**.

Sequence Data

SID	sequence
10	<a(abc)(ac)d(cf)>
20	<(ad)c(bc)(ae)>
30	<(ef)(ab)(df)cb>
40	<eg(af)cbc>

Given *support threshold* *min_sup* =2, <(ab)c> is a *sequential pattern*

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Applications

Applications of Sequential Pattern Mining

- Customer shopping sequences:
  - First buy computer, then CD-ROM, and then digital camera, within 3 months.
- Medical treatments, natural disasters (e.g., earthquakes), science & eng. processes, stocks and markets, etc.
- Telephone calling patterns
- Weblog click streams
- DNA sequences and gene structures

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Example: Sequential Pattern Mining

Sequence ID	Timestamp	Events
A	1	1,2,4
A	2	2,3
A	3	5
B	1	1,2
B	2	2,3,4
C	1	1,2
C	2	2,3,4
C	3	2,4,5
D	1	2
D	2	3,4
D	3	4,5
E	1	1,3
E	2	2,4,5

Frequent Subsequences:

sup<(1,2)> = 60%
sup<(2,3)> = 60%
sup<(2,4)> = 80%
sup<(3)(5)> = 80%
sup<(1)(2)> = 80%
sup<(2)(2)> = 60%
sup<(1)(2,3)> = 60%
sup<(2)(2,3)> = 60%
sup<(1,2)(2,3)> = 60%

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Extracting Sequential Patterns

- Given n items: $i_1, i_2, i_3, \ldots, i_n$
- Candidate 1-subsequences:

  $\langle(i_1)\rangle, \langle(i_2)\rangle, \langle(i_3)\rangle, \ldots, \langle(i_n)\rangle$
- Candidate 2-subsequences:

  $\langle(i_1, i_2)\rangle, \langle(i_1, i_3)\rangle, \ldots,$
  $\langle(i_1)\ (i_1)\rangle, \langle(i_1)\ (i_2)\rangle, \ldots, \langle(i_n)\ (i_n)\rangle$
- Candidate 3-subsequences:

  $\langle(i_1, i_2, i_3)\rangle, \langle(i_1, i_2, i_4)\rangle, \ldots,$
  $\langle(i_1, i_2)\ (i_1)\rangle, \langle(i_1, i_2)\ (i_2)\rangle, \ldots,$
  $\langle(i_1)\ (i_1, i_2)\rangle, \langle(i_1)\ (i_1, i_3)\rangle, \ldots,$
  $\langle(i_1)\ (i_1)\ (i_1)\rangle, \langle(i_1)\ (i_1)\ (i_2)\rangle, \ldots$

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Challenges on Sequential Pattern Mining

- A huge number of possible sequential patterns are hidden in dataset
- A mining algorithm should
  - find the complete set of patterns, when possible, satisfying the minimum support (frequency) threshold
  - be highly efficient, scalable, involving only a small number of database scans
  - be able to incorporate various kinds of user-specific constraints

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Generalized Sequential Pattern (GSP)

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The Apriori Property of Sequential Patterns

A basic property: Apriori (Agrawal & Sirkant'94)

- If a sequence S is not frequent, then none of the super-sequences of S is frequent.

Example: <abc> is infrequent ➔ <abcde> and <(abc)(de)> are infrequent too!

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Methods for Sequential Pattern Mining

- Apriori-based Approaches
  - GSP
  - SPADE
- Pattern-Growth-based Approaches
  - FreeSpan
  - PrefixSpan

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Recall The Apriori Algorithm

C_k: Candidate itemsets of size k
L_k: Frequent itemsets of size k

sup_{min} = 2

Tid	Items
10	A, C, D
20	B, C, E
30	A, B, C, E
40	B, E

1st scan →

C_1

Itemset	sup
{A}	2
{B}	3
{C}	3
{D}	1
{E}	3

→ L_1

Itemset	sup
{A}	2
{B}	3
{C}	3
{E}	3

1st Iteration

C_2

Itemset
{A, B}
{A, C}
{A, E}
{B, C}
{B, E}
{C, E}

2nd scan →

C_2

Itemset	sup
{A, B}	1
{A, C}	2
{A, E}	1
{B, C}	2
{B, E}	3
{C, E}	2

→ L_2

Itemset	sup
{A, C}	2
{B, C}	2
{B, E}	3
{C, E}	2

2nd Iteration

Why no {A, D} or {B, D}?

C_3

Itemset
{B, C, E}

3rd scan →

Itemset	sup
{B, C, E}	2

→ L_3

Itemset	sup
{B, C, E}	2

3rd Iteration

Why no {A, C, E}?

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Generalized Sequential Pattern (GSP) Mining

- Step 1
  - Make the first pass over the sequence database D to yield all candidate of length-1
- Step 2 (Repeat until no new frequent sequences are found)
  - Candidate Generation:
    - Merge pairs of frequent subsequences found in the (k-1)th pass to generate candidate sequences that contain k items
  - Candidate Pruning:
    - Prune candidate length k sequences that contain infrequent length (k-1) subsequences
  - Support Counting:
    - Make a new pass over the sequence database D to find the support for these candidate sequences
  - Candidate Elimination:
    - Eliminate candidate length k sequences whose actual support is less than minsup

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Example: Finding Length-1 Sequential Patterns

- Initial candidates: all singleton sequences: <a>, <b>, <c>, <d>, <e>, <f>, <g>, <h>
- Scan dataset once, count support for candidate.

min_sup =2

SID	sequence
10	<(bd)cb(ac)>
20	<(bf)(ce)b(fg)>
30	<(ah)(bf)abf>
40	<(be)(ce)d>
50	<a(bd)bcb(ade)>

Cand	Sup
<a>	3
<b>	5
<c>	4
<d>	3
<e>	3
<f>	2
~~<g>~~	~~1~~
~~<h>~~	~~1~~

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Example: Finding Length-2 Sequential Patterns

- Now, generate length-2 candidates.

	<a>	<b>	<c>	<d>	<e>	<f>
<a>	<aa>	<ab>	<ac>	<ad>	<ae>	<af>
<b>	<ba>	<bb>	<bc>	<bd>	<be>	<bf>
<c>	<ca>	<cb>	<cc>	<cd>	<ce>	<cf>
<d>	<da>	<db>	<dc>	<dd>	<de>	<df>
<e>	<ea>	<eb>	<ec>	<ed>	<ee>	<ef>
<f>	<fa>	<fb>	<fc>	<fd>	<fe>	<ff>

	<a>	<b>	<c>	<d>	<e>	<f>
<a>		<(ab)>	<(ac)>	<(ad)>	<(ae)>	<(af)>
<b>			<(bc)>	<(bd)>	<(be)>	<(bf)>
<c>				<(cd)>	<(ce)>	<(cf)>
<d>					<(de)>	<(df)>
<e>						<(ef)>
<f>						

- 51 length-2 candidates!!!
- Without Apriori property, 8*8+8*7/2 = 92 candidates

➔ Apriori prunes 44.6% candidates!

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Generating Candidates in General

From L_{k-1} to C_k

- Step 1: Join
  - Sequences α and β can join, if dropping first item in α is the same as dropping the last item in β

  Examples:

  <(12)3> join <(2)34> ➔ <(12)34>

  <(12)3> join <(2)(34)> ➔ <(12)(34)>
- Step 2: Pruning
  - Check whether all length k-1 subsequences of a candidate is contained in L_{k-1}

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The GSP Mining Process

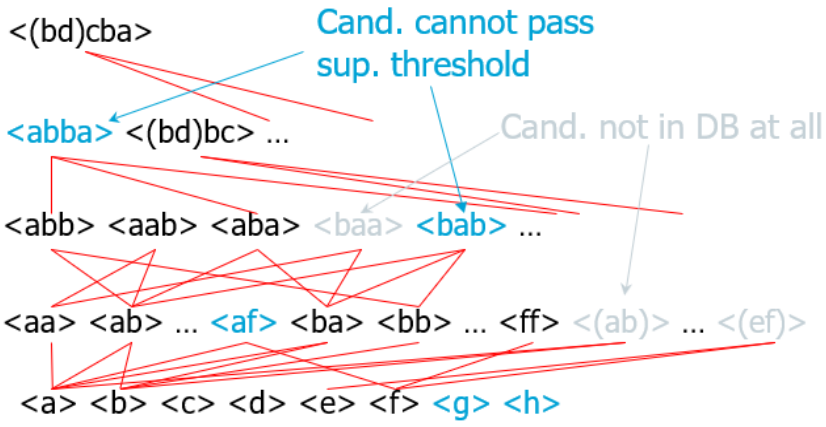
5th scan: 1 cand. 1 length-5 seq. pat.

4th scan: 8 cand. 7 length-4 seq. pat.

3rd scan: 46 cand. 20 length-3 seq. pat. (20 cand. not in DB at all)

2nd scan: 51 cand. 19 length-2 seq. pat. (10 cand. not in DB at all)

1st scan: 8 cand. 6 length-1 seq. pat.



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Drawbacks of GSP

- A huge set of candidate sequences generated (especially 2-item candidate sequence)
- Multiple scans of database needed. The length of each candidate grows by one at each database scan.
- Inefficient for mining long sequential patterns.
  - A long pattern grow up from short patterns
  - # of short patterns is exponential to the length of mined patterns.

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Bottlenecks of GSP

- A huge set of candidates could be generated
  - 1,000 frequent length-1 sequences generates huge number of length-2 candidates!
- Multiple scans of database in mining
- Breadth-first search
- Mining long sequential patterns
  - Needs an exponential number of short candidates
  - A length-100 sequential pattern needs ~10^{30} candidate sequences!
- Some of these can be addressed with PrefixSpan

$$1000 \times 1000 + \frac{1000 \times 999}{2} = 1{,}499{,}500$$

$$\sum_{i=1}^{100} \binom{100}{i} = 2^{100} - 1 \approx 10^{30}$$

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Sequential Pattern Mining

with PrefixSpan

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Prefix and Suffix

- Assume a pre-specified order on items, e.g., alphabetical order

  <a>, <aa>, <a(ab)>, <a(abc)> are prefixes of sequence <a(abc)(ac)d(cf)>

  Note: <a(ac)> is not a prefix of <a(abc)(ac)d(cf)>

- Given sequence <a(abc)(ac)d(cf)>:

Prefix	Suffix
<a>	<(abc)(ac)d(cf)>
<aa>	<(_bc)(ac)d(cf)>
<a(ab)>	<(_c)(ac)d(cf)>

- (_bc) means: the last element in the prefix together with (bc) form one element

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Prefix-based Projection

Given a sequence, α, let α' be subsequence of α:

- α' is called a projection of α with respect to prefix β, if and only if
  - α' has prefix β, and
  - α' is the maximum subsequence of α with prefix β

SID	Sequence
10	<a(abc)(ac)d(cf)>
20	<(ad)c(bc)(ae)>
30	<(ef)(ab)(df)cb>
40	<eg(af)cbc>

Example:

<(abc)(ac)d(cf)> is a projection of <a(abc)(ac)d(cf)> w.r.t. prefix <a>

<ad(cf)> is a projection of <a(abc)(ac)d(cf)> w.r.t. prefix <ad>

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Projected (Suffix) Database

- Let β be a sequential pattern, β-projected database is the collection of suffixes of projections of sequences in the database w.r.t. prefix β
- Examples:
  - <a>-projected database
    - <(abc)(ac)d(cf)>
    - <(_d)c(bc)(ae)>
    - <(_b)(df)cb>
    - <(_f)cbc>
  - <ab>-projected database
    - <(_c)(ac)d(cf)> (<a(bc)(ac)d(cf)> is the projection of <a(abc)(ac)d(cf)> w.r.t. prefix <ab>)
    - <(_c)(ae)> (<a(bc)(ae)> is the projection of <(ad)c(bc)(ae)> w.r.t. prefix <ab>)
    - <c> (<abc> is the projection of <eg(af)cbc> w.r.t prefix <ab>)

SID	Sequence
10	<a(abc)(ac)d(cf)>
20	<(ad)c(bc)(ae)>
30	<(ef)(ab)(df)cb>
40	<eg(af)cbc>

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Examples

SID	Sequence
10	<a(abc)(ac)d(cf)>
20	<(ad)c(bc)(ae)>
30	<(ef)(ab)(df)cb>
40	<eg(af)cbc>

Starting with <a>-projected database
- <(abc)(ac)d(cf)>
- <(_d)c(bc)(ae)>
- <(_b)(df)cb>
- <(_f)cbc>

<aa>-projected database
- <(_bc)(ac)d(cf)>
- <(_e)>

<ab>-projected database
- <(_c)(ac)d(cf)>
- <(_c)(ae)>
- <c>

<(ab)>-projected database
- <(_c)(ac)d(cf)>
- <(df)cb>

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Mining Sequential Patterns by Prefix Projections

- Step 1: Find length-1 sequential patterns.
  <a>, <b>, <c>, <d>, <e>, <f>
- Step 2: Divide search space. The complete set of sequential patterns can be partitioned into 6 subsets:
  The ones having prefix <a>
  The ones having prefix <b>
  …
  The ones having prefix <f>
- Step 3: Mine each subset recursively via corresponding projected databases.

SID	Sequence
10	<a(abc)(ac)d(cf)>
20	<(ad)c(bc)(ae)>
30	<(ef)(ab)(df)cb>
40	<eg(af)cbc>

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Finding Sequential Patterns with Prefix <a>

- Consider only projections w.r.t. <a>:
  - <a>-projected (suffix) database:
    <(abc)(ac)d(cf)>
    <(_d)c(bc)(ae)>
    <(_b)(df)cb>
    <(_f)cbc>

SID	Sequence
10	<a(abc)(ac)d(cf)>
20	<(ad)c(bc)(ae)>
30	<(ef)(ab)(df)cb>
40	<eg(af)cbc>

- Find all the length-2 sequential patterns having prefix <a>:
  <aa>, <ab>, <(ab)>, <ac>, <ad>, <af>
  - Further partition into 6 subsets:
    Having prefix <aa>
    …
    Having prefix <af>

Why are those 6 subsets?

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Finding Sequential Patterns with Prefix <a>

- By scanning the <a>-projected database once, its locally frequent items are identified as
  a : 2, b : 4, _b : 2, c : 4, d : 2, and f : 2.
- Thus, all the length-2 sequential patterns prefixed with <a> are found:
  <aa>: 2, <ab>: 4, <(ab)>: 2, <ac> : 4, <ad>: 2, and <af>: 2.

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Completeness of PrefixSpan

SID	sequence
10	<a(abc)(ac)d(cf)>
20	<(ad)c(bc)(ae)>
30	<(ef)(ab)(df)cb>
40	<eg(af)cbc>

Length-1 sequential patterns
<a>, <b>, <c>, <d>, <e>, <f>

Having prefix <a>

Having prefix <b>

Having prefix <c>, …, <f>

<a>-projected database
<(abc)(ac)d(cf)>
<(_d)c(bc)(ae)>
<(_b)(df)cb>
<(_f)cbc>

Length-2 sequential patterns
<aa>, <ab>, <(ab)>,
<ac>, <ad>, <af>

<b>-projected database
<(_c)(ac)d(cf)>
<(_c)(ae)>
<(df)cb>
<c>

…

Having prefix <aa>

Having prefix <af>

<aa>-proj. db … <af>-proj. db … …

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Projected Databases & Sequential Patterns

SID	Sequence
10	<a(abc)(ac)d(cf)>
20	<(ad)c(bc)(ae)>
30	<(ef)(ab)(df)cb>
40	<eg(af)cbc>

prefix	projected (suffix) database	sequential patterns
$\langle a \rangle$	$\langle (abc)(ac)d(cf) \rangle$, $\langle (\_d)c(bc)(ae) \rangle$, $\langle (\_b)(df)cb \rangle$, $\langle (\_f)cbc \rangle$	$\langle a \rangle$, $\langle aa \rangle$, $\langle ab \rangle$, $\langle a(bc) \rangle$, $\langle a(bc)a \rangle$, $\langle aba \rangle$, $\langle abc \rangle$, $\langle (ab) \rangle$, $\langle (ab)c \rangle$, $\langle (ab)d \rangle$, $\langle (ab)f \rangle$, $\langle (ab)dc \rangle$, $\langle ac \rangle$, $\langle aca \rangle$, $\langle acb \rangle$, $\langle acc \rangle$, $\langle ad \rangle$, $\langle adc \rangle$, $\langle af \rangle$
$\langle b \rangle$	$\langle (\_c)(ac)d(cf) \rangle$, $\langle (\_c)(ae) \rangle$, $\langle (df)cb \rangle$, $\langle c \rangle$	$\langle b \rangle$, $\langle ba \rangle$, $\langle bc \rangle$, $\langle (bc) \rangle$, $\langle (bc)a \rangle$, $\langle bd \rangle$, $\langle bdc \rangle$, $\langle bf \rangle$
$\langle c \rangle$	$\langle (ac)d(cf) \rangle$, $\langle (bc)(ae) \rangle$, $\langle b \rangle$, $\langle bc \rangle$	$\langle c \rangle$, $\langle ca \rangle$, $\langle cb \rangle$, $\langle cc \rangle$
$\langle d \rangle$	$\langle (cf) \rangle$, $\langle c(bc)(ae) \rangle$, $\langle (\_f)cb \rangle$	$\langle d \rangle$, $\langle db \rangle$, $\langle dc \rangle$, $\langle dcb \rangle$
$\langle e \rangle$	$\langle (\_f)(ab)(df)cb \rangle$, $\langle (af)cbc \rangle$	$\langle e \rangle$, $\langle ea \rangle$, $\langle eab \rangle$, $\langle eac \rangle$, $\langle eacb \rangle$, $\langle eb \rangle$, $\langle ebc \rangle$, $\langle ec \rangle$, $\langle ecb \rangle$, $\langle ef \rangle$, $\langle efb \rangle$, $\langle efc \rangle$, $\langle efcb \rangle$.
$\langle f \rangle$	$\langle (ab)(df)cb \rangle$, $\langle cbc \rangle$	$\langle f \rangle$, $\langle fb \rangle$, $\langle fbc \rangle$, $\langle fc \rangle$, $\langle fcb \rangle$

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Efficiency of PrefixSpan

- No candidate sequence needs to be generated
- Projected databases keep shrinking
- Major cost of PrefixSpan: Constructing projected databases
- Can be improved by pseudo-projections (virtual projection in memory)

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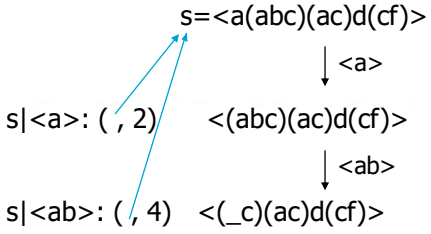
Speed-up by Pseudo-Projection

- Major cost of PrefixSpan: projection
  - Postfixes of sequences often appear repeatedly in recursive projected databases
- When (projected) database can be held in main memory, use pointers to form projections:
  - Pointer to the sequence
  - Offset of the postfix



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Pseudo-Projection vs. Physical Projection

- Pseudo-projection avoids physically copying postfixes
  - Efficient in running time and space when database can be held in main memory
- However, it is not efficient when database cannot fit in main memory
  - Disk-based random accessing is very costly
- Suggested Approach:
  - Integration of physical and pseudo-projection
  - Swapping to pseudo-projection when the data set fits in memory

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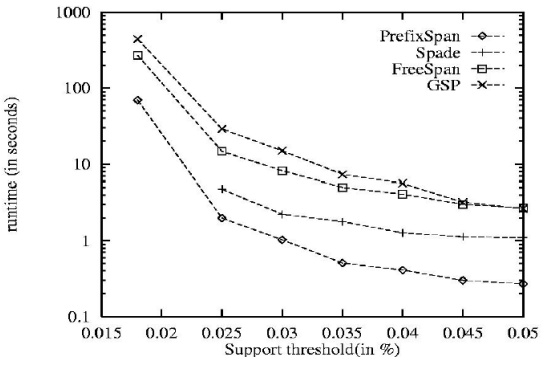
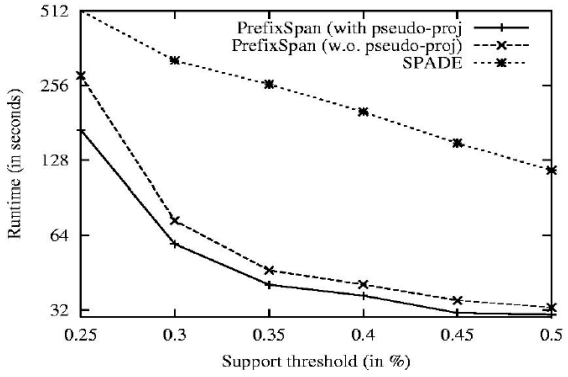
Performance Comparison





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Sequential Rules

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Sequential Pattern Implications

- <af> is a sequential pattern in the dataset:

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10	<a(abc)(ac)d(cf)>
20	<(ad)c(bc)(ae)>
30	<(ef)(ab)(df)cb>
40	<eg(af)cbc>

- Does it imply that if someone buys <a>, he'll also buy <f> later?
- Not really, <a> implies <f> only 50% of the time! ➔ Need sequential rules!

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Sequential Rules

Two main types of sequential rules

- Standard Sequential rules
- Partially-ordered Sequential rules

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Standard Sequential Rules

Standard Sequential Rules: Rules of the form $X \rightarrow Y$, where X and Y are sequential patterns.

Example: <a(bc)> → <de>

- Two thresholds must be set by the user:
  - minimum support > 0
  - minimum confidence > 0
- Support: how many sequences contain the rule
- Confidence: how many sequences contain the rule divided by how many sequences contain its antecedent

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Example: Standard Sequential Rule

- Consider the standard sequential rule: <ab> → <f>

SID	sequence
10	<a(abc)(ac)d(cf)>
20	<(ad)c(bc)(ae)>
30	<(ef)(ab)(df)cb>
40	<eg(af)cbc>

Support: 1 sequence (25%)

Confidence: 1 / 4 = 0.25 (25%)

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Order Matters?

- Notice that some sequential rules that are very similar but have only some small ordering variations.
- Example: <ab> → <f> and <ba> → <f>

SID	sequence
10	<a(abc)(ac)d(cf)>
20	<(ad)c(bc)(ae)>
30	<(ef)(ab)(df)cb>
40	<eg(af)cbc>

SID	sequence
10	<a(abc)(ac)d(cf)>
20	<(ad)c(bc)(ae)>
30	<(ef)(ab)(df)cb>
40	<eg(af)cbc>

Rule	Support	Confidence
<ab> → <f>	25%	25%
<ba> → <f>	25%	50%

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Partially-Ordered Sequential Rules

Partially-Ordered Sequential Rules: Rules of the form $X \rightarrow Y$, where X and Y are itemsets that are unordered.

Example: {a,b} → {f}

- If we observe a and b (in any order), they will be followed by f

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Partially-Ordered Sequential Rules

- This type of rule is often more interesting because it can summarize many standard sequential rules
- Example:

$\{Vivaldi\}, \{Mozart\}, \{Handel\} \Rightarrow \{Berlioz$
$\{Mozart\}, \{Vivaldi\}, \{Handel\} \Rightarrow \{Berlioz\},$
$\{Handel\}, \{Vivaldi\}, \{Mozart\} \Rightarrow \{Berlioz\},$
$\{Handel, Vivaldi\}, \{Mozart\} \Rightarrow \{Berlioz\},$
$\{Handel\}, \{Vivaldi, Mozart\} \Rightarrow \{Berlioz\},$
$\{Handel, Vivaldi, Mozart\} \Rightarrow \{Berlioz\}.$

Standard sequential rules

$\{Mozart, Vivaldi, Handel\} \Rightarrow \{Berlioz\}$

Partially-ordered sequential rules

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Partially-Ordered Sequential Rules

- A partially-ordered sequential rule X → Y is a relationship between two disjoint and non-empty itemsets X , Y .
- A sequential rule X → Y has two properties:
  - Support: #of sequences where X occurs before Y, divided by the number of sequences.
  - Confidence: # of sequences where X occurs before Y, divided by the number of sequences where X occurs.
- Need to find all valid rules, rules with a support and confidence not less than user-defined thresholds minSup and minConf

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Example: Mining Partially-Ordered Sequential Rules

- Consider the following sequence database with minSup= 0.5 and minConf= 0.5

ID	Sequences
seq1	{*a*, *b*},{*c*},{*f*},{*g*},{*e*}
seq2	{*a*, *d*},{*c*},{*b*},{*a*, *b*, *e*, *f*}
seq3	{*a*},{*b*},{*f*},{*e*}
seq4	{*b*},{*f*, *g*}

ID	Rule	Support	Confidence
r1	{*a*, *b*, *c*}⇒{*e*}	0.5	1.0
r2	{*a*}→{*c*, *e*, *f*}	0.5	0.66
r3	{*a*, *b*} →{*e*, *f*}	0.75	1.0
r4	{*b*} →{*e*, *f*}	0.75	0.75
r5	{*a*} →{*e*, *f*}	0.75	1.0
r6	{*c*} →{*f*}	0.5	1.0
r7	{*a*}→{*b*}	0.5	0.66
…	…	…	…

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Summary

- Basic concepts: sequential patterns etc
- Scalable sequential pattern mining methods
  - GSP
  - PrefixSpan
- Sequential Rules: Standard and Partially-Ordered