DATA - 255 Deep-Learning Technologies
HW 1 — Fall 24

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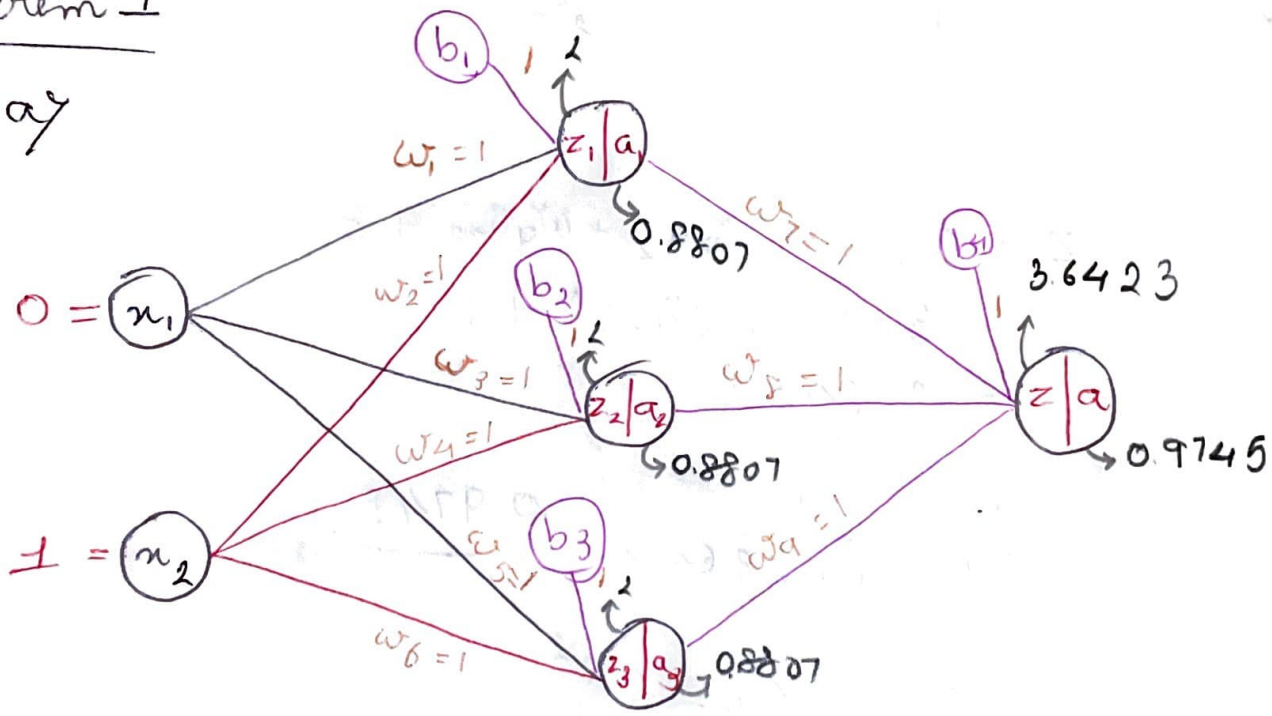
Problem 1

a)



Given

$x_1 = 0$ all weights & bias are = 1

$x_2 = 1$ $\alpha = 0.1$, $y = 1$

$$MSE = \frac{1}{2}(y - \hat{y})^2$$

activation = sigmoid.

No. of sample (n) = 1

$z_1 = w_1 x_1 + w_2 x_2 + b_1$

$= (1 \times 0) + (1 \times 1) + 1$

$= 2$

$a_1 = \frac{1}{1 + e^{-2}} = 0.8807$

$z_2 = w_3 x_1 + w_4 x_2 + b_2$

$= (1 \times 0) + (1 \times 1) + 1$

$= 2.$

$a_2 = \frac{1}{1 + e^{-2}} = 0.8807$

$z_3 = w_5 x_1 + w_6 x_2 + b_3$

$= (1 \times 0) + (1 \times 1) + 1$

$= 2$

$a_3 = \frac{1}{1 + e^{-2}} = 0.8807$

$$Z = w_7 a_1 + w_8 a_2 + w_9 a_3 + b$$

$$= 3 \times (1 \times 0.8807) + 1$$

$$= 3.6423$$

$$a/\hat{y} = \frac{1}{1 + e^{-3.6423}} = 0.9745.$$

$$L = \frac{1}{n} \times \frac{1}{2} (y - \hat{y})^2$$

$$= \frac{1}{2} (1 - 0.9745)^2 = 0.00032$$

Back propagation

$\rightarrow w_7 = 1$

$$w_7' = w_7 - \alpha \frac{\partial L}{\partial w_7}$$

$$\frac{\partial L}{\partial w_7} = \frac{\partial L}{\partial a} \times \frac{\partial a}{\partial z} \times \frac{\partial z}{\partial w_7}$$

$$\frac{\partial L}{\partial a} = \frac{1}{2}(y-a)^2$$
$$= \frac{2}{2}(y-a) - 1$$
$$= -(1 - 0.9745)$$
$$\frac{\partial L}{\partial a} = -0.0255$$

$$\frac{\partial z}{\partial w_7} = (w_7 a_1 + w_8 a_2 + w_9 a_3 + b_4)$$
$$= a_1 = 0.8807$$

$$\frac{\partial a}{\partial z} = a(1-a)$$
$$= 0.9745(1-0.9745)$$
$$= 0.0248.$$

$\Rightarrow w_7' = 1 - 0.1(0.8807 \times 0.0248 \times -0.025)$

$\boxed{w_7' = 1.000055}$

$\rightarrow w_8 = 1$

$$w_8' = w_8 - \alpha \frac{\partial L}{\partial w_8}$$

$$\frac{\partial L}{\partial w_8} = \frac{\partial L}{\partial a} \times \frac{\partial a}{\partial z} \times \frac{\partial z}{\partial w_8}$$

same

$$\frac{\partial z}{\partial w_8} = (w_7 a_1 + w_8 a_2 + w_9 a_3 + b_4)$$
$$= a_2 = 0.8807$$

$\Rightarrow w_8' = 1 - 0.1(5.57 \times 10^{-4})$

$\boxed{w_8' = 1.000055}$

$\rightarrow w_9 = 1$

$$w_9' = w_9 - \alpha \frac{\partial L}{\partial w_9}$$

$$\frac{\partial z}{\partial w_9} = a_3 = 0.8807$$

$$\frac{\partial L}{\partial w_9} = \frac{\partial L}{\partial a} \times \frac{\partial a}{\partial z} \times \frac{\partial z}{\partial w_9}$$

same

$\boxed{w_9' = 1.000055}$

→ $w_1 = 1$

$$w_1' = w_1 - \alpha \frac{\partial L}{\partial w_1}$$

$$\frac{\partial L}{\partial w_1} = \underbrace{\frac{\partial L}{\partial a} \times \frac{\partial a}{\partial z}}_{\text{same.}} \times \frac{\partial z}{\partial a_1} \times \frac{\partial a_1}{\partial z_1} \times \frac{\partial z_1}{\partial w_1}$$

$$\frac{\partial z}{\partial a_1} = w_7 = 1 \; ; \; \frac{\partial a_1}{\partial z_1} = a_1(1-a_1) = 0.1051 \; ; \; \frac{\partial z_1}{\partial w_1} = x_1 = 0$$

$$w_1' = w_1 - 0.1(-0.0255 \times 0.0248 \times 1 \times 0.1051 \times 0)$$

$$\boxed{w_1' = w_1 = 1}$$

→ $w_2 = 1$

$$w_2' = w_2 - \alpha \frac{\partial L}{\partial w_2}$$

$$\frac{\partial L}{\partial w_2} = \underbrace{\frac{\partial L}{\partial a} \times \frac{\partial a}{\partial z} \times \frac{\partial z}{\partial a_1} \times \frac{\partial a_1}{\partial z_1}}_{\text{same.}} \times \frac{\partial z_1}{\partial w_2}$$

$$\frac{\partial z_1}{\partial w_2} = x_2 = 1$$

$$w_2' = 1 - 0.1(-0.0255 \times 0.0248 \times 1 \times 0.1051 \times 1)$$

$$\boxed{w_2' = 1.0000005}$$

→ $w_3 = 1$

$$w_3' = w_3 - \alpha \frac{\partial L}{\partial w_3}$$

$$\frac{\partial L}{\partial w_3} = \underbrace{\frac{\partial L}{\partial a} \times \frac{\partial a}{\partial z}}_{\text{same}} \times \frac{\partial \varepsilon}{\partial a_2}^{1} \times \frac{\partial a_2}{\partial z_2} \times \frac{\partial z_2}{\partial w_3}^{0} = 0$$

$\frac{\partial a_2}{\partial z_2} \rightarrow 0.1051$

$$w_3' = w_3 - 0$$

$$\boxed{w_3' = w_3 = 1}$$

$\rightarrow w_4 = 1$

$$w_4' = w_4 - \alpha \frac{\partial L}{\partial w_4}$$

$$\frac{\partial L}{\partial w_4} = \underbrace{\frac{\partial L}{\partial a} \times \frac{\partial a}{\partial z}}_{\text{same}} \times \frac{\partial z}{\partial a_2}\uparrow^{1} \times \frac{\partial a_2}{\partial z_2} \times \frac{\partial z_2}{\partial w_4}\uparrow^{1}$$

$\hookrightarrow 0.1051$

$$\boxed{w_4' = 1.0000005}$$

$\rightarrow w_5 = 1$

$$w_5' = w_5 - \alpha \frac{\partial L}{\partial w_5}$$

$$\frac{\partial L}{\partial w_5} = \underbrace{\frac{\partial L}{\partial a} \times \frac{\partial a}{\partial z}}_{\text{same}} \times \frac{\partial z}{\partial a_3}\uparrow^{1} \times \frac{\partial a_3}{\partial z_3} \times \frac{\partial z_3}{\partial w_5}\uparrow^{0}$$

$\hookrightarrow 0.1051$

$$w_5' = w_5 - \alpha(0)$$

$$\boxed{w_5' = w_5 = 1}$$

$\rightarrow w_6 = 1$

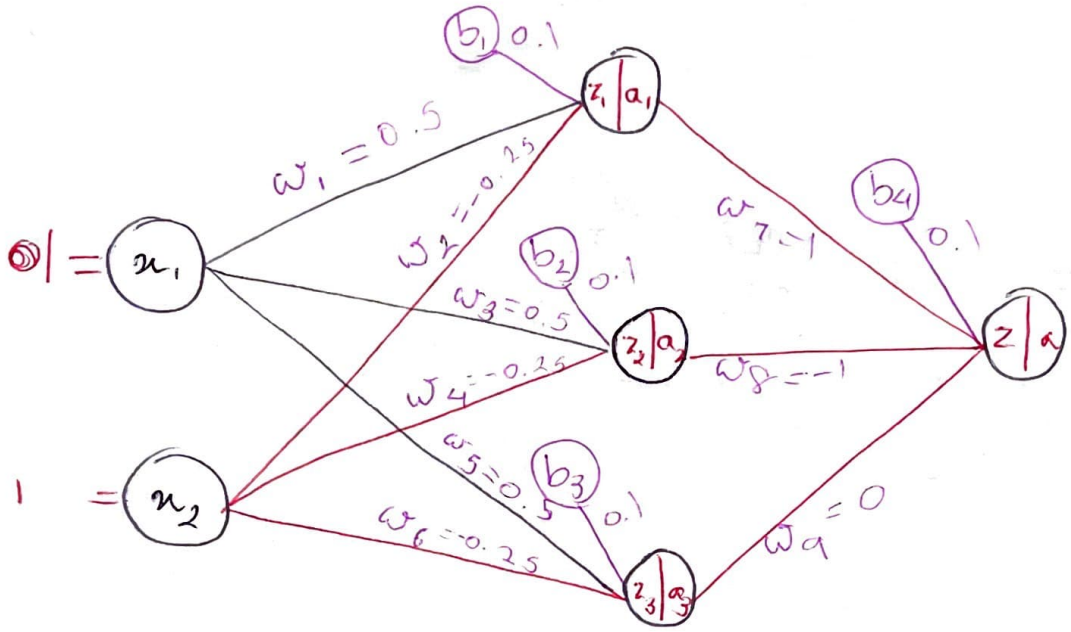
$$w_6' = w_6 - \alpha \frac{\partial L}{\partial w_6}$$

$$\frac{\partial L}{\partial w_6} = \underbrace{\frac{\partial L}{\partial a} \times \frac{\partial a}{\partial z}}_{\text{same}} \times \frac{\partial z}{\partial a_3}\uparrow^{1} \times \frac{\partial a_3}{\partial z_3} \times \frac{\partial z_3}{\partial w_6}\uparrow^{1}$$

$\hookrightarrow 0.1051$

$$\boxed{w_6' = 1.0000005}$$

b)



$$z_1 = (w_1 \times x_1) + (w_2 \times x_2) + b_1$$
$$= (0.5 \times 1) + (-0.25 \times 1) + 0.1$$
$$= 0.35$$

$$a_1 = \frac{1}{1 + e^{-0.35}} = 0.5866$$

$$z_2 = (w_3 \times x_1) + (w_4 \times x_2) + b_2$$
$$= (0.5 \times 1) + (-0.25 \times 1) + 0.1$$
$$= 0.35$$

$$a_2 = \frac{1}{1 + e^{-0.35}} = 0.5866$$

$$z_3 = (w_5 \times x_1) + (w_6 \times x_2) + b_3$$
$$= (0.5 \times 1) + (-0.25 \times 1) + 0.1$$
$$= 0.35$$

$$a_3 = \frac{1}{1 + e^{-0.35}} = 0.5866$$

$$z = w_7 a_1 + w_8 a_2 + w_9 a_3 + b_4$$
$$= (1 \times 0.5866) + (0 \times 0.5866) + (-1 \times 0.5866) + 0.1$$
$$= 0.1$$

$$a = \frac{1}{1 + e^{-0.1}} = 0.5249$$

$$h = \frac{1}{1} \times \frac{1}{2} (y - a)^2$$
$$= \frac{1}{2} (1 - 0.5249)^2 = \underline{0.1128}$$

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Old → w_1, w_5, w_7, w_9

1) $w_9 = 0$

$$w_9' = w_9 - \alpha \frac{\partial h}{\partial w_9}$$

$$\frac{\partial h}{\partial w_9} = \frac{\partial h}{\partial a} \times \frac{\partial a}{\partial z} \times \frac{\partial z}{\partial w_9}$$

$$\frac{\partial h}{\partial a} = \frac{\partial}{\partial a}\left[\frac{1}{2}(y-a)^2\right]$$
$$= -(y-a)$$
$$= -(1-0.5249)$$
$$= -0.4751$$

$$\frac{\partial z}{\partial w_9} = \frac{\partial}{\partial w_9}(w_7 a_1 + w_8 a_2 + w_9 a_3 + b_4)$$
$$= a_3 = 0.5866$$

$$\frac{\partial a}{\partial z} = a(1-a)$$
$$= 0.5249(1-0.5294)$$
$$= 0.249$$

$$w_9' = 0 - 0.1(-0.4751 \times 0.249 \times 0.5866)$$

$$\boxed{w_9' = 0.00693}$$

2) $w_7 = 1$

$$w_7' = w_7 - \alpha \frac{\partial h}{\partial w_7}$$

$$\frac{\partial z}{\partial w_7} = (w_7 a_1 + w_8 a_2 + w_9 a_3 + b_4)$$
$$= a_1 = 0.5866$$

$$\frac{\partial h}{\partial w_7} = \frac{\partial h}{\partial a} \times \frac{\partial a}{\partial z} \times \frac{\partial z}{\partial w_7}$$

same

$$w_7' = w_7 - \alpha(-0.4751 \times 0.249 \times 0.5866)$$

$$\boxed{w_7' = 1.00693}$$

3) $w_5 = 0.5$

$$w_5' = w_5 - \alpha \frac{\partial L}{\partial w_5}$$

$$\frac{\partial L}{\partial w_5} = \underbrace{\frac{\partial L}{\partial a} \times \frac{\partial a}{\partial z}}_{\text{same}} \times \frac{\partial z}{\partial a_3} \times \frac{\partial a_3}{\partial z_3} \times \frac{\partial z_3}{\partial w_5}$$

$$\frac{\partial z}{\partial a_3} = w_4 = 0, \quad \frac{\partial a_3}{\partial z_3} = a_3(1-a_3) = 0.2425, \quad \frac{\partial z_3}{\partial w_5} = x_1$$

$$w_5' = 0.5 - 0.1(-0.4751 \times 0.249 \times 0 \times 0.2425 \times 1)$$

$$\boxed{w_5' = 0.5}$$

4) $w_1 = 0.5$

$$w_1' = w_1 - \alpha \frac{\partial L}{\partial w_1}$$

$$\frac{\partial L}{\partial w_1} = \underbrace{\frac{\partial L}{\partial a} \times \frac{\partial a}{\partial z}}_{\text{same}} \times \frac{\partial z}{\partial a_1} \times \frac{\partial a_1}{\partial z_1} \times \frac{\partial z_1}{\partial w_1}$$

$$\frac{\partial z}{\partial a_1} = w_7 = 1, \quad \frac{\partial a_1}{\partial z_1} = a_1(1-a_1) = 0.2425, \quad \frac{\partial z_1}{\partial w_1} = x_1$$

$$w_1' = 0.5 - 0.1(-0.4751 \times 0.249 \times 1 \times 0.2425 \times 1)$$

$$\boxed{w_1' = 0.50286}$$