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DATA 220: Mathematical Methods for Data Analytics

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Conditional Probability

- Independent probability – the outcome of one event does not depend on the outcome of another event
  - Tossing coin
- Dependent probability – the probability of one evet is influenced by the occurrence of another events
  - Probability of drawing a red card from a deck of cards after drawing a black card
- Conditional probability – the probability of an event occurring given that another event has occurred
  - Probability of an email being spam given that it contain certain words

$P(R) = 0.05$; where R is the event that it rains on a randomly selected day

$P(R \mid C) = ?$; what is the probability that it rains given that it is cloudy ➔ **conditional probability**

Conditional Probability

- Example: Assume you role a fair die where A is the event that the outcome is odd number and B is the event that the outcome is less than or equal to 3.
  - $P(A) \ =?$
  - $P(A|B) \ = ?$

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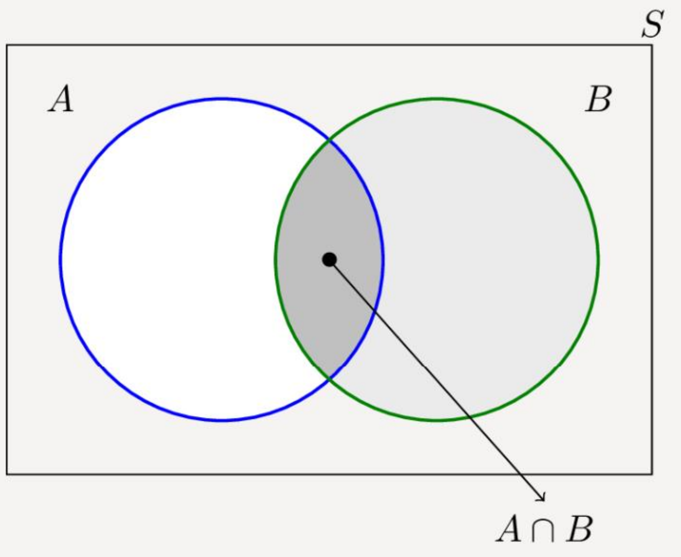
Conditional Probability

- Area inside the Blue circle – represents the total probability of A
- Area inside the Red circle – represents the total probability of B
- The intersection of A and B – represents the joint probability of both events occurring
- If A and B are two events in a sample space S, then the conditional probability of A given B is defined as follows:

$$P(A|B) = \frac{P(A \cap B)}{P(B)};$$

when $P(B) > 0$



$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Fig. 1.21 - Venn diagram for conditional probability, $P(A|B)$.

https://www.probabilitycourse.com/chapter1/1_4_0_conditional_probability.php

Conditional Probability

- Axiom 1: $P\ (A|B) \geq\ 0$
- Axiom 2: $P\ (B|B) = 1$
- Axiom 3: $P\ (A_1 \cup A_2 \ldots .|B) = P(A_1|B) + P(A_2|B) +\ \ldots$
  - Where $A_1, A_2, \ldots$ are disjoint events

https://www.probabilitycourse.com/chapter1/1_4_0_conditional_probability.php

Conditional Probability

- Conditional Independence
  - Two events A and B are independent if
    - $P(A \cap B) = P\ (A)\ P(B);$
      - Equivalently: $P(A|B) = P\ (A)$
  - Two events A and B are conditionally independent given an event C if
    - $P(\ (A \cap B)|C) = P\ (A|C)\ P(B|C)$

https://www.probabilitycourse.com/chapter1/1_4_0_conditional_probability.php

Conditional Probability

- Law of Total Probability
  - Suppose $B_1, B_2, \ldots B_k$ are mutually exclusive and exhaustive events in a sample space . Then for any event A in that sample space:

$$P(A) = P(A \cap B_1) + \ldots + P(A \cap B_k)$$

$$P(A) = P(B_1)P(A|\, B_1) + \ldots + P(B_k)P(A|\, B_k) = \sum_{i=1}^{k} P(B_i)P(A|B_i)$$

https://www.probabilitycourse.com/chapter1/1_4_0_conditional_probability.php

Conditional Probability

- Law of Total Probability
  - Example: There are three bags each containing 100 marbles
    - Bag1: 75 Red & 25 Blue
    - Bag2: 60 Red & 40 Blue
    - Bag3: 45 Red & 55 Blue

  You choose one of the bags at random and then pick a marble from the chosen bag, also at random. What is the probability that the chosen marble is red?

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Conditional Probability

- Law of Total Probability
  - Example: Three machines make parts at a factory. Suppose we know the following about the manufacturing process:
    - M1 makes 60%, M2 makes 30% and M3 makes 10% of the parts
    - Of the parts M1 makes, 7% are defective
    - Of the parts M2 makes, 15% are defective
    - Of the parts M3 makes, 30% are defective

  You randomly selected a part, what is the probability that the part is defective?

https://www.probabilitycourse.com/chapter1/1_4_0_conditional_probability.php

Conditional Probability

$$P(A|B) \neq P(B|A)$$

$$P(A \cap B) = P(B \cap A)$$

- $P(A \cap B) = P(B)\ P(A|B)$
- $P(B \cap A) = P(A)\ P(B|A)$
- $P(B)\ P(A|B) = P(A)\ P(B|A)$

Bayes Theorem

For any two events A and B, where $P(B) \neq 0$, *we have*

$$P(A \mid B) = \frac{P(B \mid A)\, P(A)}{P(B)}$$

Where,
$P(A)$ is an unconditional probability: prior probability
$P(B)$ is an unconditional probability: marginal probability
$P(B|A)$ is a conditional probability: likelihood of event A given a fixed B
$P(A\,|B)$ is a conditional probability: posterior probability

Bayes Theorem

$For\ any\ two\ events\ A\ and\ B,\ where\ P(B) \neq 0, we\ have$

$$P(A \mid B) = \frac{P(B \mid A)\, P(A)}{P(B)}$$

Example: suppose we observe that the chosen marble is red, what is the probability that bag 1 was chosen?

Where,
$P(A)$ is an unconditional probability: prior probability
$P(B)$ is an unconditional probability: marginal probability
$P(B|A)$ is a conditional probability: likelihood of event A given a fixed B
$P(A\ |B)$ is a conditional probability: posterior probability

Bayes Theorem

$$P(A \mid B) = \frac{P(B \mid A)\, P(A)}{P(B)}$$

Even if 100% of patients with pancreatic cancer have a certain symptom, when someone has the same symptom, it does not mean that this person has a 100% chance of getting pancreatic cancer. Assuming the incidence rate of pancreatic cancer is 1/100000, while 10/99999 healthy individuals have the same symptoms worldwide, what is the probability of having pancreatic cancer given the symptoms?

Naïve Bayes Classifier

$$P(A \mid B) = \frac{P(B \mid A)\, P(A)}{P(B)}$$

Outlook	Temperature	Humidity	Windy	PlayTennis
Sunny	Hot	High	False	No
Sunny	Hot	High	True	No
Overcast	Hot	High	False	Yes
Rainy	Mild	High	False	Yes
Rainy	Cool	Normal	False	Yes
Rainy	Cool	Normal	True	No
Overcast	Cool	Normal	True	Yes
Sunny	Mild	High	False	No
Sunny	Cool	Normal	False	Yes
Rainy	Mild	Normal	False	Yes
Sunny	Mild	Normal	True	Yes
Overcast	Mild	High	True	Yes
Overcast	Hot	Normal	False	Yes
Rainy	Mild	High	True	No

https://en.wikipedia.org/wiki/Bayes'_theorem