Data -220 Mathematical Method for DA

Home - work 4

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Question 1

$f(x,y) = x^3 - 12xy + 8y^3$

$f_x = 3x^2 - 12y$ & $f_y = -12x + 24y$

$\Rightarrow x^2 = 4y$ & $\Rightarrow 24y = 12x$

$\Rightarrow x = 2y^2$

By applying formula theorem

$\Rightarrow 4y^4 = 4y$

$\Rightarrow y^4 = y$

$\Rightarrow y^4 - y = 0$

$\Rightarrow y(y^3 - 1) = 0$

$\Rightarrow y = 0$ & $y = 1$

So $x = 0, y = 0$

$x = 2, y = 1$

$f_{xx} = 6x$, $f_{yy} = 48y$, $f_{xy} = f_{yx} = -12$

cp $\Rightarrow (0,0)$ & $(2,1)$

$D(x,y) = f_{xx} f_{yy} - f_{xy}^2$

for cp $-(0,0)$

$D(0,0) = 0 \times 0 - (-12)^2$

$= -144$

$D < 0$; it is a saddle point

for CP(2,1)

$D(2,1) = 12 \times 48 - (-12)^2$

$= 432$

$D(2,1) > 0$ & $f_{xx} > 0$, critical point is local minimum (2,1)

Question 2

$f(x,y) = x^2 + xy + y^2$

s.t. ; $x^2 + y^2 = 8$

$\nabla f = k \nabla g$

$\langle 2x+y, x+2y \rangle = k \langle 2x, 2y \rangle$

$2x + y = 2kx$ — ① $x + 2y = 2ky$ — ②

divide ① by x

$2 + \frac{y}{x} = 2k$

divide ② by y

$\frac{x}{y} + 2 = 2k$

$2 + \frac{y}{x} = \frac{x}{y} + 2$

$y^2 = x^2$ — ③

$x^2 + y^2 = 8$

$2x^2 = 8$

$x^2 = 4$

$x = \pm 2$

(2, -2) (-2, 2)

(2, 2) (-2, -2)

$f(2,-2) = 4$
$f(-2,2) = 4$ } minimum p.t

$f(-2,-2) = 12$
$f(2,2) = 12$ } maximum pt

Question 3

$x_0 = 0.5$, $\alpha = 0.01$, $f(x) = x^4 + x^3 - 2x^2$

$$f'(x) = 4x^3 + 3x^2 - 4x$$

$$x_1 = x_0 - \alpha f'(0.5)$$
$$= 0.5 - 0.01 \times (-0.75)$$
$$= 0.5075$$

$$x_1 = x_0 - \alpha f'(0.5075)$$
$$= 0.5075 - 0.01(-0.73449)$$
$$= 0.5148449$$