SJSU SAN JOSÉ STATE UNIVERSITY

Hypothesis Test – Nonparametric Test

- Wilcoxon signed-rank test aka Signed-rank test
  - Nonparametric test: test the location of a population based on sample data, or to compare the locations of two populations using two matched (paired) samples
  - alternative to the one- or two-sample (paired) t-test
  - No assumptions about distribution of the data required
    - Testing data ranks instead of data means
    - Tests for differences in medians instead of differences in means

Assumptions for Signed -rank test:
- Data should be ordinal or continuous level
- Paired or matched data
- Paired observations should be independent to each other

https://en.wikipedia.org/wiki/Wilcoxon_signed-rank_test

SJSU SAN JOSÉ STATE UNIVERSITY

Hypothesis Test – Nonparametric Test

- Wilcoxon signed-rank test process
  - **Calculate Differences:** Find the differences between each pair of observations
  - **Rank Absolute Differences:** Rank the absolute values of these differences, ignoring the sign.
  - **Separate Ranks:** Divide ranks into two groups and calculate W+ and W-
  - **Determine W:** The test statistic W is defined as the smaller of W+ and W-.
  - **Hypothesis Testing:**
    - Null Hypothesis (H0): Assumes no difference between paired observations.
    - Alternative Hypothesis (H1): Assumes a difference exists between paired observations.
  - **Compare W to Critical Value (find C.V. from W table):** If $W \leq$ critical value => reject the null hypothesis (H0), otherwise fail to reject H0

SJSU SAN JOSÉ STATE UNIVERSITY

A study is run to evaluate the effectiveness of an exercise program in reducing systolic blood pressure in patients with pre-hypertension (defined as a systolic blood pressure between 120-139 mmHg or a diastolic blood pressure between 80-89 mmHg). A total of 15 patients with pre-hypertension enroll in the study, and their systolic blood pressures are measured. Each patient then participates in an exercise training program where they learn proper techniques and execution of a series of exercises. Patients are instructed to do the exercise program 3 times per week for 6 weeks. After 6 weeks, systolic blood pressures are again measured. The data are shown below.

Patient	Systolic Blood Pressure Before Exercise Program	Systolic Blood Pressure After Exercise Program	Difference (Before-After)
1	125	118	7
2	132	134	-2
3	138	130	8
4	120	124	-4
5	125	105	20
6	127	130	-3
7	136	130	6
8	139	132	7
9	131	123	8
10	132	128	4
11	135	126	9
12	136	140	-4
13	128	135	-7
14	127	126	1
15	130	132	-2

https://sphweb.bumc.bu.edu/otlt/mph-modules/bs/bs704_nonparametric/BS704_Nonparametric6.html

SJSU SAN JOSÉ STATE UNIVERSITY

The final step is to attach the signs of the observed differences to each rank

Observed Differences		Ordered Absolute Values of Differences	Ranks
7		1	1
-2		-2	2.5
8		-2	2.5
-4		-3	4
20		-4	6
-3		-4	6
6		4	6
7		6	8
8		-7	10
4		7	10
9		7	10
-4		8	12.5
-7		8	12.5
1		9	14
-2		20	15

Signed ranks
1
-2.5
-2.5
-4
.
.
.
.

$W+ = 89$
$W- =$

Conclusion?

https://sphweb.bumc.bu.edu/otlt/mph-modules/bs/bs704_nonparametric/BS704_Nonparametric6.html

SJSU SAN JOSÉ STATE UNIVERSITY

Critical Values for Wilcoxon Signed Rank Test

n	Two-Tailed Test		One-Tailed Test	
	α = .05	α = .01	α = .05	α = .01
5	--	--	0	--
6	0	--	2	--
7	2	--	3	0
8	3	0	5	1
9	5	1	8	3
10	8	3	10	5
11	10	5	13	7
12	13	7	17	9
13	17	9	21	12
14	21	12	25	15
15	25	15	30	19
16	29	19	35	23
17	34	23	41	27

For p-value:

$$Z = \frac{W - \frac{n(n+1)}{4}}{\sqrt{\frac{n(n+1)(2n+1)}{24}}}$$

https://sphweb.bumc.bu.edu/otlt/mph-modules/bs/bs704_nonparametric/B

```
[4]  1 import numpy as np
     2 from scipy.stats import wilcoxon
     3 # paired observations
     4 before = [125, 132, 138, 120, 125, 127, 136, 139, 131, 132, 135, 136, 128, 127, 130]
     5 after = [118, 134, 130, 124, 105, 130, 130, 132, 123, 128, 126, 140, 135, 126, 132]
     6 # Perform Wilcoxon signed-rank test
     7 w_statistic, p_value = wilcoxon(before, after)
     8
     9 print("Wilcoxon W statistic:", w_statistic)
    10 print("P-value:", p_value)

Wilcoxon W statistic: 31.0
P-value: 0.10699462890625
```

SJSU SAN JOSÉ STATE UNIVERSITY

Hypothesis Test – Nonparametric Test

- Mann-Whitney U test aka Wilcoxon rank-sum test
  - Nonparametric test alternative to the independent two-samples t-test
  - Test if there is a significant difference between two independent groups

Assumptions for Wilcoxon rank-sum test:
- Ordinal or continuous level data
- Observations between two groups (samples) should be **independent** to each other

https://en.wikipedia.org/wiki/Wilcoxon_signed-rank_test

SJSU SAN JOSÉ STATE UNIVERSITY

Hypothesis Test – Nonparametric Test

- Mann-Whitney U test aka Wilcoxon rank-sum test process
  - Rank all the observations from both samples together
  - Calculate the sum of the ranks for each group separately
  - Compute the test statistic U, which is the smaller of the two sums of ranks
  - Compare the test statistic to critical value and conclude

U statistic [edit]

Let $X_1, \ldots, X_{n_1}$ be an i.i.d. sample from X, and $Y_1, \ldots, Y_{n_2}$ an i.i.d. sample from Y, and both samples independent of each other. The corresponding Mann–Whitney *U* statistic is defined as the smaller of:

$$U_1 = n_1 n_2 + \frac{n_1(n_1+1)}{2} - R_1, U_2 = n_1 n_2 + \frac{n_2(n_2+1)}{2} - R_2$$

with

R_1, R_2 being the sum of the ranks in groups 1 and 2, respectively. [7]

https://en.wikipedia.org/wiki/Mann%E2%80%93Whitney_U_test

SJSU SAN JOSÉ STATE UNIVERSITY

Hypothesis Test – Nonparametric Test

Consider a Phase II clinical trial designed to investigate the effectiveness of a new drug to reduce symptoms of asthma in children. A total of n=10 participants are randomized to receive either the new drug or a placebo. Participants are asked to record the number of episodes of shortness of breath over a 1 week period following receipt of the assigned treatment. The data are shown below.

Placebo	7	5	6	4	12
New Drug	3	6	4	2	1

Is there a difference in the number of episodes of shortness of breath over a 1 week period in participants receiving the new drug as compared to those receiving the placebo? By inspection, it appears that participants receiving the placebo have more episodes of shortness of breath, but is this statistically significant?

https://sphweb.bumc.bu.edu/otlt/mph-modules/bs/bs704_nonparametric/bs704_nonparametric4.html

SJSU SAN JOSÉ STATE UNIVERSITY

Hypothesis Test – Nonparametric Test

		Total Sample (Ordered Smallest to Largest)		Ranks	
Placebo	New Drug	Placebo	New Drug	Placebo	New Drug
7	3		1		1
5	6		2		2
6	4		3		3
4	2	4	4	4.5	4.5
12	1	5		6	
		6	6	7.5	7.5
		7		9	
		12		10	

$$U1 = 5 * 5 + \frac{5 * 6}{2} - 37 = 3$$
$$U2 = 22$$

https://sphweb.bumc.bu.edu/otlt/mph-modules/bs/bs704_nonparametric/bs704_nonparametric4.html

SJSU SAN JOSÉ STATE UNIVERSITY

Hypothesis Test – Nonparametric Test

$$U1 = 5 * 5 + \frac{5 * 6}{2} - 37 = 3$$
$$U2 = 22$$

C.V. = ?

the decision rule is to reject H_0 if U ≤ C.V.

Critical Values of the Mann-Whitney U
(Two-Tailed Testing)

n_2	α	n_1 3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
3	.05	--	0	0	1	1	2	2	3	3	4	4	5	5	6	6	7	7	8
	.01	--	0	0	0	0	0	0	0	0	1	1	1	2	2	2	2	3	3
4	.05	--	0	1	2	3	4	4	5	6	7	8	9	10	11	11	12	13	14
	.01	--	--	0	0	0	1	1	2	2	3	3	4	5	5	6	6	7	8
5	.05	0	1	2	3	5	6	7	8	9	11	12	13	14	15	17	18	19	20
	.01	--	--	0	1	1	2	3	4	5	6	7	7	8	9	10	11	12	13
6	.05	1	2	3	5	6	8	10	11	13	14	16	17	19	21	22	24	25	27
	.01	--	0	1	2	3	4	5	6	7	9	10	11	12	13	15	16	17	18
7	.05	1	3	5	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34
	.01	--	0	1	3	4	6	7	9	10	12	13	15	16	18	19	21	22	24
8	.05	2	4	6	8	10	13	15	17	19	22	24	26	29	31	34	36	38	41
	.01	--	1	2	4	6	7	9	11	13	15	17	18	20	22	24	26	28	30
9	.05	2	4	7	10	12	15	17	20	23	26	28	31	34	37	39	42	45	48
	.01	0	1	3	5	7	9	11	13	16	18	20	22	24	27	29	31	33	36
10	.05	3	5	8	11	14	17	20	23	26	29	33	36	39	42	45	48	52	55
	.01	0	2	4	6	9	11	13	16	18	21	24	26	29	31	34	37	39	42
11	.05	3	6	9	13	16	19	23	26	30	33	37	40	44	47	51	55	58	62
	.01	0	2	5	7	10	13	16	18	21	24	27	30	33	36	39	42	45	48
12	.05	4	7	11	14	18	22	26	29	33	37	41	45	49	53	57	61	65	69
	.01	1	3	6	9	12	15	18	21	24	27	31	34	37	41	44	47	51	54
13	.05	4	8	12	16	20	24	28	33	37	41	45	50	54	59	63	67	72	76
	.01	1	3	7	10	13	17	20	24	27	31	34	38	42	45	49	53	56	60
14	.05	5	9	13	17	22	26	31	36	40	45	50	55	59	64	67	74	78	83
	.01	1	4	7	11	15	18	22	26	30	34	38	42	46	50	54	58	63	67
15	.05	5	10	14	19	24	29	34	39	44	49	54	59	64	70	75	80	85	90
	.01	2	5	8	12	16	20	24	29	33	37	42	46	51	55	60	64	69	73
16	.05	6	11	15	21	26	31	37	42	47	53	59	64	70	75	81	86	92	98
	.01	2	5	9	13	18	22	27	31	36	41	45	50	55	60	65	70	74	79
17	.05	6	11	17	22	28	34	39	45	51	57	63	67	75	81	87	93	99	105
	.01	2	6	10	15	19	24	29	34	39	44	49	54	60	65	70	75	81	86
18	.05	7	12	18	24	30	36	42	48	55	61	67	74	80	86	93	99	106	112
	.01	2	6	11	16	21	26	31	37	42	47	53	58	64	70	75	81	87	92
19	.05	7	13	19	25	32	38	45	52	58	65	72	78	85	92	99	106	113	119
	.01	3	7	12	17	22	28	33	39	45	51	56	63	69	74	81	87	93	99
20	.05	8	14	20	27	34	41	48	55	62	69	76	83	90	98	105	112	119	127
	.01	3	8	13	18	24	30	36	42	48	54	60	67	73	79	86	92	99	105

https://sphweb.bumc.bu.edu/otlt/mph-modules/bs/bs704_nonparametric/bs704_nonparametric4.html

Hypothesis Test – Nonparametric Test

```python
import scipy.stats as stats

placebo = [7, 5, 6, 4, 12]
new_drug = [3, 6, 4, 2, 1]

# Perform the Mann-Whitney U test
U_statistic, p_value = stats.ranksums(placebo, new_drug)

# Output the results
print("Mann-Whitney U Statistic:", U_statistic)
print("p-value:", p_value)
```

```
Mann-Whitney U Statistic: 1.9844852778949553
p-value: 0.04720176769014221
```

https://sphweb.bumc.bu.edu/otlt/mph-modules/bs/bs704_nonparametric/bs704_nonparametric4.html