Data 220 Mathematical for Data analytics

homework - 3

Name :- Prayog Nikul Purani
SJSU ID :- 017416737

Problem 1

a) $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x-y \\ x+y \\ 4x \end{bmatrix}$

homogeneity :-

iff. $T(cu) = cT(u)$

$u = \begin{bmatrix} x \\ y \end{bmatrix}$

LHS $\Rightarrow T(cu) = T\left(c\begin{bmatrix} x \\ y \end{bmatrix}\right) = T\left(\begin{bmatrix} cx \\ cy \end{bmatrix}\right)$

$\Rightarrow \begin{bmatrix} cx - cy \\ cx + cy \\ 4cx \end{bmatrix}$

RHS $\Rightarrow cT(u) = cT\left(\begin{bmatrix} x \\ y \end{bmatrix}\right)$

$\Rightarrow c\begin{bmatrix} x-y \\ x+y \\ 4x \end{bmatrix}$

$\Rightarrow \begin{bmatrix} cx - cy \\ cx + cy \\ 4cx \end{bmatrix}$

so, LHS = RHS

additivity

$u = \begin{bmatrix} x_1 \\ y_1 \end{bmatrix}$; $v = \begin{bmatrix} x_2 \\ y_2 \end{bmatrix}$

iff $T(u+v) = T(u) + T(v)$

LHS $\Rightarrow T\left(\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} + \begin{bmatrix} x_2 \\ y_2 \end{bmatrix}\right) = T\left(\begin{bmatrix} x_1+x_2 \\ y_1+y_2 \end{bmatrix}\right)$

$$\Rightarrow \begin{bmatrix} (x_1+x_2)-(y_1+y_2) \\ (x_1+x_2)+(y_1+y_2) \\ 4(x_1+x_2) \end{bmatrix}$$

RHS $\Rightarrow T(u) + T(v) = T\left(\begin{bmatrix} x_1 \\ y_1 \end{bmatrix}\right) + T\left(\begin{bmatrix} x_2 \\ y_2 \end{bmatrix}\right)$

$$\Rightarrow \begin{bmatrix} x_1-y_1 \\ x_1+y_1 \\ 4x_1 \end{bmatrix} + \begin{bmatrix} x_2-y_2 \\ x_2+y_2 \\ 4x_2 \end{bmatrix} = \begin{bmatrix} (x_1+x_2)-(y_1+y_2) \\ (x_1+x_2)+(y_1+y_2) \\ 4(x_1+x_2) \end{bmatrix}$$

so, LHS = RHS

So, we can say that both the properties are satisfied so the transformation is linear.

b) $T(x, y) = \begin{bmatrix} 3x+2y \\ 4x-y \end{bmatrix}$

homogenity

$u = \begin{bmatrix} x \\ y \end{bmatrix}$ iff $T(cu) = cT(u)$

LHS $\Rightarrow T(cu) = T\begin{bmatrix} cx \\ cy \end{bmatrix} = \begin{bmatrix} 3cx + 2cy \\ 4cx - cy \end{bmatrix}$

RHS $\Rightarrow cT(u) = cT\begin{bmatrix} x \\ y \end{bmatrix} = c\begin{bmatrix} 3x+2y \\ 4x-y \end{bmatrix}$

$$= \begin{bmatrix} 3cx+2cy \\ 4cx - cy \end{bmatrix}$$

LHS = RHS

additivity

$u = \begin{bmatrix} x_1 \\ y_1 \end{bmatrix}$ $v = \begin{bmatrix} x_2 \\ y_2 \end{bmatrix}$

$T(u+v) = T(u) + T(v)$

LHS $\Rightarrow T\left(\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} + \begin{bmatrix} x_2 \\ y_2 \end{bmatrix}\right) = T\left(\begin{bmatrix} x_1+x_2 \\ y_1+y_2 \end{bmatrix}\right)$

$\Rightarrow \begin{bmatrix} 3(x_1+x_2) + 2(y_1+y_2) \\ 4(x_1+x_2) - (y_1+y_2) \end{bmatrix}$

RHS $\Rightarrow T(u) + T(v)$

$T\left(\begin{bmatrix} x_1 \\ y_1 \end{bmatrix}\right) + T\left(\begin{bmatrix} x_2 \\ y_2 \end{bmatrix}\right)$

$\begin{bmatrix} 3x_1 + 2y_1 \\ 4x_1 - y_1 \end{bmatrix} + \begin{bmatrix} 3x_2 + 2y_2 \\ 4x_2 - y_2 \end{bmatrix} \Rightarrow \begin{bmatrix} 3(x_1+x_2) + 2(y_1+y_2) \\ 4(x_1+x_2) - (y_1+y_2) \end{bmatrix}$

LHS = RHS

So, as it satisfy both of the properties so, we can tell that the function transformation is linear

c) $T(a,b) = \begin{bmatrix} x^2 \\ y^2 \end{bmatrix}$

homogenity

$a = \begin{bmatrix} x \\ y \end{bmatrix}$ $T(cu) = cT(u)$

LHS $\Rightarrow T(cu) = T\left(c\begin{bmatrix} x \\ y \end{bmatrix}\right) = T\begin{pmatrix} cx \\ cy \end{pmatrix}$

$\Rightarrow \begin{bmatrix} (cx)^2 \\ (cy)^2 \end{bmatrix} = \begin{bmatrix} c^2x^2 \\ c^2y^2 \end{bmatrix}$

RHS $\Rightarrow cT(u) = cT\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = c\begin{bmatrix} x^2 \\ y^2 \end{bmatrix}$

$= \begin{bmatrix} cx^2 \\ cy^2 \end{bmatrix}$

LHS $\neq$ RHS

additivity

$u = \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} \quad v = \begin{bmatrix} x_2 \\ y_2 \end{bmatrix}$

if :- $T(u+v) = T(u) + T(v)$

LHS $\Rightarrow T(u+v) = T\left(\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} + \begin{bmatrix} x_2 \\ y_2 \end{bmatrix}\right)$

$T\left(\begin{bmatrix} x_1 + x_2 \\ y_1 + y_2 \end{bmatrix}\right)$

$\begin{bmatrix} (x_1 + x_2)^2 \\ (y_1 + y_2)^2 \end{bmatrix}$

RHS $\Rightarrow T(u) + T(v) = T\left(\begin{bmatrix} x_1 \\ y_1 \end{bmatrix}\right) + T\left(\begin{bmatrix} x_2 \\ y_2 \end{bmatrix}\right)$

$\begin{bmatrix} x_1^2 + x^2 \\ y_1^2 + y_2^2 \end{bmatrix}$

LHS $\neq$ RHS

so, as it don't satisfy the condition of linear transformation so the transformation is non-linear.