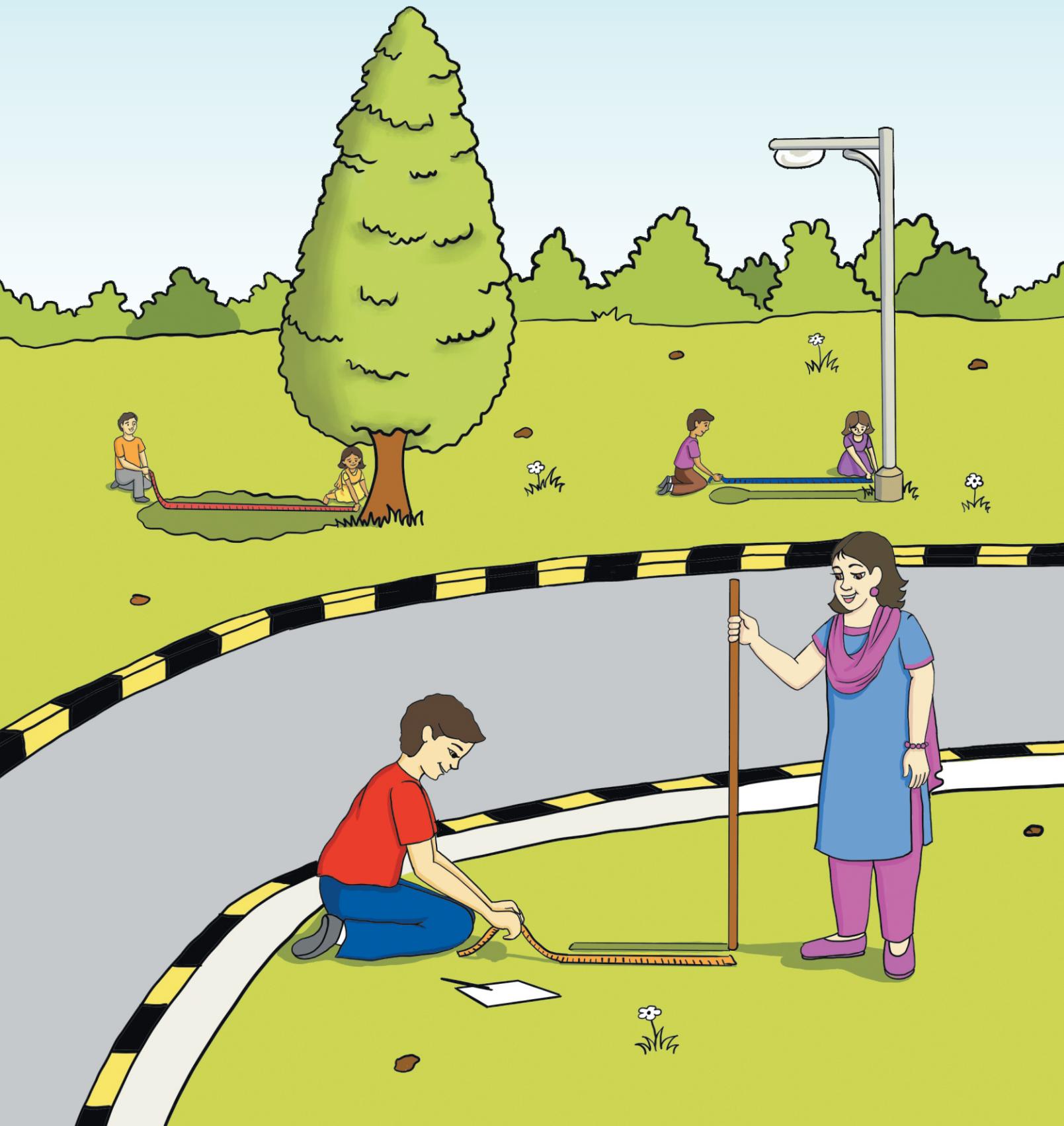


MATHEMATICS

Part - II

STANDARD NINE



The coordination committee formed by GR No. Abhyas - 2116/(Pra.Kra.43/16) SD - 4
Dated 25.4.2016 has given approval to prescribe this textbook in its meeting held on 3.3.2017

MATHEMATICS

Part-II

STANDARD NINE

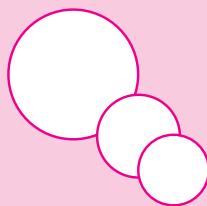


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Cover and Illustrations :

Dhanashri Mokashi

Computer Drawings :

Sandeep Koli, Mumbai

Co-ordination : Ujjwala Godbole
I/C Special Officer for Mathematics

Translation : Dr Jayashri Atre
Shri. V. D. Godbole
Smt. Mrinalini Desai

Scrutiny : Smt. Prajakti Gokhale
Smt. Taruben Popat

Co-ordination :
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Academic Secretary for Languages
Santosh Pawar
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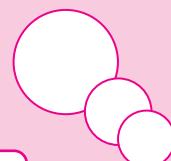
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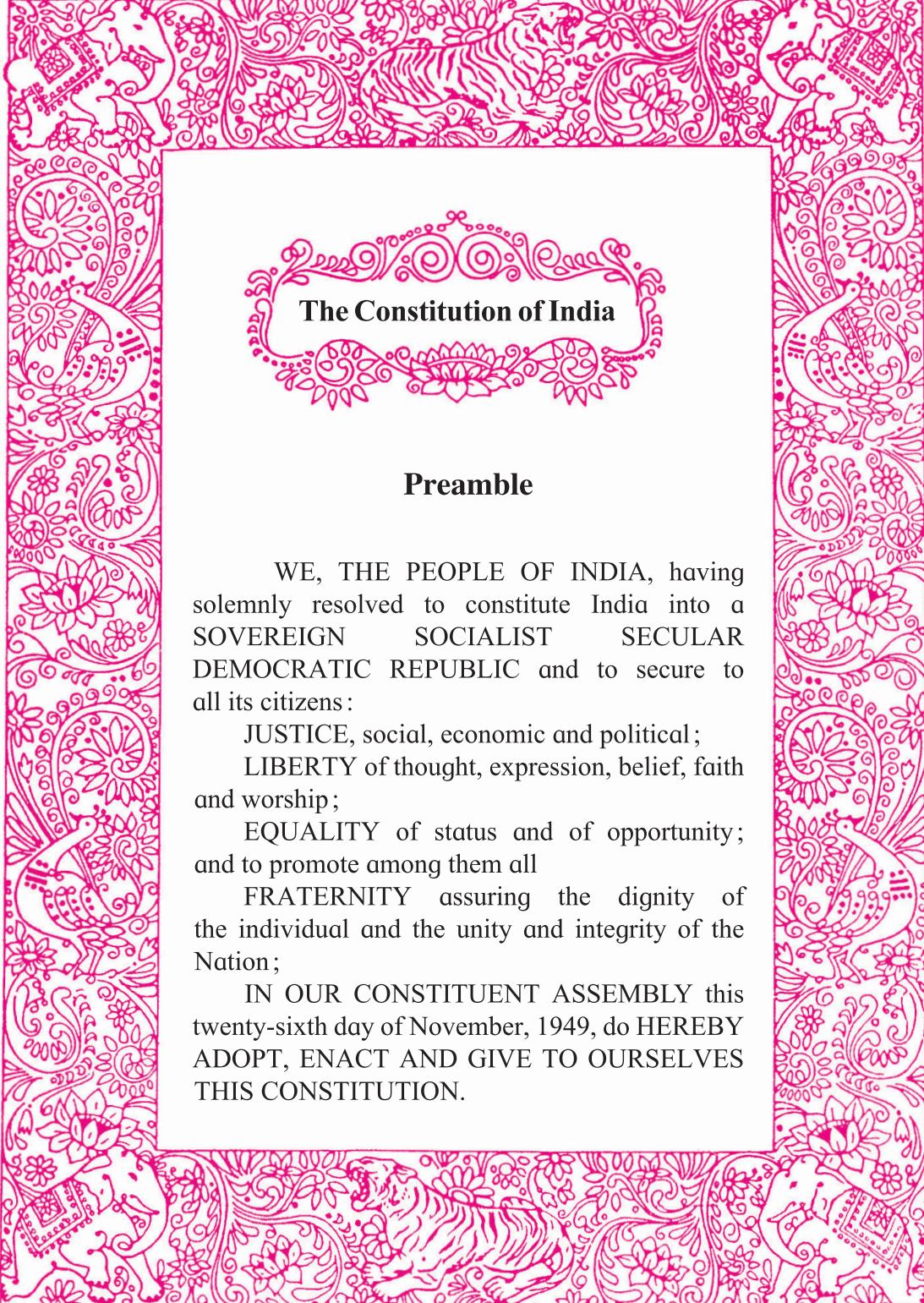
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The Constitution of India

Preamble

WE, THE PEOPLE OF INDIA, having solemnly resolved to constitute India into a SOVEREIGN SOCIALIST SECULAR DEMOCRATIC REPUBLIC and to secure to all its citizens:

JUSTICE, social, economic and political;

LIBERTY of thought, expression, belief, faith and worship;

EQUALITY of status and of opportunity; and to promote among them all

FRATERNITY assuring the dignity of the individual and the unity and integrity of the Nation;

IN OUR CONSTITUENT ASSEMBLY this twenty-sixth day of November, 1949, do HEREBY ADOPT, ENACT AND GIVE TO OURSELVES THIS CONSTITUTION.

NATIONAL ANTHEM

Jana-gana-mana-adhināyaka jaya hē
Bhārata-bhāgya-vidhātā,

Panjāba-Sindhu-Gujarāta-Marāthā
Drāvida-Utkala-Banga

Vindhya-Himāchala-Yamunā-Gangā¹
uchchala-jaladhi-taranga

Tava subha nāmē jāgē, tava subha āsisa māgē,
gāhē tava jaya-gāthā,

Jana-gana-mangala-dāyaka jaya hē
Bhārata-bhāgya-vidhātā,

Jaya hē, Jaya hē, Jaya hē,
Jaya jaya jaya, jaya hē.

PLEDGE

India is my country. All Indians
are my brothers and sisters.

I love my country, and I am proud
of its rich and varied heritage. I shall
always strive to be worthy of it.

I shall give my parents, teachers
and all elders respect, and treat
everyone with courtesy.

To my country and my people,
I pledge my devotion. In their
well-being and prosperity alone lies
my happiness.

Preface

Dear Students,

Welcome to the ninth standard!

You are now going to begin your studies at the secondary level after completing your primary education curriculum. You had only one Mathematics textbook up to the eighth standard, now you will use two textbooks – Mathematics Part-I and Mathematics Part-II.

Up to the eighth standard you have verified the properties of lines, triangles, quadrilaterals, circles, etc. given in the textbook. Now you are going to give logical proofs of these and some more properties. The skill of logical reasoning is of utmost importance in all fields of life. This textbook gives you an opportunity to learn the skill gradually.

Different activities are given in the textbook to help you understand different concepts. Other activities have been provided for revision and additional practice. You are expected to do all these and learn the proofs of properties. Discuss the reason behind every step of a proof and learn the property.

In this textbook, Mathematics-Part II, two new topics namely Trigonometry and Co-ordinate Geometry are introduced. These topics will provide a foundation for higher studies. The study of Surface Area and Volume will be useful in day to day life.

Use of internet will also help you to understand the subject. You will get through the course joyfully if you follow the three point plan of – a deep study of the textbook, activity-based learning and ample practice.

So come on! Let us study Mathematics in the company of our teachers, parents, friends and the internet. Best wishes to you for your studies!



(Dr Sunil Magar)

Director

Maharashtra State Bureau of Textbook
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Pune

Date : 28 April, 2017

Akshaya Tritiya

Indian Solar Year :

8 Vaishakh 1939

It is expected that students will develop the following competencies after studying Mathematics Part II syllabus in Standard IX

Area	Topic	Competency statement
1. Geometry	1.1 Euclidean Geometry 1.2 Parallel lines and pairs of angles 1.3 Theorems on angles and sides of a triangle. 1.4 Similar triangles 1.5 Circle 1.6 Geometric constructions 1.7 Quadrilateral	The students will be able to – <ul style="list-style-type: none"> • write ‘what is given’ and ‘what is to be proved’ from the given statement. • write the proof of the given statements by using logical conclusions. • identify the pairs of angles made by a transversals of parallel lines. • understand the properties of pairs of angles and make use of them. • write ‘Given’ ‘To prove’ and ‘proof’ of the statements. • identify similar triangles and write the ratios of corresponding sides. • prove the properties of chord of circle using tests of congruence of triangles. • draw incircle and circumcircle. • construct triangles if different type of information is given. • write proofs of the properties of different types of quadrilaterals. • use ICT tools to verify the properties of triangle, quadrilateral and circle.
2. Co-ordinate Geometry	2.1 Basics of co-ordinate Geometry	<ul style="list-style-type: none"> • explain the meaning of co-ordinates of a point in a plane. • describe a point by its co-ordinates. • use ICT tools to find the co-ordinates of a point.
3. Mensuration	3.1 Surface area and Volume	<ul style="list-style-type: none"> • find the surface area and volume of a sphere and a cone.
4. Trigonometry	4.1 Introduction to trigonometry	<ul style="list-style-type: none"> • tell the different trigonometric ratios using similar triangles and Pythagoras theorem and make use of it.

Instructions for teachers

It is expected that the teachers should go through the textbook of Mathematics Part-II for std IX thoroughly. The book contains many activities and practicals. Try to understand the purpose behind them.

The activities are of two types, (1) to write the proofs and (2) practical verification of properties and theorems. A teacher should make use of discussion, question-answers, group activities etc. to carry out the activities and make the text book more useful. A teacher is also expected to encourage the students to do the activities in the book and help them to invent new ones.

It is more important to write the proofs pursuing logical thinking than doing them by heart. The text book contains a variety of examples to enhance students' logical thinking. Teachers should construct more such examples with the help of students. Examples, which require a little higher thinking ability, are star-marked. Teachers should encourage the students who write proofs logically correct but thinking in a different way.

In the process of evaluation, it is advised to make use of open ended questions and of activity-sheets. Teachers should endeavour to develop such methods of evaluation.

The list of practicals given in the text book should be considered as specimen. Teachers can frame different practicals as well as teaching aids of their own using available material. Different activities given in the text book are included in the practicals. We hope that the evaluation method based on all these will be helpful to develop different competencies for further studies.

List of some practicals (specimen)

- (1) To find the distance between two points on a number line.
- (2) To verify the properties of angles made by a transversal of parallel lines.
- (3) To verify the properties of sides and angles of a triangle using Geometric instruments.
- (4) To verify the property of median on hypotenuse of a right angled triangle.
- (5) To do the construction of a triangle with given specific conditions.
- (6) An activity is given in the book to derive the formula of the surface area of a cone. Using the same activity, derive the formula for the area of a circle which is πr^2 .
- (7) To draw proportionate map of a room on a graph paper by considering the measurements of the things inside the room.
- (8) By drawing X and Y-axes on the school ground, ask students to tell the co-ordinates of a students' positions on the ground.
- (9) To find the volume of a cylindrical vessel using formula. Then fill the vessel completely with water and find the volume of the water. Compare both the measurements.

Similar activities can be done for different three dimensional objects.

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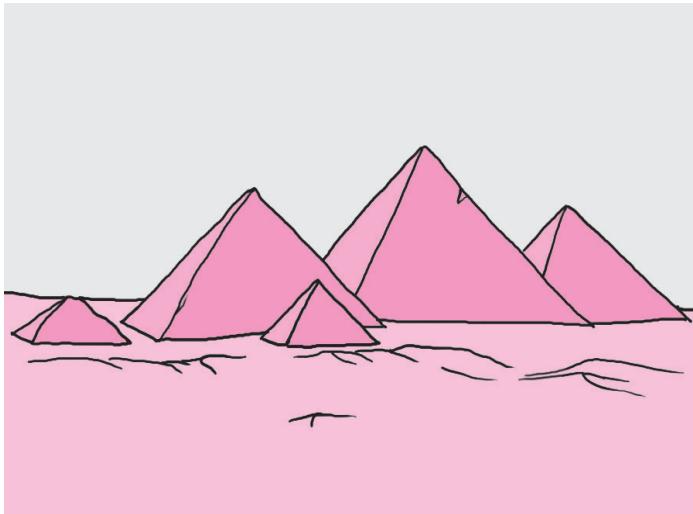
Chapters	Pages
1. Basic Concepts in Geometry	1 to 12
2. Parallel Lines	13 to 23
3. Triangles	24 to 50
4. Constructions of Triangles	51 to 56
5. Quadrilaterals	57 to 75
6. Circle	76 to 87
7. Co-ordinate Geometry	88 to 99
8. Trigonometry	100 to 113
9. Surface Area and Volume	114 to 123
● Answers	124 to 128



Let's study.

- Point, line and plane
 - Co-ordinates of a points and distance
 - Betweenness
 - Conditional statements
 - Proof

Did you recognise the adjacent picture ? It is a picture of pyramids in Egypt, built 3000 years before Christian Era. How the people were able to build such huge structures in so old time ? It is not possible to build such huge structures without developed knowledge of Geometry and Engineering



The word Geometry itself suggests the origin of the subject. It is generated from the Greek words Geo (Earth) and Metria (measuring). So

it can be guessed that the subject must have evolved from the need of measuring the Earth, that is land.

Geometry was developed in many nations in different periods and for different constructions. The first Greek mathematician, Thales, had gone to Egypt. It is said that he determined height of a pyramid by measuring its shadow and using properties of similar triangles.

Ancient Indians also had deep knowledge of Geometry. In vedic period, people used geometrical properties to build altars. The book shulba-sutra describes how to build different shapes by taking measurements with the help of a string. In course of time, the mathematicians Aaryabhat, Varahamihir, Bramhagupta, Bhaskaracharya and many others have given valuable contribution to the subject of Geometry.



Let's learn.

Basic concepts in geometry (Point, Line and Plane)

We do not define numbers. Similarly we do not define a point, line and plane also. These are some basic concepts in Geometry. Lines and planes are sets of points. Keep in mind that the word ‘line’ is used in the sense ‘straight line’.

Co-ordinates of points and distance

Observe the following number line.

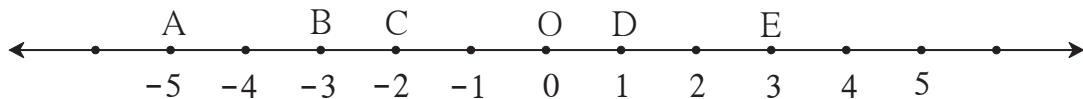


Fig. 1.1

Here, the point D on the number line denotes the number 1. So, it is said that 1 is the co-ordinate of point D. The point B denotes the number -3 on the line. Hence the co-ordinate of point B is -3 . Similarly the co-ordinates of point A and E are -5 and 3 respectively.

The point E is 2 unit away from point D. It means the distance between points D and E is 2. Thus, we can find the distance between two points on a number line by counting number of units. The distance between points A and B on the above number line is also 2.

Now let us see how to find distance with the help of co-ordinates of points.

To find the distance between two points, consider their co-ordinates and subtract the smaller co-ordinate from the larger.

The co-ordinates of points D and E are 1 and 3 respectively. We know that $3 > 1$.

Therefore, distance between points E and D = $3 - 1 = 2$

The distance between points E and D is denoted as $d(E,D)$. This is the same as $l(ED)$, that is, the length of the segment ED.

$$d(E, D) = 3 - 1 = 2$$

$$\therefore l(\text{ED}) = 2$$

$$d(E, D) = l(ED) = 2$$

Similarly $d(D, E) = 2$

$$d(\text{C}, \text{D}) = 1 - (-2)$$

$$= 1 + 2 = 3$$

$$\therefore d(C, D) = l(CD) = 3$$

Similarly $d(D, C) = 3$

Now, let us find $d(A,B)$. The co-ordinate of A is -5 and that of B is -3 ; $-3 > -5$

$$\therefore d(A, B) = -3 - (-5) = -3 + 5 = 2.$$

From the above examples it is clear that the distance between two distinct points is always a positive number.

Note that, if the two points are not distinct then the distance between them is zero.



Remember this !

- The distance between two points is obtained by subtracting the smaller co-ordinate from the larger co-ordinate.
 - The distance between any two points is a non-negative real number.



Let's learn.

Betweenness

If P , Q , R are three distinct collinear points, there are three possibilities.



Fig. 1.2

- (i) Point Q is between P and R (ii) Point R is between P and Q (iii) Point P is between R and Q

If $d(P, Q) + d(Q, R) = d(P, R)$ then it is said that point Q is between P and R. The betweenness is shown as P - Q - R.

Solved examples

Ex (1) On a number line, points A, B and C are such that

$$d(A, B) = 5, d(B, C) = 11 \text{ and } d(A, C) = 6.$$

Which of the points is between the other two ?

Solution : Which of the points A, B and C is between the other two, can be decided as follows.

$$d(B,C) = 11 \dots \quad (I)$$

$$d(A,B) + d(A,C) = 5+6 = 11 \dots \text{(II)}$$

$\therefore d(B, C) = d(A, B) + d(A, C) \dots$ [from (I) and (II)]

Point A is between point B and point C.

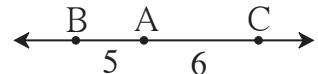


Fig. 1.3

Ex (2) U, V and A are three cities on a straight road. The distance between U and A is 215 km, between V and A is 140 km and between U and V is 75 km. Which of them is between the other two ?

Solution : $d(U,A) = 215$; $d(V,A) = 140$; $d(U,V) = 75$

$$d(U,V) + d(V,A) = 75 + 140 = 215; \quad d(U,A) = 215$$

$$\therefore d(U,A) = d(U,V) + d(V,A)$$

∴ The city V is between the cities U and A.

Ex (3) The co-ordinate of point A on a number line is 5. Find the co-ordinates of points on the same number line which are 13 units away from A.

Solution : As shown in the figure, let us take points T and D to the left and right of A respectively, at a distance of 13 units.

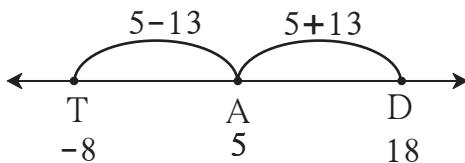


Fig. 1.4

The co-ordinate of point T, which is to the left of A ,will be $5 - 13 = -8$

The co-ordinate of point D, which is to the right of A, will be $5 + 13 = 18$

∴ the co-ordinates of points 13 units away from A will be -8 and 18 .

Verify your answer : $d(A,D) = d(A,T) = 13$

Activity

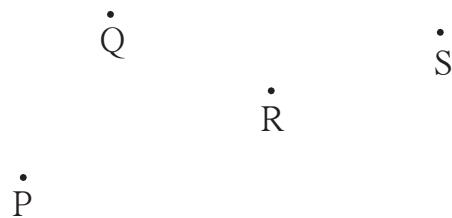
- (1) Points A, B, C are given aside. Check, with a stretched thread, whether the three points are collinear or not. If they are collinear, write which one of them is between the other two.

(2) Given aside are four points P, Q, R, and S. Check which three of them are collinear and which three are non collinear. In the case of three collinear points, state which of them is between the other two.

(3) Students are asked to stand in a line for mass drill. How will you check whether the students standing are in a line or not ?

(4) How had you verified that light rays travel in a straight line ?

Recall an experiment in science which you have done in a previous standard.



Practice set 1.1

1. Find the distances with the help of the number line given below.

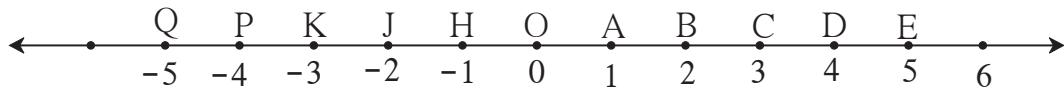


Fig. 1.5

- (i) $d(B, E)$ (ii) $d(J, A)$ (iii) $d(P, C)$ (iv) $d(J, H)$
 (v) $d(K, O)$ (vi) $d(O, E)$ (vii) $d(P, J)$ (viii) $d(Q, B)$
2. If the co-ordinate of A is x and that of B is y , find $d(A, B)$.
 (i) $x = 1, y = 7$ (ii) $x = 6, y = -2$ (iii) $x = -3, y = 7$
 (iv) $x = -4, y = -5$ (v) $x = -3, y = -6$ (vi) $x = 4, y = -8$
3. From the information given below, find which of the point is between the other two.
 If the points are not collinear, state so.
 (i) $d(P, R) = 7$, $d(P, Q) = 10$, $d(Q, R) = 3$
 (ii) $d(R, S) = 8$, $d(S, T) = 6$, $d(R, T) = 4$
 (iii) $d(A, B) = 16$, $d(C, A) = 9$, $d(B, C) = 7$
 (iv) $d(L, M) = 11$, $d(M, N) = 12$, $d(N, L) = 8$
 (v) $d(X, Y) = 15$, $d(Y, Z) = 7$, $d(X, Z) = 8$
 (vi) $d(D, E) = 5$, $d(E, F) = 8$, $d(D, F) = 6$
4. On a number line, points A, B and C are such that $d(A, C) = 10$, $d(C, B) = 8$
 Find $d(A, B)$ considering all possibilities.
5. Points X, Y, Z are collinear such that $d(X, Y) = 17$, $d(Y, Z) = 8$, find $d(X, Z)$.
6. Sketch proper figure and write the answers of the following questions.
 (i) If A - B - C and $l(AC) = 11$, $l(BC) = 6.5$, then $l(AB) = ?$
 (ii) If R - S - T and $l(ST) = 3.7$, $l(RS) = 2.5$, then $l(RT) = ?$
 (iii) If X - Y - Z and $l(XZ) = 3\sqrt{7}$, $l(XY) = \sqrt{7}$, then $l(YZ) = ?$
7. Which figure is formed by three non-collinear points?



Let's learn.

In the book, Mathematics - Part I for std IX, we have learnt union and intersection of sets in the topic on sets. Now, let us describe a segment, a ray and a line as sets of points.

(1) Line segment :

The union set of point A, point B and points between

A and B is called segment AB. Segment AB is

written as seg AB in brief. Seg AB means seg BA.

Point A and point B are called the end points of seg AB.

The distance between the end points of a segment

is called the length of the segment. That is $l(AB) = d(A,B)$

$l(AB) = 5$ is also written as $AB = 5$.



Fig. 1.6

(2) Ray AB :

Suppose, A and B are two distinct points. The union set of all points on seg AB and the points P such that A - B - P, is called ray AB.

Here point A is called the starting point of ray AB.



Fig. 1.7

(3) Line AB :

The union set of points on ray AB and opposite ray of ray AB is called line AB.

The set of points of seg AB is a subset of points of line AB.

(4) Congruent segments :

If the length of two segments is equal then the two segments are congruent.

If $l(AB) = l(CD)$ then $\text{seg } AB \cong \text{seg } CD$

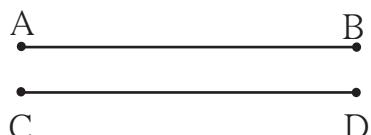


Fig. 1.8

(5) Properties of congruent segments :

- (i) Reflexivity : $\text{seg AB} \cong \text{seg AB}$
 - (ii) Symmetry : If $\text{seg AB} \cong \text{seg CD}$ then $\text{seg CD} \cong \text{seg AB}$
 - (iii) Transitivity : If $\text{seg AB} \cong \text{seg CD}$ and $\text{seg CD} \cong \text{seg EF}$ then $\text{seg AB} \cong \text{seg EF}$

(6) Midpoint of a segment :

If A-M-B and $\overline{AM} \cong \overline{MB}$, then M is called the midpoint of \overline{AB} .

Every segment has one and only one midpoint.



Fig. 1.9

(7) Comparison of segments :

If length of segment AB is less than the length of segment CD, it is written as $\text{seg } AB < \text{seg } CD$ or $\text{seg } CD > \text{seg } AB$.

The comparison of segments depends upon their lengths.

(8) Perpendicularity of segments or rays :

If the lines containing two segments, two rays or a ray and a segment are perpendicular to each other then the two segments, two rays or the segment and the ray are said to be perpendicular to each other.

In the figure 1.11, seg AB \perp line CD,
seg AB \perp ray CD.

(9) Distance of a point from a line :

If $\text{seg } CD \perp \text{line } AB$ and the point D lies on line AB then the length of seg CD is called the distance of point C from line AB.

The point D is called the foot of the perpendicular. If $l(CD) = a$, then the point C is at a distance of ' a ' from the line AB.

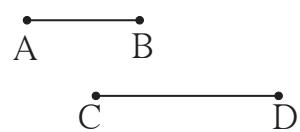


Fig. 1.10

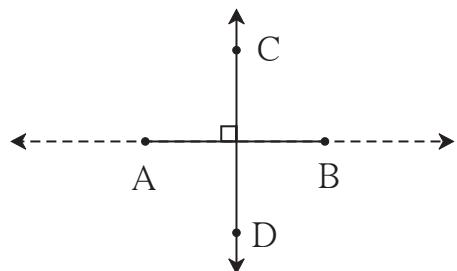


Fig. 1.11

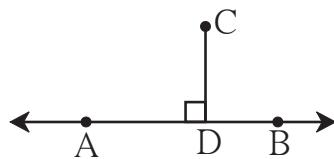


Fig. 1.12

Practice set 1.2

1. The following table shows points on a number line and their co-ordinates. Decide whether the pair of segments given below the table are congruent or not.

Point	A	B	C	D	E
Co-ordinate	-3	5	2	-7	9

- (i) seg DE and seg AB (ii) seg BC and seg AD (iii) seg BE and seg AD

2. Point M is the midpoint of seg AB. If AB = 8 then find the length of AM.

3. Point P is the midpoint of seg CD. If CP = 2.5, find $l(CD)$.

4. If AB = 5 cm, BP = 2 cm and AP = 3.4 cm, compare the segments.

5. Write the answers to the following questions with reference to figure 1.13.

- (i) Write the name of the opposite ray of ray RP

(ii) Write the intersection set of ray PQ and ray RP.

(iii) Write the union set of seg PQ and seg QR.

(iv) State the rays of which seg QR is a subset.

(v) Write the pair of opposite rays with common end point R.

(vi) Write any two rays with common end point S.

(vii) Write the intersection set of ray SP and ray ST.



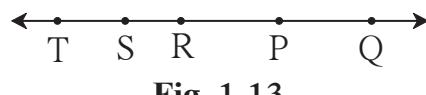


Fig. 1.13

6. Answer the questions with the help of figure 1.14.

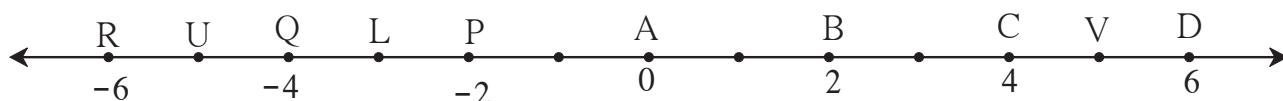


Fig. 1.14

- (i) State the points which are equidistant from point B.
 - (ii) Write a pair of points equidistant from point Q.
 - (iii) Find $d(U,V)$, $d(P,C)$, $d(V,B)$, $d(U,L)$.



Let's learn.

Conditional statements and converse

The statements which can be written in the ‘If-then’ form are called conditional statements. The part of the statement following ‘If’ is called the antecedent, and the part following ‘then’ is called the consequent.

For example, consider the statement : The diagonals of a rhombus are perpendicular bisectors of each other.

The statement can be written in the conditional form as, ‘If the given quadrilateral is a rhombus then its diagonals are perpendicular bisectors of each other.’

If the antecedent and consequent in a given conditional statement are interchanged, the resulting statement is called the **converse** of the given statement.

If a conditional statement is true, its converse is not necessarily true. Study the following examples.

Conditional statement : If a quadrilateral is a rhombus then its diagonals are perpendicular bisectors of each other.

Converse : If the diagonals of a quadrilateral are perpendicular bisectors of each other then it is a rhombus.

In the above example, the statement and its converse are true.

Now consider the following example,

Conditional statement : If a number is a prime number then it is even or odd.

Converse : If a number is even or odd then it is a prime number.

In this example, the statement is true, but its converse is false.



Proofs

We have studied many properties of angles, triangles and quadrilaterals through activities.

In this standard we are going to look at the subject of Geometry with a different point of view, which was originated by the Greek mathematician Euclid, who lived in the third century before Christian Era. He gathered the knowledge of Geometry prevailing at his time and streamlined it. He took for granted some self evident geometrical statements which were accepted by all and called them **Postulates**. He showed that on the basis of the postulates some more properties can be proved logically.

Properties proved logically are called **Theorems**.

Some of Euclid's postulates are given below.

- (1) There are infinite lines passing through a point.
 - (2) There is one and only one line passing through two points.
 - (3) A circle of given radius can be drawn taking any point as its centre.
 - (4) All right angles are congruent with each other.
 - (5) If two interior angles formed on one side of a transversal of two lines add up to less than two right angles then the lines produced in that direction intersect each other



Euclid

We have verified some of these postulates through activities.

A property is supposed to be true if it can be proved logically. It is then called a **Theorem**. The logical argument made to prove a theorem is called its **proof**.

When we are going to prove that a conditional statement is true, its antecedent is called ‘Given part’ and the consequent is called ‘the part to be proved’.

There are two types of proofs, **Direct** and **Indirect**.

Let us give a direct proof of the property of angles made by two intersecting lines.

Theorem : The opposite angles formed by two intersecting lines are of equal measures.

Given : Line AB and line CD intersect at point O such that A - O - B, C - O - D.

To prove : (i) $\angle AOC = \angle BOD$

(ii) $\angle BOC = \angle AOD$

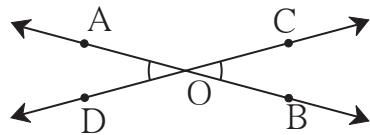


Fig. 1.15

Proof : $\angle AOC + \angle BOC = 180^\circ$ (I) (angles in linear pair)

$$\angle BOC + \angle BOD = 180^\circ \dots \dots \dots \text{(II) (angles in linear pair)}$$

$$\angle AOC + \angle BOC = \angle BOC + \angle BOD \dots \dots \text{[from (I) and (II)]}$$

$\therefore \angle AOC = \angle BOD$ eliminating $\angle BOC$.

Similarly, it can be proved that $\angle BOC = \angle AOD$.

Indirect proof :

This type of proof starts with an assumption that the consequence is false. Using it and the properties accepted earlier, we start arguing step by step and reach a conclusion. The conclusion is contradictory with the antecedent or a property which is already accepted. Hence, the assumption that the consequent is false goes wrong. So it is accepted that the consequent is true.

Study the following example.

Statement : A prime number greater than 2 is odd.

Conditional statement : If p is a prime number greater than 2 then it is odd.

Given : p is a prime number greater than 2. That is, 1 and p are the only divisors of p .

To prove : p is an odd number.

Proof : Let us suppose that p is not an odd number.

So p is an even number

\therefore a divisor of p is 2 (I)

But it is given that p is a prime number greater than 2(given)

\therefore 1 and p are the only divisors of p (II)

Statements (I) and (II) are contradictory.

\therefore the assumption , that p is not odd is false.

This proves that a prime number greater than 2 is odd.

Practice set 1.3

1. Write the following statements in ‘if-then’ form.
 - (i) The opposite angles of a parallelogram are congruent.
 - (ii) The diagonals of a rectangle are congruent.
 - (iii) In an isosceles triangle, the segment joining the vertex and the mid point of the base is perpendicular to the base.
 2. Write converses of the following statements.
 - (i) The alternate angles formed by two parallel lines and their transversal are congruent.
 - (ii) If a pair of the interior angles made by a transversal of two lines are supplementary then the lines are parallel.
 - (iii) The diagonals of a rectangle are congruent.

Problem set 1

- 1.** Select the correct alternative from the answers of the questions given below.

(i) How many mid points does a segment have ?
(A) only one (B) two (C) three (D) many

(ii) How many points are there in the intersection of two distinct lines ?
(A) infinite (B) two (C) one (D) not a single

(iii) How many lines are determined by three distinct points ?
(A) two (B) three (C) one or three (D) six

(iv) Find $d(A, B)$, if co-ordinates of A and B are -2 and 5 respectively.
(A) -2 (B) 5 (C) 7 (D) 3

(v) If $P - Q - R$ and $d(P,Q) = 2$, $d(P,R) = 10$, then find $d(Q,R)$.
(A) 12 (B) 8 (C) $\sqrt{96}$ (D) 20

2. On a number line, co-ordinates of P, Q, R are $3, -5$ and 6 respectively. State with reason whether the following statements are true or false.

(i) $d(P,Q) + d(Q,R) = d(P,R)$ (ii) $d(P,R) + d(R,Q) = d(P,Q)$
(iii) $d(R,P) + d(P,Q) = d(R,Q)$ (iv) $d(P,Q) - d(P,R) = d(Q,R)$

3. Co-ordinates of some pairs of points are given below. Hence find the distance between each pair.

(i) $3, 6$ (ii) $-9, -1$ (iii) $-4, 5$ (iv) $0, -2$
(v) $x+3, x-3$ (vi) $-25, -47$ (vii) $80, -85$

- 4.** Co-ordinate of point P on a number line is -7 . Find the co-ordinates of points on the number line which are at a distance of 8 units from point P.

5. Answer the following questions.

 - If $A - B - C$ and $d(A,C) = 17$, $d(B,C) = 6.5$ then $d(A,B) = ?$
 - If $P - Q - R$ and $d(P,Q) = 3.4$, $d(Q,R) = 5.7$ then $d(P,R) = ?$

6. Co-ordinate of point A on a number line is 1 . What are the co-ordinates of points on the number line which are at a distance of 7 units from A ?

7. Write the following statements in conditional form.

 - Every rhombus is a square.
 - Angles in a linear pair are supplementary.
 - A triangle is a figure formed by three segments.
 - A number having only two divisors is called a prime number.

8. Write the converse of each of the following statements.

 - If the sum of measures of angles in a figure is 180° , then the figure is a triangle.
 - If the sum of measures of two angles is 90° then they are complement of each other.
 - If the corresponding angles formed by a transversal of two lines are congruent then the two lines are parallel.
 - If the sum of the digits of a number is divisible by 3 then the number is divisible by 3.

9. Write the antecedent (given part) and the consequent (part to be proved) in the following statements.

 - If all sides of a triangle are congruent then its all angles are congruent.
 - The diagonals of a parallelogram bisect each other.

10*. Draw a labelled figure showing information in each of the following statements and write the antecedent and the consequent.

 - Two equilateral triangles are similar.
 - If angles in a linear pair are congruent then each of them is a right angle.
 - If the altitudes drawn on two sides of a triangle are congruent then those two sides are congruent.





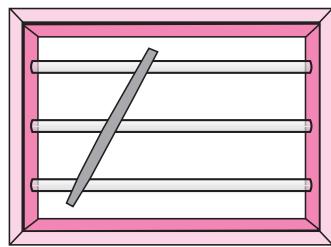
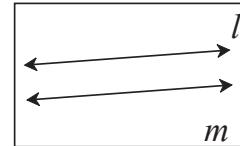
Let's study.

- Properties of angles formed by parallel lines and its transversal
- Tests of parallelness of two lines
- Use of properties of parallel lines



Let's recall.

Parallel lines : The lines which are coplanar and do not intersect each other are called parallel lines.



Hold a stick across the horizontal parallel bars of a window as shown in the figure.
How many angles are formed ?

- Do you recall the pairs of angles formed by two lines and their transversal ?

In figure 2.1, line n is a transversal of line l and line m .

Here, in all 8 angles are formed. Pairs of angles formed out of these angles are as follows :

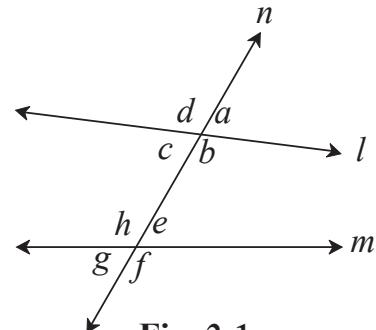


Fig. 2.1

Pairs of corresponding angles

- (i) $\angle d, \angle h$
- (ii) $\angle a, \boxed{\quad}$
- (iii) $\angle c, \boxed{\quad}$
- (iv) $\angle b, \boxed{\quad}$

Pairs of alternate interior angles

- (i) $\angle c, \angle e$ (ii) $\angle b, \angle h$

Pairs of alternate exterior angles

- (i) $\angle d, \angle f$
- (ii) $\angle a, \angle g$

Pairs of interior angles on the same side of the transversal

- (i) $\angle c, \angle h$
- (ii) $\angle b, \angle e$

Some important properties :

- (1) When two lines intersect, the pairs of opposite angles formed are congruent.
- (2) The angles in a linear pair are supplementary.

- (3) When one pair of corresponding angles is congruent, then all the remaining pairs of corresponding angles are congruent.
 - (4) When one pair of alternate angles is congruent, then all the remaining pairs of alternate angles are congruent.
 - (5) When one pair of interior angles on one side of the transversal is supplementary, then the other pair of interior angles is also supplementary.

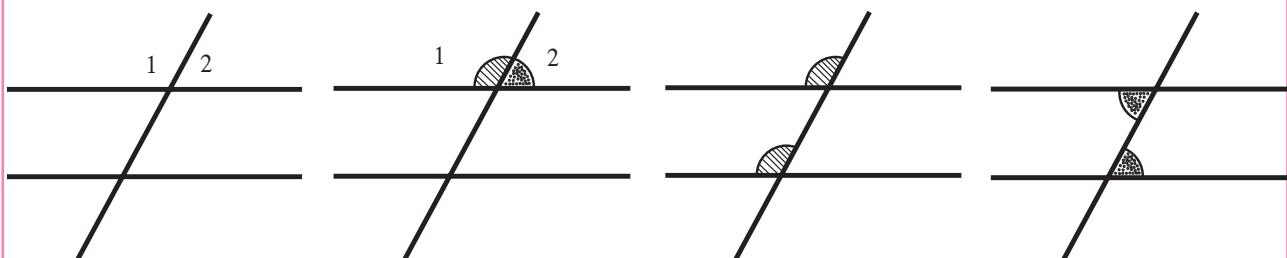


Properties of parallel lines

Activity

To verify the properties of angles formed by a transversal of two parallel lines.

Take a piece of thick coloured paper. Draw a pair of parallel lines and a transversal on it. Paste straight sticks on the lines. Eight angles will be formed. Cut pieces of coloured paper, as shown in the figure, which will just fit at the corners of $\angle 1$ and $\angle 2$. Place the pieces near different pairs of corresponding angles, alternate angles and interior angles and verify their properties.





Let's learn.

We have verified the properties of angles formed by a transversal of two parallel lines. Let us now prove the properties using Euclid's famous fifth postulate given below.

If sum of two interior angles formed on one side of a transversal of two lines is less than two right angles then the lines produced in that direction intersect each other.

Interior angle theorem

Theorem : If two parallel lines are intersected by a transversal, the interior angles on either side of the transversal are supplementary.

Given : line $l \parallel$ line m and line n is their transversal. Hence as shown in the figure $\angle a, \angle b$ are interior angles formed on one side and $\angle c, \angle d$ are interior angles formed on other side of the transversal.

To prove : $\angle a + \angle b = 180^\circ$
 $\angle d + \angle c = 180^\circ$

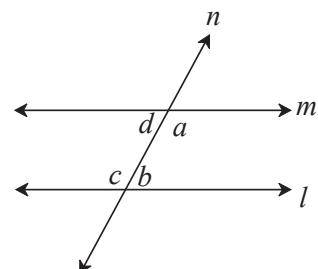


Fig. 2.2

Proof : Three possibilities arise regarding the sum of measures of $\angle a$ and $\angle b$.

- (i) $\angle a + \angle b < 180^\circ$ (ii) $\angle a + \angle b > 180^\circ$ (iii) $\angle a + \angle b = 180^\circ$

Let us assume that the possibility (i) $\angle a + \angle b < 180^\circ$ is true.

Then according to Euclid's postulate, if the line l and line m are produced will intersect each other on the side of the transversal where $\angle a$ and $\angle b$ are formed.

But line l and line m are parallel linesgiven

$\therefore \angle a + \angle b < 180^\circ$ impossible(I)

Now let us suppose that $\angle a + \angle b > 180^\circ$ is true.

$\therefore \angle a + \angle b \geq 180^\circ$

But $\angle a + \angle d \equiv 180^\circ$

and $\angle c \pm \angle b \equiv 180^\circ$ angles in linear pairs

$$\therefore \angle a + \angle d + \angle b + \angle c = 180^\circ + 180^\circ = 360^\circ$$

$$\therefore \angle C + \angle D \equiv 360^\circ - (\angle A + \angle B)$$

If $\angle a + \angle b \geq 180^\circ$ then $[360^\circ - (\angle a + \angle b)] \leq 180^\circ$

$$\therefore \angle a + \angle d \leq 180^\circ$$

∴ In that case line l and line m produced will intersect each other on the same side of the transversal where $\angle c$ and $\angle d$ are formed.

$\therefore \angle c + \angle d < 180$ is impossible.

That is $\angle a + \angle b > 180^\circ$ is impossible..... (II)

∴ the remaining possibility,

$\angle a + \angle b = 180^\circ$ is true.....from (I) and (II)

$\therefore \angle a + \angle b = 180^\circ$ Similarly, $\angle c + \angle d = 180^\circ$

Note that, in this proof, because of the contradictions we have denied the possibilities $\angle a + \angle b > 180^\circ$ and $\angle a + \angle b < 180^\circ$.

Therefore, this proof is an example of indirect proof.

Corresponding angles and alternate angles theorems

Theorem : The corresponding angles formed by a transversal of two parallel lines are of equal measure.

Given : line $l \parallel$ line m
line n is a transversal.

To prove : $\angle a = \angle b$

Proof : $\angle a + \angle c = 180^\circ$ (I) angles in linear pair

Fig. 2.3

$$\angle a \pm \angle c \equiv \angle b \pm \angle c \quad \text{from (I) and (II)}$$

$\cdot \angle a \equiv \angle b$

Theorem : The alternate angles formed by a transversal of two parallel lines are of equal measures

Given : line $l \parallel$ line m
 line n is a transversal

To prove : $\angle d = \angle b$

Proof : $\angle d + \angle$

Proof : $\angle d + \angle c = 180^\circ$ (I) angles in linear pair

$\angle c + \angle b = 180^\circ$ (II), property of interior angles of parallel line

$$\angle d + \angle c \equiv \angle c + \angle b \quad \text{from (I) and (II)}$$

$\cdot \angle d - \angle b$

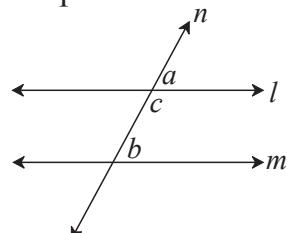


Fig. 2.3

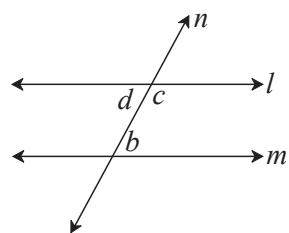


Fig. 2.4

Practice set 2.1

1. In figure 2.5, line RP || line MS and line DK is their transversal. $\angle DHP = 85^\circ$

Find the measures of following angles.

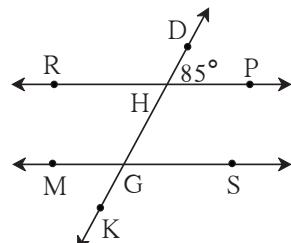


Fig. 2.5

2. In figure 2.6, line $p \parallel$ line q and line l and line m are transversals. Measures of some angles are shown. Hence find the measures of $\angle a, \angle b, \angle c, \angle d$.

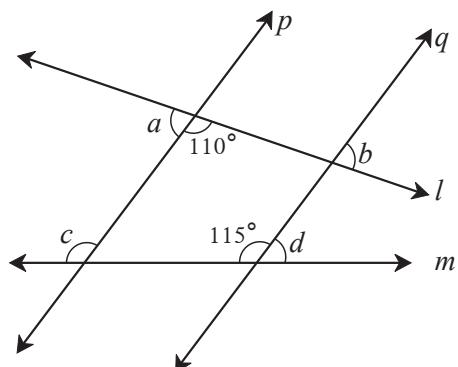


Fig. 2.6

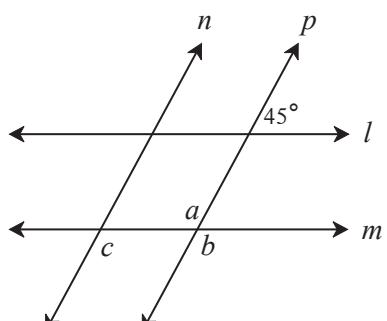


Fig. 2.7

3. In figure 2.7, line $l \parallel$ line m and line $n \parallel$ line p . Find $\angle a$, $\angle b$, $\angle c$ from the given measure of an angle.

- 4*. In figure 2.8, sides of $\angle PQR$ and $\angle XYZ$ are parallel to each other. Prove that,
 $\angle PQR \cong \angle XYZ$

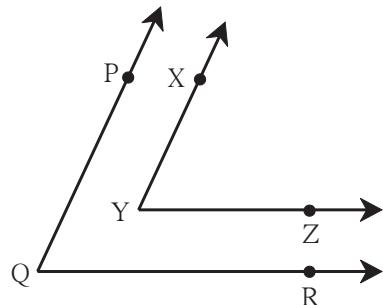


Fig. 2.8

5. In figure 2.9, line $AB \parallel$ line CD and line PQ is transversal. Measure of one of the angles is given.

Hence find the measures of the following angles.

- (i) \angle ART (ii) \angle CTQ
 (iii) \angle DTQ (iv) \angle PRB

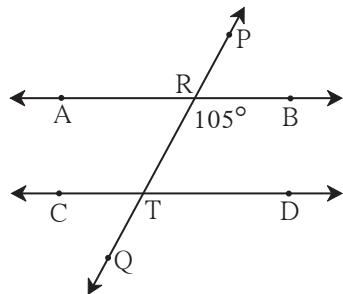


Fig. 2.9



Let's learn.

Use of properties of parallel lines

Let us prove a property of a triangle using the properties of angles made by a transversal of parallel lines.

Theorem : The sum of measures of all angles of a triangle is 180° .

Given : $\triangle ABC$ is any triangle.

To prove : $\angle ABC + \angle ACB + \angle BAC = 180^\circ$.

Construction : Draw a line parallel to seg BC and passing through A. On the line take points P and Q such that, P - A - Q.

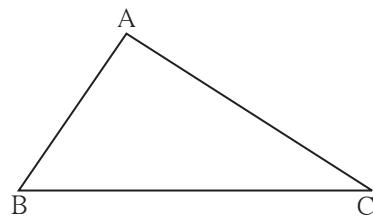


Fig. 2.10

Proof : Line PQ || line BC and seg AB is a transversal.

$\therefore \angle ABC = \angle PAB$alternate angles....(I)

line PQ || line BC and seg AC is a transversal.

$\therefore \angle ACB = \angle QAC$alternate angles....(II)

\therefore From I and II ,

$$\angle ABC + \angle ACB = \angle PAB + \angle QAC \dots \text{(III)}$$

Adding $\angle BAC$ to both sides of (III).

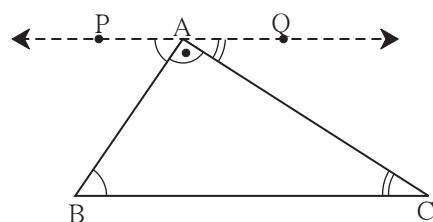


Fig. 2.11

$$\begin{aligned}
 \angle ABC + \angle ACB + \angle BAC &= \angle PAB + \angle QAC + \angle BAC \\
 &= \angle PAB + \angle BAC + \angle QAC \\
 &= \angle PAC + \angle QAC \dots (\because \angle PAB + \angle BAC = \angle PAC) \\
 &= 180^\circ \quad \dots \text{Angles in linear pair}
 \end{aligned}$$

That is, sum of measures of all three angles of a triangle is 180° .



Let's discuss.

In fig. 2.12, How will you decide whether line l and line m are parallel or not?

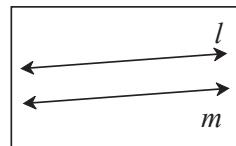


Fig. 2.12



Let's learn.

Tests for parallel lines

Whether given two lines are parallel or not can be decided by examining the angles formed by a transversal of the lines.

- (1) If the interior angles on the same side of a transversal are supplementary then the lines are parallel.
 - (2) If one of the pairs of alternate angles is congruent then the lines are parallel.
 - (3) If one of the pairs of corresponding angles is congruent then the lines are parallel.

Interior angles test

Theorem : If the interior angles formed by a transversal of two distinct lines are supplementary, then the two lines are parallel.

Given : Line XY is a transversal of line AB and line CD.

$$\angle BPQ + \angle PQD = 180^\circ$$

To prove : line AB || line CD

Proof : We are going to give an indirect proof.

Let us suppose that the statement to be proved is wrong. That is, we assume, line AB and line CD are not parallel, means line AB and CD intersect at point

So Δ POT is formed.

$\therefore \angle TPQ + \angle PQT + \angle PTQ = 180^\circ$ sum of angles of a triangle

but $\angle TPQ + \angle PQT = 180^\circ$ given

That is the sum of two angles of the triangle is 180° .

But sum of three angles of a triangle is 180° .

$$\therefore \angle PTQ = 0^\circ.$$

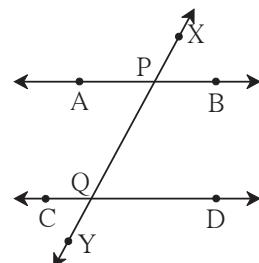


Fig. 2.13

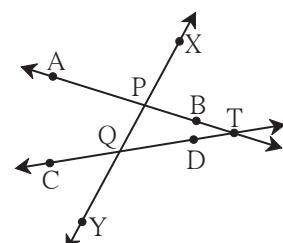


Fig. 2.14

\therefore line PT and line QT means line AB and line CD are not distinct lines.

But, we are given that line AB and line CD are distinct lines.

\therefore we arrive at a contradiction.

\therefore our assumption is wrong. Hence line AB and line CD are parallel.

Thus it is proved that if the interior angles formed by a transversal are supplementary, then the lines are parallel.

This property is called **interior angles test** of parallel lines.

Alternate angles test

Theorem : If a pair of alternate angles formed by a transversal of two lines is congruent then the two lines are parallel.

Given : Line n is a transversal of line l and line m .

$\angle a$ and $\angle b$ is a congruent pair of alternate angles.

That is, $\angle a = \angle b$

To prove : line $l \parallel$ line m

Proof : $\angle a + \angle c = 180^\circ$ angles in linear pair

$\angle a = \angle b$ given

$\therefore \angle b + \angle c = 180^\circ$

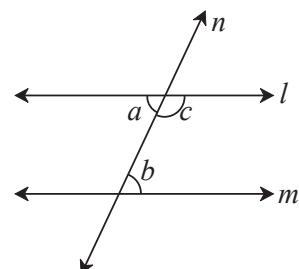


Fig. 2.15

But $\angle b$ and $\angle c$ are interior angles on the same side of the transversal.

\therefore line $l \parallel$ line m interior angles test

This property is called the **alternate angles test** of parallel lines.

Corresponding angles Test

Theorem : If a pair of corresponding angles formed by a transversal of two lines is congruent then the two lines are parallel.

Given : Line n is a transversal of line l and line m .

$\angle a$ and $\angle b$ is a congruent pair of corresponding angles.

That is, $\angle a = \angle b$

To prove : line $l \parallel$ line m

Proof : $\angle a + \angle c = 180^\circ$ angles in linear pair

$\angle a = \angle b$ given

$\therefore \angle b + \angle c = 180^\circ$

That is a pair of interior angles on the same side of the transversal is congruent.

\therefore line $l \parallel$ line m interior angles test

This property is called the **corresponding angles test** of parallel lines.

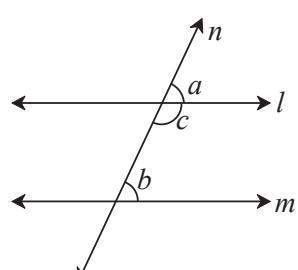


Fig. 2.16

Corollary I : If a line is perpendicular to two lines in a plane, then the two lines are parallel to each other.

Given : Line $n \perp$ line l and line $n \perp$ line m

To prove : line $l \parallel$ line m

Proof : line $n \perp$ line l and line $n \perp$ line m ...given

$$\therefore \angle a = \angle c = 90^\circ$$

$\angle a$ and $\angle c$ are corresponding angles formed by transversal n of line l and line m .

\therefore line $l \parallel$ line m corresponding angles test

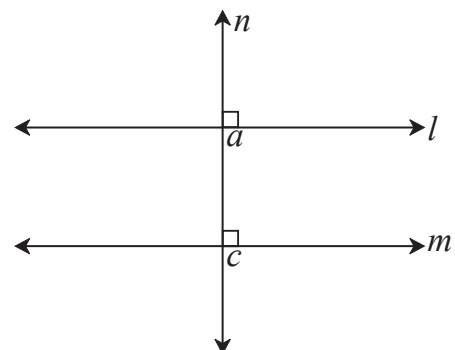


Fig. 2.17

Corollary II : If two lines in a plane are parallel to a third line in the plane then those two lines are parallel to each other. Write the proof of the corollary.

Practice set 2.2

1. In figure 2.18, $y = 108^\circ$ and $x = 71^\circ$
Are the lines m and n parallel? Justify?

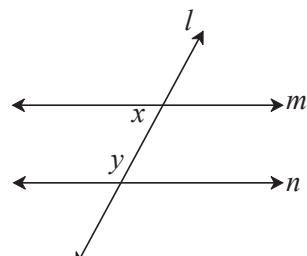


Fig. 2.18

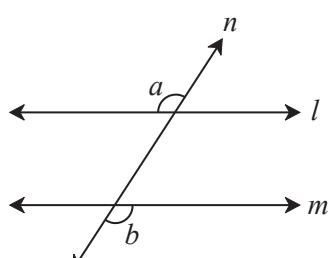


Fig. 2.19

3. In figure 2.20, if $\angle a \cong \angle b$ and $\angle x \cong \angle y$ then prove that line $l \parallel$ line n .

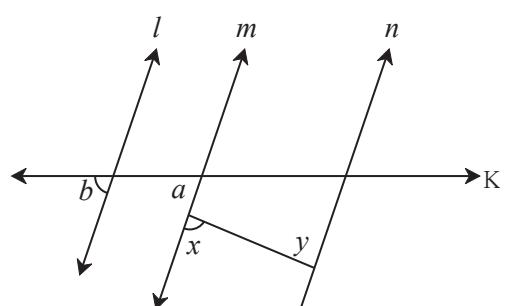


Fig. 2.20

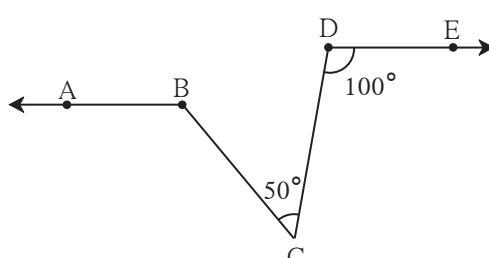


Fig. 2.21

4. In figure 2.21, if ray BA \parallel ray DE, $\angle C = 50^\circ$ and $\angle D = 100^\circ$. Find the measure of $\angle ABC$.

(Hint : Draw a line passing through point C and parallel to line AB.)

-

In figure 2.22, ray $AE \parallel$ ray BD ,
 ray AF is the bisector of $\angle EAB$ and
 ray BC is the bisector of $\angle ABD$.
 Prove that line $AF \parallel$ line BC .

Fig. 2.22

6. A transversal EF of line AB and line CD intersects the lines at point P and Q respectively. Ray PR and ray QS are parallel and bisectors of $\angle BPQ$ and $\angle PQC$ respectively.

Prove that line AB || line CD.

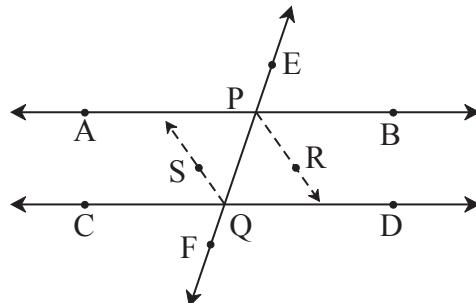


Fig. 2.23

Problem set 2

1. Select the correct alternative and fill in the blanks in the following statements.

 - If a transversal intersects two parallel lines then the sum of interior angles on the same side of the transversal is
 (A) 0° (B) 90° (C) 180° (D) 360°
 - The number of angles formed by a transversal of two lines is
 (A) 2 (B) 4 (C) 8 (D) 16
 - A transversal intersects two parallel lines. If the measure of one of the angles is 40° then the measure of its corresponding angle is
 (A) 40° (B) 140° (C) 50° (D) 180°
 - In ΔABC , $\angle A = 76^\circ$, $\angle B = 48^\circ$, $\therefore \angle C =$
 (A) 66° (B) 56° (C) 124° (D) 28°
 - Two parallel lines are intersected by a transversal. If measure of one of the alternate interior angles is 75° then the measure of the other angle is
 (A) 105° (B) 15° (C) 75° (D) 45°

2*. Ray PQ and ray PR are perpendicular to each other. Points B and A are in the interior and exterior of $\angle QPR$ respectively. Ray PB and ray PA are perpendicular to each other. Draw a figure showing all these rays and write -

 - A pair of complementary angles
 - A pair of supplementary angles.
 - A pair of congruent angles.

3. Prove that, if a line is perpendicular to one of the two parallel lines, then it is perpendicular to the other line also.

4. In figure 2.24, measures of some angles are shown. Using the measures find the measures of $\angle x$ and $\angle y$ and hence show that $l \parallel m$.

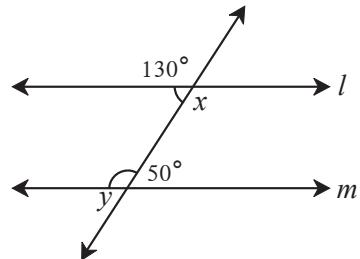


Fig. 2.24

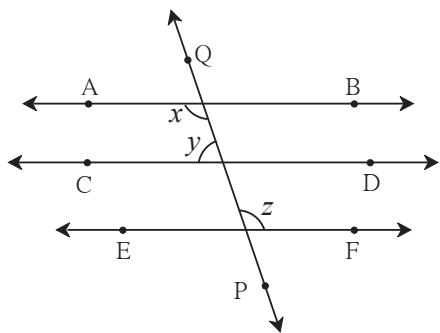


Fig. 2.25

6. In figure 2.26, if line $q \parallel$ line r , line p is their transversal and if $a = 80^\circ$ find the values of f and g .

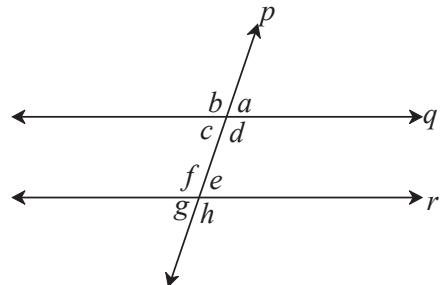


Fig. 2.26

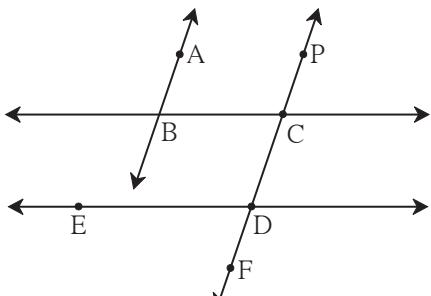


Fig. 2.27

8. In figure 2.28, line PS is a transversal of parallel line AB and line CD. If Ray QX, ray QY, ray RX, ray RY are angle bisectors, then prove that \square QXRY is a rectangle.

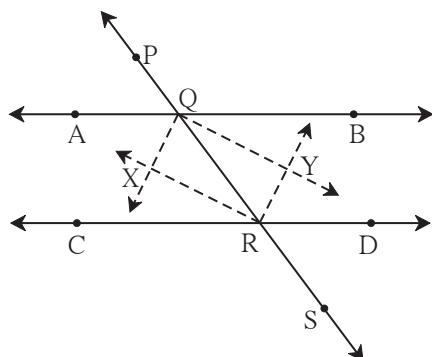


Fig. 2.28





Let's study.

- Theorem of remote interior angles of a triangle
 - Congruence of triangles
 - Theorem of an isosceles triangle
 - Property of $30^\circ - 60^\circ - 90^\circ$ angled triangle
 - Median of a triangle
 - Property of median on hypotenuse of a right angled triangle
 - Perpendicular bisector theorem
 - Angle bisector theorem
 - Similar triangles

Activity :

Draw a triangle of any measure on a thick paper. Take a point T on ray QR as shown in fig. 3.1. Cut two pieces of thick paper which will exactly fit the corners of $\angle P$ and $\angle Q$. See that the same two pieces fit exactly at the corner of $\angle PRT$ as shown in the figure.

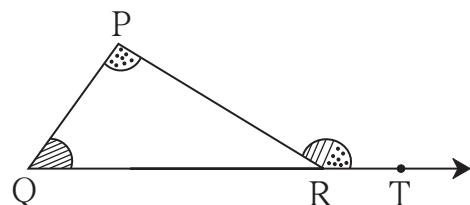


Fig. 3.1



Let's learn.

Theorem of remote interior angles of a triangle

Theorem: The measure of an exterior angle of a triangle is equal to the sum of its remote interior angles. P

Given : $\angle PRS$ is an exterior angle of $\triangle PQR$.

To prove : $\angle \text{PRS} = \angle \text{PQR} + \angle \text{QPR}$

Proof : The sum of all angles of a triangle is 180° .

$$\therefore \angle PQR + \angle QPR + \angle PRQ = 180^\circ \dots\dots\dots(I)$$

$$\angle PRO + \angle PRS = 180^\circ \dots\text{angles in linear pair} \dots \text{(II)}$$

\therefore from (I) and (II)

$$\angle POR + \angle QPR + \angle PRO = \angle PRO + \angle PRS$$

$\therefore \angle PQR + \angle QPR = \angle PRS$ eliminating $\angle PRQ$ from both sides

\therefore the measure of an exterior angle of a triangle is equal to the sum of its remote interior angles.

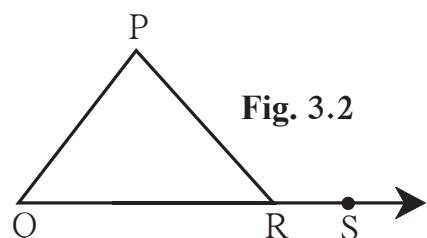


Fig. 3.2



Use your brain power!

Can we give an alternative proof of the theorem drawing a line through point R and parallel to seg PQ in figure 3.2 ?



Let's learn.

Property of an exterior angle of triangle

The sum of two positive numbers a and b , that is $(a + b)$ is greater than a and greater than b also. That is, $a + b > a$, $a + b > b$

Using this inequality we get one property related to exterior angle of a triangle.

If $\angle PRS$ is an exterior angle of $\triangle PQR$ then

$$\angle PRS > \angle P, \quad \angle PRS > \angle Q$$

∴ an exterior angle of a triangle is greater than its remote interior angle.

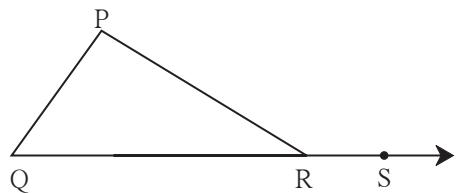


Fig. 3.3

Solved examples

Ex (1) The measures of angles of a triangle are in the ratio $5 : 6 : 7$. Find the measures.

Solution : Let the measures of the angles of a triangle be $5x$, $6x$, $7x$.

$$\therefore 5x + 6x + 7x = 180^\circ$$

$$18x = 180^\circ$$

$$x = 10^\circ$$

$$5x = 5 \times 10 = 50^\circ, \quad 6x = 6 \times 10 = 60^\circ, \quad 7x = 7 \times 10 = 70^\circ$$

\therefore the measures of angles of the triangle are 50° , 60° and 70° .

Ex (2) Observe figure 3.4 and find the measures of $\angle PRS$ and $\angle RTS$.

Solution : \angle PRS is an exterior angle of \triangle PQR.

So from the theorem of remote interior angles,

$$\begin{aligned}\angle PRS &= \angle PQR + \angle QPR \\&= 40^\circ + 30^\circ \\&\equiv 70^\circ\end{aligned}$$

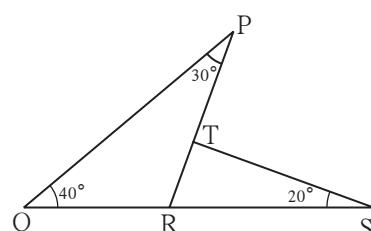


Fig. 3.4

$$\angle \text{TRS} + \angle \text{RTS} + \angle \text{TSR} = \boxed{\quad} \dots \text{sum of all angles of a triangle}$$

$$\therefore \boxed{} + \angle RTS + \boxed{} = 180^\circ$$

$$\therefore \angle RTS + 90^\circ = 180^\circ$$

$$\therefore \angle RTS = \boxed{}$$

Ex (3) Prove that the sum of exterior angles of a triangle, obtained by extending its sides in the same direction is 360° .

Given : $\angle PAB$, $\angle QBC$ and $\angle ACR$
are exterior angles of $\triangle ABC$

To prove : $\angle PAB + \angle QBC + \angle ACR = 360^\circ$

Proof : Method I

Considering exterior $\angle PAB$ of $\triangle ABC$,
 $\angle ABC$ and $\angle ACB$ are its remote interior angles.

$$\angle PAB = \angle ABC + \angle ACB \quad \text{---(I)}$$

Similarly, $\angle ACR = \angle ABC + \angle BAC$ ----(II)..theorem of remote interior angles and $\angle CBQ = \angle BAC + \angle ACB$ ---- (III)

Adding (I), (II) and (III),

$$\begin{aligned}
 & \angle PAB + \angle ACR + \angle CBQ \\
 &= \angle ABC + \angle ACB + \angle ABC + \angle BAC + \angle BAC + \angle ACB \\
 &= 2\angle ABC + 2\angle ACB + 2\angle BAC \\
 &= 2(\angle ABC + \angle ACB + \angle BAC) \\
 &= 2 \times 180^\circ \quad \dots \dots \text{sum of interior angles of a triangle} \\
 &= 360^\circ
 \end{aligned}$$

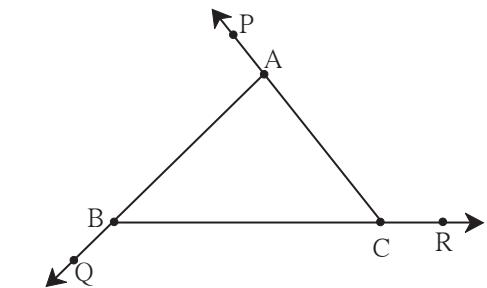


Fig. 3.5

Method II

$$\angle c + \angle f = 180^\circ \dots \text{(angles in linear pair)}$$

Also, $\angle a + \angle d = 180^\circ$

and $\angle b + \angle e = 180^\circ$

$$\therefore c^+ / f^+ / g^+ / d^+ / h^+ / e = 180^\circ \times 3 = 540^\circ$$

$$\angle f + \angle d + \angle e + (\angle a + \angle b + \angle c) = 540^\circ$$

$$\therefore \angle f + \angle d + \angle e + 180^\circ = 540^\circ$$

$$\therefore f + d + e = 540^\circ - 180^\circ$$

$$= 360^\circ$$

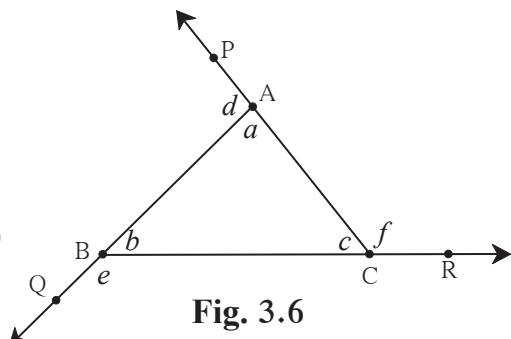


Fig. 3.6

Ex (4) In figure 3.7, bisectors of $\angle B$ and $\angle C$ of $\triangle ABC$ intersect at point P.

Prove that $\angle BPC = 90 + \frac{1}{2} \angle BAC$.

Complete the proof filling in the blanks.

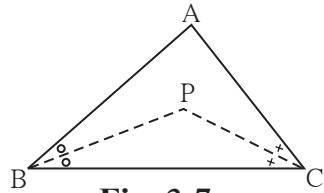


Fig. 3.7

Proof : In $\triangle ABC$,

$\angle BAC + \angle ABC + \angle ACB =$ sum of measures of angles of a triangle

$$\therefore \frac{1}{2} \angle BAC + \frac{1}{2} \angle ABC + \frac{1}{2} \angle ACB = \frac{1}{2} \times \boxed{\quad}$$

....multiplying each term by $\frac{1}{2}$

$$\therefore \frac{1}{2} \angle BAC + \angle PBC + \angle PCB = 90^\circ$$

$$\therefore \angle PBC + \angle PCB = 90^\circ - \frac{1}{2} \angle BAC \dots\dots\dots(I)$$

In Δ BPC

$$\angle BPC + \angle PBC + \angle PCB = 180^\circ \dots \text{sum of measures of angles of a triangle}$$

$$\therefore \angle BPC + \boxed{\quad} = 180^\circ \dots\text{from (I)}$$

$$\therefore \angle BPC = 180^\circ - (90^\circ - \frac{1}{2} \angle BAC)$$

$$= 180^\circ - 90^\circ + \frac{1}{2} \angle BAC$$

$$= 90^\circ + \frac{1}{2} \angle BAC$$

Practice set 3.1

1. In figure 3.8, $\angle ACD$ is an exterior angle of $\triangle ABC$.
 $\angle B = 40^\circ$, $\angle A = 70^\circ$. Find the measure of $\angle ACD$.

2. In $\triangle PQR$, $\angle P = 70^\circ$, $\angle Q = 65^\circ$ then find $\angle R$.

3. The measures of angles of a triangle are x° , $(x-20)^\circ$, $(x-40)^\circ$.
Find the measure of each angle.

4. The measure of one of the angles of a triangle is twice the measure of its smallest angle and the measure of the other is thrice the measure of the smallest angle. Find the measures of the three angles.



Fig. 3.8

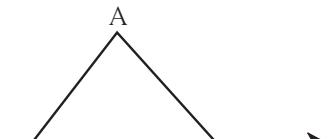


Fig. 3.8

5. In figure 3.9, measures of some angles are given. Using the measures find the values of x, y, z .

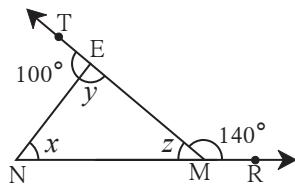


Fig. 3.9

6. In figure 3.10, line $AB \parallel$ line DE . Find the measures of $\angle DRE$ and $\angle ARE$ using given measures of some angles.

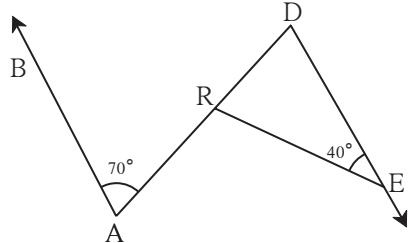


Fig. 3.10

7. In $\triangle ABC$, bisectors of $\angle A$ and $\angle B$ intersect at point O. If $\angle C = 70^\circ$. Find measure of $\angle AOB$.

8. In Figure 3.11, line $AB \parallel$ line CD and line PQ is the transversal. Ray PT and ray QT are bisectors of $\angle BPQ$ and $\angle PQD$ respectively.

Prove that $m\angle PTQ = 90^\circ$.

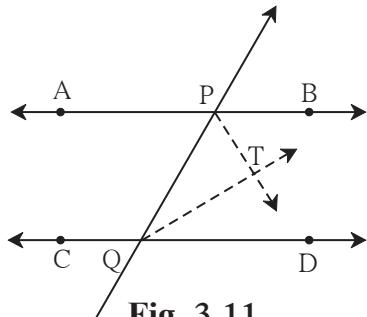


Fig. 3.11

9. Using the information in figure 3.12, find the measures of $\angle a$, $\angle b$ and $\angle c$.

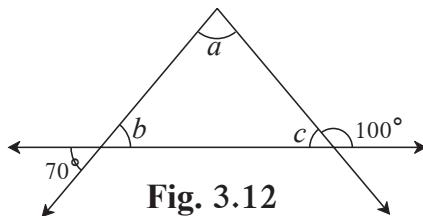


Fig. 3.12

10. In figure 3.13, line $DE \parallel$ line GF
ray EG and ray FG are bisectors of
 $\angle DEF$ and $\angle DFM$ respectively.
Prove that,

$$(i) \angle DEG = \frac{1}{2} \angle EDF \quad (ii) EF = FG.$$

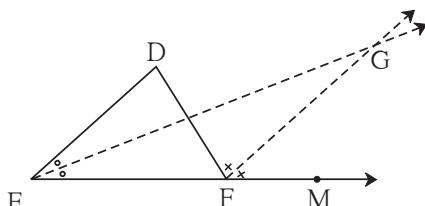


Fig. 3.13



Let's learn.

Congruence of triangles

We know that, if a segment placed upon another fits with it exactly then the two segments are congruent. When an angle placed upon another fits with it exactly then the two angles are congruent. Similarly, if a triangle placed upon another triangle fits exactly with it then the two triangles are said to be congruent. If $\triangle ABC$ and $\triangle PQR$ are congruent is written as $\triangle ABC \cong \triangle PQR$

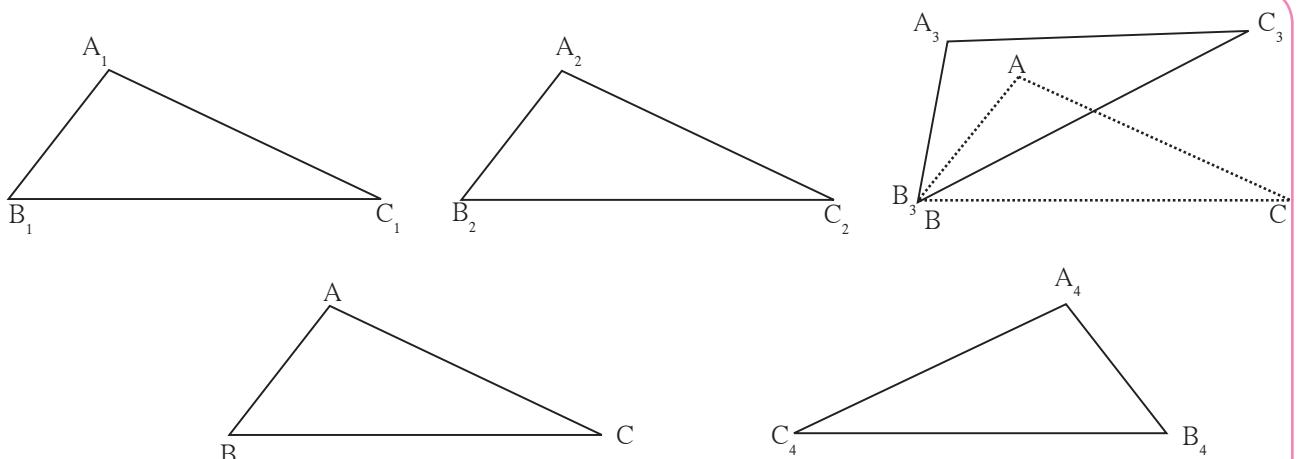


Fig. 3.14

Activity : Draw $\triangle ABC$ of any measure on a card-sheet and cut it out.

Place it on a card-sheet. Make a copy of it by drawing its border. Name it as $\Delta A_1 B_1 C_1$.

Now slide the $\triangle ABC$ which is the cut out of a triangle to some distance and make one more copy of it. Name it $\triangle A_2B_2C_2$

Then rotate the cut out of triangle ABC a little, as shown in the figure, and make another copy of it. Name the copy as $\Delta A_3B_3C_3$. Then flip the triangle ABC, place it on another card-sheet and make a new copy of it. Name this copy as $\Delta A_4B_4C_4$.

Have you noticed that each of $\Delta A_1B_1C_1$, $\Delta A_2B_2C_2$, $\Delta A_3B_3C_3$ and $\Delta A_4B_4C_4$ is congruent with ΔABC ? Because each of them fits exactly with ΔABC .

Let us verify for $\Delta A_3B_3C_3$. If we place $\angle A$ upon $\angle A_3$, $\angle B$ upon $\angle B_3$ and $\angle C$ upon $\angle C_3$, then only they will fit each other and we can say that $\Delta ABC \cong \Delta A_3B_3C_3$.

We also have $AB = A_3 B_3$, $BC = B_3 C_3$, $CA = C_3 A_3$.

Note that, while examining the congruence of two triangles, we have to write their angles and sides in a specific order, that is with a specific one-to-one correspondence.

If $\triangle ABC \cong \triangle PQR$, then we get the following six equations :

$$\angle A = \angle P, \angle B = \angle Q, \angle C = \angle R \dots \text{ (I)} \text{ and } AB = PQ, BC = QR, CA = RP \dots \text{ (II)}$$

This means, with a one-to-one correspondence between the angles and the sides of two triangles, we get three pairs of congruent angles and three pairs of congruent sides.

Given six equations above are true for congruent triangles. For this let us see three specific equations are true then all six equations become true and hence two triangles are congruent.

- (1) In a correspondence, if two angles of ΔABC are equal to two angles of ΔPQR and the sides included by the respective pairs of angles are also equal, then the two triangles are congruent.

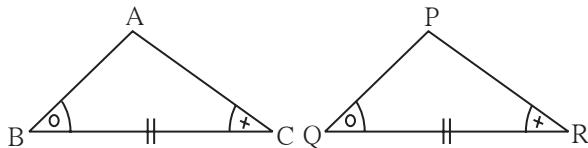


Fig. 3.15

This property is called as angle-side-angle test, which in short we write A-S-A test.

- (2) In a correspondence, if two sides of ΔABC are equal to two sides of ΔPQR and the angles included by the respective pairs of sides are also equal, then the two triangles are congruent.

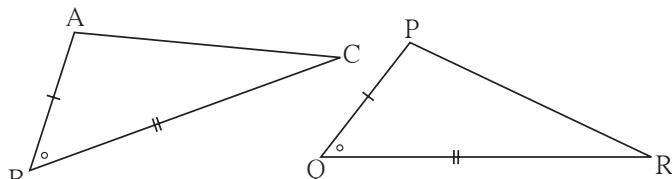


Fig. 3.16

This property is called as side-angle-side test, which in short we write S-A-S test.

- (3) In a correspondence, if three sides of ΔABC are equal to three sides of ΔPQR , then the two triangles are congruent.

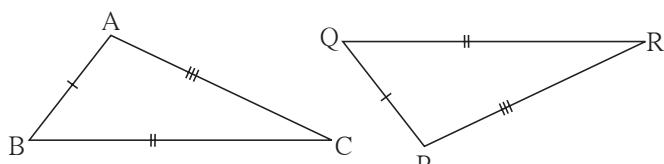


Fig. 3.17

This property is called as side-side-side test, which in short we write S-S-S test.

- (4) If in ΔABC and ΔPQR , $\angle B$ and $\angle Q$ are right angles, hypotenuses are equal and $AB = PQ$, then the two triangles are congruent.

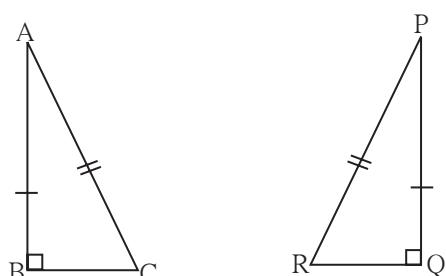


Fig. 3.18

This property is called the hypotenuse side test.



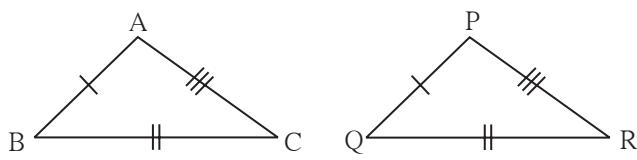
Remember this

We have constructed triangles using the given information about parts of triangles. (For example, two angles and the included side, three sides, two sides and an included angle). We have experienced that the triangle constructed with any of these information is unique. So if by some one-to-one correspondence between two triangles, these three parts of one triangle are congruent with corresponding three parts of the other triangle then the two triangles are congruent. Then we come to know that in that correspondence their three angles and three sides are congruent. If two triangles are congruent then their respective angles and respective sides are congruent. This property is useful to solve many problems in Geometry.

Practice set 3.2

1. In each of the examples given below, a pair of triangles is shown. Equal parts of triangles in each pair are marked with the same signs. Observe the figures and state the test by which the triangles in each pair are congruent.

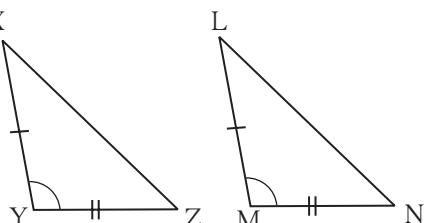
(i)



By [comment](#) test

$$\Delta \text{ ABC} \cong \Delta \text{ PQR}$$

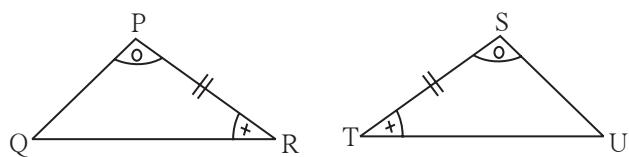
(ii)



By test

$$\Delta \text{ XYZ} \cong \Delta \text{ LMN}$$

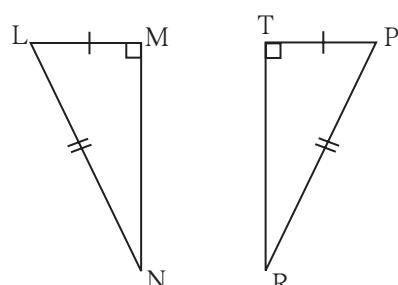
(iii)



By test

$$\Delta \text{ PRQ} \cong \Delta \text{ STU}$$

(iv)



By test

$$\Delta \text{ LMN} \cong \Delta \text{ PTR}$$

Fig. 3.19

2. Observe the information shown in pairs of triangles given below. State the test by which the two triangles are congruent. Write the remaining congruent parts of the triangles.

(i)

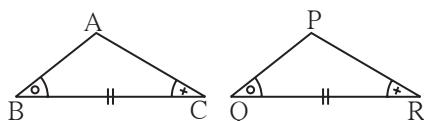


Fig. 3.20

From the information shown in the figure, in $\triangle ABC$ and $\triangle PQR$

$$\angle ABC \cong \angle PQR$$

$$\text{seg } BC \cong \text{seg } QR$$

$$\angle ACB \cong \angle PRQ$$

$$\therefore \triangle ABC \cong \triangle PQR \dots \boxed{\text{test}}$$

$$\therefore \angle BAC \cong \boxed{\quad} \dots \text{corresponding angles of congruent triangles.}$$

$$\begin{aligned} \text{seg } AB &\cong \boxed{\quad} && \text{corresponding sides of congruent triangles} \\ \text{and } \boxed{\quad} &\cong \text{seg } PR && \} \end{aligned}$$

3. From the information shown in the figure, state the test assuring the congruence of $\triangle ABC$ and $\triangle PQR$. Write the remaining congruent parts of the triangles.

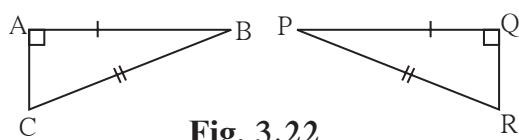


Fig. 3.22

5. In figure 3.24, $\text{seg } AB \cong \text{seg } CB$ and $\text{seg } AD \cong \text{seg } CD$.

Prove that

$$\triangle ABD \cong \triangle CBD$$

(ii)

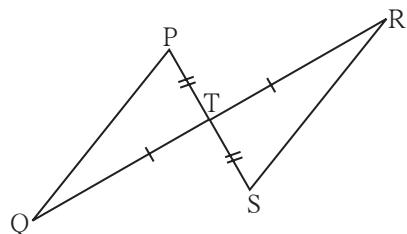


Fig. 3.21

From the information shown in the figure,,

In $\triangle PTQ$ and $\triangle STR$

$$\text{seg } PT \cong \text{seg } ST$$

$$\angle PTQ \cong \angle STR \dots \text{vertically opposite angles}$$

$$\text{seg } TQ \cong \text{seg } TR$$

$$\therefore \triangle PTQ \cong \triangle STR \dots \boxed{\text{test}}$$

$$\begin{aligned} \therefore \angle TPQ &\cong \boxed{\quad} && \} \text{corresponding} \\ \text{and } \boxed{\quad} &\cong \angle TRS && \} \text{angles of congruent} \\ &&& \text{triangles.} \end{aligned}$$

$$\text{seg } PQ \cong \boxed{\quad} \text{ corresponding sides of congruent triangles.}$$

4. As shown in the following figure, in $\triangle LMN$ and $\triangle PNM$, $LM = PN$, $LN = PM$. Write the test which assures the congruence of the two triangles. Write their remaining congruent parts.

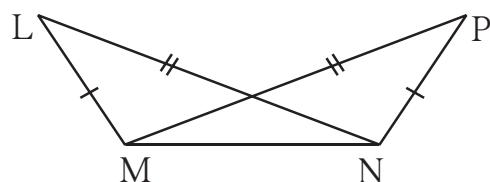


Fig. 3.23

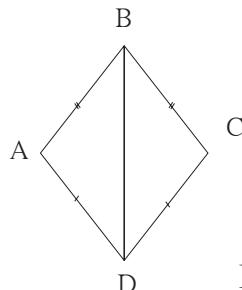


Fig. 3.24

Please note : corresponding sides of congruent triangles in short we write c.s.c.t. and corresponding angles of congruent triangles in short we write c.a.c.t.



Corollary : If three angles of a triangle are congruent then its three sides also are congruent.
(Write the proof of this corollary yourself.)

Both the above theorems are converses of each other also.

Similarly the corollaries of the theorems are converses of each other.



Use your brain power!

- (1) Can the theorem of isosceles triangle be proved doing a different construction ?
 - (2) Can the theorem of isosceles triangle be proved without doing any construction ?



Let's learn.

Property of 30° - 60° - 90° triangle

Activity I

Every student in the group should draw a right angled triangle, one of the angles measuring 30° . The choice of lengths of sides should be their own. Each one should measure the length of the hypotenuse and the length of the side opposite to 30° angle.

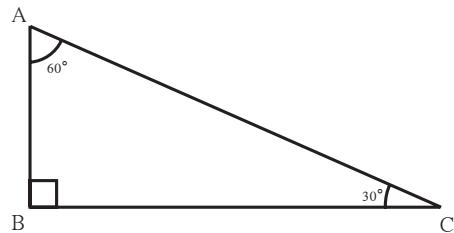


Fig. 3.28

One of the students in the group should fill in the following table.

Triangle Number	1	2	3	4
Length of the side opposite to 30° angle				
Length of the hypotenuse				

Did you notice any property of sides of right angled triangle with one of the angles measuring 30° ?

Activity II

The measures of angles of a set square in your compass box are 30° , 60° and 90° .

Verify the property of the sides of the set square.

Let us prove an important property revealed from these activities.



Theorem : If the acute angles of a right angled triangle have measures 30° and 60° , then the length of the side opposite to 30° angle is half the length of the hypotenuse.
 (Fill in the blanks and complete the proof.) A

Given : In ΔABC

$$\angle B = 90^\circ, \angle C = 30^\circ, \angle A = 60^\circ$$

To prove : $AB = \frac{1}{2} AC$

Construction : Take a point D on the extended seg AB such that AB = BD. Draw seg DC.

Proof : $\triangle ABC$ and $\triangle DBC$

$$\text{seg AB} \cong \text{seg DB} \dots$$

$$\angle ABC \cong \angle DBC \dots\dots$$

seg BC \cong seg BC

$$\therefore \triangle ABC \cong \triangle DBC \dots$$

$\therefore \angle BAC \cong \angle BDC$ (c.a.c.t.)

$$\text{In } \triangle ABC, \angle BAC = 60^\circ \therefore \angle BDC = 60^\circ$$

$$\angle DAC = \angle ADC = \angle ACD = 60^\circ \quad \therefore \text{sum of angles of } \triangle ADC \text{ is } 180^\circ$$

$\therefore \triangle ADC$ is an equilateral triangle

$\therefore AC \equiv AD \equiv DC$ corollary of converse of isosceles triangle theorem

But $AB = \frac{1}{2} AD$ construction

$$\therefore AB = \frac{1}{2} AC \dots \because AD = AC$$

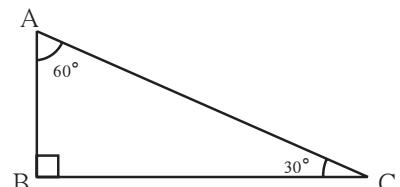


Fig. 3.29

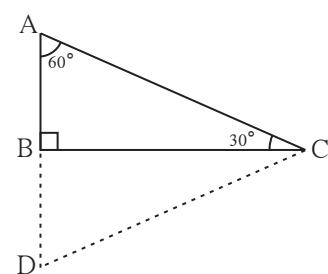


Fig. 3.30

Activity

With the help of the Figure 3.29 above fill in the blanks and complete the proof of the following theorem.

Theorem : If the acute angles of a right angled triangle have measures 30° and 60° then the length of the side opposite to 60° angle is $\frac{\sqrt{3}}{2} \times$ hypotenuse

Proof : In the above theorem we have proved $AB = \frac{1}{2} AC$

$$AB^2 + BC^2 = \boxed{} \quad \text{..... Pythagoras theorem}$$

$$\frac{1}{4} AC^2 + BC^2 = \boxed{}$$

$$\therefore BC^2 = AC^2 - \frac{1}{4} AC^2$$

$$\therefore BC^2 = \boxed{}$$

$$\therefore BC = \frac{\sqrt{3}}{2} AC$$

Activity : Complete the proof of the theorem.

Theorem : If measures of angles of a triangle are 45° , 45° , 90° then the length of each side containing the right angle is $\frac{1}{\sqrt{2}} \times \text{hypotenuse}$.

Proof : In ΔABC , $\angle B = 90^\circ$ and $\angle A = \angle C = 45^\circ$

$$\therefore BC = AB$$

By Pythagoras theorem

$$AB^2 + BC^2 = \boxed{\quad}$$

$$AB^2 + \boxed{\quad} = AC^2 \dots \because (BC = AB)$$

$$\therefore 2AB^2 = \boxed{\quad}$$

$$\therefore AB^2 = \boxed{\quad}$$

$$\therefore AB = \frac{1}{\sqrt{2}} AC$$

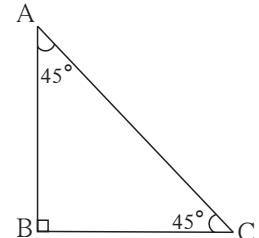


Fig. 3.31

This property is called $45^\circ - 45^\circ - 90^\circ$ theorem.



Remember this !

(1) If the acute angles of a right angled triangle are 30° , 60° then the length of side opposite to 30° angle is half of hypotenuse and the length of side opposite to 60° angle is $\frac{\sqrt{3}}{2}$ hypotenuse . This property is called $30^\circ - 60^\circ - 90^\circ$ theorem.

(2) If acute angles of a right angled triangle are 45° , 45° then the length of each side containing the right angle is $\frac{\text{hypotenuse}}{\sqrt{2}}$.

This property is called $45^\circ - 45^\circ - 90^\circ$ theorem



Let's recall.

Median of a triangle

The segment joining a vertex and the mid-point of the side opposite to it is called a **Median** of the triangle.

In Figure 3.32, point D is the mid point of side BC.

\therefore seg AD is a median of ΔABC .

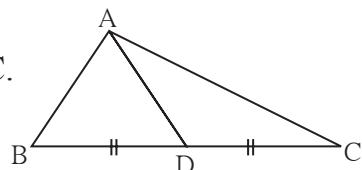


Fig. 3.32

Activity I : Draw a triangle ABC. Draw medians AD, BE and CF of the triangle. Let their point of concurrence be G, which is called the centroid of the triangle. Compare the lengths of AG and GD with a divider. Verify that the length of AG is twice the length of GD. Similarly, verify that the length of BG is twice the length of GE and the length of CG is twice the length of GF. Hence note the following property of medians of a triangle.

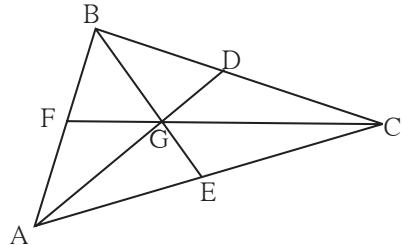


Fig. 3.33

The point of concurrence of medians of a triangle divides each median in the ratio $2 : 1$.

Activity II : Draw a triangle ABC on a card board. Draw its medians and denote their point of concurrence as G. Cut out the triangle.

Now take a pencil. Try to balance the triangle on the flat tip of the pencil. The triangle is balanced only when the point G is on the flat tip of the pencil.

This activity shows an important property of the **centroid** (point of concurrence of the medians) of the triangle.

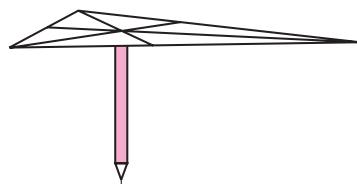


Fig. 3.34



Let's learn.

Property of median drawn on the hypotenuse of right triangle

Activity : In the figure 3.35, ΔABC is a right angled triangle. seg BD is the median on hypotenuse.

Measure the lengths of the following segments

$$AD = \dots \quad DC = \dots \quad BD = \dots$$

From the measurements verify that $BD = \frac{1}{2} AC$.

Now let us prove the property, the length of the median is half the length of the hypotenuse.

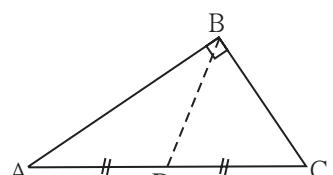


Fig. 3.35

Theorem : In a right angled triangle, the length of the median of the hypotenuse is half the length of the hypotenuse.

Given : In $\triangle ABC$, $\angle B = 90^\circ$, seg BD is the median.

To prove : $BD = \frac{1}{2} AC$

Construction : Take point E on the ray BD such that B - D - E

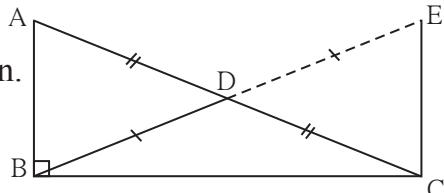


Fig. 3.36

Proof : (Main steps are given. Write the steps in between with reasons and complete the proof.)

$\Delta ADB \cong \Delta CDE$ by S-A-S test

line AB || line ECby test of alternate angles

$\Delta ABC \cong \Delta ECB$ by S-A-S test

$$BD = \frac{1}{2} AC$$



Remember this !

In a right angled triangle, the length of the median on its hypotenuse is half the length of the hypotenuse.

Practice set 3.3

- Find the values of x and y using the information shown in figure 3.37.
Find the measure of $\angle ABD$ and $m\angle ACD$.

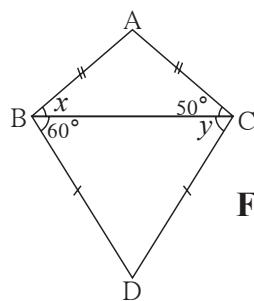
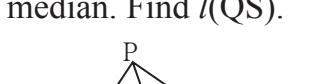


Fig. 3.37

2. The length of hypotenuse of a right angled triangle is 15. Find the length of median of its hypotenuse.

3. In $\triangle PQR$, $\angle Q = 90^\circ$, $PQ = 12$, $QR = 5$ and QS is a median. Find $l(QS)$.

4. In figure 3.38, point G is the point of concurrence of the medians of $\triangle PQR$. If $GT = 2.5$, find the lengths of PG and PT .



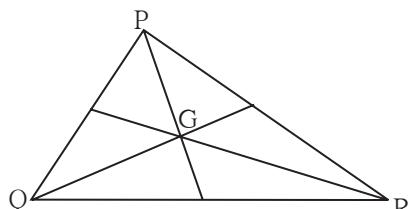


Fig. 3.38



Let's recall.

Activity : Draw a segment AB of convenient length. Label its mid-point as M. Draw a line l passing through the point M and perpendicular to seg AB.

Did you notice that the line l is the perpendicular bisector of seg AB ?

Now take a point P anywhere on line l . Compare the distance PA and PB with a divider. What did you find? You should have noticed that $PA = PB$. This observation shows that any point on the perpendicular bisector of a segment is equidistant from its end points.

Now with the help of a compass take any two points like C and D, which are equidistant from A and B. Did all such points lie on the line l ? What did you notice from the observation? Any point equidistant from the end points of a segment lies on the perpendicular bisector of the segment.

These two properties are two parts of the perpendicular bisector theorem. Let us now prove them.

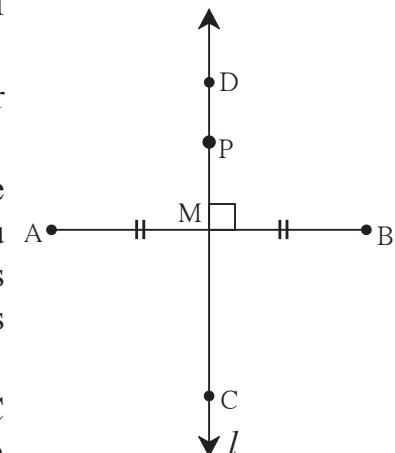


Fig. 3.39



Let's learn.

Perpendicular bisector theorem

Part I : Every point on the perpendicular bisector of a segment is equidistant from the end points of the segment.

Given : line l is the perpendicular bisector of seg AB at point M.

Point P is any point on l .

To prove: $PA = PB$

Construction : Draw seg AP and seg BP.

Proof : In ΔPMA and ΔPMB

seg PM \cong seg PM common side

$\angle \text{PMA} \cong \angle \text{PMB}$ each is a right angle

$\text{seg } AM \cong \text{ seg } BM$ given

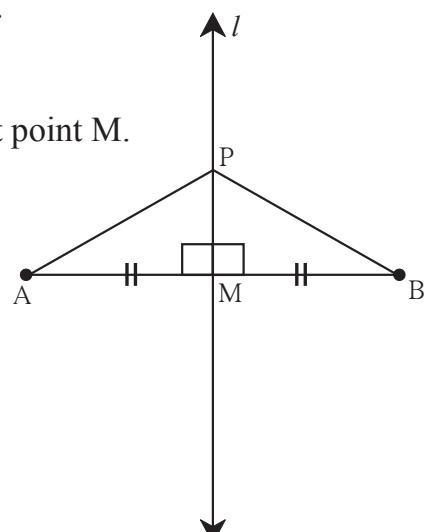
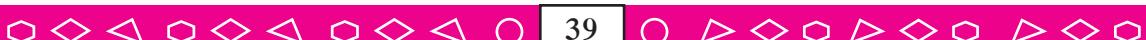


Fig. 3.40



$\therefore \Delta \text{PMA} \cong \Delta \text{PMB}$ S-A-S test

$\therefore \text{seg PA} \cong \text{seg PB}$ c.s.c.t.

$$\therefore l(\text{PA}) = l(\text{PB})$$

Hence every point on the perpendicular bisector of a segment is equidistant from the end points of the segment.

Part II : Any point equidistant from the end points of a segment lies on the perpendicular bisector of the segment.

Given : Point P is any point equidistant from the end points of seg AB. That is, $PA = PB$.

To prove: Point P is on the perpendicular bisector of seg AB.

Construction : Take mid-point M of

$\text{soc RA} \approx \text{soc RB}$

$\text{seg } \overline{AM} \cong \text{seg } \overline{BM}$

seg PM \cong common side

$\therefore \Delta \text{ PAM} \cong \Delta \text{ PBM}$ test.

$\therefore \angle PMA \cong \angle PMB$c.a.c.t.

$$\text{But } \angle \text{PMA} + \boxed{} = 180^\circ$$

$$\angle PMA + \angle PMA = 180^\circ \dots\dots (\because \angle PMB = \angle PMA)$$

$$2 \angle \text{PMA} =$$

$$\therefore \angle PMA = 90^\circ$$

$$\therefore \text{seg PM} \perp \text{seg AB} \quad \dots\dots(1)$$

But Point M is the midpoint of seg AB.construction (2)

\therefore line PM is the perpendicular bisector of seg AB. So point P is on the perpendicular bisector of seg AB

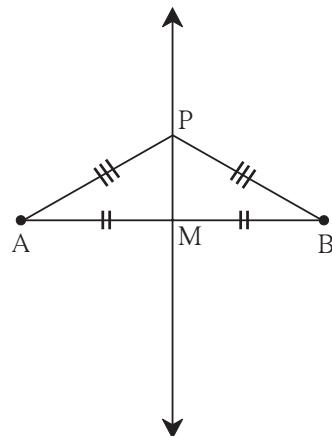


Fig. 3.41

Angle bisector theorem

Part I : Every point on the bisector of an angle is equidistant from the sides of the angle.

Given : Ray QS is the bisector of $\angle PQR$.

Point A is any point on ray QS

$$\text{seg AB} \perp \text{ray OP} \quad \text{seg AC} \perp \text{ray QR}$$

To prove : $\text{seg AB} \cong \text{seg AC}$

Proof : Write the proof using test of congruence of triangles

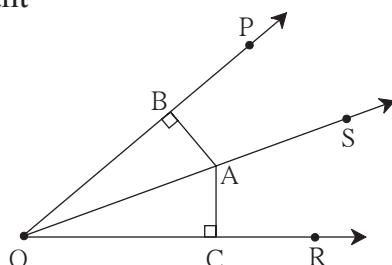


Fig. 3.42

Part II : Any point equidistant from sides of an angle is on the bisector of the angle.

Given : A is a point in the interior of $\angle PQR$.

seg AC \perp ray QR seg AB \perp ray QP
and AB = AC

To prove : Ray QA is the bisector of $\angle PQR$.

That is $\angle BQA = \angle CQA$

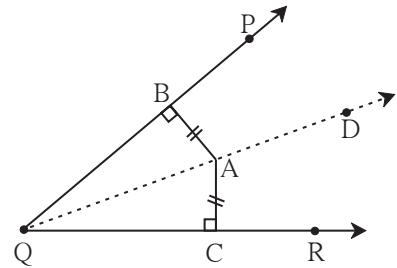


Fig. 3.43

Proof : Write the proof using proper test of congruence of triangles.



Let's recall.

Activity

As shown in the figure, draw $\triangle XYZ$ such that $XZ >$ side XY

Find which of $\angle Z$ and $\angle Y$ is greater.

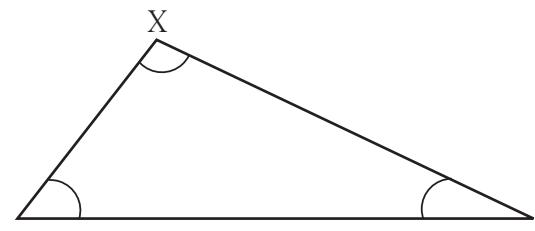


Fig. 3.44



Let's learn.

Properties of inequalities of sides and angles of a triangle

Theorem : If two sides of a triangle are unequal, then the angle opposite to the greater side is greater than angle opposite to the smaller side.

Given : In $\triangle XYZ$, side $XZ >$ side XY

To prove : $\angle XYZ > \angle XZY$

Construction : Take point P on side XZ such that $XY = XP$, Draw seg YP.

Proof : In ΔXYP

$XY = XP$ construction

$\therefore \angle XYP = \angle XPY$isosceles triangle theorem(I)

$\angle XPY$ is an exterior angle of $\triangle YPZ$.

$\therefore \angle XPY > \angle PZY$ exterior angle theorem

$\therefore \angle XYP > \angle PZY$ from (I)

$\therefore \angle XYP + \angle PYZ > \angle PZY$ If $a > b$ and $c > 0$ then $a + c > b$

$\therefore \angle XYZ > \angle PZY$, that is $\angle XYZ > \angle XZY$

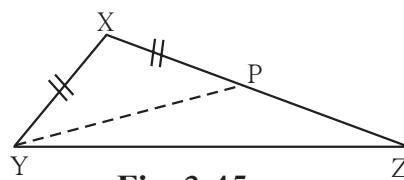


Fig. 3.45

Theorem : If two angles of a triangle are unequal then the side opposite to the greater angle is greater than the side opposite to smaller angle.

The theorem can be proved by indirect proof. Complete the following proof by filling in the blanks.

Given : In ΔABC , $\angle B > \angle C$

To prove : $AC > AB$

Proof : There are only three possibilities regarding the lengths of side AB and side AC of ΔABC

(i) $AC < AB$

(ii)

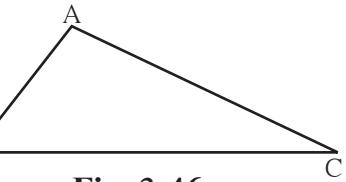


Fig. 3.46

(iii)

(i) Let us assume that $AC < AB$.

If two sides of a triangle are unequal then the angle opposite to greater side is .

$$\therefore \angle C > \boxed{\quad}$$

But $\angle C < \angle B$ (given)

This creates a contradiction.

$$\therefore \boxed{\quad} < \boxed{\quad} \text{ is wrong.}$$

(ii) If $AC = AB$

then $\angle B = \angle C$

But $>$ (given)

This also creates a contradiction.

$$\therefore \boxed{\quad} = \boxed{\quad} \text{ is wrong}$$

$\therefore AC > AB$ is the only remaining possibility.

$$\therefore AC > AB$$

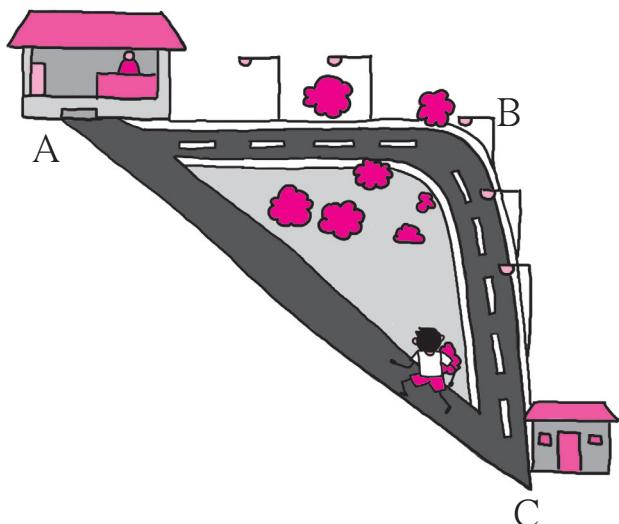


Let's recall.

As shown in the adjacent picture, there is a shop at A. Sameer was standing at C. To reach the shop, he choose the way $C \rightarrow A$ instead of $C \rightarrow B \rightarrow A$, because he knew that the way $C \rightarrow A$ was shorter than the way $C \rightarrow B \rightarrow A$. So which property of a triangle had he realised ?

The sum of two sides of a triangle is greater than its third side.

Let us now prove the property.



Theorem : The sum of any two sides of a triangle is greater than the third side.

Given : $\triangle ABC$ is any triangle.

To prove : $AB + AC > BC$

$$AB + BC \geq AC$$

$$AC + BC \geq AB$$

Construction : Take a point D on ray BA such that $AD = AC$.

Proof : In $\triangle ACD$, $AC = AD$ construction

$\cdot \angle ACD \equiv \angle ADC$ *c.a.s.t.*

$$\therefore \angle ACD + \angle ACB = \angle ADC$$

$\vdash \langle \text{BCD} \rangle \prec \langle \text{ADC} \rangle$

∴ side BD > side BC the side opposite to greater angle is greater

$$\therefore BA + AD \geq BC \quad \therefore BD = BA + AD$$

$$BA + AC \geq BC \quad \therefore AD \equiv AC$$

Similarly we can prove that $AB \pm BC \geq AC$.

and $BC + AC > AB$

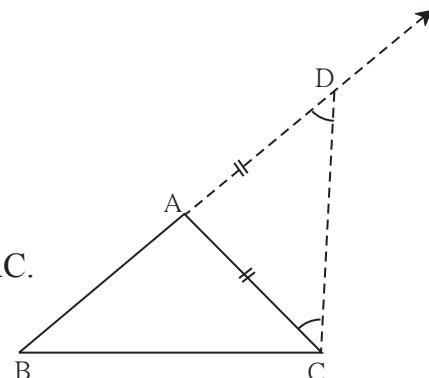


Fig. 3.47

Practice set 3.4

1. In figure 3.48, point A is on the bisector of $\angle XYZ$.

If $AX = 2$ cm then find AZ .

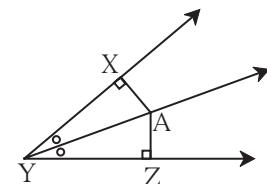


Fig. 3.48

2.

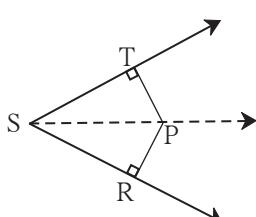


Fig. 3.49

In figure 3.49, $\angle RST = 56^\circ$, seg PT \perp ray ST,

$\text{seg PR} \perp \text{ray SR}$ and $\text{seg PR} \cong \text{seg PT}$

Find the measure of $\angle RSP$.

State the reason for your answer.

3. In $\triangle PQR$, $PQ = 10 \text{ cm}$, $QR = 12 \text{ cm}$, $PR = 8 \text{ cm}$. Find out the greatest and the smallest angle of the triangle.
 4. In $\triangle FAN$, $\angle F = 80^\circ$, $\angle A = 40^\circ$. Find out the greatest and the smallest side of the triangle. State the reason.
 5. Prove that an equilateral triangle is equiangular.

6. Prove that, if the bisector of $\angle BAC$ of ΔABC is perpendicular to side BC, then ΔABC is an isosceles triangle.

7. In figure 3.50, if $\text{seg } PR \cong \text{seg } PQ$, show that $\text{seg } PS > \text{seg } PQ$.

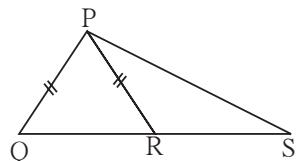


Fig. 3.50

8. In figure 3.51, in ΔABC , $\text{seg } AD$ and $\text{seg } BE$ are altitudes and $AE = BD$.

Prove that $\text{seg } AD \cong \text{seg } BE$

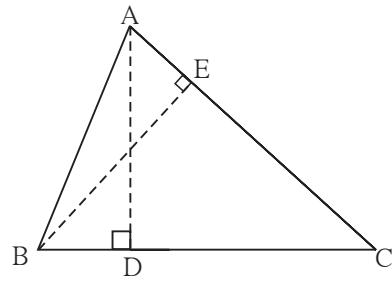


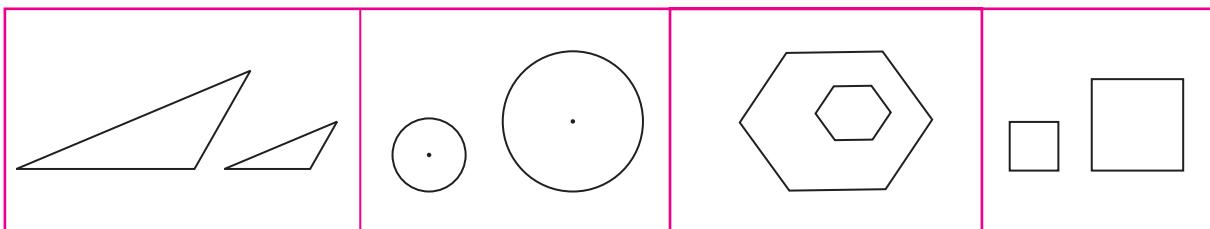
Fig. 3.51



Let's learn.

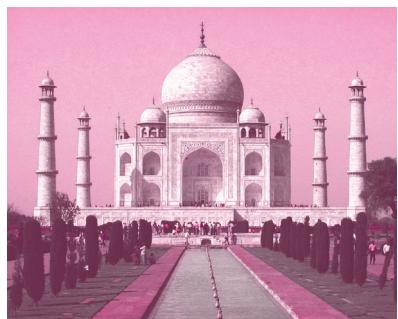
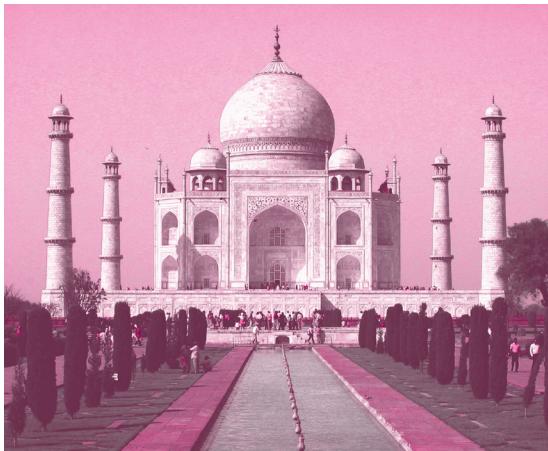
Similar triangles

Observe the following figures.



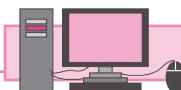
The pairs of figures shown in each part have the same shape but their sizes are different. It means that they are not congruent.

Such like looking figures are called similar figures.



We find similarity in a photo and its enlargement, also we find similarity between a road-map and the roads.

The proportionality of all sides is an important property of similarity of two figures. But the angles in the figures have to be of the same measure. If the angle between this roads is not the same in its map, then the map will be misleading.



ICT Tools or Links

Take a photograph on a mobile or a computer. Recall what you do to reduce it or to enlarge it. Also recall what you do to see a part of the photograph in detail.

Now we shall learn properties of similar triangles through an activity.

Activity : On a card-sheet, draw a triangle of sides 4 cm, 3 cm and 2 cm. Cut it out. Make 13 more copies of the triangle and cut them out from the card sheet.

Note that all these triangular pieces are congruent. Arrange them as shown in the following figure and make three triangles out of them.

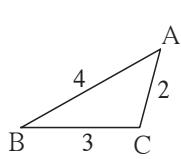


Fig. 3.52

1 triangle

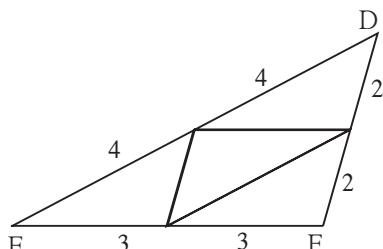


Fig. 3.53

4 triangles

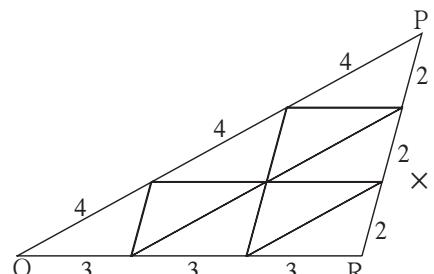


Fig. 3.54

9 triangles

$\triangle ABC$ and $\triangle DEF$ are similar in the correspondence $ABC \leftrightarrow DEF$.

$$\angle A \cong \angle D, \angle B \cong \angle E, \angle C \cong \angle F$$

$$\text{and } \frac{AB}{DE} = \frac{4}{8} = \frac{1}{2}; \quad \frac{BC}{EF} = \frac{3}{6} = \frac{1}{2}; \quad \frac{AC}{DF} = \frac{2}{4} = \frac{1}{2},$$

.....the corresponding sides are in proportion.

Similarly, consider $\triangle DEF$ and $\triangle PQR$. Are their angles congruent and sides proportional in the correspondence $DEF \leftrightarrow PQR$?



Similarity of triangles

In $\triangle ABC$ and $\triangle PQR$, If (i) $\angle A = \angle P$, $\angle B = \angle Q$, $\angle C = \angle R$ and

(ii) $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$; then $\triangle ABC$ and $\triangle PQR$ are called similar triangles.

' \triangle ABC and \triangle PQR are similar' is written as ' \triangle ABC \sim \triangle PQR'.

Let us learn the relation between the corresponding angles and corresponding sides of similar triangles through an activity.

Activity : Draw a triangle $\Delta A_1 B_1 C_1$ on a card-sheet and cut it out. Measure $\angle A_1, \angle B_1, \angle C_1$.

Draw two more triangles $\Delta A_2 B_2 C_2$ and $\Delta A_3 B_3 C_3$ such that

$$\angle A_1 = \angle A_2 = \angle A_3, \quad \angle B_1 = \angle B_2 = \angle B_3, \quad \angle C_1 = \angle C_2 = \angle C_3$$

and $B_1 C_1 > B_2 C_2 > B_3 C_3$. Now cut these two triangles also. Measure the lengths of the three triangles. Arrange the triangles in two ways as shown in the figure.

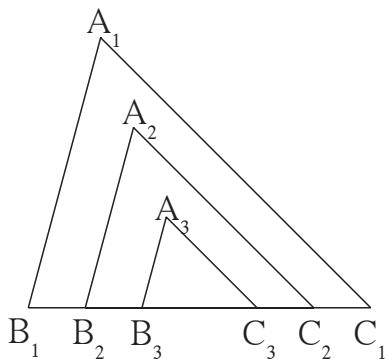


Fig. 3.55

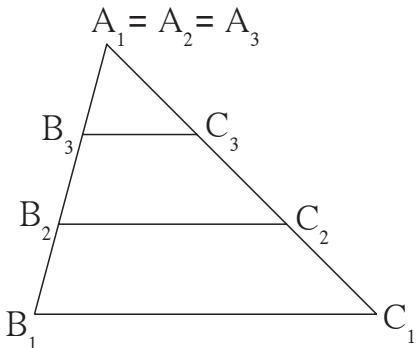


Fig. 3.56

Fig. 3.55
Check the ratios $\frac{A_1 B_1}{A_2 B_2}$, $\frac{B_1 C_1}{B_2 C_2}$, $\frac{A_1 C_1}{A_2 C_2}$. You will notice that the ratios are equal.

Similarly, see whether the ratios $\frac{A_1 C_1}{A_3 C_3}, \frac{B_1 C_1}{B_3 C_3}, \frac{A_1 B_1}{A_3 B_3}$ are equal.

From this activity note that, when corresponding angles of two triangles are equal, the ratios of their corresponding sides are also equal. That is, their corresponding sides are in the same proportion.

We have seen that, in $\triangle ABC$ and $\triangle PQR$ if

(i) $\angle A = \angle P$, $\angle B = \angle Q$, $\angle C = \angle R$, then (ii) $\frac{AB}{PO} = \frac{BC}{QR} = \frac{AC}{PR}$

This means, if corresponding angles of two triangles are equal then the corresponding sides are in the same proportion.

This rule can be proved elaborately. We shall use it to solve problems.



Remember this !

- If corresponding angles of two triangles are equal then the two triangles are similar.
 - If two triangles are similar then their corresponding sides are in proportion and corresponding angles are congruent.

Ex. Some information is shown in $\triangle ABC$ and $\triangle PQR$ in figure 3.57. Observe it. Hence find the lengths of side AC and PQ.

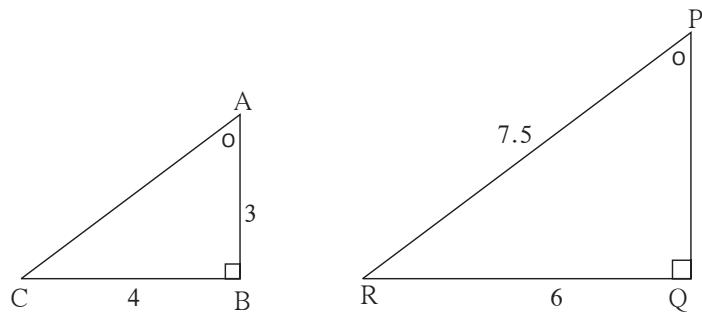


Fig. 3.57

Solution : The sum of all angles of a triangle is 180° .

It is given that,

$$\angle A = \angle P \text{ and } \angle B = \angle Q \quad \therefore \angle C = \angle R$$

$\therefore \triangle ABC$ and $\triangle PQR$ are equiangular triangles.

\therefore these sides are proportional.

$$\therefore \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$$

$$\therefore \frac{3}{PQ} = \frac{4}{6} = \frac{AC}{7.5}$$

$$\therefore 4 \times PQ = 18$$

$$\therefore PQ = \frac{18}{4} = 4.5$$

Similarly $6 \times AC = 7.5 \times 4$

$$\therefore AC = \frac{7.5 \times 4}{6} = \frac{30}{6} = 5$$

Practice set 3.5

1. If $\Delta XYZ \sim \Delta LMN$, write the corresponding angles of the two triangles and also write the ratios of corresponding sides.
 2. In ΔXYZ , $XY = 4$ cm, $YZ = 6$ cm, $XZ = 5$ cm, If $\Delta XYZ \sim \Delta PQR$ and $PQ = 8$ cm then find the lengths of remaining sides of ΔPQR .
 3. Draw a sketch of a pair of similar triangles. Label them. Show their corresponding angles by the same signs. Show the lengths of corresponding sides by numbers in proportion.



Let's recall.

While preparing a map of a locality, you have to show the distances between different spots on roads with a proper scale. For example, 1 cm = 100 m, 1 cm = 50 m etc. Did you think of the properties of triangle? Keep in mind that side opposite to greater angle is greater.

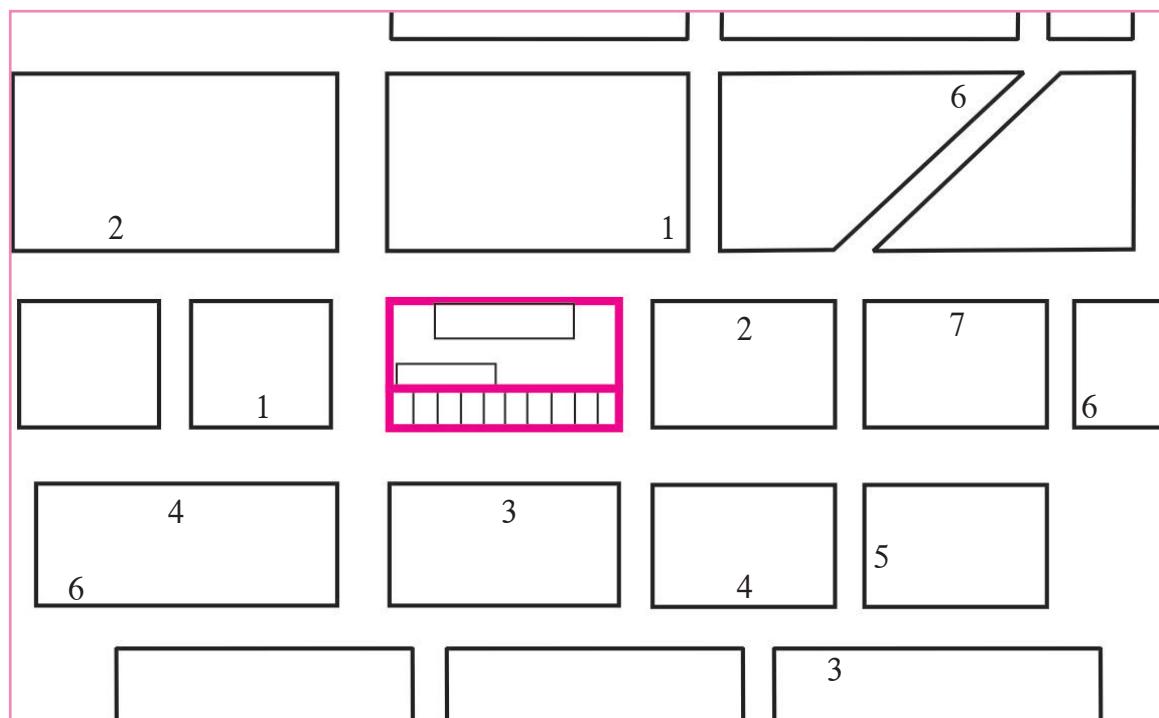
Project :

Prepare a map of road surrounding your school or home, upto a distance of about 500 metre.

How will you measure the distance between two spots on a road ?

While walking, count how many steps cover a distance of about two metre. Suppose, your three steps cover a distance of 2 metre. Considering this proportion 90 steps means 60 metre. In this way you can judge the distances between different spots on roads and also the lengths of roads. You have to judge the measures of angles also where two roads meet each other. Choosing a proper scale for lengths of roads, prepare a map. Try to show shops, buildings, bus stops, rickshaw stand etc. in the map.

A sample map with legend is given below



Legend: 1. Book store 2. Bus stop 3. Stationery shop 4. Bank
5. Medical store 6. Restaurant 7. Cycle shop

Problem set 3

1. Choose the correct alternative answer for the following questions.

(i) If two sides of a triangle are 5 cm and 1.5 cm, the length of its third side cannot be

(A) 3.7 cm (B) 4.1 cm (C) 3.8 cm (D) 3.4 cm

(ii) In $\triangle PQR$, If $\angle R > \angle Q$ then

(A) $QR > PR$ (B) $PQ > PR$ (C) $PQ < PR$ (D) $QR < PR$

(iii) In $\triangle TPQ$, $\angle T = 65^\circ$, $\angle P = 95^\circ$ which of the following is a true statement ?

(A) $PQ < TP$ (B) $PQ < TQ$ (C) $TQ < TP < PQ$ (D) $PQ < TP < TQ$

2. $\triangle ABC$ is isosceles in which $AB = AC$. Seg BD and seg CE are medians. Show that $BD = CE$.

3. In $\triangle PQR$, If $PQ > PR$ and bisectors of $\angle Q$ and $\angle R$ intersect at S. Show that $SQ > SR$.

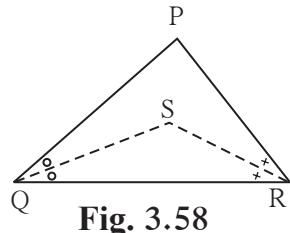


Fig. 3.58

4. In figure 3.59, point D and E are on side BC of $\triangle ABC$, such that $BD = CE$ and $AD = AE$. Show that $\triangle ABD \cong \triangle ACE$.

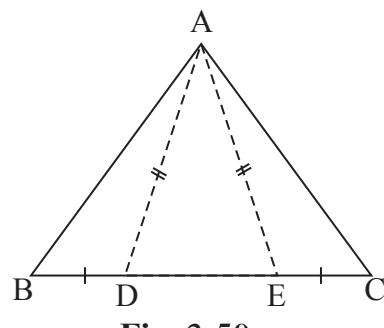


Fig. 3.59

5. In figure 3.60, point S is any point on side QR of $\triangle PQR$
Prove that : $PO + OR + RP > 2PS$

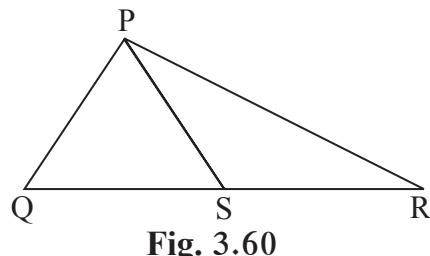


Fig. 3.60

6. In figure 3.61, bisector of $\angle BAC$ intersects side BC at point D.
Prove that $AB \geq BD$

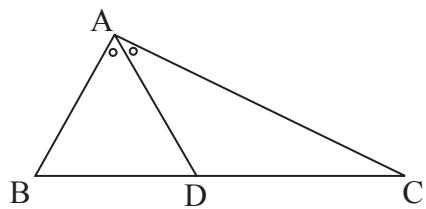


Fig. 3.61

7.

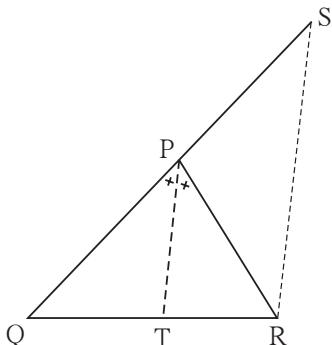


Fig. 3.62

8. In figure 3.63, $\overline{AD} \perp \overline{BC}$.
 \overline{AE} is the bisector of $\angle CAB$ and
 $C - E - D$.

Prove that

$$\angle DAE = \frac{1}{2} (\angle C - \angle B)$$

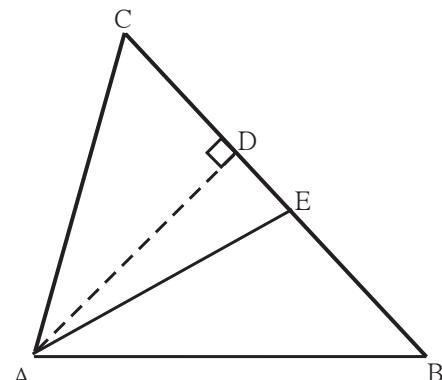


Fig. 3.63



Use your brain power!

We have learnt that if two triangles are equiangular then their sides are in proportion. What do you think if two quadrilaterals are equiangular ? Are their sides in proportion? Draw different figures and verify.

Verify the same for other polygons.





Let's study.

To construct a triangle, if following information is given.

- Base, an angle adjacent to the base and sum of lengths of two remaining sides.
 - Base, an angle adjacent to the base and difference of lengths of remaining two sides.
 - Perimeter and angles adjacent to the base.



Let's recall.

In previous standard we have learnt the following triangle constructions.

- * To construct a triangle when its three sides are given.
 - * To construct a triangle when its base and two adjacent angles are given.
 - * To construct a triangle when two sides and the included angle are given.
 - * To construct a right angled triangle when its hypotenuse and one side is given.

Perpendicular bisector Theorem

- Every point on the perpendicular bisector of a segment is equidistant from its end points.
 - Every point equidistant from the end points of a segment is on the perpendicular bisector of the segment.

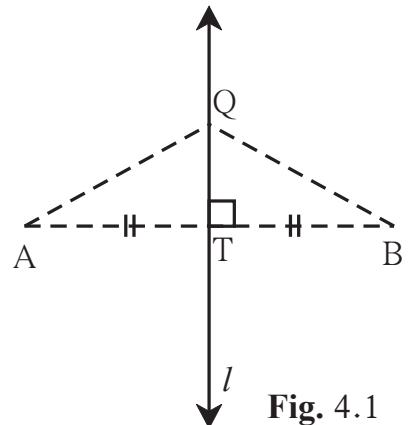


Fig. 4.1



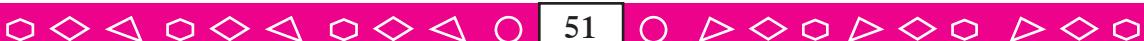
Let's learn.

Constructions of triangles

To construct a triangle, three conditions are required. Out of three sides and three angles of a triangle two parts and some additional information about them is given, then we can construct a triangle using them.

We frequently use the following property in constructions.

If a point is on two different lines then it is the intersection of the two lines.



Construction I

To construct a triangle when its base, an angle adjacent to the base and the sum of the lengths of remaining sides is given.

Ex. Construct $\triangle ABC$ in which $BC = 6.3 \text{ cm}$, $\angle B = 75^\circ$ and $AB + AC = 9 \text{ cm}$.

Solution : Let us first draw a rough figure of expected triangle.

Explanation : As shown in the rough figure, first we draw seg $BC = 6.3 \text{ cm}$ of length. On the ray making an angle of 75° with seg BC , mark point D such that

$$BD = AB + AC = 9 \text{ cm}$$

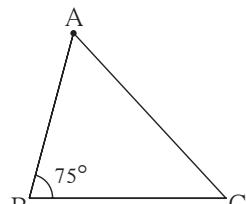
Now we have to locate point A on ray BD .

$$BA + AD = BA + AC = 9$$

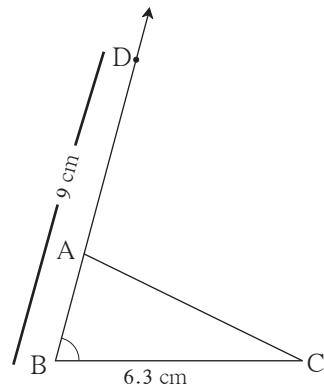
$$\therefore AD = AC$$

\therefore point A is on the perpendicular bisector of seg CD .

\therefore the point of intersection of ray BD and the perpendicular bisector of seg CD is point A .



Rough figure 4.2

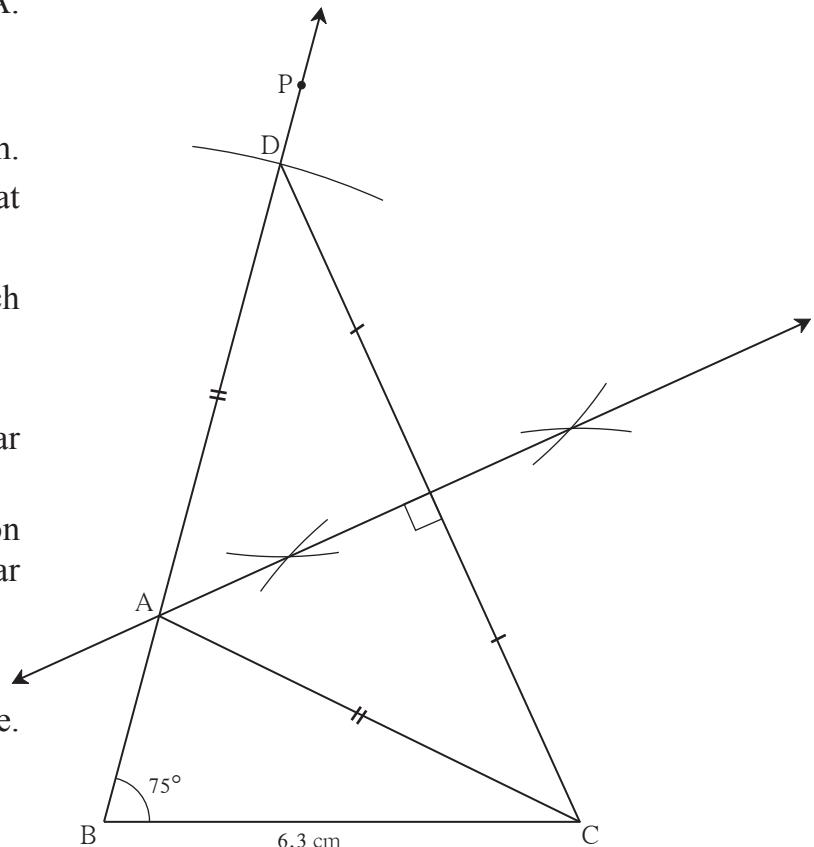


Rough figure 4.3

Steps of construction

- (1) Draw seg BC of length 6.3 cm .
- (2) Draw ray BP such that $m\angle PBC = 75^\circ$.
- (3) Mark point D on ray BP such that $d(B,D) = 9 \text{ cm}$
- (4) Draw seg DC .
- (5) Construct the perpendicular bisector of seg DC .
- (6) Name the point of intersection of ray BP and the perpendicular bisector of CD as A .
- (7) Draw seg AC .

$\triangle ABC$ is the required triangle.



Fair fig. 4.4

Practice set 4.1

1. Construct $\triangle PQR$, in which $QR = 4.2$ cm, $m\angle Q = 40^\circ$ and $PQ + PR = 8.5$ cm
 2. Construct $\triangle XYZ$, in which $YZ = 6$ cm, $XY + XZ = 9$ cm. $\angle XYZ = 50^\circ$
 3. Construct $\triangle ABC$, in which $BC = 6.2$ cm, $\angle ACB = 50^\circ$, $AB + AC = 9.8$ cm
 4. Construct $\triangle ABC$, in which $BC = 3.2$ cm, $\angle ACB = 45^\circ$ and perimeter of $\triangle ABC$ is 10 cm

Construction II

To construct a triangle when its base, angle adjacent to the base and difference between the remaining sides is given.

Ex (1) Construct $\triangle ABC$, such that $BC = 7.5$ cm, $\angle ABC = 40^\circ$, $AB - AC = 3$ cm.

Solution : Let us draw a rough figure.

Explanation : $AB - AC = 3 \text{ cm} \therefore AB > AC$

Draw seg BC. We can draw the ray BL such that $\angle LBC = 40^\circ$. We have to locate point A on ray BL. Take point D on ray BL such that $BD = 3\text{ cm}$.

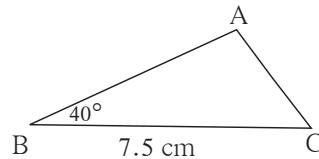
Now , B-D-A and $BD = AB - AD = 3.$

It is given that $AB - AC = 3$

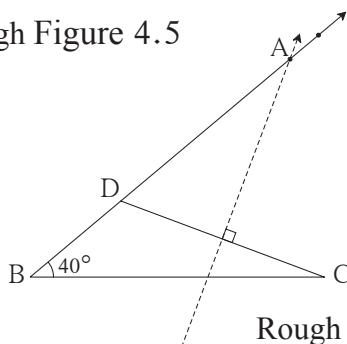
$$\therefore AD = AC$$

∴ point A is on the perpendicular bisector of seg DC.

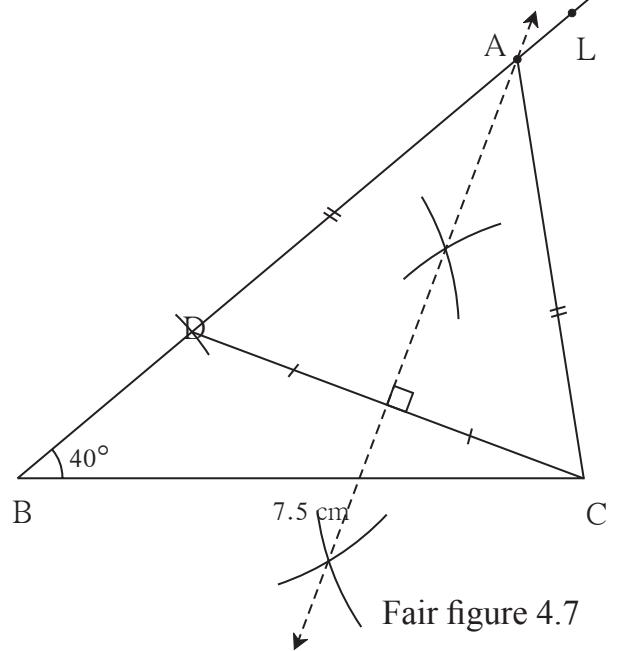
∴ point A is the intersection of ray BL and the perpendicular bisector of seg DC.



Rough Figure 4.5



Rough figure 4.6



Fair figure 4.7

Ex. 2 Construct $\triangle ABC$, in which side $BC = 7 \text{ cm}$, $\angle B = 40^\circ$ and $AC - AB = 3 \text{ cm}$.

Solution : Let us draw a rough figure.

seg $BC = 7 \text{ cm}$. $AC > AB$.

We can draw ray BT such that

$$\angle TBC = 40^\circ$$

Point A is on ray BT . Take point D on opposite ray of ray BT such that

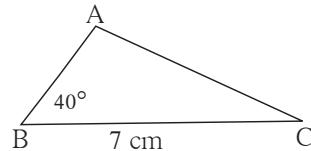
$$BD = 3 \text{ cm}.$$

Now $AD = AB + BD = AB + 3 = AC$

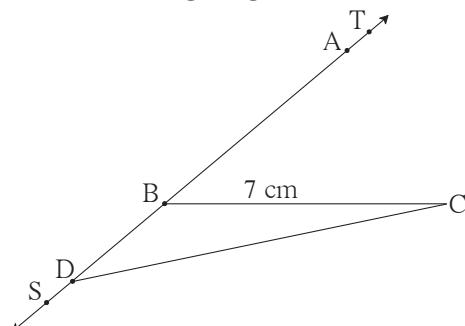
$$(\because AC - AB = 3 \text{ cm.})$$

$$\therefore AD = AC$$

\therefore point A is on the perpendicular bisector of seg CD.



Rough figure 4.8



Rough figure 4.9

Steps of construction

(1) Draw BC of length 7 cm.

(2) Draw ray BT such that

$$\angle TBC = 40^\circ$$

(3) Take point D on the opposite ray BS of ray BT such that

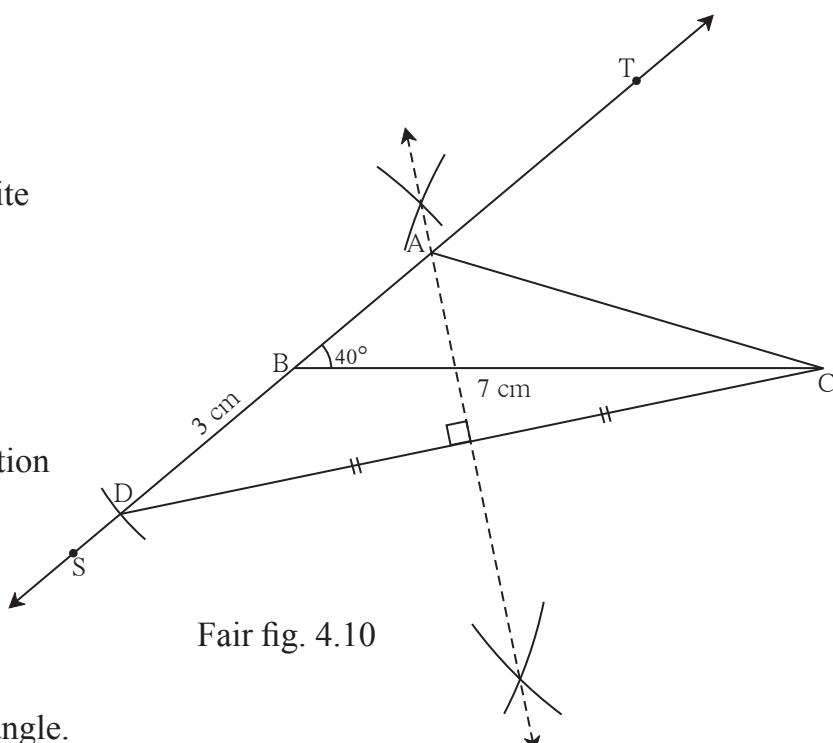
$$BD = 3 \text{ cm}.$$

(4) Construct perpendicular bisector of seg DC.

(5) Name the point of intersection of ray BT and the perpendicular bisector of DC as A.

(6) Draw seg AC.

$\triangle ABC$ is the required triangle.



Fair fig. 4.10

Practice set 4.2

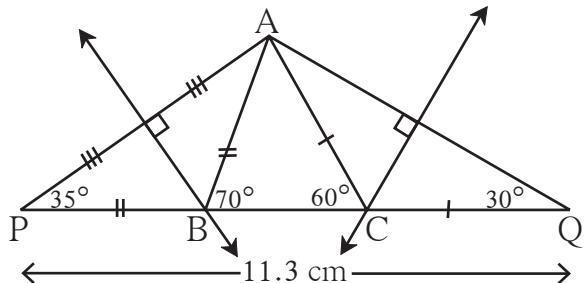
- Construct $\triangle XYZ$, such that $YZ = 7.4 \text{ cm}$, $\angle XYZ = 45^\circ$ and $XY - XZ = 2.7 \text{ cm}$.
- Construct $\triangle PQR$, such that $QR = 6.5 \text{ cm}$, $\angle PQR = 60^\circ$ and $PQ - PR = 2.5 \text{ cm}$.
- Construct $\triangle ABC$, such that $BC = 6 \text{ cm}$, $\angle ABC = 100^\circ$ and $AC - AB = 2.5 \text{ cm}$.

Construction III

To construct a triangle, if its perimeter, base and the angles which include the base are given.

Ex. Construct $\triangle ABC$ such that $AB + BC + CA = 11.3$ cm, $\angle B = 70^\circ$, $\angle C = 60^\circ$.

Solution : Let us draw a rough figure.



Rough Fig. 4.11

Explanation : As shown in the figure, points P and Q are taken on line BC such that,

$$PB = AB, \quad CQ = AC$$

$$\therefore PQ = PB + BC + CQ = AB + BC + AC = 11.3 \text{ cm.}$$

Now in Δ PBA, $PB = BA$

$\therefore \angle APB = \angle PAB$ and $\angle APB + \angle PAB = \text{exterior angle } ABC = 70^\circ$

.....theorem of remote interior angles

$\therefore \angle APB = \angle PAB = 35^\circ$ Similarly, $\angle CQA = \angle CAQ = 30^\circ$

Now we can draw $\triangle PAQ$, as its two angles and the included side is known.

Since $BA = BP$, point B lies on the perpendicular bisector of seg AP.

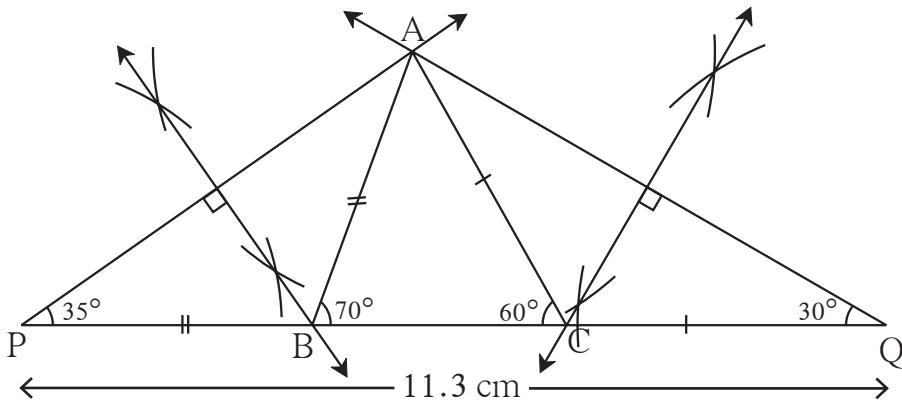
Similarly, $CA = CQ$, therefore point C lies on the perpendicular bisector of seg AQ

∴ by constructing the perpendicular bisectors of seg AP and AQ we can get points B and C, where the perpendicular bisectors intersect line PQ.

Steps of construction

- (1) Draw seg PQ of 11.3 cm length.
 - (2) Draw a ray making angle of 35° at point P.
 - (3) Draw another ray making an angle of 30° at point Q.
 - (4) Name the point of intersection of the two rays as A.
 - (5) Draw the perpendicular bisector of seg AP and seg AQ. Name the points as B and C respectively where the perpendicular bisectors intersect line PQ.
 - (6) Draw seg AB and seg AC.

$\triangle ABC$ is the required triangle.



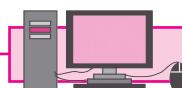
Final Fig. 4.12

Practice set 4.3

1. Construct $\triangle PQR$, in which $\angle Q = 70^\circ$, $\angle R = 80^\circ$ and $PQ + QR + PR = 9.5$ cm.
 2. Construct $\triangle XYZ$, in which $\angle Y = 58^\circ$, $\angle X = 46^\circ$ and perimeter of triangle is 10.5 cm.
 3. Construct $\triangle LMN$, in which $\angle M = 60^\circ$, $\angle N = 80^\circ$ and $LM + MN + NL = 11$ cm.

Problem set 4

1. Construct $\triangle XYZ$, such that $XY + XZ = 10.3$ cm, $YZ = 4.9$ cm, $\angle XYZ = 45^\circ$.
 2. Construct $\triangle ABC$, in which $\angle B = 70^\circ$, $\angle C = 60^\circ$, $AB + BC + AC = 11.2$ cm.
 3. The perimeter of a triangle is 14.4 cm and the ratio of lengths of its side is $2 : 3 : 4$.
Construct the triangle.
 4. Construct $\triangle PQR$, in which $PQ - PR = 2.4$ cm, $QR = 6.4$ cm and $\angle PQR = 55^\circ$.



ICT Tools or Links

Do constructions of above types on the software Geogebra and enjoy the constructions. The third type of construction given above is shown on Geogebra by a different method. Study that method also.





Let's study.

- Parallelogram
 - Rectangles
 - Mid point theorem
 - Tests of parallelogram
 - Square
 - Rhombus
 - Trapezium



Let's recall.

A quadrilateral with vertices labeled A, B, C, and D. Vertex A is at the bottom left, B is at the top, C is at the top right, and D is at the bottom right. The quadrilateral is drawn with black lines.

Fig. 5.1

Write the following pairs considering $\square ABCD$

Pairs of adjacent sides:

(1) ... , ... (2) ... , ...

(3) ... , ... (4) ... , ...

Pairs of adjacent angles :

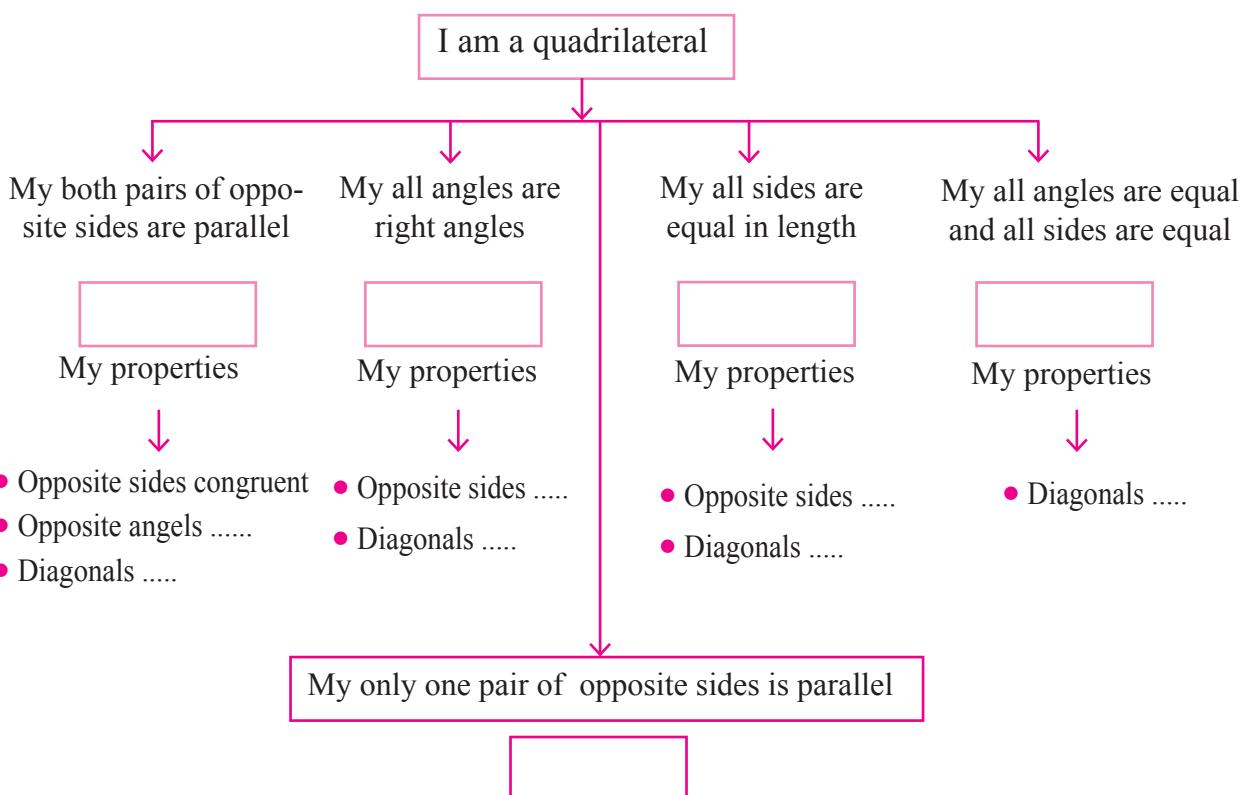
(1) ... , ... (2) ... , ...

(3) ... , ... (4) ... , ...

Pairs of opposite sides (1) , (2) ,

Pairs of opposite angles (1) , (2) ,

Let's recall types of quadrilaterals and their properties .



You know different types of quadrilaterals and their properties. You have learned them through different activities like measuring sides and angles, by paper folding method etc. Now we will study these properties by giving their logical proofs.

A property proved logically is called a proof.

In this chapter you will learn that how a rectangle, a rhombus and a square are parallelograms. Let us start our study from parallelogram.



Parallelogram

A quadrilateral having both pairs of opposite sides parallel is called a parallelogram.

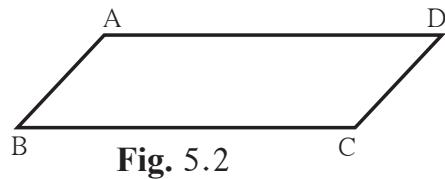
For proving the theorems or for solving the problems we need to draw figure of a parallelogram frequently. Let us see how to draw a parallelogram.

Suppose we have to draw a parallelogram $\square ABCD$.

Method I :

- Let us draw seg AB and seg BC of any length and making an angle of any measure with each other.
- Now we want seg AD and seg BC parallel to each other. So draw a line parallel to seg BC through the point A.
- Similarly we will draw line parallel to AB through the point C. These lines will intersect in point D.

So constructed quadrilateral ABCD will be a parallelogram.

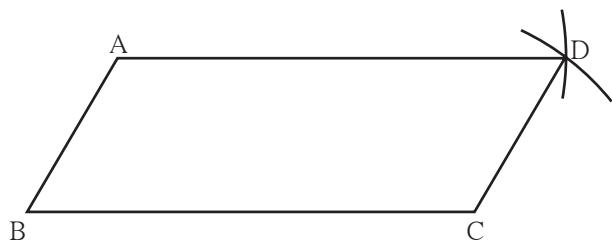


Method II :

- Let us draw seg AB and seg BC of any length and making angle of any measure between them.
- Draw an arc with compass with centre A and radius BC.
- Similarly draw an arc with centre C and radius AB intersecting the arc previously drawn.
- Name the point of intersection of two arcs as D.

Draw seg AD and seg CD.

Quadrilateral so formed is a parallelogram ABCD



In the second method we have actually drawn $\square ABCD$ in which opposite sides are equal. We will prove that a quadrilateral whose opposite sides are equal, is a parallelogram.

Activity I Draw five parallelograms by taking various measures of lengths and angles.

For the proving theorems on parallelogram, we use congruent triangles. To understand how they are used, let's do the following activity.

Activity II

- Draw a parallelogram ABCD on a card sheet. Draw diagonal AC. Write the names of vertices inside the triangle as shown in the figure. Then cut it out.
 - Fold the quadrilateral on the diagonal AC and see whether $\triangle ADC$ and $\triangle CBA$ match with each other or not.

and separate ΔADC and ΔCBA . By rotating and flipping ΔCBA , check whether it matches exactly with ΔADC .

What did you find? Which sides of ΔCBA match with which sides of ΔADC ?

Which angles of $\triangle CBD$ match with which angles of $\triangle ADC$?

Side DC matches with side AB and side AD matches with side CB. Similarly $\angle B$ matches with $\angle D$.

So we can see that opposite sides and angles of a parallelogram are congruent.

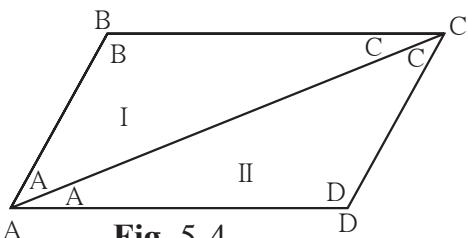


Fig. 5.4

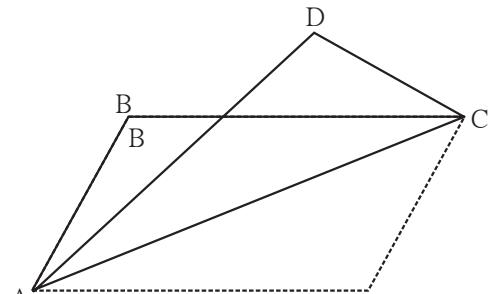


Fig. 5.5

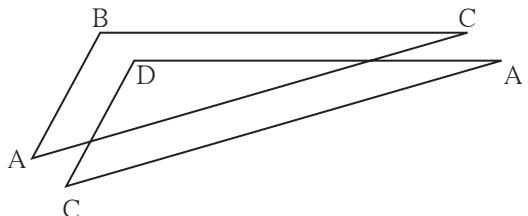


Fig. 5.6

We will prove these properties of a parallelogram.

Theorem 1. Opposite sides and opposite angles of a parallelogram are congruent.

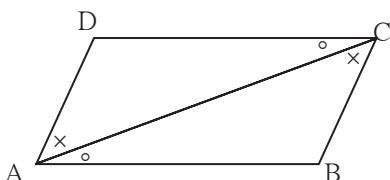


Fig. 5.7

Given : $\square ABCD$ is a parallelogram.

It means side AB || side DC,
side AD || side BC.

To prove : $\text{seg } AD \cong \text{seg } BC$; $\text{seg } DC \cong \text{seg } AB$
 $\angle ADC \cong \angle CBA$, and $\angle DAB \cong \angle BCD$.

Construction : Draw diagonal AC.

Proof : seg DC || seg AB and diagonal AC is a transversal.

$$\therefore \angle DCA \cong \angle BAC \quad \dots \dots \dots (1)$$

and $\angle DAC \cong \angle BCA$ (2) }
 Now in $\triangle ADC$ and $\triangle CBA$ }..... alternate angles

Now, in $\triangle ADC$ and $\triangle CBA$,

$$\angle DAC \cong \angle BCA \quad \dots \text{from (2)}$$

$$\angle DCA \cong \angle BAC \quad \dots \text{from (1)}$$

$\text{seg } AC \cong \text{seg } CA$ common side

$\therefore \Delta\text{ADC} \cong \Delta\text{CBA}$ ASA test

\therefore side AD \cong side CB c.s.c.t.

and side DC \cong side AB c.s.c.t.,

Also, $\angle ADC \cong \angle CBA$ c.a.c.t.

Similarly we can prove $\angle DAB \cong \angle BCD$



Use your brain power!

In the above theorem, to prove $\angle DAB \cong \angle BCD$, is any change in the construction needed ? If so, how will you write the proof making the change ?

To know one more property of a parallelogram let us do the following activity.

Activity : Draw a parallelogram PQRS. Draw diagonals PR and QS. Denote the intersection of diagonals by letter O. Compare the two parts of each diagonal with a divider. What do you find ?

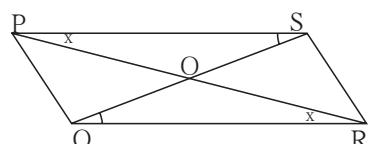


Fig. 5.8

Theorem : Diagonals of a parallelogram bisect each other.

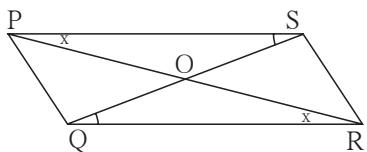


Fig. 5.9

Given : $\square PQRS$ is a parallelogram. Diagonals PR and QS intersect in point O .

To prove : seg PO \cong seg RO,
seg SO \cong seg QO.

Proof : In Δ POS and Δ ROQ

$\angle OPS \cong \angle ORQ$ alternate angles

side PS \cong side RQ opposite sides of parallelogram

$$\angle \text{PSO} \cong \angle \text{RQO} \dots \text{ alternate angles}$$

$\therefore \Delta\text{POS} \cong \Delta\text{ROO}$ ASA test

$$\begin{aligned} \therefore \text{seg PO} &\cong \text{seg RO} \\ \text{and seg SO} &\cong \text{seg QO} \end{aligned} \quad \left. \begin{array}{l} \dots \\ \dots \end{array} \right\} \begin{array}{l} \text{corresponding sides of} \\ \text{congruent triangles} \end{array}$$



Remember this !

- Adjacent angles of a parallelogram are supplementary.
 - Opposite sides of a parallelogram are congruent.
 - Opposite angles of a parallelogram are congruent.
 - Diagonals of a parallelogram bisect each other.

Solved Examples

Ex (1) $\square PQRS$ is a parallelogram. $PQ = 3.5$, $PS = 5.3$, $\angle Q = 50^\circ$ then find the lengths of remaining sides and measures of remaining angles.

Solution : \square PORS is a parallelogram.

$$\begin{aligned}\therefore \angle Q + \angle P &= 180^\circ \quad \dots \text{interior angles are} \\ \therefore 50^\circ + \angle P &= 180^\circ \quad \text{supplementary.} \\ \therefore \angle P &\equiv 180^\circ - 50^\circ \equiv 130^\circ\end{aligned}$$

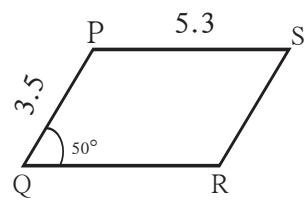


Fig. 5.10

Now , $\angle P = \angle R$ and $\angle Q = \angle S$ opposite angles of a parallelogram.

$$\therefore \angle R = 130^\circ \text{ and } \angle S = 50^\circ$$

Similarly, $PS = OR$ and $PO = SR$ opposite sides of a parallelogram.

$\therefore \text{OR} = 5.3$ and $\text{SR} = 3.5$

Ex (2) $\square ABCD$ is a parallelogram. If $\angle A = (4x + 13)^\circ$ and $\angle D = (5x - 22)^\circ$ then find the measures of $\angle B$ and $\angle C$.

Solution : Adjacent angles of a parallelogram are supplementary.

$\angle A$ and $\angle D$ are adjacent angles.

$$\therefore (4x + 13)^\circ + (5x - 22)^\circ = 180$$

$$\therefore 9x - 9 = 180$$

$$\therefore 9x = 189$$

$$\therefore x = 21$$

$$\therefore \angle A = 4x + 13 = 4 \times 21 + 13 = 84 + 13 = 97^\circ$$

$$\angle D = 5x - 22 = 5 \times 21 - 22 = 105 - 22 = 83^\circ \quad \therefore \angle B = 83^\circ$$

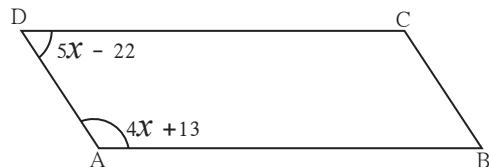


Fig. 5.11

Practice set 5.1

1. Diagonals of a parallelogram WXYZ intersect each other at point O. If $\angle XYZ = 135^\circ$ then what is the measure of $\angle XWZ$ and $\angle YZW$?
If $l(OY) = 5 \text{ cm}$ then $l(WY) = ?$
 2. In a parallelogram ABCD, If $\angle A = (3x + 12)^\circ$, $\angle B = (2x - 32)^\circ$ then find the value of x and then find the measures of $\angle C$ and $\angle D$.
 3. Perimeter of a parallelogram is 150 cm. One of its sides is greater than the other side by 25 cm. Find the lengths of all sides.
 4. If the ratio of measures of two adjacent angles of a parallelogram is 1 : 2, find the measures of all angles of the parallelogram.
 - 5*. Diagonals of a parallelogram intersect each other at point O. If $AO = 5$, $BO = 12$ and $AB = 13$ then show that $\square ABCD$ is a rhombus.

6. In the figure 5.12, $\square PQRS$ and $\square ABCR$ are two parallelograms.
If $\angle P = 110^\circ$ then find the measures of all angles of $\square ABCR$.

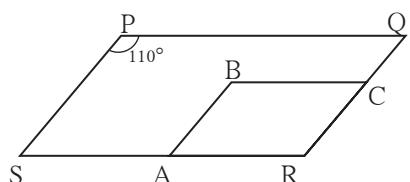


Fig. 5.12

7. In figure 5.13 $\square ABCD$ is a parallelogram. Point E is on the ray AB such that $BE = AB$ then prove that line ED bisects seg BC at point F.

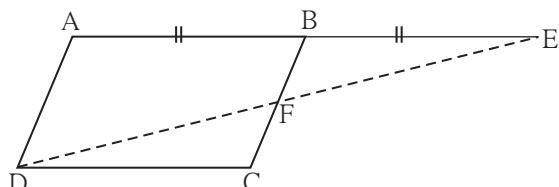


Fig. 5.13



Let's recall.

Tests for parallel lines

1. If a transversal intersects two lines and a pair of corresponding angles is congruent then those lines are parallel.
 2. If a transversal intersects two lines and a pair of alternate angles is congruent then those two lines are parallel.
 3. If a transversal intersects two lines and a pair of interior angles is supplementary then those two lines are parallel.



Let's learn.

Tests for parallelogram

Suppose, in $\square PQRS$, $PS = QR$ and $PQ = SR$ and we have to prove that $\square PQRS$ is a parallelogram. To prove it, which pairs of sides of $\square PQRS$ should be shown parallel ?

Which test can we use to show the sides parallel? Which line will be convenient as a transversal to obtain the angles necessary to apply the test?

Theorem : If pairs of opposite sides of a quadrilateral are congruent then that quadrilateral is a parallelogram.

Given : In $\square PQRS$

side PS \cong side OR

$$\text{side PQ} \cong \text{side SR}$$

To prove : $\square PQRS$ is a parallelogram.

Construction : Draw diagonal PR

Proof : In Δ SPR and Δ QRP

side PS \cong side QRgiven

side SR \cong side QP given

side PR \cong side RP common side

$\therefore \Delta \text{ SPR} \cong \Delta \text{ ORP}$ sss test

$\therefore \angle \text{SPR} \cong \angle \text{QRP}$ c.a.c.t.

Similarly, $\angle \text{PRS} \cong \angle \text{RPO}$ c.a.c.t.

$\angle SPR$ and $\angle QRP$ are alternate angles formed by the transversal PR of seg PS and seg QR.

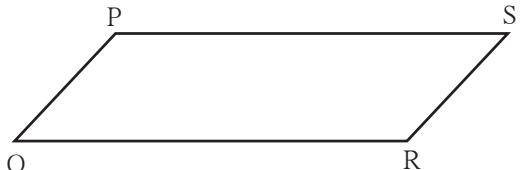


Fig. 5.14

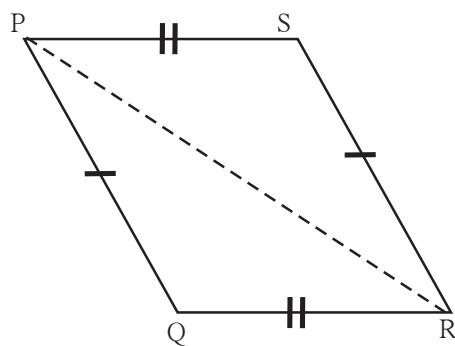


Fig. 5.15

\therefore side PS || side QR(I) alternate angles test for parallel lines.

Similarly $\angle PRS$ and $\angle RPQ$ are the alternate angles formed by transversal PR of seg PQ and seg SR.

\therefore side PQ \parallel side SR(II)alternate angle test

\therefore from (I) and (II) $\square PQRS$ is a parallelogram.

On page 56, two methods to draw a parallelogram are given. In the second method actually we have drawn a quadrilateral of which opposite sides are equal. Did you now understand why such a quadrilateral is a parallelogram ?

Theorem : If both the pairs of opposite angles of a quadrilateral are congruent then it is a parallelogram.

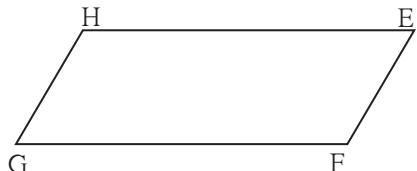


Fig. 5.16

Given : In $\square EFGH$ $\angle E \cong \angle G$
 and $\angle \dots \cong \angle \dots$

To prove : \square EFGH is a

Proof : Let $\angle E = \angle G = x$ and $\angle H = \angle F = y$

Sum of all angles of a quadrilateral is

$$\therefore \angle E + \angle G + \angle H + \angle F = \dots$$

$$\therefore x + y + \dots + \dots = \dots$$

$$\therefore \square x + \square y = \dots$$

$$\therefore x + y = 180^\circ$$

$$\therefore \angle G + \angle H = \dots$$

$\angle G$ and $\angle H$ are interior angles formed by transversal HG of seg HE

and seg GF.

\therefore side HE \parallel side GF (I) interior angle test for parallel lines.

Similarly, $\angle G + \angle F = \dots$

\therefore side \parallel side (II) interior angle test for parallel lines.

∴ From (I) and (II), $\square EFGH$ is a

Theorem : If the diagonals of a quadrilateral bisect each other then it is a parallelogram.

Given : Diagonals of $\square ABCD$ bisect each other in the point E.

It means $\text{seg AE} \cong \text{seg CE}$

and $\text{seg BE} \cong \text{seg DE}$

To prove : $\square ABCD$ is a parallelogram.

Proof : Find the answers for the following questions and write the proof of your own.

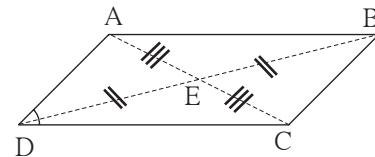


Fig. 5.17

1. Which pair of alternate angles should be shown congruent for proving $\text{seg } AB \parallel \text{seg } DC$?
Which transversal will form a pair of alternate angles ?
 2. Which triangles will contain the alternate angles formed by the transversal?
 3. Which test will enable us to say that the two triangles congruent ?
 4. Similarly, can you prove that $\text{seg } AD \parallel \text{seg } BC$?

The three theorems above are useful to prove that a given quadrilateral is a parallelogram. Hence they are called as tests of a parallelogram.

One more theorem which is useful as a test for parallelogram is given below.

Theorem : A quadrilateral is a parallelogram if a pair of its opposite sides is parallel and congruent.

Given : In $\square ABCD$

$\text{seg CB} \cong \text{seg DA}$ and $\text{seg CB} \parallel \text{seg DA}$

To prove : $\square ABCD$ is a parallelogram.

Construction : Draw diagonal BD.

Write the complete proof which is given in short.

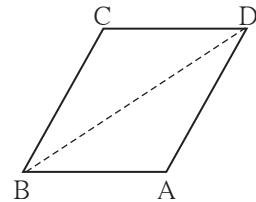


Fig. 5.18

Δ CBD \cong Δ ADBSAS test

$\therefore \angle CDB \cong \angle ABD$ c.a.c.t.

\therefore seg CD \parallel seg BA alternate angle test for parallel lines



Remember this !

- A quadrilateral is a parallelogram if its pairs of opposite angles are congruent.
 - A quadrilateral is a parallelogram if its pairs of opposite sides are congruent.
 - A quadrilateral is a parallelogram if its diagonals bisect each other.
 - A quadrilateral is a parallelogram if a pair of its opposite sides is parallel and congruent.

These theorems are called tests for parallelogram



Let's recall.

Lines in a note book are parallel. Using these lines how can we draw a parallelogram ?



Solved examples -

Ex (1) $\square PQRS$ is parallelogram. M is the midpoint of side PQ and N is the mid point of side RS. Prove that $\square PMNS$ and $\square MQRN$ are parallelograms.

Given : \square PQRS is a parallelogram.
M and N are the midpoints of side PQ and side RS respectively.

To prove : $\square PMNS$ is a parallelogram.
 $\square MQRN$ is a parallelogram

Proof : side PQ || side SR

\therefore side PM \parallel side SN (\because P-M-Q; S-N-R)(I)
 side PQ \cong side SR.

$$\therefore \frac{1}{2} \text{ side } PQ = \frac{1}{2} \text{ side } SR$$

\therefore side PM \cong side SN (\because M and N are midpoints.).....(II)

\therefore From (I) and (II), $\square PMNQ$ is a parallelogram,

Similarly, we can prove that $\square MQRN$ is parallelogram.

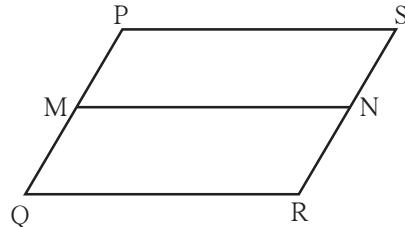


Fig. 5.19

Ex (2) Points D and E are the midpoints of side AB and side AC of $\triangle ABC$ respectively.

Point F is on ray ED such that $ED = DF$. Prove that $\square AFBE$ is a parallelogram.

For this example write ‘given’ and ‘to prove’ and complete the proof given below.

Given : -----

To prove : -----

Proof : seg AB and seg EF are _____ of $\square AFBE$.

$$\text{seg } AD \cong \text{seg } DB \dots$$

seg \cong seg construction.

∴ Diagonals of $\square AFBE$ each other.

$\therefore \square AFBE$ is a parallelogram ...by _____ test.

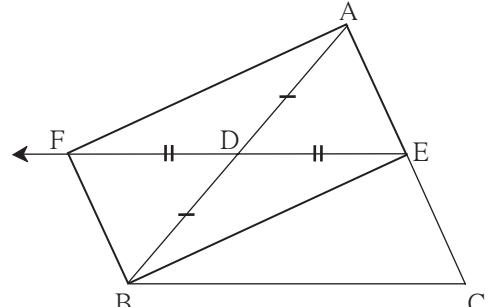


Fig. 5.20

Ex (3) Prove that every rhombus is a parallelogram.

Given : $\square ABCD$ is a rhombus

To prove : $\square ABCD$ is parallelogram.

Proof : $\text{seg AB} \cong \text{seg BC} \cong \text{seg CD} \cong \text{seg DA}$ (given)

\therefore side AB \cong side CD and side BC \cong side AD

$\therefore \square ABCD$ is a parallelogram opposite side test for parallelogram

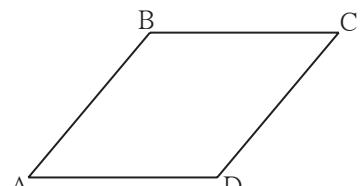


Fig. 5.21

Practice set 5.2

1. In figure 5.22, $\square ABCD$ is a parallelogram, P and Q are midpoints of side AB and DC respectively, then prove $\square APCQ$ is a parallelogram.

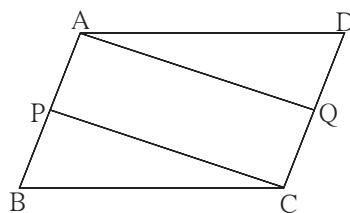


Fig. 5.22

- 2.** Using opposite angles test for parallelogram, prove that every rectangle is a parallelogram.

3. In figure 5.23, G is the point of concurrence of medians of $\triangle DEF$. Take point H on ray DG such that $D-G-H$ and $DG = GH$, then prove that $\square GEHF$ is a parallelogram.

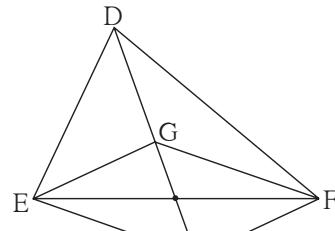


Fig. 5.23

4. Prove that quadrilateral formed by the intersection of angle bisectors of all angles of a parallelogram is a rectangle. (Figure 5.24)

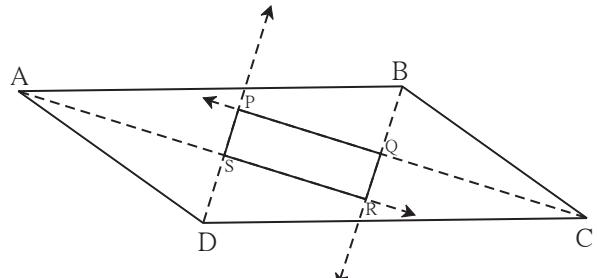


Fig. 5.24

5. In figure 5.25, if points P, Q, R, S are on the sides of parallelogram such that $AP = BQ = CR = DS$ then prove that $\square PORS$ is a parallelogram.

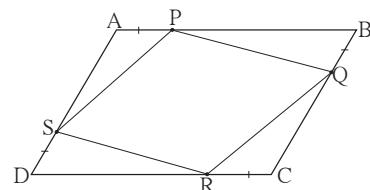


Fig. 5.25



Let's learn.

Properties of rectangle, rhombus and square

Rectangle, rhombus and square are also parallelograms. So the properties that opposite sides are equal, opposite angles are equal and diagonals bisect each other hold good in these types of quadrilaterals also. But there are some more properties of these quadrilaterals.

Proofs of these properties are given in brief. Considering the steps in the given proofs, write the proofs in detail.

Theorem : Diagonals of a rectangle are congruent.

Given : $\square ABCD$ is a rectangle

To prove : Diagonal $AC \cong$ diagonal BD

Proof : Complete the proof by giving suitable reasons.

$$\Delta ADC \cong \Delta DAB \dots\dots \text{SAS test}$$

$$\therefore \text{diagonal } AC \cong \text{diagonal } BD \dots\dots \text{c.s.c.t.}$$

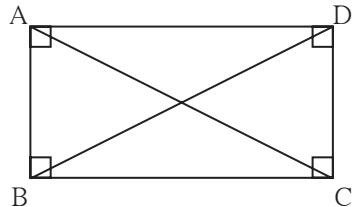


Fig. 5.26

Theorem : Diagonals of a square are congruent.

Write ‘Given’, ‘To prove’ and ‘proof’ of the theorem.

Theorem : Diagonals of a rhombus are perpendicular bisectors of each other.

Given : $\square EFGH$ is a rhombus

To prove : (i) Diagonal EG is the perpendicular bisector of diagonal HF .

(ii) Diagonal HF is the perpendicular bisector of diagonal EG .

Proof : (i) $\text{seg } EF \cong \text{seg } EH \quad \left. \begin{array}{l} \\ \end{array} \right\} \text{ given}$
 $\text{seg } GF \cong \text{seg } GH \quad \left. \begin{array}{l} \\ \end{array} \right\} \text{ given}$

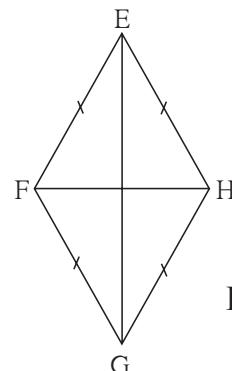


Fig. 5.27

Every point which is equidistant from end points of a segment is on the perpendicular bisector of the segment.

\therefore point E and point G are on the perpendicular bisector of $\text{seg } HF$.

One and only one line passes through two distinct points.

\therefore line EG is the perpendicular bisector of diagonal HF .

\therefore diagonal EG is the perpendicular bisector of diagonal HF .

(ii) Similarly, we can prove that diagonal HF is the perpendicular bisector of EG .

Write the proofs of the following statements.

- Diagonals of a square are perpendicular bisectors of each other.
- Diagonals of a rhombus bisect its opposite angles.
- Diagonals of a square bisect its opposite angles.



Remember this !

- Diagonals of a rectangle are congruent.
- Diagonals of a square are congruent.
- Diagonals of a rhombus are perpendicular bisectors of each other.
- Diagonals of a rhombus bisect the pairs of opposite angles.
- Diagonals of a square are perpendicular bisectors of each other.
- Diagonals of a square bisect opposite angles.

Practice set 5.3

1. Diagonals of a rectangle ABCD intersect at point O. If $AC = 8$ cm then find the length of BO and if $\angle CAD = 35^\circ$ then find the measure of $\angle ACB$.
 2. In a rhombus PQRS if $PQ = 7.5$ then find the length of QR.
If $\angle QPS = 75^\circ$ then find the measure of $\angle PQR$ and $\angle SRQ$.
 3. Diagonals of a square IJKL intersect at point M, Find the measures of $\angle IMJ$, $\angle JIK$ and $\angle LJK$.
 4. Diagonals of a rhombus are 20 cm and 21 cm respectively, then find the side of rhombus and its perimeter.
 5. State with reasons whether the following statements are ‘true’ or ‘false’.
 - (i) Every parallelogram is a rhombus.
 - (ii) Every rhombus is a rectangle.
 - (iii) Every rectangle is a parallelogram.
 - (iv) Every square is a rectangle.
 - (v) Every square is a rhombus.
 - (vi) Every parallelogram is a rectangle.



Let's learn.

Trapezium

When only one pair of opposite sides of a quadrilateral is parallel then the quadrilateral is called a trapezium.

In the adjacent figure only side AB and side DC of $\square ABCD$ are parallel to each other. So this is a trapezium. $\angle A$ and $\angle D$ is a pair of adjacent angles and so is the pair of $\angle B$ and $\angle C$. Therefore by property of parallel lines both the pairs are supplementary.

If non-parallel sides of a trapezium are congruent then that quadrilateral is called as an **Isoceles trapezium**.

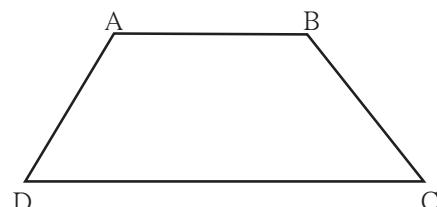


Fig. 5.28

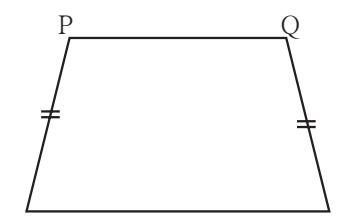
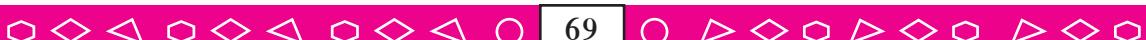


Fig. 5.29

The segment joining the midpoints of non parallel sides of a trapezium is called the median of the trapezium



Solved examples

Ex (1) Measures of angles of $\square ABCD$ are in the ratio $4 : 5 : 7 : 8$. Show that $\square ABCD$ is a trapezium.

Solution : Let measures of $\angle A$, $\angle B$, $\angle C$ and $\angle D$ are $(4x)^\circ$, $(5x)^\circ$, $(7x)^\circ$, and $(8x)^\circ$ respectively.

Sum of all angles of a quadrilateral is 360° .

$$\therefore 4x + 5x + 7x + 8x = 360$$

$$\therefore 24x = 360 \quad \therefore x = 15$$

$$\angle A \equiv 4 \times 15 \equiv 60^\circ \quad \angle B \equiv 5 \times 15 \equiv 75^\circ \quad \angle C \equiv 7 \times 15 \equiv 105^\circ$$

and $\angle D = 8 \times 15 = 120^\circ$

$$\text{Now } \angle B + \angle C = 75^\circ + 105^\circ = 180^\circ$$

$$\therefore \text{side } CD \parallel \text{side } BA \quad (\text{I})$$

$$\text{But } \angle B + \angle A = 75^\circ + 60^\circ = 135^\circ \neq 180^\circ$$

• side BC and side AD are not parallel (II)

$\therefore \square ABCD$ is a trapezium. [from (I) and (II)]

Ex (2) In $\square PQRS$, side $PS \parallel$ side QR and side $PQ \cong$ side SR ,
side $OR \geq$ side PS then prove that $\angle POR \cong \angle SRO$

Given : In $\square PQRS$, side $PS \parallel$ side QR ,
side $PO \cong$ side SR and side $OR >$ side PS .

To prove : $\angle PQR \cong \angle SRQ$

Construction : Draw the segment parallel to side PQ through the point S which intersects :

Proof : In $\square PQRS$,

seg PS || seg QTgiven

seg PQ || seg STconstruction

$\therefore \square PQTS$ is a parallelogram

$\therefore \angle \text{PQT} \cong \angle \text{STR}$ corresponds

and seg PQ \cong seg STopposite sides of para

But $\text{seg } PO \cong \text{seg } SR$ given

$\vdash \text{seg ST} \cong \text{seg SR}$

$$\cdot \angle \text{STR} \cong \angle \text{SRT}$$

$\therefore \angle \text{POT} \approx \angle \text{SBT}$ [from (I) and (II)]

$$\therefore \angle BQR \approx \angle SBO$$

Hence it is proved that

¹¹ Hence, it is proved that base angles of all isosceles triangles are congruent.

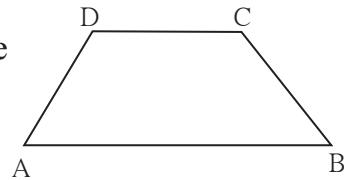


Fig. 5.30

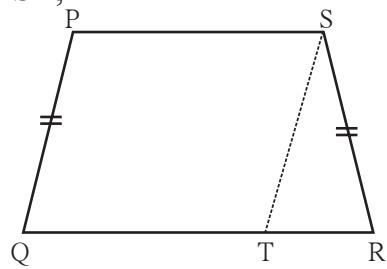


Fig. 5.31

Practice set 5.4

1. In $\square IJKL$, side $IJ \parallel$ side KL $\angle I = 108^\circ$ $\angle K = 53^\circ$ then find the measures of $\angle J$ and $\angle L$.
 2. In $\square ABCD$, side $BC \parallel$ side AD , side $AB \cong$ side DC If $\angle A = 72^\circ$ then find the measures of $\angle B$, and $\angle D$.
 3. In $\square ABCD$, side $BC <$ side AD (Figure 5.32)
 side $BC \parallel$ side AD and if
 side $BA \cong$ side CD
 then prove that $\angle ABC \cong \angle DCB$.

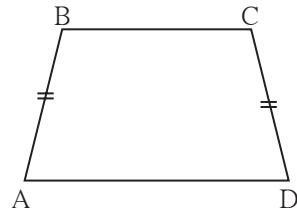


Fig. 5.32



Let's learn.

Theorem of midpoints of two sides of a triangle

Statement : The segment joining midpoints of any two sides of a triangle is parallel to the third side and half of it.

Given : In $\triangle ABC$, point P is the midpoint of seg AB and point Q is the midpoint of seg AC

To prove : seg PQ || seg BC

$$\text{and } PQ = \frac{1}{2} BC$$

Construction : Produce \overline{PQ} upto R such that $PQ = QR$
Draw seg RC.

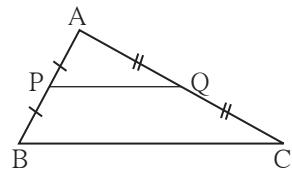


Fig. 5.33

Proof : In $\triangle AQP$ and $\triangle CQR$

$\text{seg } PQ \cong \text{seg } QR$ construction

$\text{seg } \text{AQ} \cong \text{seg } \text{QC}$ given

$\angle AQP \cong \angle CQR$ vertically opposite angles.

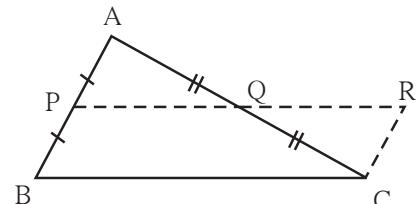


Fig. 5.34

$\therefore \Delta \text{ AOP} \cong \Delta \text{ COR}$ SAS test

$$\angle \text{PAO} \cong \angle \text{RCO} \dots \quad (1) \text{ c.a.c.t.}$$

$\therefore \text{seg AP} \cong \text{seg CR}$ (2) c.s.c.t.

From (1) line AB || line CR.....alternate angle test

from (2) $\text{seg AP} \cong \text{seg CR}$

Now, $\overline{AP} \cong \overline{PB} \cong \overline{CR}$ and $\overline{PB} \parallel \overline{CR}$

$\therefore \square PBCR$ is a parallelogram.

\therefore seg PO \parallel seg BC and PR = BC ... opposite sides are



$$PQ = \frac{1}{2} PR \quad \dots \text{(construction)}$$

$$\therefore PQ = \frac{1}{2} BC \quad \therefore PR = BC$$

Converse of midpoint theorem

Theorem : If a line drawn through the midpoint of one side of a triangle and parallel to the other side then it bisects the third side.

For this theorem ‘Given’, ‘To prove’, ‘construction’ is given below. Try to write the proof.

Given : Point D is the midpoint of side AB of $\triangle ABC$. Line l passing through the point D and parallel to side BC intersects side AC in point E.

To prove : AE = EC

Construction : Take point F on line l such that D-E-F and $DE = EF$.
Draw seg CF.

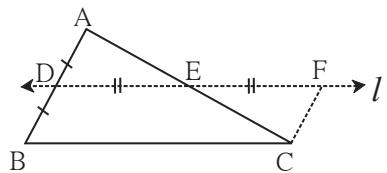


Fig. 5.35

Proof : Use the construction and line $l \parallel \text{seg } BC$ which is given. Prove $\triangle ADE \cong \triangle CFE$ and complete the proof.

Ex (1) Points E and F are mid points of seg AB and seg AC of $\triangle ABC$ respectively. If $EF = 5.6$ then find the length of BC.

Solution : In $\triangle ABC$, point E and F are midpoints of side AB and side AC respectively.

$$EF = \frac{1}{2} BC \dots\dots\text{midpoint theorem}$$

$$5.6 = \frac{1}{2} BC \quad \therefore BC = 5.6 \times 2 = 11.2$$

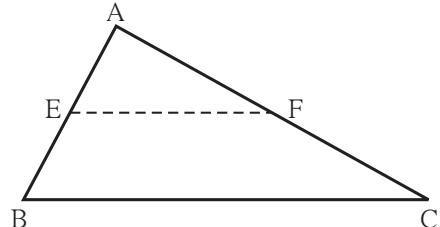


Fig. 5.36

Ex (2) Prove that the quadrilateral formed by joining the midpoints of sides of a quadrilateral in order is a parallelogram.

Given : $\square ABCD$ is a quadrilateral.

P, Q, R, S are midpoints of the sides AB, BC, CD and AD respectively.

To prove : $\square PQRS$ is a parallelogram.

Construction : Draw diagonal BD

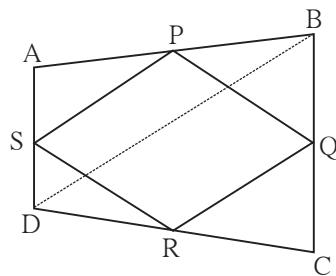


Fig. 5.37

Proof : In $\triangle ABD$, the midpoint of side AD is S and the midpoint of side AB is P .
 \therefore by midpoint theorem, $PS \parallel DB$ and $PS = \frac{1}{2} BD$ (1)

In $\triangle DBC$ point Q and R are midpoints of side BC and side DC respectively.

$\therefore QR \parallel BD$ and $QR = \frac{1}{2} BD$ by midpoint theorem (2)

$\therefore PS \parallel QR$ and $PS = QR$ from (1) and (2)

$\therefore \square PQRS$ is a parallelogram.

Practice set 5.5

1. In figure 5.38, points X, Y, Z are the midpoints of side AB, side BC and side AC of $\triangle ABC$ respectively. $AB = 5 \text{ cm}$, $AC = 9 \text{ cm}$ and $BC = 11 \text{ cm}$. Find the length of XY, YZ, XZ.

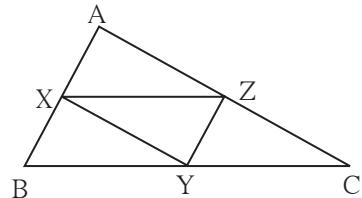


Fig. 5.38

2. In figure 5.39, $\square PQRS$ and $\square MNRL$ are rectangles. If point M is the midpoint of side PR then prove that, (i) $SL = LR$, (ii) $LN = \frac{1}{2}SQ$.

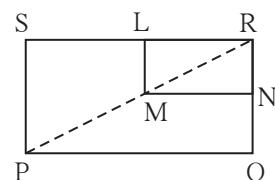


Fig. 5.39

3. In figure 5.40, ΔABC is an equilateral triangle. Points F,D and E are midpoints of side AB, side BC, side AC respectively. Show that ΔFED is an equilateral triangle.

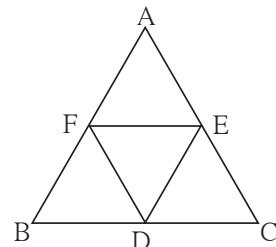


Fig. 5.40

4. In figure 5.41, seg PD is a median of $\triangle PQR$. Point T is the mid point of seg PD. Produced QT intersects PR at M. Show that $\frac{PM}{PR} = \frac{1}{3}$.
 [Hint : draw DN \parallel QM.]

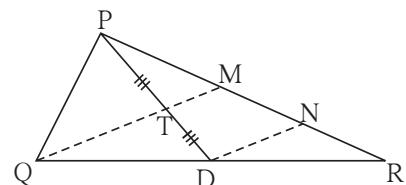


Fig. 5.41

Problem set 5

1. Choose the correct alternative answer and fill in the blanks.

(i) If all pairs of adjacent sides of a quadrilateral are congruent then it is called
(A) rectangle (B) parallelogram (C) trapezium, (D) rhombus

- (ii) If the diagonal of a square is $12\sqrt{2}$ cm then the perimeter of square is
(A) 24 cm (B) $24\sqrt{2}$ cm (C) 48 cm (D) $48\sqrt{2}$ cm

(iii) If opposite angles of a rhombus are $(2x)^\circ$ and $(3x - 40)^\circ$ then value of x is ...
(A) 100° (B) 80° (C) 160° (D) 40°

2. Adjacent sides of a rectangle are 7 cm and 24 cm. Find the length of its diagonal.

3. If diagonal of a square is 13 cm then find its side.

4. Ratio of two adjacent sides of a parallelogram is 3 : 4, and its perimeter is 112 cm. Find the length of its each side.

5. Diagonals PR and QS of a rhombus PQRS are 20 cm and 48 cm respectively. Find the length of side PQ.

6. Diagonals of a rectangle PQRS are intersecting in point M. If $\angle QMR = 50^\circ$ then find the measure of $\angle MPS$.

7. In the adjacent Figure 5.42, if

7. In the adjacent Figure 5.42, if

$\text{seg AB} \parallel \text{seg PQ}$, $\text{seg AB} \cong \text{seg PQ}$,

$\text{seg AC} \parallel \text{seg PR}$, $\text{seg AC} \cong \text{seg PR}$

then prove that,

$\text{seg BC} \parallel \text{seg QR}$ and $\text{seg BC} \cong \text{seg QR}$.

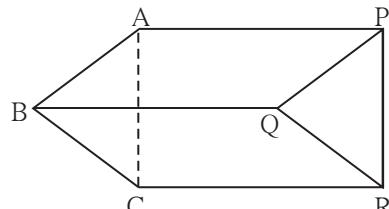


Fig. 5.42

8*. In the Figure 5.43, $\square ABCD$ is a trapezium.

$AB \parallel DC$. Points P and Q are midpoints of seg AD and seg BC respectively.

Then prove that, $PQ \parallel AB$ and

$$PQ = \frac{1}{2}(AB + DC).$$

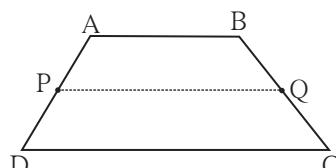


Fig. 5.43

9. In the adjacent figure 5.44, $\square ABCD$ is a trapezium. $AB \parallel DC$. Points M and N are midpoints of diagonal AC and DB respectively then prove that $MN \parallel AB$.

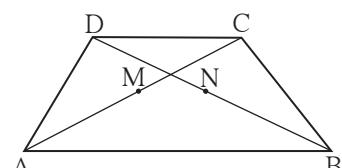


Fig. 5.44

Activity

To verify the different properties of quadrilaterals

Material : A piece of plywood measuring about 15 cm× 10 cm, 15 thin screws, twine, scissor.

Note : On the plywood sheet, fix five screws in a horizontal row keeping a distance of 2cm between any two adjacent screws. Similarly make two more rows of screws exactly below the first one. Take care that the vertical distance between any two adjacent screws is also 2cm.

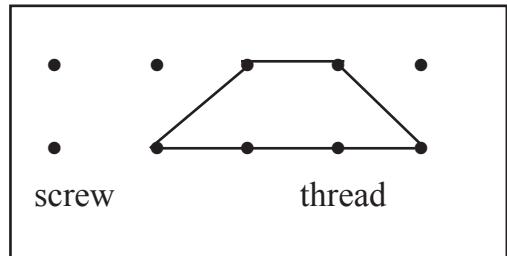


Fig. 5.45

With the help of the screws, make different types of quadrilaterals of twine. Verify the properties of sides and angles of the quadrilaterals.

Additional information

You know the property that the point of concurrence of medians of a triangle divides the medians in the ratio $2 : 1$. Proof of this property is given below.

Given : seg AD and seg BE are the medians of $\triangle ABC$ which intersect at point G.

To prove: $AG : GD = 2 : 1$

Construction : Take point F on ray AD such that
 $GD = DF$

Proof : Diagonals of $\square BGCF$ bisect each other

.... given and construction

$\therefore \square BGCF$ is a parallelogram.

$\therefore \text{seg BE} \parallel \text{set FC}$

Now point E is the midpoint of side AC of $\triangle AFC$ given

$\overline{\text{EB}} \parallel \overline{\text{FC}}$

Fig. 5.46

Line passing through midpoint of one side and parallel to the other side bisects the third side

\therefore point G is the midpoint of side AF.

$$\therefore AG = GF$$

But $GF = 2GD$ construction

$$\therefore AG = 2 GD$$

$$\therefore \frac{AG}{GD} = \frac{2}{1} \text{ i.e. } AG : GD = 2 : 1$$





Let's learn.

Properties of chord

Activity I : Every student in the group will do this activity. Draw a circle in your notebook. Draw any chord of that circle. Draw perpendicular to the chord through the centre of the circle. Measure the lengths of the two parts of the chord. Group leader will prepare a table and other students will write their observations in it.

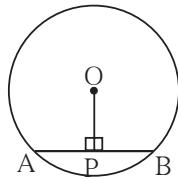


Fig. 6.3

Student Length	1	2	3	4	5	6
l (AP) cm					
l (PB) cm					

Write the property which you have observed.

Let us write the proof of this property.

Theorem : A perpendicular drawn from the centre of a circle on its chord bisects the chord.

Given : seg AB is a chord of a circle with centre O.

seg OP \perp chord AB

To prove : $\text{seg AP} \cong \text{seg BP}$

Proof : Draw seg OA and seg OB

In Δ OPA and Δ OPB

$$\angle QPA \cong \angle QPB \quad \text{seg } OP \perp \text{chord } AB$$

$\text{seg OP} \cong \text{seg OP}$ common side

hypotenuse OA \cong hypotenuse OB radii of the same circle

$\therefore \triangle QPA \cong \triangle QPB$ hypotenuse side theorem

$\text{seg PA} \cong \text{seg PB}$

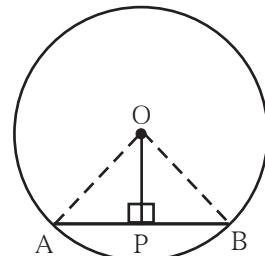


Fig. 6.4

Activity II : Every student from the group will do this activity. Draw a circle in your notebook. Draw a chord of the circle. Join the midpoint of the chord and centre of the circle. Measure the angles made by the segment with the chord. Discuss about the measures of the angles with your friends.

Which property do the observations suggest?

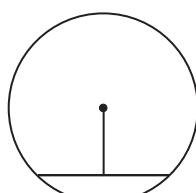


Fig. 6.5

Theorem : The segment joining the centre of a circle and the midpoint of its chord is perpendicular to the chord.

Given : seg AB is a chord of a circle with centre O and P is the midpoint of chord AB of the circle. That means $\text{seg } AP \cong \text{seg } PB$.

To prove : $\text{seg } OP \perp \text{chord } AB$

Proof : Draw seg OA and seg OB.

In ΔAOP and ΔBOP

$\text{seg } OA \cong \text{seg } OB$ radii of the same circle

$\text{seg } OP \cong \text{seg } OP$ common sides

$\text{seg } AP \cong \text{seg } BP$ given

$\therefore \Delta AOP \cong \Delta BOP$ SSS test

$\therefore \angle OPA \cong \angle OPB$ c.a.c.t.(I)

Now $\angle OPA + \angle OPB = 180^\circ$... angles in linear pair

$\therefore \angle OPB + \angle OPB = 180^\circ$ from (I)

$$\therefore 2 \angle OPB = 180^\circ$$

$$\therefore \angle OPB = 90^\circ$$

$\therefore \text{seg } OP \perp \text{chord } AB$

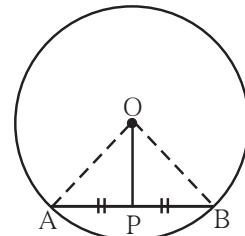


Fig. 6.6

Solved examples

Ex (1) Radius of a circle is 5 cm. The length of a chord of the circle is 8 cm. Find the distance of the chord from the centre.

Solution :

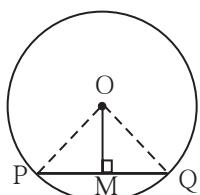


Fig. 6.7

We know that a perpendicular drawn from the centre of a circle on its chord bisects the chord.

$$\therefore PM = MQ = 4 \text{ cm}$$

Radius of the circle is 5 cm, means $OQ = 5 \text{ cm}$ given

In the right angled ΔOMQ using Pythagoras' theorem,

$$OM^2 + MQ^2 = OQ^2$$

$$\therefore OM^2 + 4^2 = 5^2$$

$$\therefore OM^2 = 5^2 - 4^2 = 25 - 16 = 9 = 3^2$$

$$\therefore OM = 3$$

Hence distance of the chord from the centre of the circle is 3 cm.

Ex (2) Radius of a circle is 20 cm. Distance of a chord from the centre of the circle is 12 cm. Find the length of the chord.

Solution : Let the centre of the circle be O. Radius = OD = 20 cm.

Distance of the chord CD from O is 12 cm. $\overline{OP} \perp \overline{CD}$

$$\therefore OP = 12 \text{ cm}$$

Now $CP = PD$ perpendicular drawn from the centre bisects the chord

In the right angled $\triangle OPD$, using Pythagoras' theorem

$$OP^2 + PD^2 = OD^2$$

$$(12)^2 + PD^2 = 20^2$$

$$PD^2 = 20^2 - 12^2$$

$$PD^2 = (20+12)(20-12)$$

$$= 32 \times 8 = 256$$

• PD \equiv 16 • CP \equiv 16

$$CD \equiv CP \pm PD \equiv 16 \pm 16 \equiv 32$$

∴ the length of the chord is 32 cm.

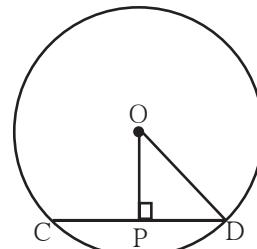


Fig. 6.8

Practice set 6.1

- Distance of chord AB from the centre of a circle is 8 cm. Length of the chord AB is 12 cm. Find the diameter of the circle.
 - Diameter of a circle is 26 cm and length of a chord of the circle is 24 cm. Find the distance of the chord from the centre.
 - Radius of a circle is 34 cm and the distance of the chord from the centre is 30 cm, find the length of the chord.
 - Radius of a circle with centre O is 41 units. Length of a chord PQ is 80 units, find the distance of the chord from the centre of the circle.
 - In figure 6.9, centre of two circles is O. Chord AB of bigger circle intersects the smaller circle in points P and Q. Show that $AP = BQ$
 - Prove that, if a diameter of a circle bisects two chords of the circle then those two chords are parallel to each other.

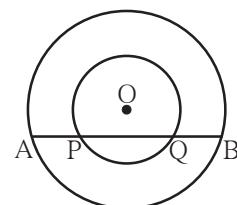


Fig. 6.9

Activity I

- (1) Draw circles of convenient radii.
 - (2) Draw two equal chords in each circle.
 - (3) Draw perpendicular to each chord from the centre.
 - (4) Measure the distance of each chord from the centre.

$OP = 4 \text{ cm}$, means distance of AB from the centre O is 4 cm.

The congruent chords of a circle are equidistant from the centre of the circle.

$$\therefore OQ = 4 \text{ cm}$$

Practice set 6.2

- Radius of circle is 10 cm. There are two chords of length 16 cm each. What will be the distance of these chords from the centre of the circle ?
 - In a circle with radius 13 cm, two equal chords are at a distance of 5 cm from the centre. Find the lengths of the chords.
 - Seg PM and seg PN are congruent chords of a circle with centre C. Show that the ray PC is the bisector of $\angle NPM$.



In previous standard we have verified the property that the angle bisectors of a triangle are concurrent. We denote the point of concurrence by letter I.



Incircle of a triangle

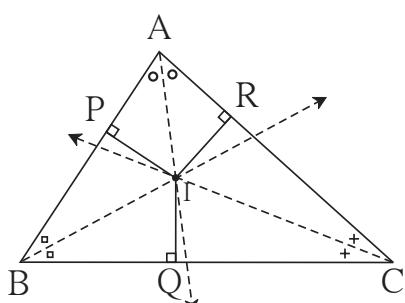


Fig. 6.13

In fig. 6.13, bisectors of all angles of a $\triangle ABC$ meet in the point I. Perpendiculars on three sides are drawn from the point of concurrence.

$$\text{IP} \perp \text{AB}, \quad \text{IO} \perp \text{BC}, \quad \text{IR} \perp \text{AC}$$

We know that, every point on the angle bisector is equidistant from the sides of the angle.

Point I is on the bisector of $\angle B$ $\therefore IP \equiv IQ$

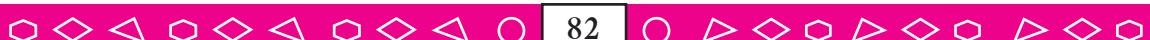
Point J is on the bisector of $\angle C$. $\therefore \text{J}O \equiv \text{JR}$

• $\text{IP} \equiv \text{IO} \equiv \text{IR}$

That is point I is equidistant from all the sides of $\triangle ABC$.

∴ if we draw a circle with centre I and radius IP, it will touch the sides AB, AC, BC of $\triangle ABC$ internally.

This circle is called the Incircle of the triangle ABC.





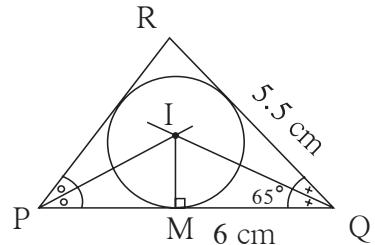
Let's learn.

To construct the incircle of a triangle

Ex. Construct $\triangle PQR$ such that $PQ = 6 \text{ cm}$, $\angle Q = 35^\circ$, $QR = 5.5 \text{ cm}$. Draw incircle of $\triangle PQR$.

Draw a rough figure and show all measures in it.

- (1) Construct $\triangle PQR$ of given measures.
 - (2) Draw bisectors of any two angles of the triangle.
 - (3) Denote the point of intersection of angle bisectors as I.
 - (4) Draw perpendicular IM from the point I to the side PQ.
 - (5) Draw a circle with centre I and radius IM.



Rough fig. 6.14

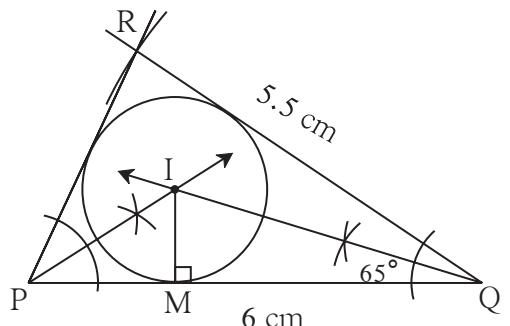


Fig. 6.15



Remember this !

The circle which touches all the sides of a triangle is called incircle of the triangle and the centre of the circle is called the incentre of the triangle.



Let's recall.

In previous standards we have verified the property that perpendicular bisectors of sides of a triangle are concurrent. That point of concurrence is denoted by the letter C.

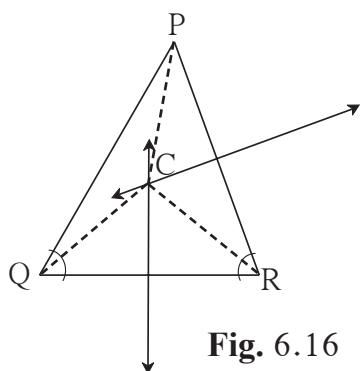


Fig. 6.16



Let's learn.

In fig. 6.16, the perpendicular bisectors of sides of $\triangle PQR$ are intersecting at point C. So C is the point of concurrence of perpendicular bisectors.

Circumcircle

Point C is on the perpendicular bisectors of the sides of triangle PQR. Join PC, QC and RC. We know that, every point on the perpendicular bisector is equidistant from the end points of the segment.

Point C is on the perpendicular bisector of seg PQ. \therefore $PC = QC \dots \dots \text{I}$

Point C is on the perpendicular bisector of seg QR. \therefore $QC = RC \dots \dots \text{II}$

$\therefore PC = QC = RC \dots \dots$ From I and II

\therefore the circle with centre C and radius PC will pass through all the vertices of $\triangle PQR$. This circle is called as the circumcircle.



Remember this !

Circle passing through all the vertices of a triangle is called circumcircle of the triangle and the centre of the circle is called the circumcentre of the triangle.



Let's learn.

To draw the circumcircle of a triangle

Ex. Construct $\triangle DEF$ such that $DE = 4.2 \text{ cm}$, $\angle D = 60^\circ$, $\angle E = 70^\circ$ and draw circumcircle of it. Draw rough figure. Write the given measures.

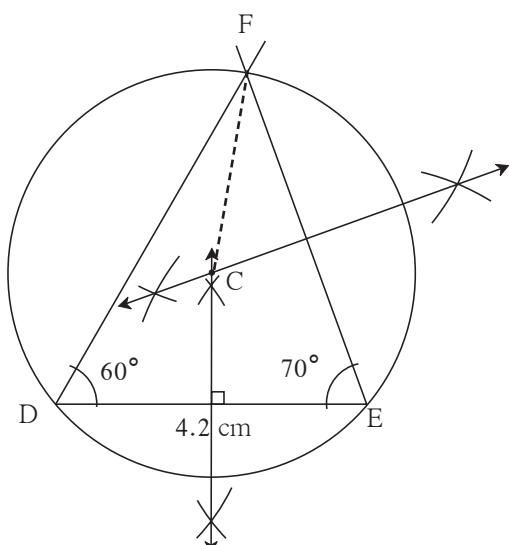


Fig. 6.18

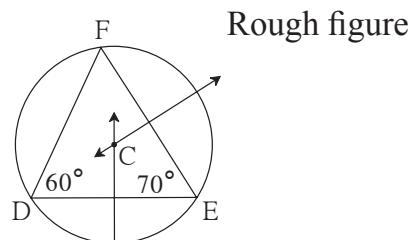


Fig. 6.17

Steps of construction :

- (1) Draw $\triangle DEF$ of given measures.
- (2) Draw perpendicular bisectors of any two sides of the triangle.
- (3) Name the point of intersection of perpendicular bisectors as C.
- (4) Join seg CF.
- (5) Draw circle with centre C and radius CF.

Activity :

Draw different triangles of different measures and draw incircles and circumcircles of them. Complete the table of observations and discuss.

Type of triangle	Equilateral triangle	Isosceles triangles	Scalene triangle
Position of incenter	Inside the triangle	Inside the triangle	Inside the triangle
Position of circumcentre	Inside the triangle	Inside, outside on the triangle	

Type of triangle	Acute angled triangle	Right angled triangle	Obtuse angled triangle
Position of incentre			
Position of circumcircle		Midpoint of hypotenuse	



Remember this !

- Incircle of a triangle touches all sides of the triangle from inside.
 - For construction of incircle of a triangle we have to draw bisectors of any two angles of the triangle.
 - Circumcircle of a triangle passes through all the vertices of a triangle.
 - For construction of a circumcircle of a triangle we have to draw perpendicular bisectors of any two sides of the triangle.
 - Circumcentre of an acute angled triangle lies inside the triangle.
 - Circumcentre of a right angled triangle is the midpoint of its hypotenuse.
 - Circumcentre of an obtuse angled triangle lies in the exterior of the triangle.
 - Incentre of any triangle lies in the interior of the triangle.

Activity : Draw any equilateral triangle. Draw incircle and circumcircle of it. What did you observe while doing this activity ?

- (1) While drawing incircle and circumcircle, do the angle bisectors and perpendicular bisectors coincide with each other ?
 - (2) Do the incentre and circumcenter coincide with each other ? If so, what can be the reason of it ?
 - (3) Measure the radii of incircle and circumcircle and write their ratio.



Remember this !

- The perpendicular bisectors and angle bisectors of an equilateral triangle are coincident.
 - The incentre and the circumcentre of an equilateral triangle are coincident.
 - Ratio of radius of circumcircle to the radius of incircle of an equilateral triangle is $2 : 1$

Practice set 6.3

1. Construct $\triangle ABC$ such that $\angle B = 100^\circ$, $BC = 6.4$ cm, $\angle C = 50^\circ$ and construct its incircle.
 2. Construct $\triangle PQR$ such that $\angle P = 70^\circ$, $\angle R = 50^\circ$, $QR = 7.3$ cm. and construct its circumcircle.
 3. Construct $\triangle XYZ$ such that $XY = 6.7$ cm, $YZ = 5.8$ cm, $XZ = 6.9$ cm. Construct its incircle.
 4. In $\triangle LMN$, $LM = 7.2$ cm, $\angle M = 105^\circ$, $MN = 6.4$ cm, then draw $\triangle LMN$ and construct its circumcircle.
 5. Construct $\triangle DEF$ such that $DE = EF = 6$ cm, $\angle F = 45^\circ$ and construct its circumcircle.

Problem set 6

1. Choose correct alternative answer and fill in the blanks.

(i) Radius of a circle is 10 cm and distance of a chord from the centre is 6 cm. Hence the length of the chord is
(A) 16 cm (B) 8 cm (C) 12 cm (D) 32 cm

(ii) The point of concurrence of all angle bisectors of a triangle is called the
(A) centroid (B) circumcentre (C) incentre (D) orthocentre

(iii) The circle which passes through all the vertices of a triangle is called
(A) circumcircle (B) incircle (C) congruent circle (D) concentric circle

(iv) Length of a chord of a circle is 24 cm. If distance of the chord from the centre is 5 cm, then the radius of that circle is
(A) 12 cm (B) 13 cm (C) 14 cm (D) 15 cm

(v) The length of the longest chord of the circle with radius 2.9 cm is
(A) 3.5 cm (B) 7 cm (C) 10 cm (D) 5.8 cm

(vi) Radius of a circle with centre O is 4 cm. If $l(OP) = 4.2$ cm, say where point P will lie.
(A) on the centre (B) Inside the circle (C) outside the circle(D) on the circle

(vii) The lengths of parallel chords which are on opposite sides of the centre of a circle are 6 cm and 8 cm. If radius of the circle is 5 cm, then the distance between these chords is
(A) 2 cm (B) 1 cm (C) 8 cm (D) 7 cm

2. Construct incircle and circumcircle of an equilateral Δ DSP with side 7.5 cm. Measure the radii of both the circles and find the ratio of radius of circumcircle to the radius of incircle.
 3. Construct Δ NTS where $NT = 5.7$ cm, $TS = 7.5$ cm and $\angle NTS = 110^\circ$ and draw incircle and circumcircle of it.

4. In the figure 6.19, C is the centre of the circle.
seg QT is a diameter
 $CT = 13$, $CP = 5$, find the length of chord RS.

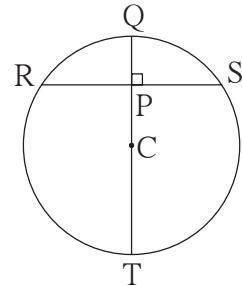


Fig. 6.19

5. In the figure 6.20, P is the centre of the circle. chord AB and chord CD intersect on the diameter at the point E
If $\angle AEP \cong \angle DEP$
then prove that AB = CD.

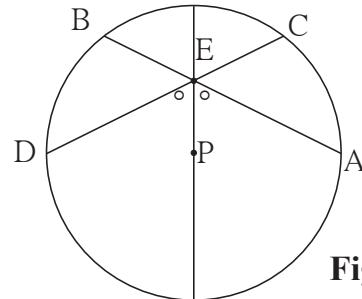


Fig. 6.20

6. In the figure 6.21, CD is a diameter of the circle with centre O. Diameter CD is perpendicular to chord AB at point E. Show that $\triangle ABC$ is an isosceles triangle.

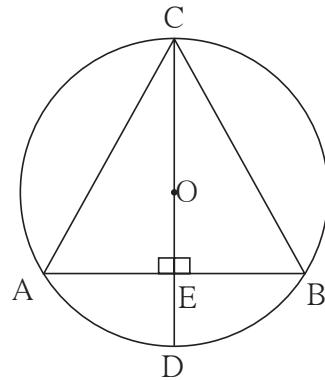


Fig. 6.21



ICT Tools or Links

Draw different circles with Geogebra software. Verify and experience the properties of chords. Draw circumcircle and incircle of different triangles. Using ‘Move Option’ experience how the incentre and circumcentre changes if the size of a triangle is changed.





Lets study

- Axis, Origin, Quadrant
 - Co-ordinates of a point in a plane.
 - To plot a point.
 - Line parallel to X-axis.
 - Line parallel to Y-axis.
 - Equation of a line.

Chintu and his friends were playing cricket on the ground in front of a big building, when a visitor arrived.

Visitor : Hey Chintu, Dattabhai lives here, doesn't he ?

Chintu : Yes, on the second floor. See that window ? Thats his flat.

Visitor : But there are five windows on the second floor. It could be any of them !

Chintu : His window is the third one from the left, on the second floor..



Chintu's description of the location of Dattabhau's flat is in fact, based on the most basic concept in Co-ordinate Geometry.

It did not suffice to give only the floor number to locate the house. Its serial number from the left or from the right also needed to be given. That is two numbers had to be given in a specific sequence. Two **ordinal numbers** namely, **second** from the ground and **third** from the left had to be used.

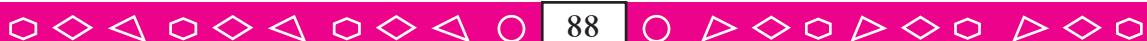


Let's Learn

Axes, origin, quadrants

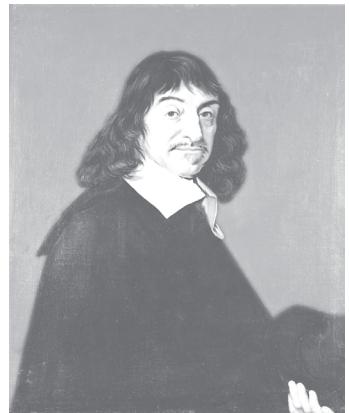
We could give the location of Dattabhau's house using two ordinal numbers. Similarly, the location of a point can be fully described using its distances from two mutually perpendicular lines.

To locate a point in a plane, a horizontal number line is drawn in the plane. This number line is called the X-axis.



Rene Descartes (1596-1650)

Rene Descartes, a French mathematician of the 17th Century, proposed the co-ordinate system to describe the position of a point in a plane accurately. It is called the Cartesian co-ordinate system. Obviously the word Cartesian is derived from his name. He brought about a revolution in the field of mathematics by establishing the relationship between Algebra and Geometry.



The Cartesian co-ordinate system is the foundation of Analytical Geometry. La Geometric was Descartes' first book on mathematics. In it, he used algebra for the study of geometry and proposed that a point in a plane can be represented by an ordered pair of real numbers. This ordered pair is the 'Cartesian Co-ordinates' of a point.

Co-ordinate geometry has been used in a variety of fields such as Physics, Engineering, Nautical Science, Seismology and Art. It plays an important role in the development of technology in Geogebra. We see the inter-relationship between Algebra and Geometry quite clearly in the software Geogebra; the very name being a combination of the words ‘Geometry’ and ‘Algebra’.

Another number line intersecting the X-axis at point marked O and perpendicular to the X-axis, is the Y-axis. Generally, the number O is represented by the same point on both the number lines. This point is called the origin and is shown by the letter O.

On the X-axis, positive numbers are shown on the right of O and negative numbers on the left.

On the Y-axis, positive numbers are shown above O and negative numbers below it.

The X and Y axes divide the plane into four parts, each of which is called a **Quadrant**. As shown in the figure, the quadrants are numbered in the anti-clockwise direction.

The points on the axes are not included in the quadrants.

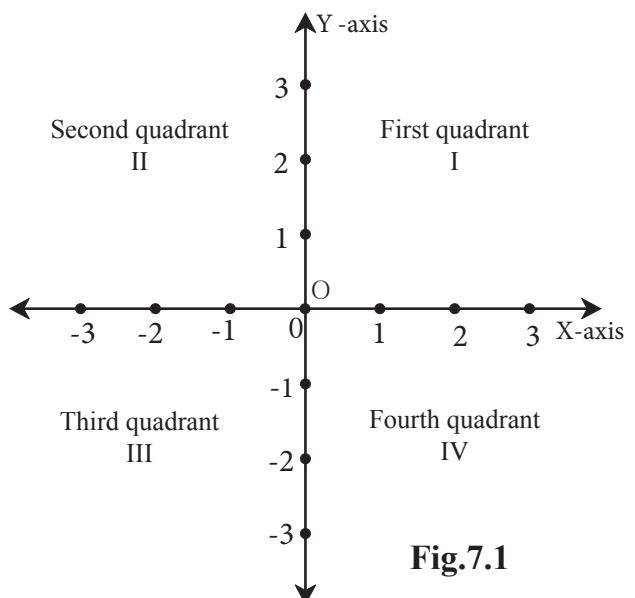


Fig. 7.1

The Co-ordinates of a point in a plane

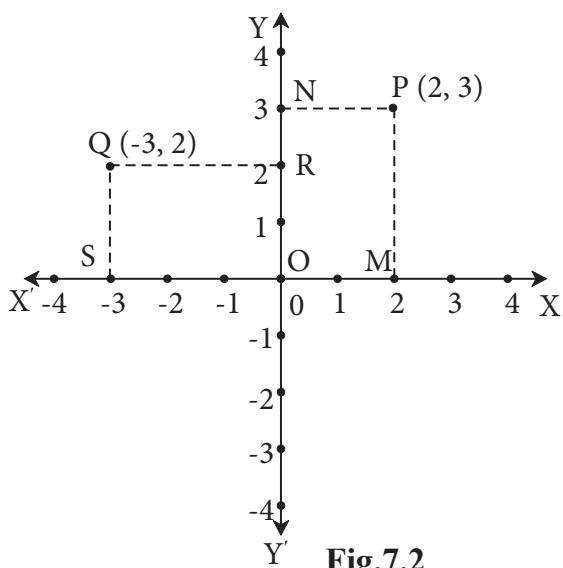


Fig.7.2

x co-ordinate first. According to this convention the order of co-ordinates of point P is decided as 2, 3. The position of the point P in brief, is described by the pair (2, 3)

The order of the numbers in the pair (2, 3) is important. Such a pair of numbers is called an ordered pair.

To describe the position of point Q, we draw seg QS \perp X-axis and seg QR \perp Y-axis. The co-ordinate of point Q on the X-axis is -3 and the co-ordinate on the Y-axis is 2 . Hence the co-ordinates of point Q are $(-3, 2)$.

Ex. Write the co-ordinates of points E, F, G, T in the figure alongside.

Solution :

- The co-ordinates of point E are $(2, 1)$
- The co-ordinates of point F are $(-3, 3)$
- The co-ordinates of point G are $(-4, -2)$.
- The co-ordinates of point T are $(3, -1)$

The point P is shown in the plane determined by the X-axis and the Y-axis. Its position can be determined by its distance from the two axes. To find these distances, we draw seg PM \perp X-axis and seg PN \perp Y-axis.

Co-ordinate of point M on X-axis is 2 and co-ordinate of point N on Y-axis.

Therefore x co-ordinate of point P is 2 and y co-ordinate of point P is 3..

The convention for describing the position of a point is to mention

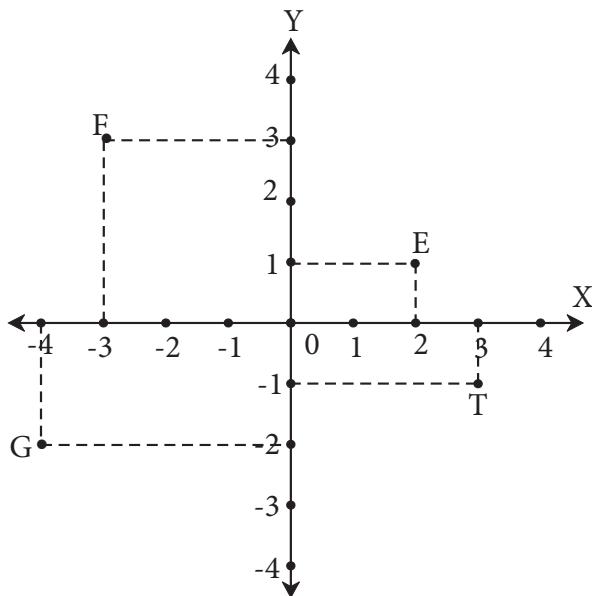


Fig.7.3





Let's learn.

Co-ordinates of points on the axes

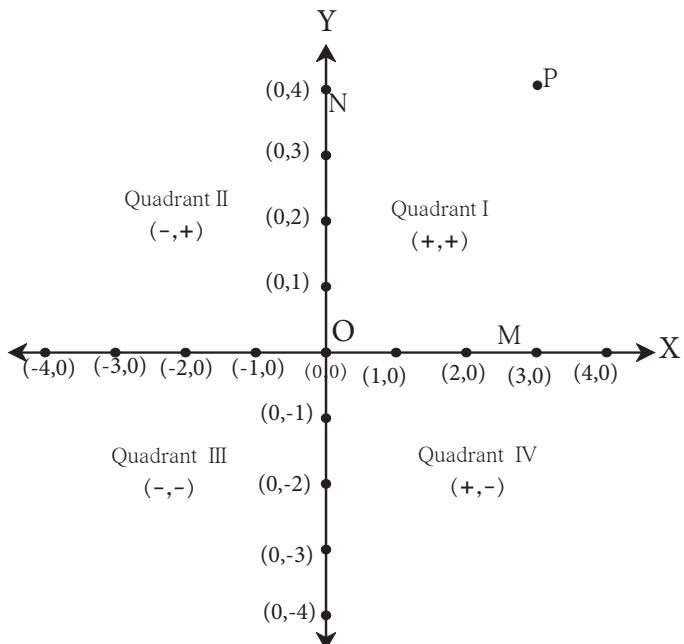


Fig.7.4

Now the origin ‘O’ is on X-axis as well as on Y-axis. Hence, its distance from X-axis and Y-axis is zero. Therefore, the co-ordinates of O are (0, 0).

One and only one pair of co-ordinates (ordered pair) is associated with every point in a plane.

The x co-ordinate of point M is its distance from the Y-axis. The distance of point M from the X-axis is zero. Hence, the y co-ordinate of M is 0.

Thus, the co-ordinates of point M on the X-axis are (3,0).

The y co-ordinate of point N on the Y-axis is 4 units from the X-axis because N is at a distance of 4. Its x co-ordinate is 0 because its distance from the Y-axis is zero.

Hence, the co-ordinates of point N on the Y-axis are $(0, 4)$.



Let's Remember

- The y co-ordinate of every point on the X-axis is zero.
 - The x co-ordinate of every point on the Y-axis is zero.
 - The coordinates of the origin are $(0, 0)$.

Ex. In which quadrant or on which axis are the points given below ?

$$A(5,7), \quad B(-6,4), \quad C(4,-7), \quad D(-8,-9), \quad P(-3,0), \quad Q(0,8)$$

Solution : The x co-ordinate of A (5, 7) is positive and its y co-ordinate is positive...

∴ point A is in the first quadrant.

The x co-ordinate of B (-6, 4) is negative and y co-ordinate is positive.

\therefore point B is in the second quadrant.

The x co-ordinate of C (4, -7) is positive and y co-ordinate is negative.

∴ point C is in the fourth quadrant.

The x co-ordinate of D $(-8, -7)$ is negative and y co-ordinate is negative.

\therefore point D is in the third quadrant.

The y co-ordinate of P(-3,0) is zero \therefore point P is on the X-axis.

The x co-ordinate of Q (0,8) is zero \therefore point Q is on the Y-axis.

Activity As shown in fig. 7.5, ask girls to sit in lines so as to form the X-axis and Y-axis.

- Ask some boys to sit at the positions marked by the coloured dots in the four quadrants.
 - Now, call the students turn by turn using the initial letter of each student's name. As his or her initial is called, the student stands and gives his or her own co-ordinates. For example Rajendra (2, 2) and Kirti (-1, 0)
 - Even as they have fun during this field activity, the students will learn how to state the position of a point in a plane.

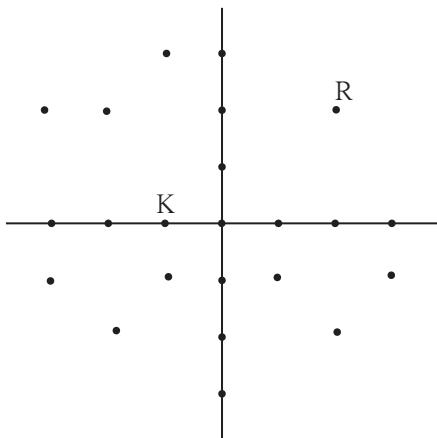


Fig. 7.5



Let's learn.

To plot the points of given co-ordinates

Suppose we have to plot the points

P (4,3) and Q (-2,2)

Steps for plotting the points

- (i) Draw X-axis and Y-axis on the plane.
Show the origin.

(ii) To find the point P (4,3), draw a line parallel to the Y-axis through the point on X axis which represents the number 4.
Through the point on Y-axis which represents the number 3 draw a line parallel to the X-axis .

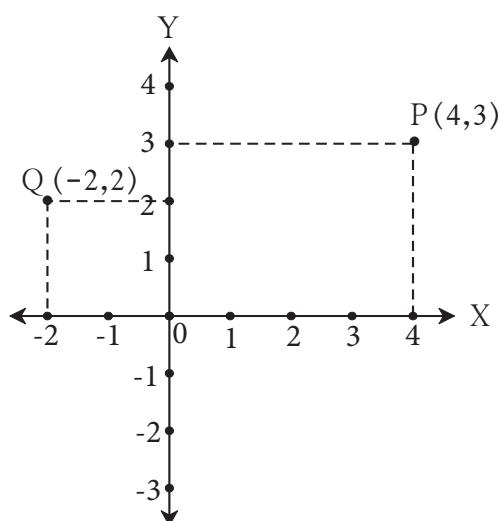


Fig. 7.6

- (iii) The point of intersection of these two lines parallel to the Y and X-axis respectively, is the point $P(4,3)$. In which quadrant does this point lie ?

(iv) In the same way, plot the point $Q(-2, 2)$. Is this point in the second quadrant ?
Using the same method, plot the points $R(-3, -4)$, $S(3, -1)$

Ex. In which quadrants or on which axis are the points given below ?

- | | | | |
|----------------|------------------|------------------|----------------------|
| (i) $(5, 3)$ | (ii) $(-2, 4)$ | (iii) $(2, -5)$ | (iv) $(0, 4)$ |
| (v) $(-3, 0)$ | (vi) $(-2, 2.5)$ | (vii) $(5, 3.5)$ | (viii) $(-3.5, 1.5)$ |
| (ix) $(0, -4)$ | (x) $(2, -4)$ | | |

Solution :

	co-ordinates	Quadrant / axis
(i)	(5,3)	Quadrant I
(ii)	(-2,4)	Quadrant II
(iii)	(2,-5)	Quadrant IV
(iv)	(0,4)	Y-axis
(v)	(-3,0)	X-axis

	co-ordinates	Quadrant / axis
(vi)	(-2, -2.5)	Quadrant III
(vii)	(5,3.5)	Quadrant I
(viii)	(-3.5,1.5)	Quadrant II
(ix)	(0, -4)	Y-axis
(x)	(2,-4)	Quadrant IV

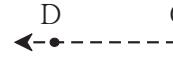
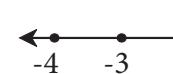
Practice set 7.1

- State in which quadrant or on which axis do the following points lie.
 - A(-3, 2), B(-5, -2), K(3.5, 1.5), D(2, 10),
 - E(37, 35), F(15, -18), G(3, -7), H(0, -5),
 - M(12, 0), N(0, 9), P(0, 2.5), Q(-7, -3)
 - In which quadrant are the following points ?
 - whose both co-ordinates are positive.
 - whose both co-ordinates are negative.
 - whose x co-ordinate is positive, and the y co-ordinate is negative.
 - whose x co-ordinate is negative and y co-ordinate is positive.
 - Draw the co-ordinate system on a plane and plot the following points.
L(-2, 4), M(5, 6), N(-3, -4), P(2, -3), Q(6, -5), S(7, 0), T(0, -5)



Let's learn.

Lines parallel to the X-axis

- On a graph paper, plot the following points
A (5, 4), B (2, 4), C (-2, 4), D (-4, 4), E (0, 4), F (3, 4)
 - Observe the co-ordinates of the given points.
 - Did you notice that the y co-ordinates of all the points are equal ?
 - All the points are collinear.
 - To which axis is this line parallel ?
 - The y co-ordinate of every point on the line DA is 4. It is constant. Therefore the line DA is described by the equation $y = 4$. If the y co-ordinate of any point is 4, will be on the line DA.


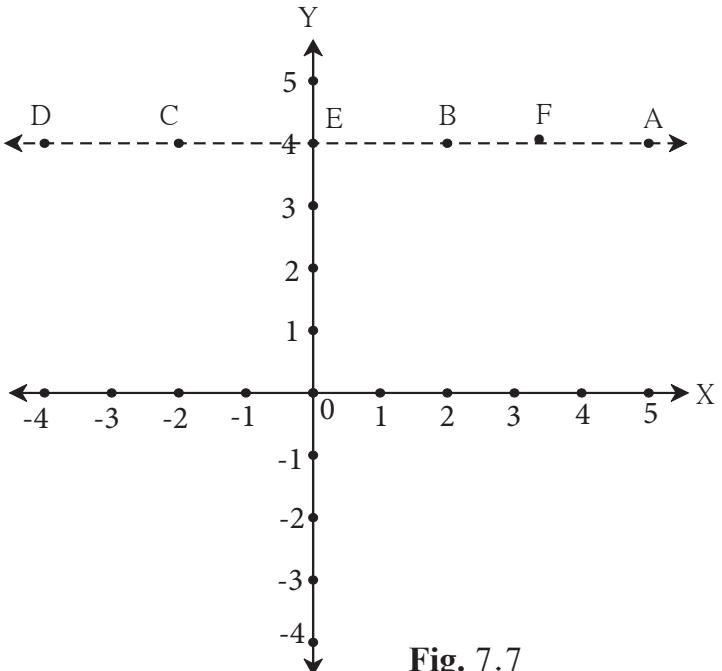


Fig. 7.7



Let's discuss.

- Can we draw a line parallel to the X-axis at a distance of 6 units from it and below the X-axis ?
 - Will all of the points $(-3, -6)$, $(10, -6)$, $(\frac{1}{2}, -6)$ be on that line ?
 - What would be the equation of this line ?



Remember this !

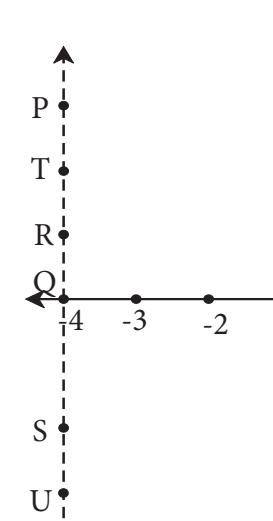
If $b > 0$, and we draw the line $y = b$ through the point $(0, b)$, it will be above the X-axis and parallel to it. If $b < 0$, then the line $y = b$ will be below the X-axis and parallel to it.

The equation of a line parallel to the X-axis is in the form $y = b$.



Let's learn.

Lines parallel to the Y-axis

- On a graph paper, plot the following points
 $P(-4, 3)$, $Q(-4, 0)$, $R(-4, 1)$, $S(-4, -2)$, $T(-4, 2)$, $U(-4, -3)$
 - Observe the co-ordinates of the points.
 - Did you notice that the x co-ordinate of all the points are the same ?
 - Are all the points collinear ?
 - To which axis is this line parallel ?
 - The x co-ordinate of every point on the line PS is -4 . It is constant. Therefore, the line PS can be described by the equation $x = -4$. Every point whose x co-ordinate is -4 lies on the line PS. The equation of the line parallel to the Y-axis at a distance of 4 units and to the left of Y-axis is $y = -4$.

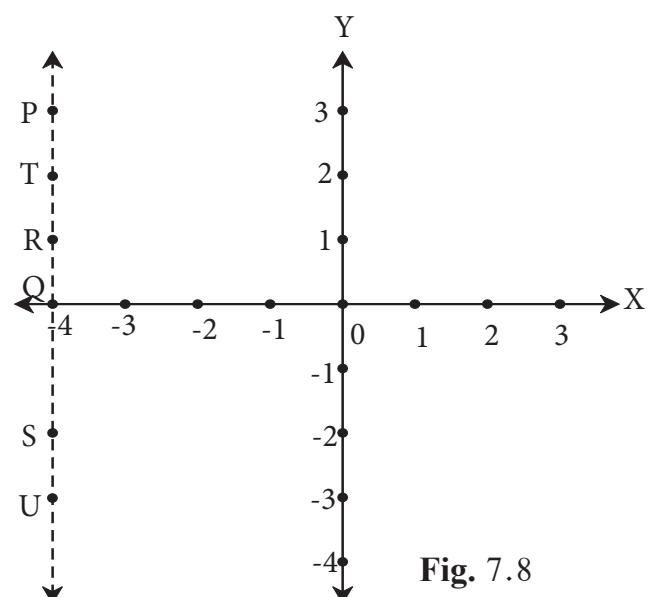


Fig. 7.8



Let's discuss

- Can we draw a line parallel to the Y-axis at a distance of 2 units from it and to its right ?
 - Will all of the points $(2,10)$, $(2,8)$, $(2, -\frac{1}{2})$ be on that line ?
 - What would be the equation of this line ?



Remember this !

If we draw the line $x = a$ parallel to the Y-axis passing through the point $(a, 0)$ and if $a > 0$ then the line will be to the right of the Y-axis. If $a < 0$, then the line will be to the left of the Y-axis.

The equation of a line parallel to the Y-axis is in the form $x = a$.



Remember this !

- (1) The y co-ordinate of every point on the X-axis is zero. Conversely, every point whose y co-ordinate is zero is on the X-axis. Therefore, the equation of the X axis is $y = 0$.
- (2) The x co-ordinate of every point on the Y-axis is zero. Conversely, every point whose x co-ordinate is zero is on the Y-axis. Therefore, the equation of the Y-axis is $x = 0$.



Let's learn.

Graph of a linear equations

Ex. Draw the graphs of the equations
 $x = 2$ and $y = -3$.

Solution : (i) On a graph paper draw the X-axis and the Y-axis.

(ii) Since it is given that $x = 2$, draw a line on the right of the Y-axis at a distance of 2 units from it and parallel to it.

(iii) Since it is given that $y = -3$, draw a line below the X-axis at a distance of 3 units from it and parallel to it.

(iv) These lines, parallel to the two axes, are the graphs of the given equations.

(v) Write the co-ordinates of the point P, the point of intersection of these two lines.

(vi) Verify that the co-ordinates of the point P are $(2, -3)$

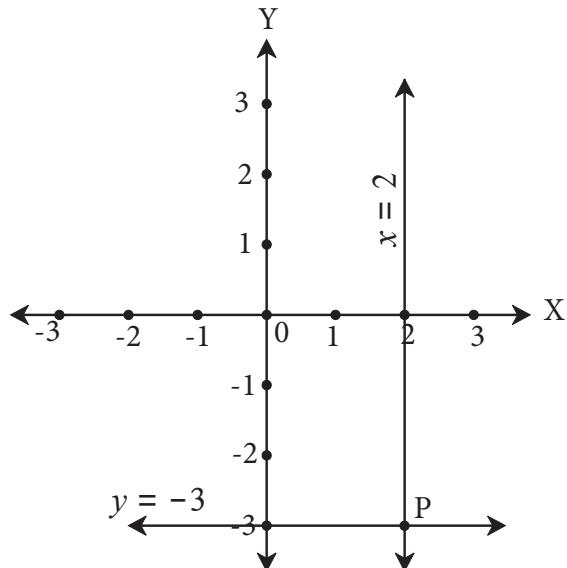


Fig. 7.9

The graph of a linear equation in the general form.

Activity : On a graph paper, plot the points $(0,1)$ $(1,3)$ $(2,5)$. Are they collinear ? If so, draw the line that passes through them.

- Through which quadrants does this line pass ?
- Write the co-ordinates of the point at which it intersects the Y-axis.
- Show any point in the third quadrant which lies on this line. Write the co-ordinates of the point.

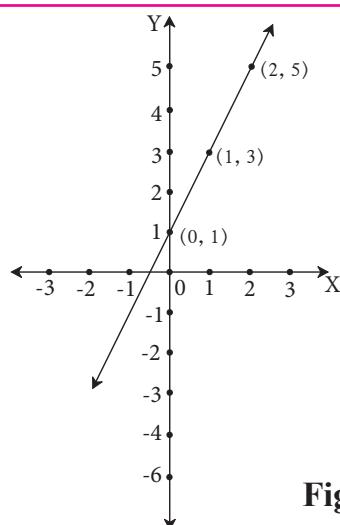


Fig. 7.10



Ex. $2x - y + 1 = 0$ is a linear equation in two variables in general form. Let us draw the graph of this equation.

Solution : $2x - y + 1 = 0$ means $y = 2x + 1$

Let us assume some values of x and find the corresponding values of y .

For example, If $x = 0$, then substituting this value of x in the equation we get $y = 1$.

Similarly, let us find the values of y when $0, 1, 2, \frac{1}{2}, -2$ are some values of x and write these values in the table below in the form of ordered pairs.

x	0	1	2	$\frac{1}{2}$	-2
y	1	3	5	2	-3
(x, y)	(0,1)	(1,3)	(2,5)	$(\frac{1}{2}, 2)$	(-2, -3)

Now, let us plot these points. Let us verify that these points are collinear. Let us draw that line. The line is the graph of the equation $2x - y + 1 = 0$.



ICT Tools or Links

Use the Software Geogebra to draw the X and Y-axis. Plot several points.

Find and study the co-ordinates of the points in ‘Algebraic view’. Read the equations of lines that are parallel to the axes. Use the ‘move’ option to vary the positions of the lines.

What are the equations of the X-axis and the Y-axis ?

Practice set 7.2

1. On a graph paper plot the points A(3,0), B(3,3), C(0,3). Join A, B and B, C. What is the figure formed ?
 2. Write the equation of the line parallel to the Y-axis at a distance of 7 units from it to its left.
 3. Write the equation of the line parallel to the X-axis at a distance of 5 units from it and below the X-axis.
 4. The point Q(-3, -2) lies on a line parallel to the Y-axis. Write the equation of the line and draw its graph.

5. X-axis and line $x = -4$ are parallel lines. What is the distance between them?

6. Which of the equations given below have graphs parallel to the X-axis, and which ones have graphs parallel to the Y-axis ?

(i) $x = 3$ (ii) $y - 2 = 0$ (iii) $x + 6 = 0$ (iv) $y = -5$

7. On a graph paper, plot the points A(2, 3), B(6, -1) and C(0, 5). If those points are collinear then draw the line which includes them. Write the co-ordinates of the points at which the line intersects the X-axis and the Y-axis.

8. Draw the graphs of the following equations on the same system of co-ordinates. Write the co-ordinates of their points of intersection.

$$x + 4 = 0, \quad y - 1 = 0, \quad 2x + 3 = 0, \quad 3y - 15 = 0$$

9. Draw the graphs of the equations given below

Problem set 7

1. Choose the correct alternative answer for the following questions.

(i) What is the form of co-ordinates of a point on the X-axis ?
(A) (b, b) (B) $(0, b)$ (C) $(a, 0)$ (D) (a, a)

(ii) Any point on the line $y = x$ is of the form
(A) (a, a) (B) $(0, a)$ (C) $(a, 0)$ (D) $(a, -a)$

(iii) What is the equation of the X-axis ?
(A) $x = 0$ (B) $y = 0$ (C) $x + y = 0$ (D) $x = y$

(iv) In which quadrant does the point $(-4, -3)$ lie ?
(A) First (B) Second (C) Third (D) Fourth

(v) What is the nature of the line which includes the points $(-5,5), (6,5), (-3,5), (0,5)$?
(A) Passes through the origin,, (B) Parallel to Y-axis.
(C) Parallel to X-axis (D) None of these

(vi) Which of the points P $(-1,1)$, Q $(3,-4)$, R $(1,-1)$, S $(-2,-3)$, T $(-4,4)$ lie in the fourth quadrant?
(A) P and T (B) Q and R (C) only S (D) P and R

2. Some points are shown in the figure 7.11

With the help of it answer the following questions :

- (i) Write the co-ordinates of the points Q and R.
 - (ii) Write the co-ordinates of the points T and M.
 - (iii) Which point lies in the third quadrant ?
 - (iv) Which are the points whose x and y co-ordinates are equal ?

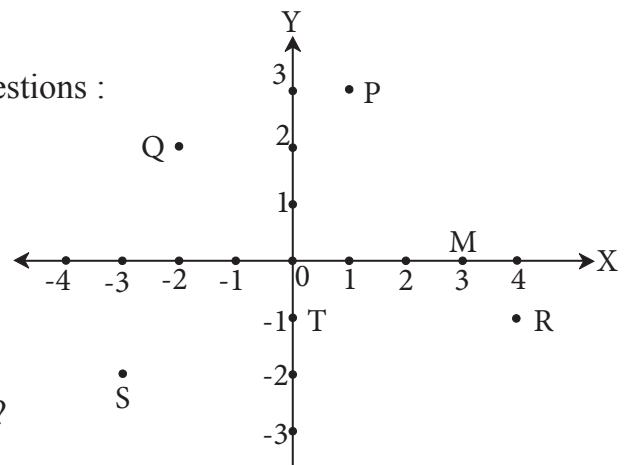


Fig.7.11

3. Without plotting the points on a graph, state in which quadrant or on which axis do the following point lie.

- (i) $(5, -3)$ (ii) $(-7, -12)$ (iii) $(-23, 4)$
 (iv) $(-9, 5)$ (v) $(0, -3)$ (vi) $(-6, 0)$

4. Plot the following points on the one and the same co-ordinate system.

$$A(1, 3), B(-3, -1), C(1, -4), \\ D(-2, 3), E(0, -8), F(1, 0)$$

5. In the graph alongside, line LM is parallel to the Y-axis. (Fig. 7.12)

- (i) What is the distance of line LM from the Y-axis ?
 - (ii) Write the co-ordinates of the points P, Q and R.
 - (iii) What is the difference between the x co-ordinates of the points L and M?

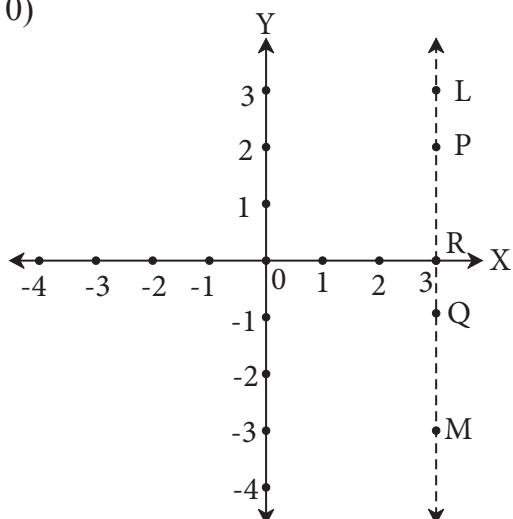
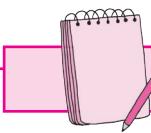


Fig.7.12

6. How many lines are there which are parallel to X-axis and having a distance 5 units?
7*. If ‘ a ’ is a real number, what is the distance between the Y-axis and the line $x = a$?

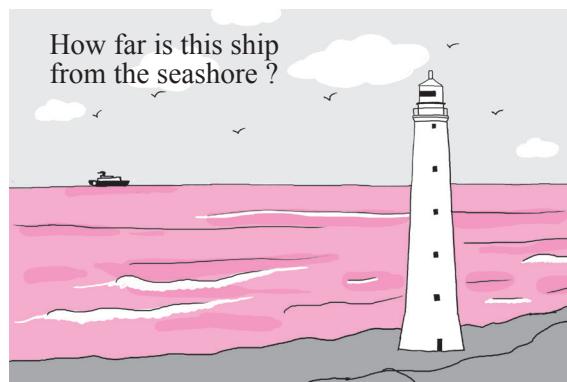
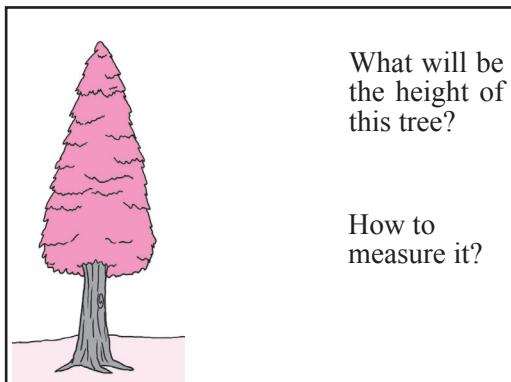




Let's study.

- Introduction of Trigonometry
 - Relations among Trigonometric Ratios
 - Trigonometric Ratios
 - Trigonometric Ratios of Particular Angles

Introduction to Trigonometry



We can measure distances by using a rope or by walking on ground, but how to measure the distance between a ship and a light house? How to measure the height of a tall tree?

Observe the above pictures. Questions in the pictures are related to mathematics. Trigonometry, a branch of mathematics, is useful to find answers to such questions. Trigonometry is used in different branches of Engineering, Astronomy, Navigation etc.

The word Trigonometry is derived from three Greek words ‘Tri’ means three, ‘gona’ means sides and ‘metron’ means measurements.



Let's recall.

We have studied triangle. The subject trigonometry starts with right angled triangle, theorem of Pythagoras and similar triangles, so we will recall these topics.

- In ΔABC , $\angle B$ is a right angle and side AC opposite to $\angle B$, is hypotenuse. Side opposite to $\angle A$ is BC and side opposite to $\angle C$ is AB.
Using Pythagoras' theorem, we can write the following statement for this triangle.

$$(AB)^2 + (BC)^2 = (AC)^2$$

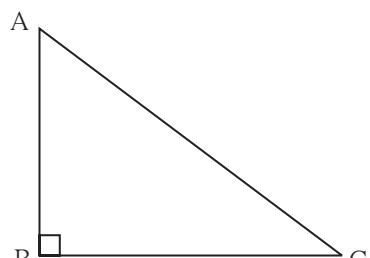


Fig. 8.1

- If $\Delta ABC \sim \Delta PQR$ then their corresponding sides are in the same proportions.

$$\text{So } \frac{AB}{PO} = \frac{BC}{QR} = \frac{AC}{PR}$$

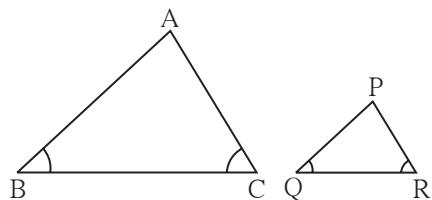


Fig. 8.2

Let us see how to find the height of a tall tree using properties of similar triangles.

Activity : This experiment can be conducted on a clear sunny day.

Look at the figure given alongside.

Height of the tree is QR, height of the stick is BC.

Thrust a stick in the ground as shown in the figure. Measure its height and length of its shadow. Also measure the length of the shadow of the tree. Rays of sunlight are parallel. So $\triangle PQR$ and $\triangle ABC$ are equiangular, means similar triangles. Sides of similar triangles are proportional.

So we get $\frac{QR}{PR} = \frac{BC}{AC}$.

Therefore, we get an equation,

$$\text{height of the tree} = QR = \frac{BC}{AC} \times PR$$

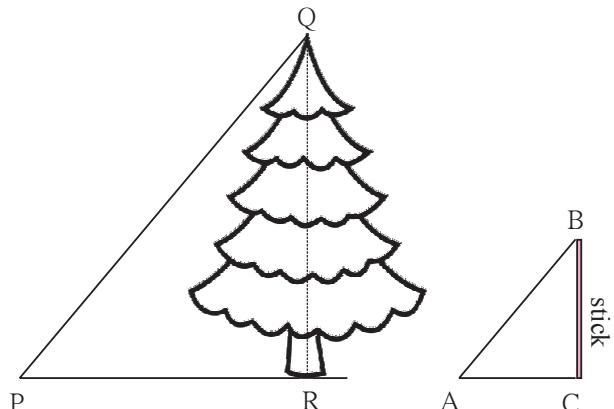


Fig.8.3

We know the values of PR, BC and AC. Substituting these values in this equation, we get length of QR, means height of the tree.



Use your brain power !

It is convenient to do this experiment between 11:30 am and 1:30 pm instead of doing it in the morning at 8' O clock. Can you tell why ?

Activity : You can conduct this activity and find the height of a tall tree in your surrounding. If there is no tree in the premises then find the height of a pole.

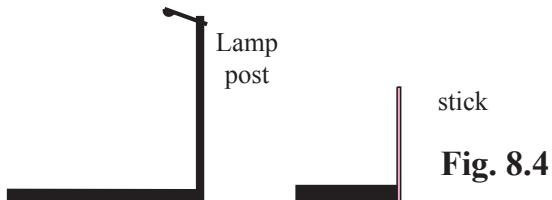


Fig. 8.4



Terms related to right angled triangle

In right angled ΔABC , $\angle B = 90^\circ$, $\angle A$ and $\angle C$ are acute angles.

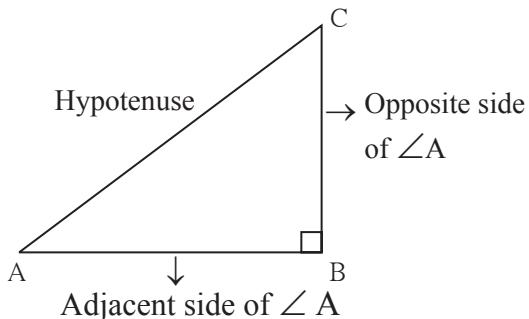


Fig. 8.5

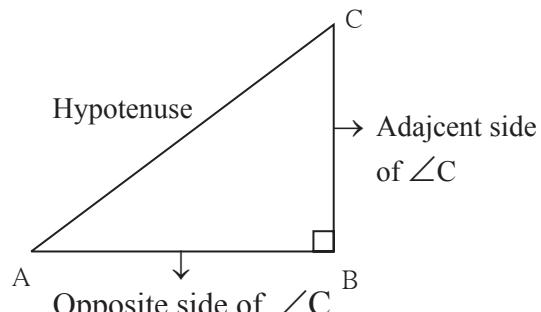


Fig. 8.6

Ex. In the figure 8.7, ΔPQR is a right angled triangle. Write-

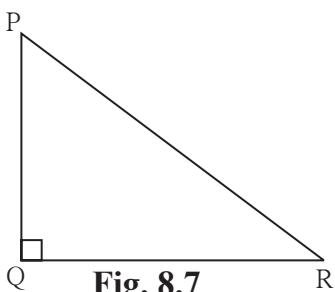


Fig. 8.7

side opposite to $\angle P$ =.....

side opposite to $\angle R$ =.....

side adjacent to $\angle P$ =

side adjacent to $\angle R$ =

Trigonometric ratios

In the adjacent Fig.8.8 some right angled triangles are shown. $\angle B$ is their common angle. So all right angled triangles are similar.

$$\Delta PQB \sim \Delta ACB$$

$$\therefore \frac{PB}{AB} = \frac{PQ}{AC} = \frac{BQ}{BC}$$

$$\therefore \frac{PQ}{AC} = \frac{PB}{AB} \quad \therefore \frac{PQ}{PB} = \frac{AC}{AB} \dots \text{alternando}$$

$$\frac{QB}{BC} = \frac{PB}{AB} \quad \therefore \frac{QB}{PB} = \frac{BC}{AB} \dots \text{alternando}$$

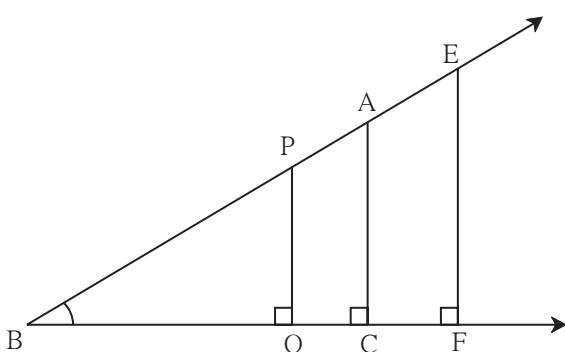


Fig. 8.8

The figures of triangles in 8.9 and 8.10 are of the triangles separated from the figure 8.8

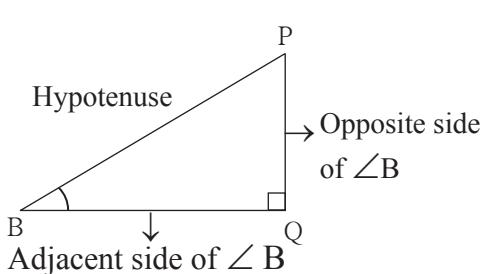


Fig.8.9

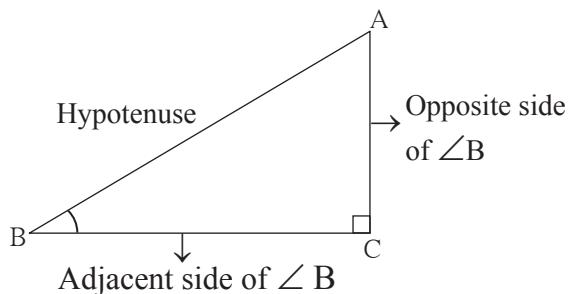


Fig.8.10

- (i) In Δ PQB,

$$\frac{PQ}{PB} = \frac{\text{Opposite side of } \angle B}{\text{Hypotenuse}}$$

The ratios $\frac{PQ}{PB}$ and $\frac{AC}{AB}$ are equal.

$$\therefore \frac{PQ}{PB} = \frac{AC}{AB} = \frac{\text{Opposite side of } \angle B}{\text{Hypotenuse}}$$

In $\triangle ACB$,

$$\frac{AC}{AB} = \frac{\text{Opposite side of } \angle B}{\text{Hypotenuse}}$$

This ratio is called the ‘sine’ ratio of $\angle B$, and is written in brief as ‘ $\sin B$ ’.

- (ii) In $\triangle PQB$ and $\triangle ACB$,

$$\frac{BQ}{PB} = \frac{\text{Adjacent side of } \angle B}{\text{Hypotenuse}} \quad \text{and} \quad \frac{BC}{AB} = \frac{\text{Adjacent side of } \angle B}{\text{Hypotenuse}}$$

$$\therefore \frac{BQ}{PB} = \frac{BC}{AB} = \frac{\text{Adjacent side of } \angle B}{\text{Hypotenuse}}$$

This ratio is called as the ‘cosine’ ratio of $\angle B$, and written in brief as ‘ $\cos B$ ’.

$$(iii) \frac{PQ}{BO} = \frac{AC}{BC} = \frac{\text{Opposite side of } \angle B}{\text{Adjacent side of } \angle B}$$

This ratio is called as the tangent ratio of $\angle B$, and written in brief as $\tan B$.

Ex. i

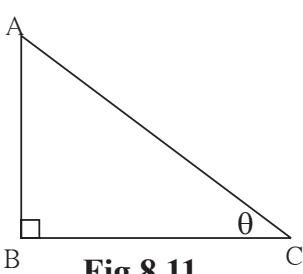


Fig.8.11

Sometimes we write measures of acute angles of a right angled triangle by using Greek letters θ (Theta), α (Alpha), β (Beta) etc.

In the adjacent figure of ΔABC , measure of acute angle C is denoted by the letter θ . So we can write the ratios $\sin C$, $\cos C$, $\tan C$ as $\sin \theta$, $\cos \theta$, $\tan \theta$ respectively.

$$\sin C = \sin \theta = \frac{AB}{AC}, \quad \cos C = \cos \theta = \frac{BC}{AC}, \quad \tan C = \tan \theta = \frac{AB}{BC}$$



Remember this !

- $\sin \text{ ratio} = \frac{\text{opposite side}}{\text{hypotenuse}}$
 - $\cos \text{ ratio} = \frac{\text{adjacent side}}{\text{hypotenuse}}$
 - $\tan \text{ ratio} = \frac{\text{opposite side}}{\text{adjcent side}}$
 - $\sin \theta = \frac{\text{opposite side of } \angle \theta}{\text{hypotenuse}}$
 - $\cos \theta = \frac{\text{adjacent side of } \angle \theta}{\text{hypotenuse}}$
 - $\tan \theta = \frac{\text{opposite side of } \angle \theta}{\text{opposite side of } \angle \theta}$

Practice set 8.1

1.

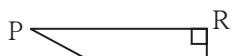


Fig. 8.12

In the Fig.8.12, $\angle R$ is the right angle of $\triangle POR$. Write the following ratios.

- (i) $\sin P$ (ii) $\cos Q$ (iii) $\tan P$ (iv) $\tan Q$

2.

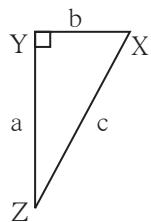


Fig. 8.13

In the right angled ΔXYZ , $\angle XYZ = 90^\circ$ and a, b, c are the lengths of the sides as shown in the figure. Write the following ratios,

- (i) $\sin X$ (ii) $\tan Z$ (iii) $\cos X$ (iv) $\tan X$.

3.

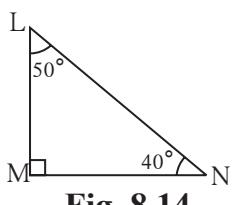


Fig. 8.14

In right angled $\triangle LMN$, $\angle LMN = 90^\circ$
 $\angle L = 50^\circ$ and $\angle N = 40^\circ$,
write the following ratios.

- (i) $\sin 50^\circ$ (ii) $\cos 50^\circ$
 (iii) $\tan 40^\circ$ (iv) $\cos 40^\circ$

4

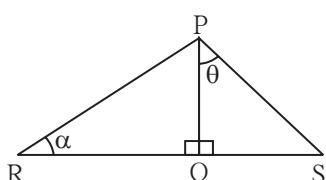


Fig. 8.15

In the figure 8.15., $\angle PQR = 90^\circ$,
 $\angle PQS = 90^\circ$, $\angle PRQ = \alpha$ and $\angle QPS = \theta$
Write the following trigonometric ratios

- (i) $\sin \alpha, \cos \alpha, \tan \alpha$
 (ii) $\sin \theta, \cos \theta, \tan \theta$



Lets Learn

Relation among trigonometric ratios

In the Fig.8.16

$\triangle PMN$ is a right angled triangle.

$\angle M = 90^\circ$, $\angle P$ and $\angle N$ are complimentary angles.

\therefore If $\angle N = \theta$ then $\angle P = 90 - \theta$

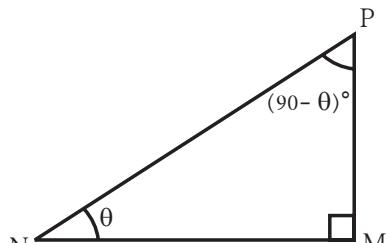


Fig.8.16

$$\sin \theta = \frac{PM}{PN} \dots\dots\dots(1)$$

$$\cos \theta = \frac{NM}{PN} \dots\dots(2)$$

$$\tan \theta = \frac{PM}{NM} \dots\dots(3)$$

$$\sin(90 - \theta) = \frac{NM}{PN} \quad \dots\dots(4)$$

$$\cos(90 - \theta) = \frac{PM}{PN} \quad \dots\dots(5)$$

$$\tan(90 - \theta) = \frac{NM}{PM} \quad \dots\dots(6)$$

$\therefore \sin \theta = \cos (90 - \theta)$ from (1) and (5)

$$\cos \theta = \sin (90 - \theta) \dots\dots \text{from (2) and (4)}$$

Also note that $\tan \theta \times \tan (90 - \theta) = \frac{PM}{NM} \times \frac{NM}{PM}$ from (3) and (6)

$$\therefore \tan \theta \times \tan (90^\circ - \theta) = 1$$

$$\text{Similarly, } \frac{\sin \theta}{\cos \theta} = \frac{\frac{PM}{PN}}{\frac{NM}{PN}} = \frac{PM}{PN} \times \frac{PN}{NM} = \frac{PM}{NM} = \tan \theta$$



Remember this !

$$\cos(90 - \theta) = \sin \theta, \quad \sin(90 - \theta) = \cos \theta$$

$$\frac{\sin \theta}{\cos \theta} = \tan \theta, \quad \tan \theta \times \tan (90^\circ - \theta) = 1$$

* For more information

$$\frac{1}{\sin \theta} = \operatorname{cosec} \theta, \quad \frac{1}{\cos \theta} = \sec \theta, \quad \frac{1}{\tan \theta} = \cot \theta$$

It means cosec θ , sec θ and cot θ are inverse ratios of sin θ , cos θ and tan θ respectively.

- $\sec \theta = \operatorname{cosec} (90 - \theta)$
 - $\operatorname{cosec} \theta = \sec (90 - \theta)$
 - $\tan \theta = \cot (90 - \theta)$
 - $\cot \theta = \tan (90 - \theta)$



Lets Recall

Theorem of 30° - 60° - 90° triangle :

We know that if the measures of angles of a triangle are $30^\circ, 60^\circ, 90^\circ$ then side opposite to 30° angle is half of the hypotenuse and side opposite to 60° angle is $\frac{\sqrt{3}}{2}$ of hypotenuse.

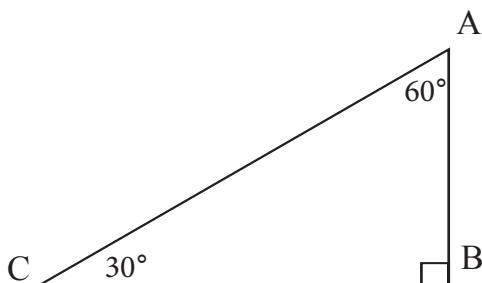


Fig. 8.17

In the Fig. 8.17, ΔABC is a right angled triangle. $\angle C = 30^\circ$, $\angle A = 60^\circ$, $\angle B = 90^\circ$.

$$\therefore AB = \frac{1}{2} AC \text{ and } BC = \frac{\sqrt{3}}{2} AC$$



Let's learn.

Trigonometric ratios of 30° and 60° angles

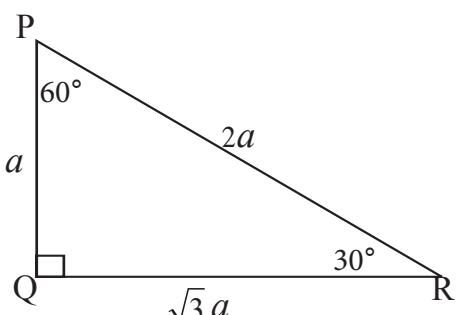


Fig. 8.18

In right angled ΔPQR if $\angle R = 30^\circ$,
 $\angle P = 60^\circ$, $\angle Q = 90^\circ$ and $PQ = a$

$$\text{then } PQ = \frac{1}{2} PR$$

$$a = \frac{1}{2} \text{ PR}$$

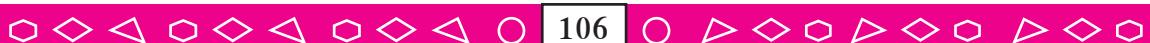
$$\therefore \text{PR} = 2a$$

$$QR = \frac{\sqrt{3}}{2} PR$$

$$QR = \frac{\sqrt{3}}{2} \times 2a$$

$$QR = \sqrt{3} a$$

\therefore If $PQ = a$, then $PR = 2a$ and $QR = \sqrt{3}a$



(I) Trigonometric ratios of the 30° angle

$$\sin 30^\circ = \frac{PQ}{PR} = \frac{a}{2a} = \frac{1}{2}$$

$$\cos 30^\circ = \frac{QR}{PR} = \frac{\sqrt{3}a}{2a} = \frac{\sqrt{3}}{2}$$

$$\tan 30^\circ = \frac{PQ}{QR} = \frac{a}{\sqrt{3}a} = \frac{1}{\sqrt{3}}$$

(II) Trigonometric ratios of 60° angle

$$\sin 60^\circ = \frac{QR}{PR} = \frac{\sqrt{3}a}{2a} = \frac{\sqrt{3}}{2}$$

$$\cos 60^\circ = \frac{PQ}{PR} = \frac{a}{2a} = \frac{1}{2}$$

$$\tan 60^\circ = \frac{QR}{PQ} = \frac{\sqrt{3}a}{a} = \sqrt{3}$$

In right angled ΔPQR , $\angle Q = 90^\circ$. Therefore $\angle P$ and $\angle R$ are complimentary angles of each other. Verify the relation between sine and cosine ratios of complimentary angles here also.

$$\sin \theta = \cos (90 - \theta)$$

$$\sin 30^\circ = \cos (90^\circ - 30^\circ) = \cos 60^\circ$$

$$\sin 30^\circ = \cos 60^\circ$$

$$\cos \theta = \sin (90 - \theta)$$

$$\cos 30^\circ = \sin (90^\circ - 30^\circ) = \sin 60^\circ$$

$$\cos 30^\circ = \sin 60^\circ$$



Remember this

$$\sin 30^\circ = \frac{1}{2}$$

$$\cos 30^\circ = \frac{\sqrt{3}}{2}$$

$$\tan 30^\circ = \frac{1}{\sqrt{3}}$$

$$\sin 60^\circ = \frac{\sqrt{3}}{2}$$

$$\cos 60^\circ = \frac{1}{2}$$

$$\tan 60^\circ = \sqrt{3}$$

(III) Trigonometric ratios of the 45° angle

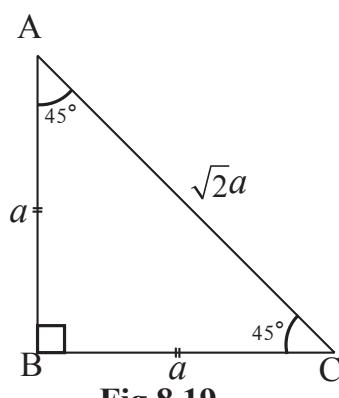


Fig.8.19

In right angled ΔABC , $\angle B = 90^\circ$, $\angle A = 45^\circ$, $\angle C = 45^\circ$. This is an isosceles triangle.

Suppose $AB = a$ then $BC = a$.

Using Pythagoras' theorem,

let us find the length

$$^2 = AB^2 +$$

$$= a^2 +$$

$$AC^2 = 2a^2$$

In the Fig. 8.19, $\angle C = 45^\circ$

$$\sin 45^\circ = \frac{AB}{AC} = \frac{a}{\sqrt{2}a} = \frac{1}{\sqrt{2}}$$

$$\cos 45^\circ = \frac{BC}{AC} = \frac{a}{\sqrt{2}a} = \frac{1}{\sqrt{2}}$$

$$\tan 45^\circ = \frac{AB}{BC} = \frac{a}{a} = 1$$



Remember this !

$$\sin 45^\circ = \frac{1}{\sqrt{2}},$$

$$\cos 45^\circ = \frac{1}{\sqrt{2}},$$

$$\tan 45^\circ = 1$$

(IV) Trigonometric ratios of the angle 0° and 90°

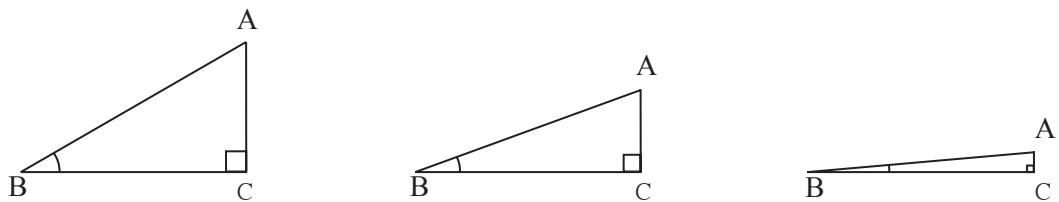


Fig.8.20

In the right angled $\triangle ACB$, $\angle C = 90^\circ$ and $\angle B = 30^\circ$. We know that $\sin 30^\circ = \frac{AC}{AB}$. Keeping the length of side AB constant, if the measure of $\angle B$ goes on decreasing the length of AC, which is opposite to $\angle B$ also goes on decreasing. So as the measure of $\angle B$ decreases, then value of $\sin \theta$ also decreases.

∴ when measure of $\angle B$ becomes 0° , then length of AC becomes 0.

$$\therefore \sin 0^\circ = \frac{AC}{AB} = \frac{0}{AB} = 0 \quad \therefore \sin 0^\circ = 0$$

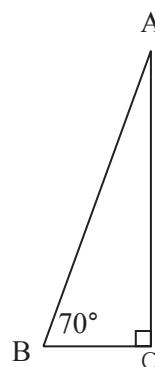
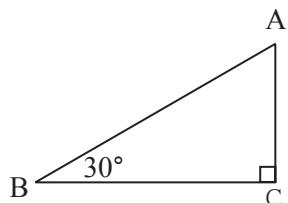


Fig.8.21

Now look at the Fig. 8.21. In this right angled triangle, as the measure of $\angle B$ increases the length of AC also increases. When measure of $\angle B$ becomes 90° , the length of AC become equal to AB

$$\therefore \sin 90^\circ = \frac{AC}{AB} \quad \therefore \sin 90^\circ = 1$$

We know the relations between trigonometric ratios of complimentary angles.

$$\sin \theta = \cos (90 - \theta) \quad \text{and} \quad \cos \theta = \sin (90 - \theta)$$

$$\therefore \cos 0^\circ = \sin (90 - 0)^\circ = \sin 90^\circ = 1$$

$$\text{and } \cos 90^\circ = \sin (90 - 90)^\circ = \sin 0^\circ = 0$$



Remember this !

$$\sin 0^\circ = 0,$$

$$\sin 90^\circ = 1,$$

$$\cos 0^\circ = 1,$$

$$\cos 90^\circ = 0$$

We know that,

$$\tan \theta = \frac{\sin \theta}{\cos \theta} \quad \therefore \tan 0 = \frac{\sin 0}{\cos 0} = \frac{0}{1} = 0$$

$$\text{But } \tan 90^\circ = \frac{\sin 90^\circ}{\cos 90^\circ} = \frac{1}{0}$$

But we can not do the division of 1 by 0. Note that θ is an acute angle. As it increases and reaches the value of 90° , $\tan \theta$ also increases indefinitely. Hence we can not find the definite value of $\tan 90$.



Remember this !

Trigonometric ratios of particular ratios.

Measures of angles		0°	30°	45°	60°	90°
Ratios						
sin		0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos		1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan		0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Undefined

Solved Examples :

Ex. (1) Find the value of $2\tan 45^\circ + \cos 30^\circ - \sin 60^\circ$

Solution : $2\tan 45^\circ + \cos 30^\circ - \sin 60^\circ$

$$\begin{aligned}
 &= 2 \times 1 + \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} \\
 &= 2 + 0 \\
 &= 2
 \end{aligned}$$

Ex. (2) Find the value of $\frac{\cos 56^\circ}{\sin 34^\circ}$

Solution : $56^\circ + 34^\circ = 90^\circ$ means 56 and 34 are the measures of complimentary angles.

$$\sin \theta = \cos (90^\circ - \theta)$$

$$\therefore \sin 34^\circ = \cos (90^\circ - 34^\circ) = \cos 56^\circ$$

$$\therefore \frac{\cos 56^\circ}{\sin 34^\circ} = \frac{\cos 56^\circ}{\cos 56^\circ} = 1$$

Ex. 3 In right angled $\triangle ACB$, If $\angle C = 90^\circ$, $AC = 3$, $BC = 4$.

Find the ratios $\sin A$, $\sin B$, $\cos A$, $\tan B$

Solution : In right angled $\triangle ACB$, using Pythagoras' theorem,

$$AB^2 = AC^2 + BC^2$$

$$= 3^2 + 4^2 = 5^2$$

$$\therefore AB = 5$$

$$\sin A = \frac{BC}{AB} = \frac{4}{5}$$

$$\cos A = \frac{AC}{AB} = \frac{3}{5}$$

$$\text{and } \sin B = \frac{AC}{AB} = \frac{3}{5}$$

$$\tan B = \frac{AC}{BC} = \frac{3}{4}$$

Ex. 4 In right angled triangle ΔPQR , $\angle Q = 90^\circ$, $\angle R = \theta$ and if $\sin \theta = \frac{5}{13}$ then find $\cos \theta$ and $\tan \theta$.

Solution :

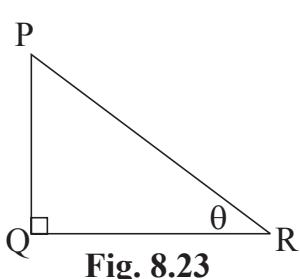


Fig. 8.23

In right angled $\triangle PQR$, $\angle R = \theta$

$$\sin \theta = \frac{5}{13}$$

$$\therefore \frac{PQ}{PR} = \frac{5}{13}$$

\therefore Let $PQ = 5k$ and $PR = 13k$

Let us find QR by using Pythagoras' theorem,

$$\begin{aligned}PQ^2 + QR^2 &= PR^2 \\(5k)^2 + QR^2 &= (13k)^2 \\25k^2 + QR^2 &= 169k^2 \\QR^2 &= 169k^2 - 25k^2 \\QR^2 &= 144k^2 \\QR &= 12k\end{aligned}$$

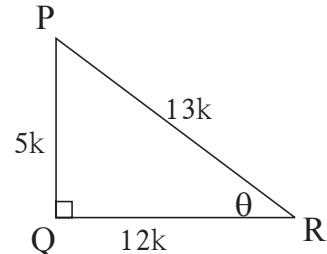


Fig. 8.24

Now, in right angled ΔPQR , $PQ = 5k$, $PR = 13k$ and $QR = 12k$

$$\therefore \cos \theta = \frac{QR}{PR} = \frac{12k}{13k} = \frac{12}{13}, \tan \theta = \frac{PQ}{OR} = \frac{5k}{12k} = \frac{5}{12}$$



Use your brain power!

- (1) While solving above example, why the lengths of PQ and PR are taken $5k$ and $13k$?
 - (2) Can we take the lengths of PQ and PR as 5 and 13 ? If so then what changes are needed in the writing of the solution.

Important Equation in Trigonometry

ΔPQR is a right angled triangle.

$$\angle PQR = 90^\circ, \angle R = \theta$$

$$\sin \theta = \frac{PQ}{PR} \dots\dots\dots(I)$$

Using Pythagoras' theorem,

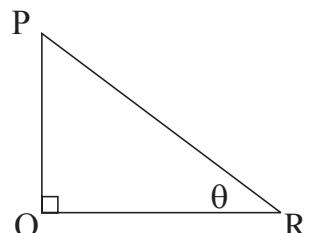


Fig.8.25

$$PQ^2 + QR^2 = PR^2$$

$$\therefore \frac{PQ^2}{PR^2} + \frac{QR^2}{PR^2} = \frac{PR^2}{PR^2} \dots \text{dividing each term by } PR^2$$

$$\therefore \left(\frac{PQ}{RR} \right)^2 + \left(\frac{QR}{RP} \right)^2 = 1$$

$\therefore (\sin \theta)^2 + (\cos \theta)^2 = 1$, from (I) & (II)



Remember this !

‘Square of’ $\sin \theta$ means $(\sin \theta)^2$. It is written as $\sin^2 \theta$.

We have proved the equation $\sin^2 \theta + \cos^2 \theta = 1$ using Pythagoras' theorem, where θ is an acute angle of a right angled triangle.

Verify that the equation is true even when $\theta = 0^\circ$ or $\theta = 90^\circ$

Since the equation $\sin^2 \theta + \cos^2 \theta = 1$ is true for any value of θ . So it is a basic trigonometrical identity.

$$(i) 0 \leq \sin \theta \leq 1, \quad 0 \leq \sin^2 \theta \leq 1 \quad (ii) 0 \leq \cos \theta \leq 1, \quad 0 \leq \cos^2 \theta \leq 1$$

Practice set 8.2

1. In the following table, a ratio is given in each column. Find the remaining two ratios in the column and complete the table.

$\sin \theta$		$\frac{11}{61}$		$\frac{1}{2}$				$\frac{3}{5}$	
$\cos \theta$	$\frac{35}{37}$				$\frac{1}{\sqrt{3}}$				
$\tan \theta$			1			$\frac{21}{20}$	$\frac{8}{15}$		$\frac{1}{2\sqrt{2}}$

2. Find the values of -

$$(i) \quad 5 \sin 30^\circ + 3 \tan 45^\circ$$

$$(ii) \frac{4}{5} \tan^2 60^\circ + 3 \sin^2 60^\circ$$

$$(iii) \quad 2 \sin 30^\circ + \cos 0^\circ + 3 \sin 90^\circ$$

$$(iv) \frac{\tan 60}{\sin 60 + \cos 60}$$

$$(v) \cos^2 45^\circ + \sin^2 30^\circ$$

$$(v) \cos 60^\circ \times \cos 30^\circ + \sin 60^\circ \times \sin 30^\circ$$

3. If $\sin \theta = \frac{4}{5}$ then find $\cos \theta$

4. If $\cos \theta = \frac{15}{17}$ then find $\sin \theta$

Problem Set 8

- 1.** Choose the correct alternative answer for following multiple choice questions.

(i) Which of the following statements is true ?

- (A) $\sin \theta = \cos (90 - \theta)$ (B) $\cos \theta = \tan (90 - \theta)$
(C) $\sin \theta = \tan (90 - \theta)$ (D) $\tan \theta = \tan (90 - \theta)$

(ii) Which of the following is the value of $\sin 90^\circ$?

- (A) $\frac{\sqrt{3}}{2}$ (B) 0 (C) $\frac{1}{2}$ (D) 1

$$(iii) 2 \tan 45^\circ + \cos 45^\circ - \sin 45^\circ = ?$$

- (A) 0 (B) 1 (C) 2 (D) 3

$$(iv) \frac{\cos 28^\circ}{\sin 62^\circ} = ?$$

- (A) 2 (B) -1 (C) 0 (D) 1

- 2.** In right angled $\triangle TSU$, $TS = 5$, $\angle S = 90^\circ$,

SU = 12 then find sin T, cos T, tan T.

Similarly find $\sin U$, $\cos U$, $\tan U$.

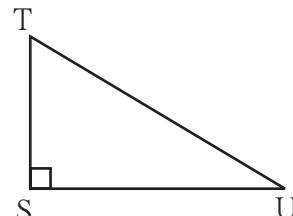


Fig. 8.26

3. In right angled $\triangle YXZ$, $\angle X = 90^\circ$, $XZ = 8 \text{ cm}$, $YZ = 17 \text{ cm}$, find $\sin Y$, $\cos Y$, $\tan Y$, $\sin Z$, $\cos Z$, $\tan Z$.

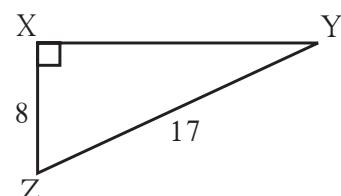


Fig. 8.27

4. In right angled ΔLMN , if $\angle N = \theta$, $\angle M = 90^\circ$,
 $\cos \theta = \frac{24}{25}$, find $\sin \theta$ and $\tan \theta$

Similarly, find $(\sin^2 \theta)$ and $(\cos^2 \theta)$.

- 5.** Fill in the blanks.

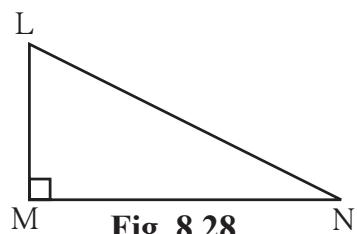


Fig. 8.28

- $$(i) \sin 20^\circ = \cos \square^\circ$$

- $$(ii) \tan 30^\circ \times \tan \square^\circ = 1$$

- $$(iii) \cos 40^\circ = \sin \square^\circ$$





Let's Study

- Surface area of a cone
 - Volume of a cone
 - Surface area of a sphere
 - Volume of a sphere



Let's recall.

We have learnt how to find the surface area and volume of a cuboid, a cube and a cylinder, in earlier standard.

Cuboid

- Length, breadth and height of a cuboid are l , b , h respectively.
 - (i) Area of vertical surfaces of a cuboid = $2(l + b) \times h$
Here we have considered only 4 surfaces into consideration.
 - (ii) Total surface area of a cuboid = $2(lb + bh + lh)$
Here we have taken all 6 surfaces into consideration.
 - (iii) Volume of a cuboid = $l \times b \times h$

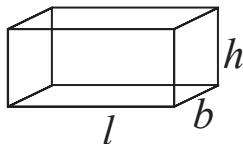


Fig.9.1

Cube

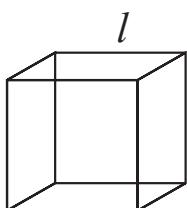


Fig.9.2

- If l is the edge of a cube,
 - Total surface area of a cube = $6l^2$
 - Area of vertical surfaces of a cube = $4l^2$
 - Volume of a cube = l^3

Cylinder

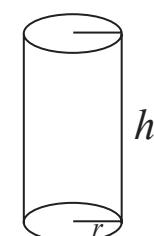


Fig.9.3

- Radius of cylinder is r and height is h .
 - (i) Curved surface area of a cylinder = $2\pi rh$
 - (ii) Total surface area of a cylinder = $2\pi r(r + h)$
 - (iii) Volume of a cylinder = $\pi r^2 h$

Practice Set 9.1

- Length, breadth and height of a cuboid shape box of medicine is 20cm, 12 cm and 10 cm respectively. Find the surface area of vertical faces and total surface area of this box.
 - Total surface area of a box of cuboid shape is 500 sq. unit. Its breadth and height is 6 unit and 5 unit respectively. What is the length of that box ?
 - Side of a cube is 4.5 cm. Find the surface area of all vertical faces and total surface area of the cube.
 - Total surface area of a cube is 5400 sq. cm. Find the surface area of all vertical faces of the cube.
 - Volume of a cuboid is 34.50 cubic metre. Breadth and height of the cuboid is 1.5m and 1.15m respectively. Find its length.
 - What will be the volume of a cube having length of edge 7.5 cm ?
 - Radius of base of a cylinder is 20cm and its height is 13cm, find its curved surface area and total surface area. ($\pi = 3.14$)
 - Curved surface area of a cylinder is 1980 cm^2 and radius of its base is 15cm. Find the height of the cylinder. ($\pi = \frac{22}{7}$).



Let's learn.

Terms related to a cone and their relation

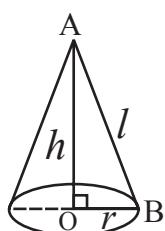


Fig.9.4

A cone is shown in the adjacent Fig.9.4. Centre of the circle, which is the base of the cone, is O and A is the vertex (apex) of the cone. Seg OB is a radius and seg OA is perpendicular to the radius at O, means AO is perpendicular height of the cone. Slant height of the cone is the length of AB, which is shown by (l).

ΔAOB is a right angled triangle.

the Pythagoras' theo

$$AB^2 = AO^2 + OB^2$$

$$\therefore l^2 \equiv h^2 + r^2$$

That is, (slant height)

¹⁰ See also 1 Cor 10:13; 1 Thess 5:13; 1 Tim 5:14; 1 Pet 4:10; 1 John 3:18.

Out of these two we can find the area of base of a cone because we know the formula for

$$\text{Area of a circle} = \pi r^2$$

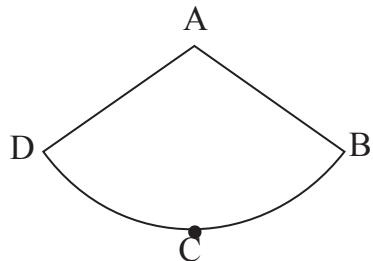


Fig.9.5

To find a formula for the curved surface area of a cone, let us see the net of the curved surface, which is a sector of a circle.

If a cone is cut along edge AB, we get its net as shown in fig.9.5.

Compare the figures 9.4 and 9.5

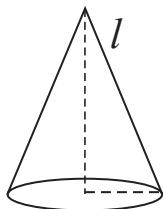
Have you noticed the following things ?

- (i) Radius AB of the sector is the same as the slant height of the cones.
 - (ii) Arc BCD of the sector is the same as circumference of the base of the cone.
 - (iii) Curved surface area of cone = Area of sector A-BCD.

It means to find the curved surface area of a cone we have to find the area of its net that is the area of the sector.

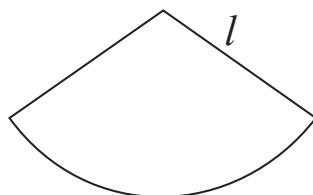
Try to understand, how it is done from the following activity.

Activity : Look at the following figures.



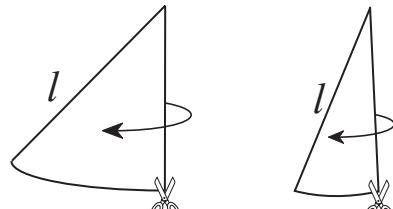
Cone

Fig. 9.6



Net of curved surface

Fig. 9.7



Pieces of the net

Fig. 9.8

Circumference of base of the circle = $2\pi r$

As shown in the Fig.9.8, make pieces of the net as small as possible. Join them as shown in the Fig.9.9.

By Joining the small pieces of net of the cone, we get a rectangle ABCD approximately.

Total length of AB and CD is $2\pi r$.

∴ length of side AB of rectangle ABCD is πr
and length of side CD is also πr :

Length of side BC of rectangle = slant height of cone = l .

Curved surface area of cone is equal to the area of the rectangle.

$$\therefore \text{curved surface area of cone} = \text{Area of rectangle} = AB \times BC = \pi r \times l = \pi r l$$

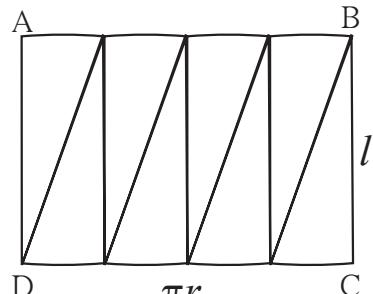


Fig. 9.9

Now, we can derive the formula for total surface area of a cone.

$$\begin{aligned}\text{Total surface area of cone} &= \text{Curved surface area} + \text{Area of base} \\ &= \pi r l + \pi r^2 \\ &= \pi r(l + r)\end{aligned}$$

Did you notice a thing ? If a cone is not closed (Just like a cap of jocker or a cap in a birthday party) it will have only one surface, which is the curved surface. Then we get the surface area of the cone by the formula $\pi r l$.

Activity : Prepare a cylinder of a card sheet, keeping one of its faces open. Prepare an open cone of card sheet which will have the same base-radius and the same height as that of the cylinder.

Pour fine sand in the cone till it just fills up the cone. Empty the cone in the cylinder. Repeat the procedure till the cylinder is just filled up with sand. Note how many coneful of sand is required to fill up the cylinder.

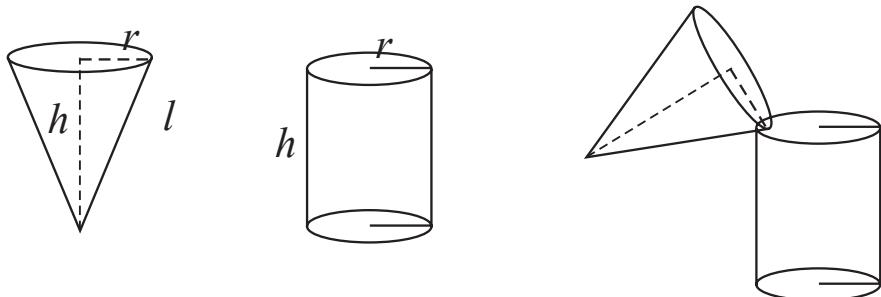


Fig. 9.10

To fill up the cylinder, three coneful of sand is required.



Let's learn.

Volume of a cone

If the base-radii and heights of a cone and a cylinder are equal then

$$3 \times \text{volume of cone} = \text{volume of cylinder}$$

$$\therefore 3 \times \text{volume of cone} = \pi r^2 h$$

$$\therefore \text{volume of cone} = \frac{1}{3} \times \pi r^2 h$$



Remember this !

Ex. (4) Area of the base of a cone is 1386 sq.cm and its height is 28 cm.

Find its surface area. ($\pi = \frac{22}{7}$)

Solution :

$$\text{Area of base of cone} = \pi r^2$$

$$\therefore 1386 = \frac{22}{7} \times r^2$$

$$\therefore \frac{1386 \times 7}{22} = r^2$$

$$\therefore 63 \times 7 = r^2$$

$$\therefore 441 = r^2$$

$r = 21 \text{ cm}$

$$\therefore l^2 = (21)^2 + (28)^2$$

$$\therefore l^2 = 441 + 784$$

$$\therefore l^2 = 1225$$

$$\therefore l = 35 \text{ cm}$$

Surface area of cone = $\pi r l$

$$= \frac{22}{7} \times 21 \times 35$$

$$= 22 \times 21 \times 5$$

$$= 2310 \text{ sq. cm.}$$

Practice set 9.2

1. Perpendicular height of a cone is 12 cm and its slant height is 13 cm. Find the radius of the base of the cone.
 2. Find the volume of a cone, if its total surface area is 7128 sq.cm and radius of base is 28 cm. ($\pi = \frac{22}{7}$)
 3. Curved surface area of a cone is 251.2 cm^2 and radius of its base is 8cm. Find its slant height and perpendicular height. ($\pi = 3.14$)
 4. What will be the cost of making a closed cone of tin sheet having radius of base 6 m and slant height 8 m if the rate of making is Rs.10 per sq.m ?
 5. Volume of a cone is 6280 cubic cm and base radius of the cone is 30 cm. Find its perpendicular height. ($\pi = 3.14$)
 6. Surface area of a cone is 188.4 sq.cm and its slant height is 10cm. Find its perpendicular height ($\pi = 3.14$)
 7. Volume of a cone is 1212 cm^3 and its height is 24cm. Find the surface area of the cone.
($\pi = \frac{22}{7}$)
 8. The curved surface area of a cone is 2200 sq.cm and its slant height is 50 cm. Find the total surface area of cone. ($\pi = \frac{22}{7}$)
 9. There are 25 persons in a tent which is conical in shape. Every person needs an area of 4 sq.m. of the ground inside the tent. If height of the tent is 18m, find the volume of the tent.

- 10.** In a field, dry fodder for the cattle is heaped in a conical shape. The height of the cone is 2.1m. and diameter of base is 7.2 m. Find the volume of the fodder. if it is to be covered by polythin in rainy season then how much minimum polythin sheet is needed ?

$$(\pi = \frac{22}{7} \text{ and } \sqrt{17.37} = 4.17.)$$



Surface area of a sphere

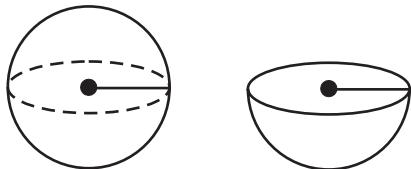


Fig. 9.11

$$\text{Surface area of a sphere} = 4\pi r^2$$

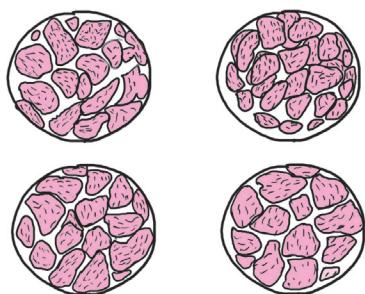
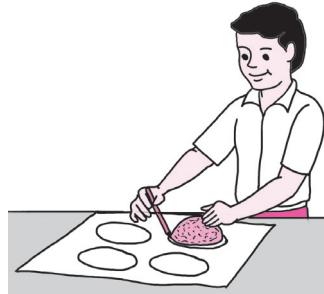
$$\therefore \text{Surface area of a hollow hemisphere} = 2\pi r^2$$

$$\begin{aligned}\text{Total surface area of a solid hemisphere} \\ &= \text{Surface area of hemisphere} + \text{Area of circle} \\ &= 2\pi r^2 + \pi r^2 = 3\pi r^2\end{aligned}$$



Take a sweet lime (Mosambe), Cut it into two equal parts.

Take one of the parts. Place its circular face on a paper. Draw its circular border. Copy three more such circles. Again, cut each half of the sweet lime into two equal parts.



Now you get 4 quarters of sweet lime. Separate the peel of a quarter part. Cut it into pieces as small as possible. Try to cover one of the circles drawn, by the small pieces.

Observe that the circle gets nearly covered.

The activity suggests that,

$$\text{Curved surface area of a sphere} = 4 \pi r^2$$

Solved Examples :

- (1) Find the surface area of a sphere having radius 7 cm. ($\pi = \frac{22}{7}$)

Solution : Surface Area of sphere = $4\pi r^2$

$$\begin{aligned}
 &= 4 \times \frac{22}{7} \times (7)^2 \\
 &= 4 \times \frac{22}{7} \times 7 \times 7 \\
 &= 88 \times 7 \\
 &\equiv 616
 \end{aligned}$$

Surface Area of sphere = 616 sq.cm.

- (2) Find the radius of a sphere having surface area 1256sq.cm. ($\pi = 3.14$)

Solution : Surface Area of Sphere = $4\pi r^2$

$$\therefore 1256 = 4 \times 3.14 \times r^2$$

$$\therefore r^2 = \frac{1256}{4 \times 3.14}$$

$$= \frac{31400}{314}$$

$$\therefore 100 = r^2$$

$$\therefore 10 = r$$

∴ radius of the sphere is 10 cm.

Activity : Make a cone and a hemisphere of cardsheet such that radii of cone and hemisphere are equal and height of cone is equal to radius of the hemisphere. Fill the cone with fine sand. Pour the sand in the hemisphere. How many cones are required to fill the hemisphere completely ?

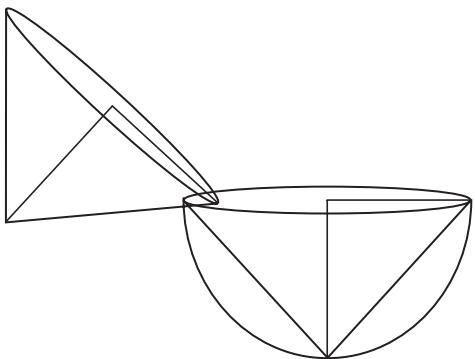
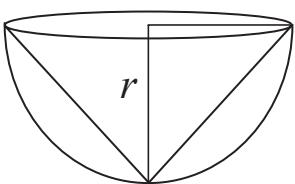
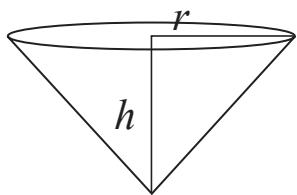


Fig. 9.12

Two cone full of sand is required to fill the hemisphere.

$$\begin{aligned}
 \therefore 2 \times \text{volume of cone} &= \text{volume of hemisphere} \\
 \therefore \text{volume of hemisphere} &= 2 \times \text{volume of cone} \\
 &= 2 \times \frac{1}{3} \times \pi r^2 h \\
 &= 2 \times \frac{1}{3} \times \pi r^2 \times r \\
 &= \frac{2}{3} \pi r^3
 \end{aligned}$$

\therefore volume of sphere

$= 2 \times$ volume of hemisphere.

$$= \frac{4}{3} \pi r^3$$

$$\therefore \text{volume of sphere} = \frac{4}{3} \pi r^3$$



Remember this !

- Volume of hemisphere = $\frac{2}{3} \pi r^3$
 - Total surface area of hemisphere = $2\pi r^2 + \pi r^2 = 3\pi r^2$

Solved Examples :

Ex. (1) Find the volume of a sphere having radius 21 cm. ($\pi = \frac{22}{7}$)

Solution : Volume of sphere = $\frac{4}{3}\pi r^3$

$$\begin{aligned}
 &= \frac{4}{3} \times \frac{22}{7} \times (21)^3 \\
 &= \frac{4}{3} \times \frac{22}{7} \times 21 \times 21 \times 21 \\
 &= 88 \times 441
 \end{aligned}$$

∴ volume of sphere = 38808 cubic cm.

Ex. (2) Find the radius of a sphere whose volume is 113040 cubic cm. ($\pi = 3.14$)

Solution : Volume of sphere = $\frac{4}{3}\pi r^3$

$$\begin{aligned} 113040 &= \frac{4}{3} \times 3.14 \times r^3 \\ \frac{113040 \times 3}{4 \times 3.14} &= r^3 \\ \frac{28260 \times 3}{3.14} &= r^3 \\ \therefore 9000 \times 3 &= r^3 \\ \therefore r^3 &= 27000 \\ \therefore r &= 30 \text{ cm} \\ \therefore \text{radius of sphere} &\text{ is } 30 \text{ cm.} \end{aligned}$$

Ex. (3) Find the volume of a sphere whose surface area is 314.sq.cm. (Take $\pi = 3.14$)

Solution : Surface area of sphere = $4\pi r^2$

$$\frac{314}{4 \times 3.14} = r^2$$

$$\frac{31400}{4 \times 314} = r^2$$

$$\therefore \frac{100}{4} = r^2$$

$$\therefore 25 = r^2$$

25

.. = 50

$$\therefore r = 5 \text{ cm}$$

$$\begin{aligned}
 \text{Volume of sphere} &= \frac{4}{3} \pi r^3 \\
 &= \frac{4}{3} \times 3.14 \times 5^3 \\
 &= \frac{4}{3} \times 3.14 \times 125 \\
 &\equiv 523.33 \text{ cubic cm}
 \end{aligned}$$

Practice Set 9.3

- Find the surface areas and volumes of spheres of the following radii.
(i) 4 cm (ii) 9 cm (iii) 3.5 cm. ($\pi = 3.14$)
 - If the radius of a solid hemisphere is 5cm, then find its curved surface area and total surface area. ($\pi = 3.14$)
 - If the surface area of a sphere is 2826 cm^2 then find its volume. ($\pi = 3.14$)
 - Find the surface area of a sphere, if its volume is 38808 cubic cm. ($\pi = \frac{22}{7}$)
 - Volume of a hemisphere is 18000π cubic cm. Find its diameter.

Problem set

- If diameter of a road roller is 0.9 m and its length is 1.4 m, how much area of a field will be pressed in its 500 rotations ?
 - To make an open fish tank, a glass sheet of 2 mm gauge is used. The outer length, breadth and height of the tank are 60.4 cm, 40.4 cm and 40.2 cm respectively. How much maximum volume of water will be contained in it ?
 - If the ratio of radius of base and height of a cone is 5:12 and its volume is 314 cubic metre. Find its perpendicular height and slant height ($\pi = 3.14$)
 - Find the radius of a sphere if its volume is 904.32 cubic cm. ($\pi = 3.14$)
 - Total surface area of a cube is 864 sq.cm. Find its volume.
 - Find the volume of a sphere, if its surface area is 154 sq.cm.
 - Total surface area of a cone is 616 sq.cm. If the slant height of the cone is three times the radius of its base, find its slant height.
 - The inner diameter of a well is 4.20 metre and its depth is 10 metre. Find the inner surface area of the well. Find the cost of plastering it from inside at the rate Rs.52 per sq.m.
 - The length of a road roller is 2.1m and its diameter is 1.4m. For levelling a ground 500 rotations of the road roller were required. How much area of ground was levelled by the road roller? Find the cost of levelling at the rate of Rs. 7 per sq. m.



Answers

1. Basic Concepts in Geometry

Practice set 1.1

Practice set 1.2

- 1.** (i) No (ii) No (iii) Yes **2. 4** **3. 5** **4.** $BP < AP < AB$

5. (i) Ray RS or Ray RT (ii) Ray PQ (iii) Seg QR (iv) Ray QR and Ray RQ etc.
 (v) Ray RQ and Ray RT etc.. (vi) Ray SR , Ray ST etc.. (vii) Point S

6. (i) Point A & Point C, Point D & Point P (ii) Point L & Point U, Point P & Point R
 (iii) $d(U,V) = 10$, $d(P,C) = 6$, $d(V,B) = 3$, $d(U,L) = 2$

Practice set 1.3

1.
 - (i) If a quadrilateral is a parallelogram then opposite angles of that quadrilateral are congruent.
 - (ii) If quadrilateral is a rectangle then diagonals are congruent.
 - (iii) If a triangle is an isosceles then segment joining vertex of a triangle and mid point of the base is perpendicular to the base
 2.
 - (i) If alternate angles made by two lines and its transversal are congruent then the lines are parallel.
 - (ii) If two parallel lines are intersected by a transversal the interior angles so formed are supplementary.
 - (iii) If the diagonals of a quadrilateral are congruent then that quadrilateral is rectangle.

Problem set 1

- 1.** (i) A (ii) C (iii) C (iv) C (v) B
2. (i) False (ii) False (iii) True (iv) False
3. (i) 3 (ii) 8 (iii) 9 (iv) 2 (v) 6 (vi) 22 (vii) 165
4. -15 and 1 **5.** (i) 10.5 (ii) 9.1 **6.** -6 and 8

2. Parallel Lines

Practice set 2.1

- (i) 95° (ii) 95° (iii) 85° (iv) 85°
 - $\angle a = 70^\circ$, $\angle b = 70^\circ$, $\angle c = 115^\circ$, $\angle d = 65^\circ$
 - $\angle a = 135^\circ$, $\angle b = 135^\circ$, $\angle c = 135^\circ$
 - (i) 75° (ii) 75° (iii) 105° (iv) 75°

Practice set 2.2

- $$1. \text{ No.} \quad 4. \angle ABC = 130^\circ$$

Problem set 2

1. (i) C (ii) C (iii) A (iv) B (v) C 4. $x = 130^\circ$ $y = 50^\circ$
 5. $x = 126^\circ$ 6. $f = 100^\circ$ $g = 80^\circ$

3. Triangles

Practice set 3.1

1. 110° 2. 45° 3. $80^\circ, 60^\circ, 40^\circ$ 4. $30^\circ, 60^\circ, 90^\circ$
 5. $60^\circ, 80^\circ, 40^\circ$ 6. $\angle DRE = 70^\circ, \angle ARE = 110^\circ$
 7. $\angle AOB = 125^\circ$ 9. $30^\circ, 70^\circ, 80^\circ$

Practice set 3.2

1. (i) SSC Test (ii) SAS Test (iii) ASA Test (iv) Hypotenuse Side Test.
 2. (i) ASA Test, $\angle BAC \cong \angle QPR$, side AB \cong side PQ, side AC \cong side PR
(ii) SAS Test, $\angle TPQ \cong \angle TSR$, $\angle TQP \cong \angle TRS$, side PQ \cong side SR
 3. Hypotenuse Side Test, $\angle ACB \cong \angle QRP$, $\angle ABC \cong \angle QPR$, side AC \cong side QR
 4. SSS Test, $\angle MLN \cong \angle MPN$, $\angle LMN \cong \angle MNP$, $\angle LNM \cong \angle PMN$

Practice set 3.3

1. $x = 50^\circ$, $y = 60^\circ$, $m\angle ABD = 110^\circ$, $m\angle ACD = 110^\circ$.
 2. 7.5 Units 3. 6.5 Units 4. $l(PG) = 5 \text{ cm}$, $l(PT) = 7.5 \text{ cm}$

Practice set 3.4

1. 2 cm 2. 28° 3. $\angle QPR, \angle PQR$ 4. greatest side NA, smallest side FN

Practice set 3.5

1. $\frac{XY}{LM} = \frac{YZ}{MN} = \frac{XZ}{LN}$, $\angle X \cong \angle L$, $\angle Y \cong \angle M$, $\angle Z \cong \angle N$

2. $l(QR) = 12 \text{ cm}$, $l(PR) = 10 \text{ cm}$

Problem set 3

1. (i) D (ii) B (iii) B

5. Quadrilaterals

Practice set 5.1

1. $m\angle XWZ = 135^\circ$, $m\angle YZW = 45^\circ$, $l(WY) = 10 \text{ cm}$
 2. $x = 40^\circ$, $\angle C = 132^\circ$, $\angle D = 48^\circ$
 3. 25 cm, 50 cm, 25 cm, 50 cm
 4. 60° , 120° , 60° , 120°
 6. $\angle A = 70^\circ$, $\angle B = 110^\circ$, $\angle C = 70^\circ$, $\angle R = 110^\circ$

Practice set 5.3

1. $BO = 4 \text{ cm}$, $\angle ACB = 35^\circ$
 2. $QR = 7.5 \text{ cm}$, $\angle PQR = 105^\circ$, $\angle SRQ = 75^\circ$
 3. $\angle IMJ = 90^\circ$, $\angle JIK = 45^\circ$, $\angle LJK = 45^\circ$
 4. side = 14.5 cm, Perimeter = 58 cm
 5. (i) False (ii) False (iii) True (iv) True (v) True (vi) False

Practice set 5.4

- $$1. \angle J = 127^\circ, \angle L = 72^\circ \quad 2. \angle B = 108^\circ, \angle D = 72^\circ$$

Practice set 5.5

- $$1. \quad XY = 4.5 \text{ cm}, YZ = 2.5 \text{ cm}, XZ = 5.5 \text{ cm}$$

Problem set 5

6. Circle

1. 20 cm 2. 5 cm 3. 32 unit 4. 9 unit

Practice set 6.2

- 1.** 12 cm **2.** 24 cm

Problem set 6

- 1.** (i) A (ii) C (iii) A (iv) B (v) D (vi) C (vii) D or B **2.** 2:1 **4.** 24 units

7. Co - ordinate Geometry

Practice set 7.1

1. point A : Quadrant II, point B : Quadrant III, point K : Quadrant I, point D : Quadrant I
point E : Quadrant I, point F : Quadrant IV, point G : Quadrant IV, point H : Y-Axis.
point M : X-Axis, point N : Y-Axis, point P : Y-Axis, point Q : Quadrant III

2. (i) Quadrant I (ii) Quadrant III (iii) Quadrant IV (iv) Quadrant II

Practice set 7.2

- 1.** Square **2.** $x = -7$ **3.** $y = -5$ **4.** $x = -3$ **5.** 4
6. (i) Y-Axis, (ii) X-axis, (iii) Y-axis, (iv) X-axis,
7. To X-axis (5,0) , To Y-axis (0,5)
8. (-4,1), (-1.5, 1), (-1.5,5), (-4,5)

Problem set 7

1. (i) C (ii) A (iii) B (iv) C (v) C (vi) B
2. (i) Q $(-2,2)$, R $(4,-1)$ (ii) T $(0,-1)$, M $(3,0)$ (iii) point S (iv) point O
3. (i) Quadrant IV (ii) Quadrant III
 (iii) Quadrant II (iv) Quadrant II (v) Y-axis (vi) X-axis
5. (i) 3 (ii) P $(3,2)$, Q $(3,-1)$, R $(3,0)$ (iii) 0 6. $y = 5$, $y = -5$ 7. $|a|$

8. Trigonometry

Practice set 8.1

1. (i) $\frac{QR}{PQ}$ (ii) $\frac{QR}{PQ}$ (iii) $\frac{QR}{PR}$ (iv) $\frac{PR}{QR}$

2. (i) $\frac{a}{c}$ (ii) $\frac{b}{a}$ (iii) $\frac{b}{c}$ (iv) $\frac{a}{b}$

3. (i) $\frac{MN}{LN}$ (ii) $\frac{LM}{LN}$ (iii) $\frac{LM}{MN}$ (iv) $\frac{MN}{LN}$

4. (i) $\frac{PQ}{PR}, \frac{RQ}{PR}, \frac{PQ}{RQ}$ (ii) $\frac{QS}{PS}, \frac{PQ}{PS}, \frac{QS}{PQ}$

Practice set 8.2

- $$1. \quad \begin{aligned} \sin \theta &: \frac{12}{37}, \frac{1}{\sqrt{2}}, \frac{\sqrt{2}}{\sqrt{3}}, \frac{21}{29}, \frac{8}{17}, \frac{1}{3}; \cos \theta : \frac{60}{61}, \frac{1}{\sqrt{2}}, \frac{\sqrt{3}}{2}, \frac{20}{29}, \frac{15}{17}, \frac{4}{5}, \frac{2\sqrt{2}}{3} \\ \tan \theta &: \frac{12}{35}, \frac{11}{60}, \frac{1}{\sqrt{3}}, \sqrt{2}, \frac{3}{4} \end{aligned}$$

- $$2. \quad \text{(i)} \frac{11}{2} \quad \text{(ii)} \frac{93}{20} \quad \text{(iii)} 5 \quad \text{(iv)} \frac{2\sqrt{3}}{\sqrt{3}+1} \quad \text{(v)} \frac{3}{4} \quad \text{(vi)} \frac{\sqrt{3}}{2}$$

Problem set 8

1. (i) A (ii) D (iii) C (iv) D

2. $\sin T = \frac{12}{13}$, $\cos T = \frac{5}{13}$, $\tan T = \frac{12}{5}$, $\sin U = \frac{5}{13}$, $\cos U = \frac{12}{13}$, $\tan U = \frac{5}{12}$

3. $\sin Y = \frac{8}{17}$, $\cos Y = \frac{15}{17}$, $\tan Y = \frac{8}{15}$, $\sin Z = \frac{15}{17}$, $\cos Z = \frac{8}{17}$, $\tan Z = \frac{15}{8}$

4. $\sin \theta = \frac{7}{25}$, $\tan \theta = \frac{7}{24}$, $\sin^2 \theta = \frac{49}{625}$, $\cos^2 \theta = \frac{576}{625}$

5. (i) 70 (ii) 60 (iii) 50

9. Surface Area and Volume

Practice set 9.1

- | | | | |
|----|-------------------------------|------------|----------------------------|
| 1. | 640 sq.cm, 1120 sq.cm. | 2. 20 Unit | 3. 81 sq.cm, 121.50 sq.cm. |
| 4. | 3600 sq.cm. | 5. 20 m | 6. 421.88 cubic cm |
| 7. | 1632.80 sq.cm, 4144.80 sq.cm. | 8. 21 cm | |

Practice set 9.2

- 1.** 5 cm **2.** 36960 cubic cm. **3.** 10 cm, 6 cm **4.** ₹ 2640
5. 15 cm **6.** 8 cm **7.** 550 sq.cm **8.** 2816 sq.cm, 9856 cubic cm
9. 600 cubic metre **10.** 28.51 cubic metre, 47.18 sq.m.

Practice Set 9.3

1. (i) 200.96 sq.cm, 267.95 cubic cm. (ii) 1017.36 sq.cm, 3052.08 cubic cm.
 (iii) 153.86 sq.m, 179.50 cubic cm.

2. 157 sq.cm, 235.5 sq.cm. 3. 14130 cubic cm. 4. 5544 sq.cm. 5. 60 cm

Problem set 9

- | | | |
|---------------|----------------------|------------------------|
| 1. 1980 sq.m. | 2. 96801.6 cubic cm. | 3. 12 m, 13 m |
| 4. 6 cm | 5. 1728 cubic cm. | 6. 179.67 cubic cm. |
| 7. 21 cm | 8. 132 sq.m., ₹ 6864 | 9. 4620 sq.m., ₹ 32340 |





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