Tutorial Sheet - III

1. The finite sheet $0 \leq x \leq 1$, $0 \leq y \leq 1$ on the z plane has the charge density $\sigma = xy(x^2 + y^2 + 25)^{3/2}$ nC/m^2. Find (a) The total charge on the sheet, (b) The electric field at (0, 0, 5) and (c) The force experienced by a -1 mC charge located at (0, 0, 5). [Ans. (a) 33.15 nC, (b) (-1.5, -1.5, 11.25) V/m, (c) (1.5, 1,5, -11.25) mN]

2. (a) Using Coulomb's law find the electric field a distance z from the center of a spherical surface of radius R, which carries a uniform charge density σ. Treat the case $z < R$ (inside) as well as $z > R$ (outside). Express your answers in terms of the total charge q on the sphere. (b) Now use the Gauss's Law to find electric fields in part (a).

3. Plane $x + 2y = 5$ carries charge $\sigma = 6$ nC/m^2. Determine $\mathbf{E}$ at (-1, 0, 1).
[Ans: $-151.7\hat{\boldsymbol{x}}$ $-303.5\hat{\boldsymbol{y}}$ V/m]

4. A thick hollow spherical shell (with inner/outer radii a/b) carries charge density $\rho = k/r^2$ in the region $a \leq r \leq b$. Find the electric field in the three regions: (i) $r < a$, (ii) $a < r < b$, (iii) $r > b$. Plot $|\mathbf{E}|$ as a function of r. Also, find the potential at the center, using infinity as your reference point.

5. Find the potential a distance s from an infinitely long straight wire that carries a uniform line charge λ. Compute the gradient of your potential, and check that it yields the correct field.

6. Find the potential inside and outside a spherical shell of radius R, which carries a uniform surface charge. Set the reference point at infinity.

7. A non-uniform line charge density $\lambda = 10/z^2$ nC/m lies along the z-axis for $|z| \geq 1$; $\lambda = 0$ for $|z| \leq 1$. Find the potential at the point $(1, \sqrt{2}, 0)$ in the free space if $V(\infty) = 0$. [Ans. 60 V].

8. If the electric field in some region is given (in spherical coordinates) by the expression $\mathbf{E}(\mathbf{r}) = \frac{A\ \hat{\mathbf{r}} + B\ \sin\theta\ \cos\phi\ \hat{\boldsymbol{\phi}}}{r}$, where A and B are constants, what is the charge density? [Ans: $\varepsilon_o(\mathrm{A} - \mathrm{B}\ \sin\phi)/\mathrm{r}^2$]

9. A charge density with spherical symmetry has density

$$\rho = \begin{cases} \frac{\rho_o r}{R}, & 0 \leq r \leq R \\ 0, & r > R \end{cases}$$

Determine $\mathbf{E}$ everywhere. [Ans: $\mathbf{E} = \rho_o r^2 \hat{\mathbf{r}}/(4\varepsilon_o R)$ for $r < R$; $\mathbf{E} = \rho_o R^3 \hat{\mathbf{r}}/(4\varepsilon_o r^2)$ for $r > R$]

10. A point charge of 5 nC is located at (-3, 4, 0) while line $y = 1, z = 1$ carries uniform charge 2 nC/m.
(a) If $V = 0$ V at O(0, 0, 0), find V at A(5, 0, 1) [Ans: 8.477 V]
(b) If $V = 100$ V at B(1, 2, 1), find V at C(-2, 5, 3) [Ans: 49.825 V]
(c) If $V = -5$ V at O, find V_{BC} [Ans: -50.175 V].

11. A charge distribution with spherical symmetry has density

$$\rho = \begin{cases} \rho_o, & 0 \leq r \leq R \\ 0, & r > R \end{cases}$$

Determine V everywhere and the energy stored in region $r < R$.

12. In an electric field $\mathbf{E} = 20\ r\ \sin\theta\hat{\mathbf{r}} + 10\ r\ \cos\theta\hat{\boldsymbol{\theta}}$ V/m, calculate the energy expended in transferring a 10 nC charge (a) From A (5, 30°, 0°) to B(5, 90°, 0°) (b) From A to C (10, 30°, 0°) (c) From A to D (5, 30°, 60°) (d) From A to E (10, 90°, 60°). [Ans: (a) -1250 nJ (b) -3750 nJ (c) zero (d) -8750 nJ]

13. A point charge Q is placed at origin. Calculate the energy stored in the region $r > a$.

14. If $V = s^2 z \sin\phi$, calculate the energy within the region defined by
$1 < s < 4$, $-2 < z < 2$, $0 < \phi < \pi\ /3$. [Ans: 6.612 nJ]

15. For the current density $\mathbf{J} = 10\ z\ \sin^2\phi\ \hat{\mathbf{s}}\ A/m^2$, find the current through the cylindrical surface $s = 2, 1 \leq z \leq 5$m. [Ans: 754 A]

16. A homogeneous dielectric ($\varepsilon_r = 2.5$) fills region 1 ($x \leq 0$) while region 2 ($x \geq 0$) is free space. **(a)** If $\mathbf{D}_1 = 12\,\hat{\boldsymbol{x}} - 10\,\hat{\boldsymbol{y}} + 4\,\hat{\boldsymbol{z}}$ nC/m^2, find $\mathbf{D}_2$ and θ_2. (b) If $E_2 = 12$ V/m and $\theta_2 = 60°$, find E_1 and θ_1. [Ans: (a) $12\,\hat{\boldsymbol{x}} - 4\,\hat{\boldsymbol{y}} + 1.6\,\hat{\boldsymbol{z}}$ nC/m^2, 19.75°, (b) 10.67 V/m, 77°]

17. A dielectric material contains 2×10^{19} polar molecules/m^3, each of dipole moment 1.8×10^{-27} C.m. Assuming that all the dipoles are aligned in the direction of the electric field $\mathbf{E} = 10^5\,\hat{\boldsymbol{x}}$ V/m, find $\mathbf{P}$ and ε_r. [Ans: $3.6\times10^{-18}\,\hat{\boldsymbol{x}}$ C/m^2, 1.04]

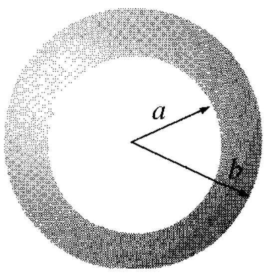
18. A large conducting cone ($\theta = 45°$) is placed on a conducting plane with a tiny gap separating it from the plane. If the cone is connected to a 50 V source, find V and $\mathbf{E}$ at $(-3, 4, 2)$.
[Ans: 22.13 V, $-11.35\,\hat{\boldsymbol{\theta}}$ V/m.]

19. A certain medium with permitivity equal to that of vacuum occupies the space between two conducting slabs located at y $= \pm2$ cm. When heated, the material emits electrons such that their charge density is given by $\rho = 50(1-\mathrm{y}^2)$ μC/m^3. If both the slabs are held at 30 kV, find the (a) potential distribution within the slabs.

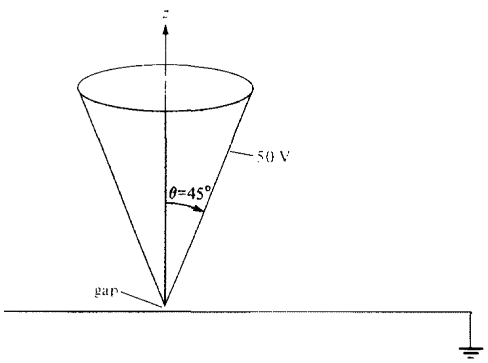
20. Consider two plane-parallel electrodes distance d apart, at voltages 0 and V_0, find the current density J if an unlimited supply of electrons at rest is supplied to the lower potential electrode. Neglect collisions. [Hint: Express the charge density in terms of the current density and electron's velocity; Use Poisson's equation.] [Ans: $J = \frac{4}{9}\varepsilon_o\left(\frac{2e}{m}\right)^{1/2}\frac{V_0{}^{3/2}}{d^2}$; m = mass of electron, e = electronic charge].

21. A sphere of radius a has a bound charge q distributed uniformly over its surface. The sphere is surrounded by dielectric fluid of permitivity ε; the fluid also contains a free charge density given by $\sigma(r) = -kV(r)$, $V(r)$ being the electrostatic potential at $\mathbf{r}$ relative to ∞ and k is some positive constant. Calculate $V(r)$ assuming the same vanishes asymptotically. [Ans: $V(r) = \frac{q}{4\pi\varepsilon r}e^{\alpha(a-r)}, \alpha^2 = |k|/\varepsilon$]

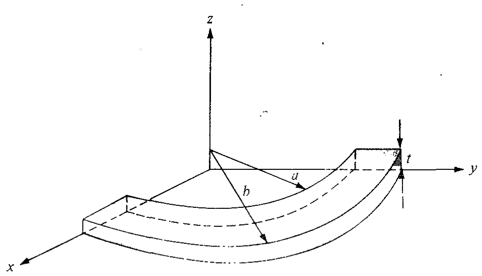
22. Show that the resistance of the bar of figure (c) between the vertical ends located at $\phi = 0$ and $\phi = \pi/2$ is R $= \frac{\pi}{2\sigma t \ln\frac{b}{a}}$



(a)Figure for problem 4



(b)Figure for problem 18



(c)Figure for problem 22

Figure for Problem 13

(Tutorial – 2, PHN-008)