# Collections, Part One

# Outline for Today

- Parameter Passing in C++
  - On xeroxes and master copies.
- Container Types
  - Holding lots of pieces of data.
- The Vector type
  - Storing sequences.
- Recursion on Vectors
  - More practice with sequences.

Parameter Passing in C++

#### Make a Prediction!

• Look over this piece of C++ code:
 void becomeWealthy(int netWorth) {
 netWorth = 1000000000;
 }

```
int main() {
   int value = 137;
   becomeWealthy(value);
   cout << value << endl; // <-- Here
   return 0;</pre>
```

• What do you think will get printed at the indicated point? Why?

Formulate a hypothesis, but don't post anything in chat just yet!

#### Make a Prediction!

• Look over this piece of C++ code:

```
void becomeWealthy(int netWorth) {
    netWorth = 1000000000;
}
int main() {
    int value = 137;
    becomeWealthy(value);
    cout << value << endl; // <-- Here
    return 0;
}</pre>
```

What do you think will get printed at the indicated point? Why?

Now, private chat me your best guess. Not sure? Just answer "??"

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int main() {
   int value = 137;
   becomeWealthy(value);
   cout << value << endl;
   return 0;
}</pre>
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  return 0;
}</pre>
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```
int main() {
   int value = 137;
   becomeWealthy(value);
   cout << value << endl;
   return 0;
}</pre>
```

```
int main() {
    void becomeWealthy(int netWorth) {
        netWorth = 1000000000;
    }
}
netWorth
```

```
int main() {
    void becomeWealthy(int netWorth) {
        netWorth = 10000000000;
    }
}
```

```
int main() {
    void becomeWealthy(int netWorth) {
        netWorth = 1000000000;
    }
    netWorth
```

```
int main() {
    void becomeWealthy(int netWorth) {
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    }
}
netWorth

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```

```
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   int value = 137;
   becomeWealthy(value);
   cout << value << endl;
   return 0;
}</pre>
```

## Parameter Passing in C++

• By default, in C++, parameters are passed by value.

```
/* This function gets a copy of the integer passed
 * into it, so we only change our local copy. The
 * caller won't see any changes.
 */
void byValue(int number) {
   number = 137;
}
```

 You can place an ampersand after the type name to take the parameter by reference.

```
/* This function takes its argument by reference, so
  * when the function returns, the int passed in will have
  * been permanently changed.
  */
void byReference(int& number) {
    number = 137;
}
```

```
int main() {
   int value = 137;
   becomeWealthy(value);
   cout << value << endl;
   return 0;
}</pre>
```

```
int main() {
  int value = 137;
  becomeWealthy(value);
  cout << value << endl;
  return 0;
}</pre>
```

```
int main() {
  int value = 137;
  becomeWealthy(value);
  cout << value << endl;
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   int value = 137;
   becomeWealthy(value);
   cout << value << endl;
   return 0;
}</pre>
```

```
int main() {
    int value = 137;
    becomeWealthy(value);
    void becomeWealthy(int& netWorth) {
        netWorth = 10000000000;
    }
}
```

```
int main() {
   int value = 137;
   becomeWealthy(value);
   void becomeWealthy(int& netWorth) {
       netWorth = 10000000000;
   }
}
```

```
int main() {
   int value = 137;
   becomeWealthy(value);
   void becomeWealthy(int& netWorth) {
       netWorth = 10000000000;
   }
}
```

```
int main() {
    int value = 137;
    becomeWealthy(value);
    void becomeWealthy(int& netWorth) {
        netWorth = 10000000000;
    }
}
```

```
int main() {
   int value = 137;
   becomeWealthy(value);
   cout << value << endl;
   return 0;
}</pre>
```

#### Make a Prediction!

• Look over this piece of C++ code:
 void gollyGee(string text) {
 text[0] = 'g';
 }
 int main() {
 string message = "wiggle";
 gollyGee(message);
 cout << message << endl; // <-- Here
 return 0;</pre>

• What do you think will get printed at the indicated point? Why?

Formulate a hypothesis, but don't post anything in chat just yet!

#### Make a Prediction!

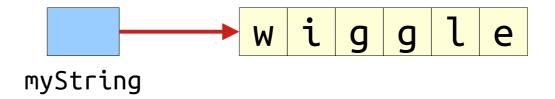
• Look over this piece of C++ code:
 void gollyGee(string text) {
 text[0] = 'g';
 }
 int main() {
 string message = "wiggle";
 gollyGee(message);
 cout << message << endl; // <-- Here
 return 0;</pre>

• What do you think will get printed at the indicated point? Why?

Now, private chat me your best guess. Not sure? Just answer "??"

# Strings in C++

• In Python, Java, and JavaScript, string variables are not the strings themselves. They're pointers to those strings.



 In C++, a variable of type string is an actual, concrete, honest-to-goodness string.

```
int main() {
    string message = "wiggle";
    gollyGee(message);
    cout << message << endl;
    return 0;
}</pre>
```

```
int main() {
    string message = "wiggle";
    gollyGee(message);
    cout << message << endl;
    return 0;
}</pre>
```

```
int main() {
    string message = "wiggle";
    gollyGee(message);
    cout << message << endl;
    return 0;
}</pre>
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int main() {
    string message = "wiggle";
    gollyGee(message);
    cout << message << endl;
    return 0;
}</pre>
```

```
int main() {
    void gollyGee(string text) {
        text[0] = 'g';
    }
    text
```

```
int main() {
    void gollyGee(string text) {
        text[0] = 'g';
    }
}
```

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int main() {
    void gollyGee(string text) {
        text[0] = 'g';
    }
}
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```
int main() {
    void gollyGee(string text) {
        text[0] = 'g';
    }
    text
```

```
int main() {
    string message = "wiggle";
    gollyGee(message);
    cout << message << endl;
    return 0;
}</pre>
```

Adding An Ampersand

```
int main() {
    string message = "wiggle";
    gollyGee(message);
    cout << message << endl;
    return 0;
}</pre>
```

```
int main() {
    string message = "wiggle";
    gollyGee(message);
    cout << message << endl;
    return 0;
}</pre>
```

```
int main() {
    string message = "wiggle";
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    cout << message << endl;
    return 0;
}</pre>
```

```
int main() {
    string message = "wiggle";
    gollyGee(message);
    cout << message << endl;
    return 0;
}</pre>
```

```
int main() {
    string message = "wiggle";
    gollyGee(message);
    roid gollyGee(string& text) {
        text[0] = 'g';
    }
```

```
int main() {
    string message = "wiggle";
    gollyGee(message);
    roid gollyGee(string& text) {
        text[0] = 'g';
    }
```

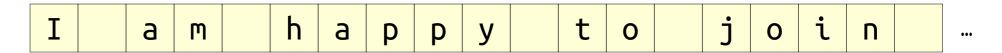
```
int main() {
    string message = "wiggle";
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    roid gollyGee(string& text) {
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    }
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int main() {
    string message = "wiggle";
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    roid gollyGee(string& text) {
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    }
```

```
int main() {
    string message = "wiggle";
    gollyGee(message);
    cout << message << endl;
    return 0;
}</pre>
```

## A Question of Speed

 When working with strings, pass-by-value is slower than pass-by-reference because of the cost of copying the string.



• *General principle:* When passing a string into a function, use pass-by-reference unless you actually want a copy of the string.

### Do You Trust Me?

 Suppose you've written the next Great American Novel and the single, sole copy is stored in the variable

```
string myMasterpiece;
```

- You see a function with this signature:
   void totallyNotSketchy(string& text);
- Would you make this call?
   totallyNotSketchy(myMasterpiece);

## Pass-by-const-Reference

- If you want to look at, but not modify, a function parameter, pass it by *const reference*:
  - The "by reference" part avoids a copy.
  - The "const" (constant) part means that the function can't change that argument.
- For example:

```
void proofreadLongEssay(const string& essay) {
   /* can read, but not change, the essay. */
}
```

This is the general Start! convention used in C++ programming. Please feel free to ask questions about this over the course Yes! Pass by Need to change of the quarter! argument? reference! Nope! Number! Object! What kind of argument? int string bool Vector<int> double Stack<char> char Queue<int> Pass by Pass by const value! reference!

**Container Types** 

## Container Types

- A collection class (also called an abstract data type or container class) is a data type used to store and organize data in some form.
  - These are things like arrays, lists, maps, dictionaries, etc.
- Our next three lectures exploring collections and how to use them appropriately.
- Later, we'll analyze their efficiencies. For now, let's just focus on how to use them.

# Vector

#### Vector

- A **Vector** is a collection class representing a list of things.
- It's similar to Java's ArrayList, JavaScript's arrays, and Python's lists.
- To make a Vector, use this syntax:

Vector<type> name;

• All elements of a Vector have to have the same type. You specify that type by placing it in <angle brackets> after the word Vector.

Vector in Action

```
"""    Python Version
v = [1, 3, 7]

v.append(271)
print(v[0])
print(v[-1])
first = v[0:2]
last = v[2:]
del v[0]
```

```
/* Java Version */
List<> v = new ArrayList<Integer>();
v.add(1); v.add(3); v.add(7);
v.add(271);
System.out.println(v.get(0));
System.out.println(v.get(v.size()-1));
List<Integer> first = v.subList(0, 2);
List<Integer> last = v.subList(2);
v.remove(0);
```

```
// JavaScript Version
let v = [1, 3, 7];

v.push(271);
console.log(v[0]);
console.log(v[v.length - 1]);

let first = v.slice(0, 2);
let last = v.slice(2);
v.splice(0, 0);
```

```
"""    Python Version
v = [1, 3, 7]

v.append(271)
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console.log(v[0]);
console.log(v[v.length - 1]);
let first = v.slice(0, 2);
let last = v.slice(2);
v.splice(0, 0);
```

```
/* Stanford C++ Version */
Vector<int> v = { 1, 3, 7 };

v += 271;

cout << v[0] << endl;

cout << v[v.size() - 1] << endl;

Vector<int> first = v.subList(0, 2);

Vector<int> last = v.subList(2);

v.remove(0);
```

```
/* Java Version */
List<> v = new ArrayList<Integer>();
v.add(1); v.add(3); v.add(7);
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System.out.println(v.get(0));
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```
"""    Python Version
v = [1, 3, 7]

v.append(271)
print(v[0])
print(v[-1])
first = v[0:2]
last = v[1:]
```

Note the use of curly braces rather than square brackets here.

```
let v = [1, 3, 7];

v.push(271);
console.log(v[0]);
console.log(v[v.length - 1]);

let first = v.slice(0, 2);
let last = v.slice(2);
v.splice(0, 0);
```

```
/* Stanford C++ Version */
Vector<int> v = { 1, 3, 7 };

v += 271; 
cout << v[0] << endl;
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Vector<int> first = v.subList(0, 2);
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v.remove(0);
```

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using the += operator.

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// JavaScript Version
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```

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/* Stanford C++ Version
Vector<int> v = { 1, 3, 7 };

v += 271;

cout << v[0] << endl;  
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Vector<int> first = v.subList(0, 2);

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```
"""    Python Version
v = [1, 3, 7]

v.append(271)
print(v[0])
print(v[-1])
first = v[0:2]
last = v[2:]
```

We select individual elements out of a Vector using square brackets. Everything is zero—indexed.

```
v.push(271);
console.log(v[0]);
console.log(v[v.length - 1]);
let first = v.slice(0, 2);
let last = v.slice(2);
v.splice(0, 0);
```

```
/* Stanford C++ Version
Vector<int> v = { 1, 3, 7 };

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cout << v[0] << endl;
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```
"""    Python Version
v = [1, 3, 7]

v.append(271)
print(v[0])
print(v[-1])
first = v[0:2]
last = v[2:]
```

C++ doesn't support negative array indices to mean "count from the back." We have to do some math to find the index of the last element.

We use the syntax v.size() to get the length of a Vector.

```
let last = v.slice(2);
v.splice(0, 0);
```

```
/* Stanford C++ Version
Vector<int> v = { 1, 3, 7 };

v += 271;
cout << v[0] << endl;
cout << v[v.size() - 1] << endl;
Vector<int> first = v.subList(0, 2);
Vector<int> last = v.subList(2);
v.remove(0);
```

```
/* Java Version */
List<> v = new ArrayList<Integer>();
v.add(1); v.add(3); v.add(7);
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List<Integer> first = v.subList(0, 2);
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v.remove(0);
```

```
""" Python Version
v = [1, 3, 7]

v.append(271)
print(v[0])
print(v[-1])
first = v[0:2]
last = v[2:]
del v[0]
```

The subList member function is used to get a subrange of the subList. Here, first will be the first two elements of the vector, and last will be the list starting at position 2.

```
v.splice(0, 0);
```

```
/* Stanford C++ Version */
Vector<int> v = { 1, 3, 7 };

v += 271;

cout << v[0] << endl;

cout << v[v.size() - 1] << endl;

Vector<int> first = v.subList(0, 2);

Vector<int> last = v.subList(2);

v.remove(0);
```

```
""" Python Version
v = [1, 3, 7]

v.append(271)
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List<Integer> first = v.subList(0, 2);
List<Integer> last = v.subList(2);
v.remove(0);
```

We can use the remove member function to remove the element at a given index.

```
console.log(v[v.length - 1]);
let first = v.slice(0, 2);
let last = v.slice(2);
v.splice(0, 0);
```

```
"""    Python Version
v = [1, 3, 7]

v.append(271)
print(v[0])
print(v[-1])
first = v[0:2]
last = v[2:]
del v[0]
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/* Java Version */
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List<Integer> first = v.subList(0, 2);
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```
// JavaScript Version
let v = [1, 3, 7];

v.push(271);
console.log(v[0]);
console.log(v[v.length - 1]);

let first = v.slice(0, 2);
let last = v.slice(2);
v.splice(0, 0);
```

```
"""    Python Version
v = ["A", "B", "C"]

# Counting for loop.
for i in range(len(v)):
    print(v[i])

# Range-based for loop.
for elem in v:
    print(elem)
```

```
/* Java Version
List<> v = new ArrayList<String>();
v.add("A"); v.add("B"); v.add("C");

/* Counting for loop. */
for (int i = 0; i < v.size(); i++) {
    System.out.println(v[i]);
}

/* Range-based for loop. */
for (String elem: v) {
    System.out.println(elem);
}</pre>
```

```
// JavaScript Version
let v = ["A", "B", "C"];

// Counting for loop.
for (let i in v) {
    console.log(v[i]);
}

// Range-based for loop.
for (let elem of v) {
    console.log(elem);
}
```

```
/* Stanford C++ Version
Vector<string> v = { "A", "B", "C" };

/* Counting for loop. */
for (int i = 0; i < v.size(); i++) {
    cout << v[i] << endl;
}

/* Range-based for loop. */
for (string elem: v) {
    cout << elem << endl;
}</pre>
```

```
""" Python Version
v = ["A", "B", "C"]

# Counting for loop.
for i in range(len(v)):
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    System.out.println(v[i]);
}

/* Range-based for loop. */
for (String elem: v) {
    System.out.println(elem);
}</pre>
```

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// JavaScript Version
let v = ["A", "B", "C"];

// Counting for loop.
for (let i in v) {
    console.log(v[i]);
}

// Range-based for loop.
for (let elem of v) {
    console.log(elem);
}
```

```
/* Stanford C++ Version
Vector<string> v = { "A", "B", "C" };

/* Counting for loop. */
for (int i = 0; i < v.size(); i++) {
    cout << v[i] << endl;
}

/* Range-based for loop. */
for (string elem: v) {
    cout << elem << endl;
}</pre>
```

```
""" Python Version
v = ["A", "B", "C"]

# Counting for loop.
for i in range(len(v)):
    print(v[i])

# Range-based for loop.
for elem in v:
    print(elem)
```

```
/* Java Version */
List<> v = new ArrayList<String>();
v.add("A"); v.add("B"); v.add("C");

/* Counting for loop. */
for (int i = 0; i < v.size(); i++) {
    System.out.println(v[i]);
}

/* Range-based for loop. */
for (String elem: v) {
    System.out.println(elem);
}</pre>
```

We can iterate over the elements of a Vector by counting upward from o (inclusive) to its size (exclusive) and accessing each element.

```
console.log(elem);
}
```

```
/* Stanford C++ Version
    Vector<string> v = { "A", "B", "C" };

/* Counting for loop. */
for (int i = 0; i < v.size(); i++) {
    cout << v[i] << endl;
}

/* Range-based for loop. */
for (string elem: v) {
    cout << elem << endl;
}</pre>
```

```
""" Python Version
v = ["A", "B", "C"]

# Counting for loop.
for i in range(len(v)):
    print(v[i])

# Range-based for loop.
for elem in v:
    print(elem)
```

```
/* Java Version */
List<> v = new ArrayList<String>();
v.add("A"); v.add("B"); v.add("C");
/* Counting for loop. */
for (int i = 0; i < v.size(); i++) {
    System.out.println(v[i]);
}
/* Range-based for loop. */
for (String elem: v) {
    System.out.println(elem);
}</pre>
```

We can also use this loop structure, which visits each element of the **vector** in the order in which they appear.

```
// Range-based for loop.
for (let elem of v) {
    console.log(elem);
}
```

```
/* Stanford C++ Version
Vector<string> v = { "A", "B", "C" };

/* Counting for loop. */
for (int i = 0; i < v.size(); i++) {
    cout << v[i] << endl;
}

/* Range-based for loop. */
for (string elem: v) {
    cout << elem << endl;
}</pre>
```

```
"""    Python Version
v = ["A", "B", "C"]

# Counting for loop.
for i in range(len(v)):
    print(v[i])

# Range-based for loop.
for elem in v:
    print(elem)
```

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/* Java Version */
List<> v = new ArrayList<String>();
v.add("A"); v.add("B"); v.add("C");

/* Counting for loop. */
for (int i = 0; i < v.size(); i++) {
    System.out.println(v[i]);
}

/* Range-based for loop. */
for (String elem: v) {
    System.out.println(elem);
}</pre>
```

```
// JavaScript Version
let v = ["A", "B", "C"];

// Counting for loop.
for (let i in v) {
    console.log(v[i]);
}

// Range-based for loop.
for (let elem of v) {
    console.log(elem);
}
```

To read more about the Vector and how to use it, check out the

Stanford C++ Library Documentation

up on the course website.

### Make a Prediction!

• Look over this piece of C++ code:

```
void dream(Vector<int> numbers) {
    numbers[1] = 1963;
}
int main() {
    Vector<int> values = { 1929, 1955, 1964 };
    dream(values);
    cout << values << endl; // <-- Here
    return 0;
}</pre>
```

What do you think will get printed at the indicated point? Why?

Formulate a hypothesis, but don't post anything in chat just yet!

#### Make a Prediction!

• Look over this piece of C++ code:

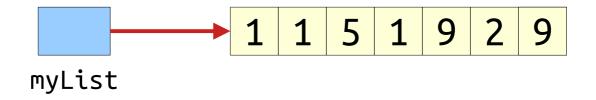
```
void dream(Vector<int> numbers) {
    numbers[1] = 1963;
}
int main() {
    Vector<int> values = { 1929, 1955, 1964 };
    dream(values);
    cout << values << endl; // <-- Here
    return 0;
}</pre>
```

What do you think will get printed at the indicated point? Why?

Now, private chat me your best guess. Not sure? Just answer "??"

## Objects in C++

 In Python, Java, and JavaScript, object variables are not the objects themselves.
 They're pointers to those objects:



• In C++, a variable of object type is an actual, concrete, honest-to-goodness object.

```
int main() {
    Vector<int> values = { 1929, 1955, 1964 };
    dream(values);
    cout << values << endl;
    return 0;
}</pre>
```

```
int main() {
   Vector<int> values = { 1929, 1955, 1964 };
   dream(values);
   cout << values << endl;
   return 0;
}</pre>
```

```
1929 1955 1964

numbers

void dream(Vector<int> numbers) {
    numbers[1] = 1963;
}
```

```
1929 1955 1964

numbers

void dream(Vector<int> numbers) {
numbers[1] = 1963;
}
```

```
1929 1963 1964

numbers

void dream(Vector<int> numbers) {
    numbers[1] = 1963;
}
```

```
1929 1963 1964

numbers

void dream(Vector<int> numbers) {
    numbers[1] = 1963;
}
```

```
int main() {
    Vector<int> values = { 1929, 1955, 1964 };
    dream(values);
    cout << values << endl;
    return 0;
}</pre>
```

```
int main() {
   Vector<int> values = { 1929, 1955, 1964 };
   dream(values);
   cout << values << endl;
   return 0;
}</pre>
```

```
int main() {
     Vector<int> values = { 1929, 1955, 1964 };
     dream(values);
     cout << values << endl;
     return 0;
}</pre>
```

This is the general Start! convention used in C++ programming. Please feel free to ask questions about this over the course Yes! Pass by Need to change of the quarter! argument? reference! Nope! Number! Object! What kind of argument? int string bool Vector<int> double Stack<char> char Queue<int> Pass by Pass by const value! reference!

Time-Out for Announcements!

### Sections

- Discussion sections start this week!
  - Didn't sign up by Sunday at 5PM? The signup link will reopen on Tuesday at 5PM, and you can choose any open section time.
  - If your section time doesn't work for you, you can switch into any section with available space starting Tuesday at 5PM. Visit cs198.stanford.edu to do this.
  - Still doesn't work for you? Ping Neel!
- You'll get your section assignment this Tuesday at 5:00PM.
- Each week we'll release a set of section problems on the course website. *These are not graded*, but we recommend you read over them before your section.

return;

Recursion on Vectors

Finding the Largest Number

# Finding the Largest Number

Our goal is to write a function

```
int maxOf(const Vector<int>& numbers);
that takes as input a Vector<int>, then
returns the largest number in the Vector.
```

- We're going to assume the Vector has at least one element in it; otherwise, it's not possible to return the largest value!
- Let's see how to do this.

# Thinking Recursively

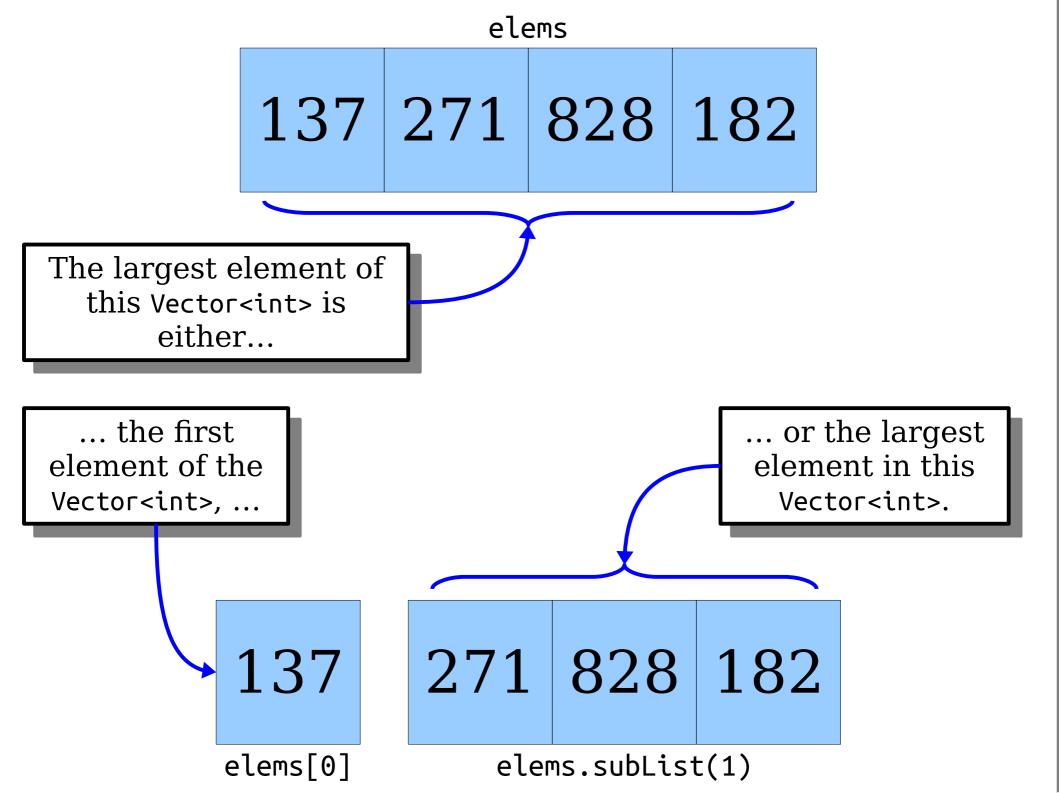
```
These simple cases
if (The problem is very simple) {
                                                  are called base
  Directly solve the problem.
                                                      cases.
  Return the solution.
} else {
  Split the problem into one or more
  smaller problems with the same
  structure as the original.
  Solve each of those smaller problems.
  Combine the results to get the overall
  solution.
  Return the overall solution.
                                                  These are the
                                                 recursive cases.
```

1 2 5 8

1 2 5

I B E X

I B E X



```
int main() {
    Vector<int> v = { 2, 7, 1 };
    cout << maxOf(v) << endl;
    return 0;
}</pre>
```

```
int main() {
    Vector<int> v = { 2, 7, 1 };
    cout << maxOf(v) << endl;
    return 0;
}</pre>
```

```
int main() {
    Vector<int> v = { 2, 7, 1 };
    cout << maxUf(v) << endl;
    return 0;
}</pre>
```

```
int main() {
    Vector<int> v = { 2, 7, 1 };
    cout << maxOf(v) << endl;
    return 0;
}</pre>
```

```
int main() {
    Vector<int> v = { 2, 7, 1 };
    cout << maxOf(v) << endl;
    return 0;
}</pre>
```

```
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];
  } else {
    int first = elems[0];
    Vector<int> rest = elems.subList(1);
    return max(first, maxOf(rest));
  }
}
```

```
int maxOf(const Vector<int>& elems) {
   if (elems.size() == 1) {
      return elems[0];
   } else {
      int first = elems[0];
      Vector<int> rest = elems.subList(1);
      return max(first, maxOf(rest));
   }
}
```

```
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];
  } else {
  int first = elems[0];
    Vector<int> rest = elems.subList(1);
    return max(first, maxOf(rest));
  }
}
```

```
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];
  }
  else {
  int first = elems[0];
    Vector<int> rest = elems.subList(1);
    return max(first, maxOf(rest));
  }
}
```

```
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];
  }
  else {
  int first = elems[0];
    Vector<int> rest = elems.subList(1);
    return max(first, maxOf(rest));
  }
}
```

```
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];
  } else {
    int first = elems[0];
    Vector<int> rest = elems.subList(1);
    return max(first, maxOf(rest));
  }
}
```

```
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];
  } else {
    int first = elems[0];
    Vector<int> rest = elems.subList(1);
    return max(first, maxOf(rest));
  }
}
```

```
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];
  } else {
    int first = elems[0];
    Vector<int> rest = elems sublist(1);
    return max(first, maxOf(rest));
  }
}
```

```
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];
  } else {
    int first = elems[0];
    Vector<int> rest = elems.subList(1);
    return max(first maxOf(rest));
  }
}
```

```
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];
  } else {
    int first = elems[0];
    Vector<int> rest = elems.subList(1);
    return max(first maxOf(rest));
  }
}

construction
    return first = first 2
    rest 7 1
```

```
nt maxOf(const Vector<int>& elems) {
int maxOf(const Vector<int>& elems) {
                                          elems
  if (elems.size() == 1) {
    return elems[0];
  } else {
    int first = elems[0];
    Vector<int> rest = elems.subList(1);
    return max(first, maxOf(rest));
```

```
nt maxOf(const Vector<int>& elems) {
int maxOf(const Vector<int>& elems) {
                                          elems
  if (elems.size() == 1) {
    return elems[0];
  } else {
    int first = elems[0];
    Vector<int> rest = elems.subList(1);
    return max(first, maxOf(rest));
```

```
nt maxOf(const Vector<int>& elems) {
int maxOf(const Vector<int>& elems) {
                                          elems
  if (elems.size() == 1) {
     return_elems[0];
    else {
     int first = elems[0];
    Vector<int> rest = elems.subList(1);
     return max(first, maxOf(rest));
```

```
nt maxOf(const Vector<int>& elems) {
int maxOf(const Vector<int>& elems) {
                                          elems
  if (elems.size() == 1) {
     return elems[0];
    else (
    int first = elems[0];
    Vector<int> rest = elems.subList(1);
    return max(first, maxOf(rest));
```

```
nt maxOf(const Vector<int>& elems) {
int maxOf(const Vector<int>& elems) {
                                          elems
  if (elems.size() == 1) {
     return elems[0];
    else {
                                          first
    int first = elems[0];
    Vector<int> rest = elems.subList(1);
    return max(first, maxOf(rest));
```

```
nt maxOf(const Vector<int>& elems) {
int maxOf(const Vector<int>& elems) {
                                          elems
  if (elems.size() == 1) {
    return elems[0];
  } else {
                                          first
    int first - elems[0];
    Vector<int> rest = elems.subList(1);
    return max(first, maxOf(rest));
```

```
nt maxOf(const Vector<int>& elems) {
int maxOf(const Vector<int>& elems) {
                                          elems
  if (elems.size() == 1) {
    return elems[0];
  } else {
                                          first
    int first - elems[0]:
    Vector<int> rest = elems.subList(1);
    return max(first, maxOf(rest));
                                          rest
```

```
nt maxOf(const Vector<int>& elems) {
int maxOf(const Vector<int>& elems) {
                                          elems
  if (elems.size() == 1) {
    return elems[0];
  } else {
                                          first
    int first = elems[0];
    Vector<int> rest = elems sublist(1);
    return max(first, max0f(rest));
                                          rest
```

```
nt maxOf(const Vector<int>& elems) {
int maxOf(const Vector<int>& elems) {
                                          elems
  if (elems.size() == 1) {
    return elems[0];
  } else {
                                          first
    int first = elems[0];
    Vector<int> rest_= elems.subList(1);
    return max first max0f(rest));
                                          rest
```

```
nt maxOf(const Vector<int>& elems) {
int maxOf(const Vector<int>& elems) {
                                         elems
  if (elems.size() == 1) {
    return elems[0];
  } else {
                                          first
    int first = elems[0];
    Vector<int> rest_= elems.subList(1);
    return max(first max0f(rest));
                                          rest
```

```
nt maxOf(const Vector<int>& elems) {
int maxOf(const Vector<int>& elems) {
                                          elems
  if (elems.size() == 1) {
    return elems[0];
  } else {
                                          first
    int first = elems[0];
    Vector<int> rest = elems sublist(1);
    return max(first, max0f(rest));
                                          rest
```

```
nt maxOf(const Vector<int>& elems) {
int maxOf(const Vector<int>& elems) {
  int maxOf(const Vector<int>& elems) {
                                           elems 1
    if (elems.size() == 1) {
      return elems[0];
    } else {
      int first = elems[0];
      Vector<int> rest = elems.subList(1);
      return max(first, maxOf(rest));
```

```
nt maxOf(const Vector<int>& elems) {
 int maxOf(const Vector<int>& elems) {
  int maxOf(const Vector<int>& elems) {
                                           elems 1
   if (elems.size() == 1) {
      return elems[0];
    } else {
      int first = elems[0];
      Vector<int> rest = elems.subList(1);
      return max(first, maxOf(rest));
```

```
nt maxOf(const Vector<int>& elems) {
int maxOf(const Vector<int>& elems) {
  int maxOf(const Vector<int>& elems) {
                                           elems
    if (elems size() == 1) {
      return elems[0];
     else {
      int first = elems[0];
      Vector<int> rest = elems.subList(1);
      return max(first, maxOf(rest));
```

```
nt maxOf(const Vector<int>& elems) {
int maxOf(const Vector<int>& elems) {
  int maxOf(const Vector<int>& elems) {
                                            elems
    if (elem<u>s size() ==</u> 1) {
      return elems[0];
    } else {
      int first = elems[0];
      Vector<int> rest = elems.subList(1);
      return max(first, maxOf(rest));
```

```
nt maxOf(const Vector<int>& elems) {
int maxOf(const Vector<int>& elems) {
  int maxOf(const Vector<int>& elems) {
                                               elems
    if (elems_size() == 1) {
  return elems[0]; 1
    } else {
      int first = elems[0];
      Vector<int> rest = elems.subList(1);
       return max(first, maxOf(rest));
```

```
nt maxOf(const Vector<int>& elems) {
int maxOf(const Vector<int>& elems) {
                                          elems
  if (elems.size() == 1) {
    return elems[0];
  } else {
                                          first
    int first = elems[0];
    Vector<int> rest = elems sublist(1);
    return max(first, max0f(rest));
                                          rest
```

```
nt maxOf(const Vector<int>& elems) {
int maxOf(const Vector<int>& elems) {
                                          elems
  if (elems.size() == 1) {
    return elems[0];
  } else {
                                          first
    int first = elems[0];
    Vector<int> rest = elems sublist(1);
     return max(first, max0f(rest));
                                          rest
```

```
nt maxOf(const Vector<int>& elems) {
int maxOf(const Vector<int>& elems) {
                                          elems
  if (elems.size() == 1) {
    return elems[0];
  } else {
                                          first
    int first = elems[0];
    Vector<int> rest = elems sublist(1);
     return max(first, max0f(rest));
                                          rest
```

```
nt maxOf(const Vector<int>& elems) {
int maxOf(const Vector<int>& elems) {
                                          elems
  if (elems.size() == 1) {
    return elems[0];
  } else {
                                          first
    int first = elems[0];
    Vector<int> rest = elems sublist(1);
    return max(first, max0f(rest));
                                          rest
```

```
int maxOf(const Vector<int>& elems) {
                                         elems
  if (elems.size() == 1) {
    return elems[0];
  } else {
                                         first
   int first = elems[0];
   Vector<int> rest = elems sublist(1);
    return max(first, max0f(rest));
                                         rest
```

```
int maxOf(const Vector<int>& elems) {
                                         elems
  if (elems.size() == 1) {
    return elems[0];
  } else {
                                         first
    int first = elems[0];
    Vector<int> rest = elems sublist(1);
    return max(first, max0f(rest));
                                         rest
```

```
int maxOf(const Vector<int>& elems) {
                                         elems
  if (elems.size() == 1) {
    return elems[0];
  } else {
                                         first
    int first = elems[0];
    Vector<int> rest = elems sublist(1);
    return max(first, max0f(rest));
                                         rest
```

```
int maxOf(const Vector<int>& elems) {
                                         elems
  if (elems.size() == 1) {
    return elems[0];
  } else {
                                         first
    int first = elems[0];
    Vector<int> rest = elems sublist(1);
    return max(first, max0f(rest));
                                         rest
```

```
int main() {
    Vector<int> v = { 2, 7, 1 };
    cout << maxOf(v) << endl;
    return 0;
}</pre>
```

## Summary from Today

- The Vector<T> type in C++ represents a sequence of elements.
- Parameters in C++ are passed by *value* by default. You can change that to use pass by *reference* if you'd like.
- Use pass-by-const-reference for objects you don't intend to change.
- Each stack frame from a recursive function gets its own copies of all the local variables.

#### Your Action Items

- Read Chapter 5.1 of the textbook.
  - It's all about Vector! There are some goodies there.
- Work on Assignment 1.
  - Aim to complete all three recursion problems by Tuesday evening.
    - Not done by then? Don't worry! Stop by the LaIR to ask questions.
  - Start working on Plotter.
- Explore the max0f example.
  - Tinker and play around with this one. See if you can get very comfortable with how it works.

#### Next Time

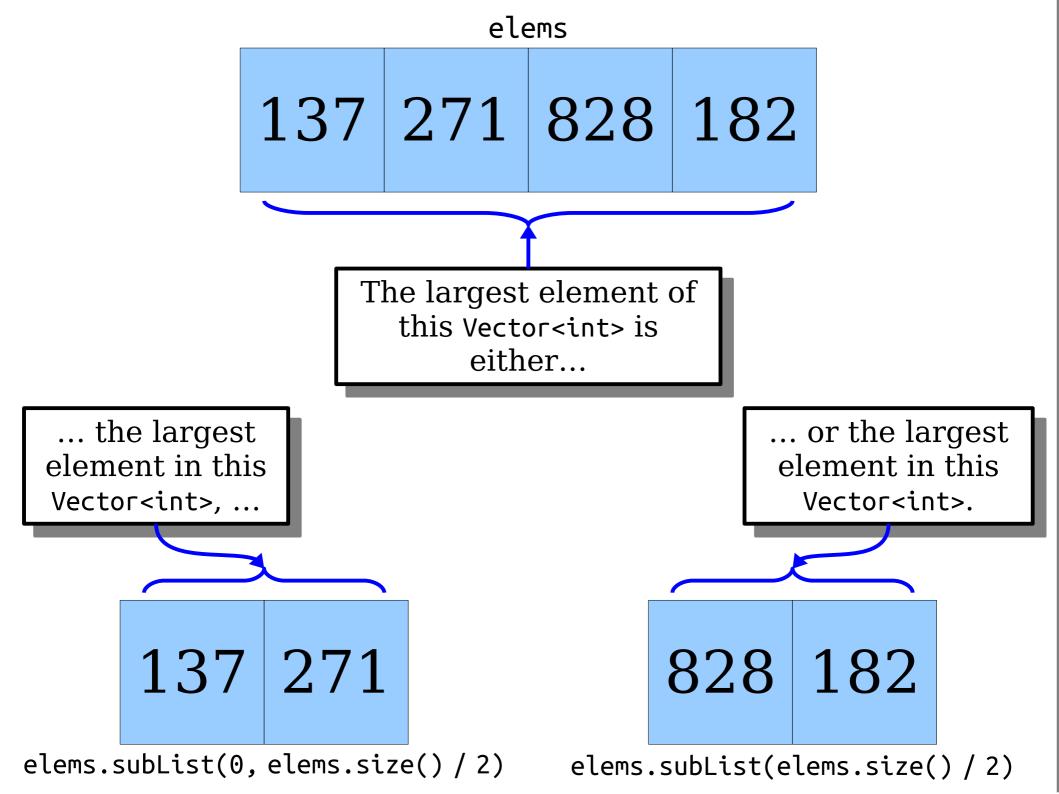
#### Stacks

How driveways relate to parentheses.

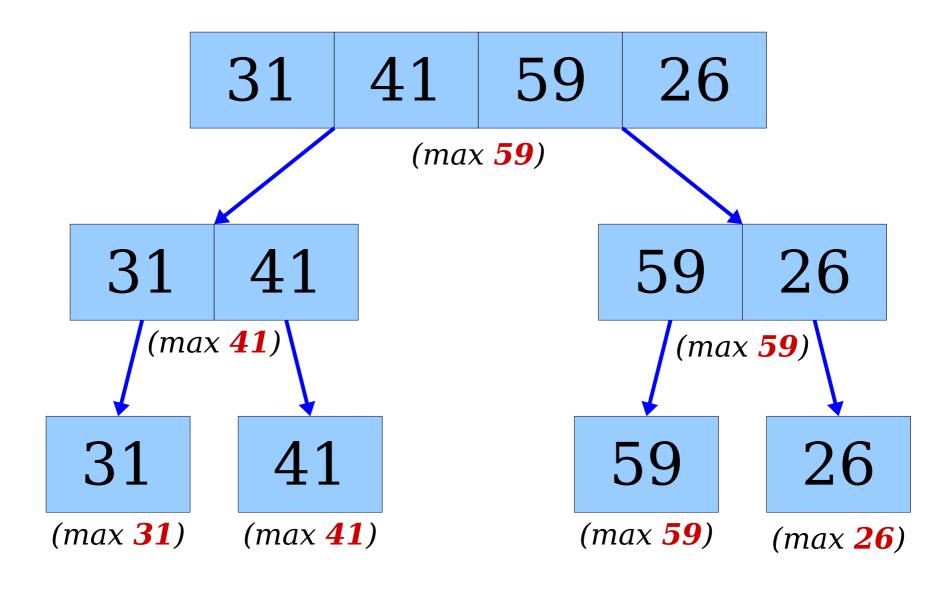
#### Queues

And a fun application.

Appendix: Finding the max, another way.



#### max0f as a Tournament



#### max0f as a Tournament

```
int maxOf(const Vector<int>& elems) {
 if (elems.size() == 1) {
    return elems[0];
 } else {
   int half = elems.size() / 2;
    /* First half of the elements. */
   Vector<int> left = elems.subList(0, half);
    /* Second half of the elements. */
   Vector<int> right = elems.subList(half);
    /* Biggest value in the overall list is either
     * the largest element in the first half or
     * the largest element in the second half.
    return max(maxOf(left), maxOf(right));
```

```
int main() {
   Vector<int> v = { 31, 41, 59, 26 };
   cout << max0f(v) << endl;</pre>
   return 0;
```

```
int main() {
   Vector<int> v = { 31, 41, 59, 26 };
   cout << maxUf(v) << endl;</pre>
   return 0;
```

```
v 31 41 59 26
int main() {
   Vector<int> v = { 31, 41, 59, 26 };
   cout << maxUf(v) << endl;</pre>
   return 0;
```

```
31 41 59 26
int main() {
   Vector<int> v = \{ 31, 41, 59, 26 \};
   cout << max0f(v) << endl;</pre>
   return 0;
```

```
31 41 59 26
int main() {
   Vector<int> v = { 31, 41, 59, 26 };
   cout << max0f(v) << endl;</pre>
   return 0;
```

```
41 59 26
                                                 31 41 59 26
                                         elems
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];
  } else {
   int half = elems.size() / 2;
   Vector<int> left = elems.subList(0, half);
   Vector<int> right = elems.subList(half);
    return max(maxOf(left), maxOf(right));
```

```
41 59 26
                                               31 41 59 26
                                       elems
pt maxOf(const Vector<int>& elems) {
if (elems.size() == 1) {
  return etems[0];
} else {
  int half = elems.size() / 2;
  Vector<int> left = elems.subList(0, half);
  Vector<int> right = elems.subList(half);
  return max(maxOf(left), maxOf(right));
```

```
41 59 26
                                                 31 41 59 26
                                         elems
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return_elems[0];
  } else {
    int half = elems.size() / 2;
   Vector<int> left = elems.subList(0, half);
   Vector<int> right = elems.subList(half);
    return max(maxOf(left), maxOf(right));
```

```
41 59 26
                                                 31 41 59 26
                                         elems
int maxOf(const Vector<int>& elems) {
 if (elems.size() == 1) {
    return elems[0];
   int half = elems.size() / 2;
    Vector<int> left = elems.subList(0, half);
   Vector<int> right = elems.subList(half);
    return max(maxOf(left), maxOf(right));
```

```
41 59 26
                                                 31 41 59 26
                                         elems
                                                 2
                                         half
int maxOf(const Vector<int>& elems) {
 if (elems.size() == 1) {
    return elems[0];
   int half = elems.size() / 2;
    Vector<int> left = elems.subList(0, half);
   Vector<int> right = elems.subList(half);
    return max(maxOf(left), maxOf(right));
```

```
41 59 26
                                                  31 | 41 | 59 | 26
                                          elems
                                                  2
                                          half
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];
  } else {
    int half - elems.size() / 2;
    Vector<int> left = elems.subList(0, half);
    vector<int> right = elems.sublist(half);
    return max(maxOf(left), maxOf(right));
```

```
41 59 26
                                                  31 | 41 | 59 | 26
                                          elems
                                          half
int maxOf(const Vector<int>& elems) {
                                                  31 41
                                          left
  if (elems.size() == 1) {
    return elems[0];
  } else {
    int half - elems.size() / 2;
    Vector<int> left = elems.subList(0, half);
    vector<int> right = elems.sublist(half);
    return max(maxOf(left), maxOf(right));
```

```
41 59 26
                                                  31 | 41 | 59 | 26
                                         elems
                                         half
int maxOf(const Vector<int>& elems) {
                                                  31 41
                                          left
  if (elems.size() == 1) {
    return elems[0];
  } else {
    int half = elems.size() / 2;
    Vactorcint laft - alone sublist(0 half).
   Vector<int> right = elems.subList(half);
    return max(maxOf(left), maxOf(right));
```

```
41 59 26
                                                  31 | 41 | 59 | 26
                                          elems
                                          half
int maxOf(const Vector<int>& elems) {
                                                  31
                                          left
                                                     41
  if (elems.size() == 1) {
    return elems[0];
                                                  59 26
                                          right
  } else {
    int half = elems.size() / 2;
    Vactorcint laft - alone sublist(0 half).
   Vector<int> right = elems.subList(half);
    return max(maxOf(left), maxOf(right));
```

```
41 59 26
                                                 31 | 41 | 59 | 26
                                         elems
                                         half
int maxOf(const Vector<int>& elems) {
                                         left
                                                 31
                                                     41
  if (elems.size() == 1) {
    return elems[0];
                                                 59 26
                                         right
  } else {
    int half = elems.size() / 2;
    Vector<int> left = elems.subList(0, half);
   Vector sint - elems sublist(half).
   return max(maxOf(left), maxOf(right));
```

```
41 59 26
                                                  31 | 41 | 59 | 26
                                          elems
                                          half
int maxOf(const Vector<int>& elems) {
                                                  31
                                          left
                                                      41
  if (elems.size() == 1) {
    return elems[0];
                                                  59 26
                                          right
  } else {
    int half = elems.size() / 2;
    Vector<int> left = elems.subList(0, half);
    Vector<int> right - elems.subList(half);
    return max(maxOf(left), maxOf(right));
```

```
59 26
                                                31 41 59 26
                                       alamc
                                                 31 41
                                         elems
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];
  } else {
    int half = elems.size() / 2;
    Vector<int> left = elems.subList(0, half);
    Vector<int> right = elems.subList(half);
    return max(maxOf(left), maxOf(right));
```

```
11 59 26
                                              31 41 59 26
                                      alamc
                                               31 41
                                       elems
nt maxOf(const Vector<int>& elems) {
if (elems.size() == 1) {
  return etems[0];
} else {
  int half = elems.size() / 2;
  Vector<int> left = elems.subList(0, half);
  Vector<int> right = elems.subList(half);
  return max(maxOf(left), maxOf(right));
```

```
59 26
                                               31 41 59 26
                                       مامر
                                                 31 41
                                         elems
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return_elems[0];
 } else {
    tnt hatf = elems.size() / 2;
    Vector<int> left = elems.subList(0, half);
    Vector<int> right = elems.subList(half);
    return max(maxOf(left), maxOf(right));
```

```
59 26
                                                  41 59 26
                                                21
                                        alamc
                                                 31 | 41
                                         elems
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];
   int half = elems.size() / 2;
    vector<int> left = elems.subList(0, half);
    Vector<int> right = elems.subList(half);
    return max(maxOf(left), maxOf(right));
```

```
59 26
                                                   41 59 26
                                        _l_mc
                                                  31 | 41
                                         elems
                                         half
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];
   int half = elems.size() / 2;
    vector<int> left = elems.subList(0, half);
    Vector<int> right = elems.subList(half);
    return max(maxOf(left), maxOf(right));
```

```
41 59 26
                                                  41 59 26
                                                21
                                                 31 | 41
                                         elems
                                         half
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];
  } else {
   int half - elems size() / 2;
   Vector<int> left = elems.subList(0, half);
    vector<int> right = elems.subList(half);
    return max(maxOf(left), maxOf(right));
```

```
41 59 26
                                                31 41 59 26
                                        alamc
                                                  31 | 41
                                         elems
                                         half
int maxOf(const Vector<int>& elems) {
                                                  31
                                         left
  if (elems.size() == 1) {
    return elems[0];
  } else {
   int half - elems size() / 2;
   Vector<int> left = elems.subList(0, half);
    vector<int> right = elems.subList(half);
    return max(maxOf(left), maxOf(right));
```

```
11 59 26
                                                31 41 59 26
                                        alamc
                                                 31 | 41
                                         elems
                                         half
int maxOf(const Vector<int>& elems) {
                                                 31
                                         left
  if (elems.size() == 1) {
    return elems[0];
  } else {
    int half = elems.size() / 2;
   Vector int left - elems sublist(0 half).
   Vector<int> right = elems.subList(half);
    return max(maxOf(left), maxOf(right));
```

```
41 59 26
                                                31 41 59 26
                                        al ems
                                                  31 | 41
                                         elems
                                         half
int maxOf(const Vector<int>& elems) {
                                                  31
                                         left
  if (elems.size() == 1) {
    return elems[0];
                                                  41
                                          right
  } else {
    int half = elems.size() / 2;
   Vector int left - elems sublist(0 half).
   Vector<int> right = elems.subList(half);
    return max(maxOf(left), maxOf(right));
```

```
59 26
                                                   41 59 26
                                                 21
                                        _l_mc
                                                  31 | 41
                                          elems
                                          half
int maxOf(const Vector<int>& elems) {
                                                  31
                                          left
  if (elems.size() == 1) {
    return elems[0];
                                                  41
                                          right
  } else {
    int half = elems.size() / 2;
    Vector<int> left = elems.subList(0, half);
    Vector < int > right = elems sublist(half)
    return max(maxOf(left), maxOf(right));
```

```
11 59 26
                                                31 41 59 26
                                        alamc
                                                  31 | 41
                                          elems
                                          half
int maxOf(const Vector<int>& elems) {
                                                  31
                                          left
  if (elems.size() == 1) {
    return elems[0];
                                                  41
                                          right
  } else {
    int half = elems.size() / 2;
    Vector<int> left = elems.subList(0, half);
    Vector<int> right = elems.subList(half);
    return max(maxOf(left), maxOf(right));
```

```
59 26
                                                 11 59 26
                                                31 /11
                                       مامسر
                                         elems
                                                 31
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];
  } else {
    int half = elems.size() / 2;
    Vector<int> left = elems.subList(0, half);
    Vector<int> right = elems.subList(half);
    return max(maxOf(left), maxOf(right));
```

```
11 59 26
                                               11 59 26
                                              31 /11
                                      _l_mc
                                       elems
                                               31
at maxOf(const Vector<int>& elems) {
if (elems.size() == 1) {
  return etems[0];
} else {
  int half = elems.size() / 2;
  Vector<int> left = elems.subList(0, half);
  Vector<int> right = elems.subList(half);
  return max(maxOf(left), maxOf(right));
```

```
11 59 26
                                                 11 59 26
                                                31 /11
                                       _l _mc
                                         elems
                                                 31
int maxOf(const Vector<int>& elems) {
  if (elems size() == 1) {
   return elems[0];
   etse {
    int half = elems.size() / 2;
    Vector<int> left = elems.subList(0, half);
    Vector<int> right = elems.subList(half);
    return max(maxOf(left), maxOf(right));
```

```
11 59 26
                                                     11 59 26
                                                    31 /11
                                           _l _mc
                                             elems
                                                     31
int maxOf(const Vector<int>& elems) {
  if (elems size() == 1) {
  return elems[0]; 31
    etse {
    int half = elems.size() / 2;
    Vector<int> left = elems.subList(0, half);
    Vector<int> right = elems.subList(half);
    return max(maxOf(left), maxOf(right));
```

```
41 59 26
                                                31 41 59 26
                                        alems
                                                  31 | 41
                                          elems
                                          half
int maxOf(const Vector<int>& elems) {
                                                  31
                                          left
  if (elems.size() == 1) {
    return elems[0];
                                                  41
                                          right
  } else {
    int half = elems.size() / 2;
    Vector<int> left = elems.subList(0, half);
    Vector<int> right = elems.subList(half);
    return max(maxOf(left), maxOf(right));
```

```
41 59 26
                                                31 41 59 26
                                        _l_mc
                                                  31 | 41
                                          elems
                                          half
int maxOf(const Vector<int>& elems) {
                                                  31
                                          left
  if (elems.size() == 1) {
    return elems[0];
                                                  41
                                          right
  } else {
    int half = elems.size() / 2;
    Vector<int> left = elems.subList(0, half);
    Vector<int> right = elems sublist(half);
    return max(maxOf(left), maxOf(right));
                   31
```

```
59 26
                                                 11 59 26
                                                31 /11
                                       مامسر
                                         elems
                                                 41
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];
  } else {
    int half = elems.size() / 2;
    Vector<int> left = elems.subList(0, half);
    Vector<int> right = elems.subList(half);
    return max(maxOf(left), maxOf(right));
```

```
11 59 26
                                               11 59 26
                                              31 /11
                                      _l_mc
                                       elems
                                                41
at maxOf(const Vector<int>& elems) {
if (elems.size() == 1) {
  return etems[0];
} else {
  int half = elems.size() / 2;
  Vector<int> left = elems.subList(0, half);
  Vector<int> right = elems.subList(half);
  return max(maxOf(left), maxOf(right));
```

```
59 26
                                                 11 59 26
                                               31 /11
                                       _l _mc
                                         elems
                                                 41
int maxOf(const Vector<int>& elems) {
  if (elems size() == 1) {
   return elems[0];
  etse
    int half = elems.size() / 2;
   Vector<int> left = elems.subList(0, half);
   Vector<int> right = elems.subList(half);
    return max(maxOf(left), maxOf(right));
```

```
11 59 26
                                                     11 59 26
                                                   31 /11
                                          _l_mc
                                            elems
                                                     41
int maxOf(const Vector<int>& elems) {
  if (elems size() == 1) {
  return elems[0]; 41
  etse
    int half = elems.size() / 2;
    Vector<int> left = elems.subList(0, half);
    Vector<int> right = elems.subList(half);
    return max(maxOf(left), maxOf(right));
```

```
41 59 26
                                                31 41 59 26
                                        alams
                                                  31 | 41
                                         elems
                                          half
int maxOf(const Vector<int>& elems) {
                                                  31
                                          left
  if (elems.size() == 1) {
    return elems[0];
                                                  41
                                          right
  } else {
    int half = elems.size() / 2;
    Vector<int> left = elems.subList(0, half);
    Vector<int> right = elems sublist(half);
    return max(max0f(left), max0f(right));
                   31
```

```
41 59 26
                                                31 41 59 26
                                        _l_mc
                                                  31 | 41
                                          elems
                                          half
int maxOf(const Vector<int>& elems) {
                                                  31
                                          left
  if (elems.size() == 1) {
    return elems[0];
                                                  41
                                          right
  } else {
    int half = elems.size() / 2;
    Vector<int> left = elems.subList(0, half);
    Vector<int> right = elems sublist(half);
    return max(maxOf(left), maxOf(right))
```

```
41 59 26
                                                31 41 59 26
                                        alamc
                                                  31 | 41
                                          elems
                                          half
int maxOf(const Vector<int>& elems) {
                                                  31
                                          left
  if (elems.size() == 1) {
    return elems[0];
                                                  41
                                          right
  } else {
    int half = elems.size() / 2;
    Vector<int> left = elems.subList(0, half);
    Vector<int> right = elems sublist(half);
    return max(max0f(left), max0f(right))
```

```
11 59 26
                                                31 41 59 26
                                        alamc
                                                  31 | 41
                                          elems
                                          half
int maxOf(const Vector<int>& elems) {
                                                  31
                                          left
  if (elems.size() == 1) {
    return elems[0];
                                                  41
                                          right
  } else {
    int half = elems.size() / 2;
    Vector<int> left = elems.subList(0, half);
    Vector<int> right = elems sublist(half);
    return max(maxOf(left), maxOf(right))
```

```
41 59 26
                                                  31 | 41 | 59 | 26
                                          elems
                                          half
int maxOf(const Vector<int>& elems) {
                                                  31
                                          left
                                                      41
  if (elems.size() == 1) {
    return elems[0];
                                                  59 26
                                          right
  } else {
    int half = elems.size() / 2;
    Vector<int> left = elems.subList(0, half);
    Vector<int> right - elems.subList(half);
    return max(maxOf(left), maxOf(right));
```

```
41 59 26
                                                  31 | 41 | 59 | 26
                                          elems
                                          half
int maxOf(const Vector<int>& elems) {
                                                  31
                                          left
                                                      41
  if (elems.size() == 1) {
    return elems[0];
                                                  59 26
                                          right
  } else {
    int half = elems.size() / 2;
    Vector<int> left = elems.subList(0, half);
    Vector<int> right = elems.subList(half);
    return max(maxOf(left), maxOf(right));
```

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59 26
                                               31 41 59 26
                                       مامر
                                                 59 26
                                         elems
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];
  } else {
    int half = elems.size() / 2;
    Vector<int> left = elems.subList(0, half);
    Vector<int> right = elems.subList(half);
    return max(maxOf(left), maxOf(right));
```

```
59 26
                                               31 41 59 26
                                       alamc
                                                 59 26
                                        elems
int maxOf(const Vector<int>& elems) {
 if (elems.size() == 1) {
   return etems[0];
 } else {
   int half = elems.size() / 2;
   Vector<int> left = elems.subList(0, half);
   Vector<int> right = elems.subList(half);
   return max(maxOf(left), maxOf(right));
```

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59 26
                                               31 41 59 26
                                       مامر
                                                 59 26
                                         elems
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return_elems[0];
 } else {
    tnt hatf = elems.size() / 2;
    Vector<int> left = elems.subList(0, half);
    Vector<int> right = elems.subList(half);
    return max(maxOf(left), maxOf(right));
```

```
59 26
                                                  41 59 26
                                               21
                                       alamc
                                                 59 26
                                         elems
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];
   int half = elems.size() / 2;
    vector<int> left = elems.subList(0, half);
    Vector<int> right = elems.subList(half);
    return max(maxOf(left), maxOf(right));
```

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59 26
                                                   41 59 26
                                        _l _mc
                                                  59 26
                                         elems
                                         half
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];
   int half = elems.size() / 2;
    vector<int> left = elems.subList(0, half);
    Vector<int> right = elems.subList(half);
    return max(maxOf(left), maxOf(right));
```

```
41 59 26
                                                  41 59 26
                                               21
                                                 59 26
                                         elems
                                         half
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];
  } else {
   int half - elems size() / 2;
   Vector<int> left = elems.subList(0, half);
    vector<int> right = elems.subList(half);
    return max(maxOf(left), maxOf(right));
```

```
41 59 26
                                                  41 59 26
                                                21
                                                 59 26
                                         elems
                                         half
int maxOf(const Vector<int>& elems) {
                                                 59
                                         left
  if (elems.size() == 1) {
    return elems[0];
  } else {
   int half - elems size() / 2;
   Vector<int> left = elems.subList(0, half);
    vector<int> right = elems.subList(half);
    return max(maxOf(left), maxOf(right));
```

```
11 59 26
                                                   41 59 26
                                        _l_mc
                                                  59 26
                                         elems
                                         half
int maxOf(const Vector<int>& elems) {
                                                  59
                                         left
  if (elems.size() == 1) {
    return elems[0];
  } else {
    int half = elems.size() / 2;
   Vector<int> right = elems.subList(half);
    return max(maxOf(left), maxOf(right));
```

```
41 59 26
                                                   41 59 26
                                        alams
                                                  59 26
                                          elems
                                          half
int maxOf(const Vector<int>& elems) {
                                                  59
                                          left
  if (elems.size() == 1) {
    return elems[0];
                                                  26
                                          right
  } else {
    int half = elems.size() / 2;
   Vector<int> right = elems.subList(half);
    return max(maxOf(left), maxOf(right));
```

```
59 26
                                                   41 59 26
                                                21
                                        _l _mc
                                                  59 26
                                         elems
                                         half
int maxOf(const Vector<int>& elems) {
                                                  59
                                         left
  if (elems.size() == 1) {
    return elems[0];
                                                  26
                                         right
  } else {
    int half = elems.size() / 2;
    Vector<int> left = elems.subList(0, half);
    Vector<int> right = elems sublist(half)
    return max(maxOf(left), maxOf(right));
```

```
11 59 26
                                                   41 59 26
                                                21
                                        _l _mc
                                                  59 26
                                         elems
                                          half
int maxOf(const Vector<int>& elems) {
                                                  59
                                          left
  if (elems.size() == 1) {
    return elems[0];
                                                  26
                                          right
  } else {
    int half = elems.size() / 2;
    Vector<int> left = elems.subList(0, half);
    Vector<int> right = elems.subList(half);
    return max(maxOf(left), maxOf(right));
```

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59 26
                                                 11 59 26
                                               59 26
                                       مامر
                                                 59
                                         elems
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];
 } else {
    int half = elems.size() / 2;
    Vector<int> left = elems.subList(0, half);
    Vector<int> right = elems.subList(half);
    return max(maxOf(left), maxOf(right));
```

```
11 59 26
                                               11 59 26
                                              59 26
                                      _l_mc
                                               59
                                       elems
ht maxOf(const Vector<int>& elems) {
if (elems.size() == 1) {
  return etems[0];
} else {
  int half = elems.size() / 2;
  Vector<int> left = elems.subList(0, half);
  Vector<int> right = elems.subList(half);
  return max(maxOf(left), maxOf(right));
```

```
11 59 26
                                                 11 59 26
                                                59 26
                                       _l_mc
                                                 59
                                         elems
int maxOf(const Vector<int>& elems) {
  if (elems size() == 1) {
   return elems[0];
   etse {
    int half = elems.size() / 2;
    Vector<int> left = elems.subList(0, half);
    Vector<int> right = elems.subList(half);
    return max(maxOf(left), maxOf(right));
```

```
11 59 26
                                                     11 59 26
                                                   59 26
                                          _l_mc
                                                     59
                                            elems
int maxOf(const Vector<int>& elems) {
  if (elems size() == 1) {
return elems[0]; 59
    etse {
    int half = elems.size() / 2;
    Vector<int> left = elems.subList(0, half);
    Vector<int> right = elems.subList(half);
    return max(maxOf(left), maxOf(right));
```

```
41 59 26
                                                31 41 59 26
                                        alamc
                                                  59 26
                                         elems
                                         half
int maxOf(const Vector<int>& elems) {
                                                  59
                                         left
  if (elems.size() == 1) {
    return elems[0];
                                                 26
                                         right
  } else {
    int half = elems.size() / 2;
    Vector<int> left = elems.subList(0, half);
    Vector<int> right = elems.subList(half);
    return max(maxOf(left), maxOf(right));
                    59
```

```
41 59 26
                                                31 41 59 26
                                        _l_mc
                                                  59 26
                                         elems
                                         half
int maxOf(const Vector<int>& elems) {
                                                  59
                                         left
  if (elems.size() == 1) {
    return elems[0];
                                                  26
                                         right
  } else {
    int half = elems.size() / 2;
    Vector<int> left = elems.subList(0, half);
    Vector<int> right = elems sublist(half);
    return max(maxOf(left), maxOf(right));
                    59
```

```
59 26
                                                 11 59 26
                                               59 26
                                       alamc
                                                 26
                                         elems
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];
  } else {
    int half = elems.size() / 2;
    Vector<int> left = elems.subList(0, half);
    Vector<int> right = elems.subList(half);
    return max(maxOf(left), maxOf(right));
```

```
11 59 26
                                               11 59 26
                                              59 26
                                      _l_mc
                                               26
                                       elems
ht maxOf(const Vector<int>& elems) {
if (elems.size() == 1) {
  return etems[0];
} else {
  int half = elems.size() / 2;
  Vector<int> left = elems.subList(0, half);
  Vector<int> right = elems.subList(half);
  return max(maxOf(left), maxOf(right));
```

```
59 26
                                                 11 59 26
                                                59 26
                                       _l_mc
                                                 26
                                         elems
int maxOf(const Vector<int>& elems) {
  if (elems size() == 1) {
   return elems[0];
   etse {
    int half = elems.size() / 2;
    Vector<int> left = elems.subList(0, half);
    Vector<int> right = elems.subList(half);
    return max(maxOf(left), maxOf(right));
```

```
11 59 26
                                                     11 59 26
                                                   59 26
                                          _l_mc
                                                     26
                                            elems
int maxOf(const Vector<int>& elems) {
  if (elems size() == 1) {
return elems[0]; 26
    etse {
    int half = elems.size() / 2;
    Vector<int> left = elems.subList(0, half);
    Vector<int> right = elems.subList(half);
    return max(maxOf(left), maxOf(right));
```

```
41 59 26
                                                31 41 59 26
                                        alams
                                                 59 26
                                         elems
                                         half
int maxOf(const Vector<int>& elems) {
                                                 59
                                         left
  if (elems.size() == 1) {
    return elems[0];
                                                 26
                                         right
  } else {
    int half = elems.size() / 2;
    Vector<int> left = elems.subList(0, half);
    Vector<int> right = elems sublist(half);
    return max(maxOf(left), maxOf(right));
                                 26
                    59
```

```
41 59 26
                                                31 41 59 26
                                        alams
                                                 59 26
                                         elems
                                         half
int maxOf(const Vector<int>& elems) {
                                                 59
                                         left
  if (elems.size() == 1) {
    return elems[0];
                                                 26
                                         right
  } else {
    int half = elems.size() / 2;
    Vector<int> left = elems.subList(0, half);
    Vector<int> right = elems sublist(half):
    return max(max0f(left), max0f(right));
                    59
                                 26
```

```
41 59 26
                                                31 41 59 26
                                        alamc
                                                  59 26
                                         elems
                                         half
int maxOf(const Vector<int>& elems) {
                                                  59
                                         left
  if (elems.size() == 1) {
    return elems[0];
                                                  26
                                         right
  } else {
    int half = elems.size() / 2;
    Vector<int> left = elems.subList(0, half);
    Vector<int> right = elems sublist(half):
    return max(max0f(left), max0f(right));
                         59
```

```
11 59 26
                                                31 41 59 26
                                        _l _mc
                                                  59 26
                                         elems
                                         half
int maxOf(const Vector<int>& elems) {
                                                  59
                                         left
  if (elems.size() == 1) {
    return elems[0];
                                                  26
                                         right
  } else {
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    return max(maxOf(left), maxOf(right));
                         59
```

```
41 59 26
                                                  31 | 41 | 59 | 26
                                          elems
                                          half
int maxOf(const Vector<int>& elems) {
                                                  31
                                          left
                                                     41
  if (elems.size() == 1) {
    return elems[0];
                                                  59 26
                                          right
  } else {
    int half = elems.size() / 2;
    Vector<int> left = elems.subList(0, half);
    Vector<int> right = elems.subList(half);
    return max(maxOf(left), maxOf(right));
                                  59
```

```
41 59 26
                                                  31 | 41 | 59 | 26
                                          elems
                                          half
int maxOf(const Vector<int>& elems) {
                                          left
                                                  31
                                                      41
  if (elems.size() == 1) {
    return elems[0];
                                                  59 26
                                          right
  } else {
    int half = elems.size() / 2;
    Vector<int> left = elems.subList(0, half);
    Vector<int> right - elems.subList(half);
    return max(maxOf(left), maxOf(right));
                                  59
```

```
41 59 26
                                                  31 | 41 | 59 | 26
                                          elems
                                          half
int maxOf(const Vector<int>& elems) {
                                          left
                                                  31
                                                      41
  if (elems.size() == 1) {
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                                                  59 26
                                          right
  } else {
    int half = elems.size() / 2;
    Vector<int> left = elems.subList(0, half);
    Vector<int> right - elems.subList(half);
    return max(maxOf(left), maxOf(right));
                        59
```

```
41 59 26
                                                 31 41 59 26
                                         elems
                                         half
int maxOf(const Vector<int>& elems) {
                                         left
                                                 31
                                                    41
  if (elems.size() == 1) {
    return elems[0];
                                                 59 26
                                         right
  } else {
   int half = elems.size() / 2;
    Vector<int> left = elems.subList(0, half);
   Vector<int> right - elems.subList(half):
   return max(maxOf(left), maxOf(right));
                        59
```

```
31 41 59 26
int main() {
   Vector\{ int > v = \{ 31, 41, 59, 26 \} ;
   cout << max0f(v) << endl;</pre>
   return 0; 59
```