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Modelling the Draganflyer four-rotor helicopter

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Abstract—**The Draganflyer is a radio-controlled helicopter. It is powered by 4 rotors and is capable of motion in air in 6 degrees of freedom and of stable hovering. Flying it requires a high degree of skill, with the operator continually making small adjustments. *In this paper, we do a theoretical analysis of the* dynamics of the Draganflyer in order to develop a model of it from which we can develop a computer control system for stable hovering and indoor flight.**

Index Terms* - *four-rotor helicopter; indoor flight, stable hovering, helicopter dynamics

I. INTRODUCTION

A number of research groups are investigating the problem of developing an indoor flying robot. This research is stimulated both by the problem of achieving indoor flight and the practical application of disaster search in partially collapsed buildings [1, 2]. They have settled on three types of flying craft: ultra-light fixed-wing planes powered by a propeller, micro-mechanical flying insects and four-rotor helicopters.

Ultra-light fixed-wing planes attempt to navigate in large indoor environments by flying very slowly [3, 4]. They require space to turn because they cannot hover, so navigating them in close quarters is very difficult. Their design involves aerodynamics at low Reynolds numbers, ultra-light weight building techniques and optimization of the motor/propeller system. Nicoud et al. [3] demonstrated a 47gram plane that can fly in a 10m x 10m room at speeds as low as 1.4m/sec. To achieve maneuverability at low speed, direction is controlled by rotating the thrust system.

Flapping wing micro-mechanical craft attempt to achieve stationary flight by controlling their attitude and vertical thrust like a hummingbird or an insect [5, 6]. Flapping wings are claimed to be the only way to reduce wingspan below 100mm. While sustained flight has been demonstrated, motion control sufficient for navigation has not been, owing to the time-varying nature of the aerodynamic forces during a wing beat. Position and velocity control are achieved with attitude control: tilting and banking the body alter the direction of the propulsion force. Altitude control is achieved with vertical thrust. Both time invariant and continuous control schemes fail.

Four-rotor helicopters (often called X4-flyers) attempt to achieve stable hovering and precise flight by balancing the forces produced by the four rotors [7, 8, 9, 10]. A single rotor helicopter (with tail rotor to oppose the induced moment) is very dangerous in an indoor environment because of the potential for the exposed rotor blades to collide with something and cause the helicopter to crash. Even skilled pilots have trouble navigating them close to the outside of buildings [11].

Four-rotor helicopters are attractive because the rotors are smaller and can be enclosed, making them safer [12]. Also, it may be possible to achieve more stationary hovering with four thrust forces acting at a distance from the centre of gravity than with one force acting through the centre of gravity.

The Draganflyer [Fig.1.] is a radio-controlled four-rotor helicopter available from RCToys [13]. It is an ideal unit for learning about the problems of flying these craft. While the advertisement claims that it is easier to fly than a helicopter and the web site shows videos of people flying it, we have not yet managed to achieve a stable hover. Close examination of the promotional videos reveals that it is being flown in large auditoriums, is continually moving in the air, and the skilled pilots are continuously making control corrections.

In this paper, we analyse the dynamics of the Draganflyer in order to understand the aerodynamic forces and torques acting on it. The model derived in this analysis will lead to a model suitable for computer-based control. While we can draw from the modeling of conventional helicopters [14, 15, 16] the physical differences result in a different model.

We have implemented the model in a flight simulator to help verify it. Pilots can attempt to fly it in simulation and watch displays of motion parameters. We crash the simulated Draganflyer just as often as the real one but with less damage.

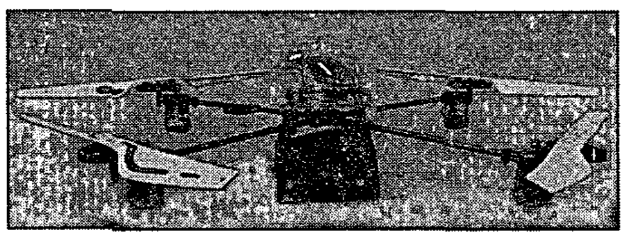

Figure 1. Draganflyer

II. DRAGANFLYER

Motion of the Draganflyer is controlled by varying the lift force produced by its four rotors. Unlike conventional helicopters, that can modify the lift force vector in both magnitude and direction by varying rotation speed, angle of blade attack (pitch angle) and cyclic pitch angle, the Draganflyer can only vary rotor speed. The pitch angle is fixed, although, as the rotors are made from flexible plastic, the air drag forces distort them causing the pitch angle to change. Thus, lift force is a function of the sum of the four rotor speeds and rotational torque is a function of their differences.

It is an under-actuated vehicle with four input forces producing motion in 6 degrees of freedom. The operator of the radio controller has four control actuations: throttle (motor speed), roll, pitch and yaw. Effectively, he is controlling it in spherical coordinates. With combinations of these actuations he can control motion in 6 degrees of freedom, though it is impossible to achieve uncoupled motion. With a combination of fine and continuously changing actuations in several dimensions a very skilled operator may get the craft to hover.

The control electronics performs three functions: receipt of the servo commands from the radio link, closed loop stabilization of roll, pitch and yaw rates, and mapping commands from spherical coordinates to four motor speeds. The commands are encoded and transmitted from the controller to the craft using pulse width modulation to perform time division multiplexing (standard in radio control). The receiver de-multiplexes these commands.

Closed loop control stabilizes roll, pitch and yaw using feedback from 3 solid-state rate gyroscopes. Closing the loop in the rotation dimensions has two effects. First, it means that the rotation of the craft is proportional to the command given by the operator, making the craft easier to fly. Second, it will attempt to correct for any external disturbance, such as a wind gust, that causes the vehicle to rotate.

Mapping the commands from control space to force space requires a model of the forces and their interactions. From observing the Draganflyer's response to commands, its electronics calculates the sum and differences of forces. This is adequate for human control, provided the craft responds as the pilot expects, because the pilot can visually observe the motion of the craft and make corrections. However, for computer control, a more detailed model is needed, particularly to achieve the precise control required for indoor flight.

It is a very dynamic vehicle because the forces opposing motion are small. It has highly coupled dynamics: a change in the speed of one rotor results in motion in at least 3 degrees of freedom. For example, reducing the speed of the right rotor will cause the craft to roll to the right due to the imbalance between left and right lift forces. It will cause the craft to yaw to the right due to the imbalance in torque between the right-left motor pair and the front-back pair. The roll will cause the craft to translate to the right, as the rotor forces are now directed toward the left as well as down. The yaw will cause the translation to change direction toward the front.

When a rotor turns it is opposed by air drag. The reactive force of the air on the rotor results in a reactive moment known as the induced moment. The induced moment acts on a rotor in the direction opposite to the rotor. A conventional helicopter uses a tail rotor to counteract the induced moment from the main rotor. In the Draganflyer the left-right pair of motors and the front-back pair of motors rotate in opposite directions to produce counter rotating torque.

In the example above, the right motor is rotating clockwise (looking down from above), producing an induced moment to cause the fuselage to turn anticlockwise (or to the left). Reducing right motor speed will reduce the anticlockwise moment and the craft will yaw to the right.

The induced torques from the four rotors cancel through the airframe, which places considerable stress on the airframe. This is a significant weakness of this design, and results in both distortion of the frame during flight and fixers coming loose due to the resultant vibrations. The small size, highly coupled dynamics, low air drag on the fuselage and high air drag on the rotors pose significant challenges in the control of four-rotor hovering robots.

III. INDOOR FLIGHT

To be useful for searching and navigating in an urban disaster environment, a hovering robot must be able to fly in restricted spaces. Restricted spaces include hallways, stairwells, and open windows. A restricted space is characterised by the width of the opening through which the robot must fly being less than twice the width of the robot. Flying in restricted indoor spaces is an unsolved problem.

The width of an opening constrains the motion. First, the robot must fly through the opening without touching the sides, which requires very stable control of flight. Second, the robot must be able to turn within the opening so that it can scan the environment with sensors, which requires very stable control of hovering. Third, the robot must navigate through a sequence of restricted spaces to achieve a surveillance task, which requires accurate localization using sensor measurements.

In recent years many researchers have worked on the problems of autonomous flight in open spaces. In these environments variation of the location of the craft from a trajectory by 50% of it's width does not result in collision with an object in the environment. Drift of the craft around a hovering location does not stop it turning, but may impact on the quality of sensor data.

The problem of flying in a restricted space can be decomposed into three sub problems: 1) flying through an opening, 2) flying along a corridor and 3) turning. The solution to each of these problems requires developments in control, dynamic models, and range and motion sensing.

To fly along a narrow corridor a robot can follow a wall, track down the centre of the corridor or follow a planned trajectory. Each approach requires the robot to follow a defined path precisely. We define "precise" as variations from the desired path being less than half the width of the robot. This is normal for robots sitting on the ground, where friction between the wheels and the ground constrains the sideways motion of the robot. The motion of a hovering robot is only constrained by air friction and gyroscopic torques. Consequently, tiny wind forces and minor variations in control forces result in considerable deviation from planned paths.

Due to this lack of opposing forces, a hovering robot is capable of rapid motion in any direction, including up against gravity. We have two control problems: 1) control of stable hovering and 2) control of directed motion. The complexity of the control results from the variable nature of the aerodynamic forces in different conditions of flight. Although aerodynamic effects are continuous, the models reported in the literature for helicopters change when they move from stationary to forward flight [15]. These models are also modified by environmental parameters, in particular the ground effect. Stable control requires the measurement and control of motion in six degrees of freedom using different models in different phases of flight.

a) Hovering Control

While stable hovering is not novel, maintaining precise position while hovering is. To turn in a restricted space, a hovering robot must be able to hover precisely in one location. Small wind or control forces will easily push it away from that spot. As air is constantly moving (even in the stillest room there are micro convection currents), the requirements on the control system are much greater than when flying in open space. Computer control of hovering in six degrees of freedom requires careful attention to decoupling control forces and to decoupling the measurement of motion. This level of control can only be achieved by sensing of robot location relative to close objects.

b) Motion Control

Trajectory following requires the precise control of directed motion. It appears that the dynamics of helicopters is simplest for maneuvers close to gliding flight. There is considerable cross coupling between the forces produced by the 4 vertical thrusters. Due to its design it is not possible to move the Draganflyer in an uncoupled way, for example to rotate while holding the craft horizontal. As a result of this under actuation, standard control techniques for actuated systems don't work well on this craft.

IV. FORCE BALANCE

Control of the Draganflyer can be thought of as achieving force and torque balance. It will hover in the air when there is no net force in any degree of freedom. The smallest force will result in linear acceleration (Newton's second law $f = ma$) and the smallest torque will result in angular acceleration (□= I□).

Force balance for a stable hover is achieved when the sum of the thrust from the 4 rotors equals the weight of the craft (Fig.2).

$$f_1 + f_2 + f_3 + f_4 = (m_0 + m_1 + m_2 + m_3 + m_4) * g \qquad (1)$$

where 0, 1, 2, 3, 4 are the centre, front, right, rear and left parts, as shown in Fig. 2.

Motion is opposed by forces from three sources: gravity, inertia and air drag. Gravity opposes vertical motion and results in the consumption of considerable energy to keep the Draganflyer in the air. The heaviest component is the battery pack. As the horizontal velocity of the Draganflyer is usually low, and it has no aerodynamic surfaces to generate lift when moving, all lift is generated by the rotors. Consequently, the time the Draganflyer can stay in the air is determined by a balance between the lift coefficient of the rotors and the weight of the batteries.

Inertia opposes linear and rotary acceleration helping to stabilize motion. The inertia due to the mass of the motors resists angular acceleration. By locating the centre of gravity below the aerodynamic centre of the craft (through placing the batteries below the intersection of the airframe), the inertia due to the mass of the batteries resists both pitch and roll.

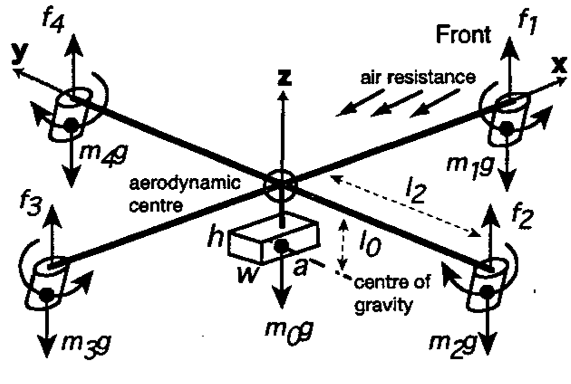

Figure 2. Force Balance

Air drag provides damping to linear and rotary motion. As the drag force is proportional to velocity, drag forces are small except for those in opposition to the rotation of the rotor. Consequently, air drag provides damping to rotor velocity and hence slows the response of the craft to external forces, such as wind gusts.

V. COORDINATE FRAMES

To develop our analysis of the aerodynamics, we use two coordinate frames: a robot frame and a coincident instantaneous frame. The x axis of the robot frame points forward and the z axis points up. The axes of the motors are parallel to the z axis, as shown in Fig.2. The robot frame is fixed to the robot and moves with the robot relative to the instantaneous frame. The instantaneous frame is fixed in the world at the current location of the robot frame.

All equations are expressed in the instantaneous frame. When absolute values are required in the world frame they are calculated in the instantaneous frame and transformed to the world frame. Introducing the instantaneous frame results in simpler equations, except that the gravitational vector must be transformed into the instantaneous frame every time it is used.

VI. INERTIA

The airframe of the Draganflyer is a carbon fibre cross that supports the four motors at the ends of the cross beams and the electronics and battery at the intersection of the cross. We can model the inertia with 5 masses attached to a centre of rotation by 5 thin rods. First, we will model the cross spars with two slender rods that intersect. As shown in Fig. 2., the arms of the cross are coincident with the x and y axes of the robot frame. The inertia of the cross for rotation about the point of intersection is:

$$I_{cx} = I_{cy} = \frac{m_c l^2}{12} + \frac{m_c d^2}{2} \qquad (2)$$

$$I_{cz} = \frac{m_c l^2}{6} \qquad (3)$$

where $l = l_1 + l_3 = l_2 + l_4$ is the length and d the radius of the cross pieces.

Second, we will model the motors as cylinders of radius r and length p that hang below the ends of the cross spars. The inertia for the front motor (1) for rotation about the point of intersection of the cross is:

$$I_{mx1} = \frac{m_1 r^2}{4} + \frac{m_1 p^2}{3} \tag{4}$$

$$I_{my1} = \frac{m_1 r^2}{4} + \frac{m_1 p^2}{3} + m_1 l_1^2 \tag{5}$$

$$I_{mz1} = \frac{m_1 r^2}{2} + m_1 l_1^2 \tag{6}$$

Third, we will model the battery hanging below the intersection with a rectangular parallelepiped with dimensions a, w, and h, and lump the mass of the electronics into it.

$$I_{bx} = \frac{m_b\left(w^2 + h^2\right)}{12} + m_b l_0^2 \tag{7}$$

$$I_{by} = \frac{m_b\left(a^2 + h^2\right)}{12} + m_b l_0^2 \tag{8}$$

$$I_{bz} = \frac{m_b\left(w^2 + a^2\right)}{12} \tag{9}$$

From the above equations, the inertia for rotation about each axis is calculated by summation. For example, the inertia for rotation about the x axis (roll) is:

$$I_x = I_{cx} + I_{mx1} + I_{mx2} + I_{mx3} + I_{mx4} + I_{bx} \tag{10}$$

VII. Gyroscopic Torques

The torques that are required to overcome the above inertias are not the only ones that have to be considered. The motors turn the rotors at speeds up to 2,500 rpm. The axes of these motors (spin axes) are parallel to the z axis of the robot frame. When the Draganflyer rolls (rotation around the x axis) or pitches (rotation around the y axis) it changes the direction of the angular momentum vectors of the four motors. The result is a gyroscopic torque that attempts to turn the spin axis so that it aligns with the precession axis. No gyroscopic torque occurs with rotation around the z axis (yaw) because the spin and precession axes are already parallel.

For a roll, the spin is around the z axis (ω_{pz}), the roll rate is around the x axis (ω_x), so the gyroscopic torque must be around the y axis.

$$\tau_y = I_{pz}\omega_{pz} \times \omega_x \tag{11}$$

Similarly, for pitch (around the y axis) the gyroscopic torque is around the x axis.

$$\tau_x = I_{pz}\omega_{pz} \times \omega_y \tag{12}$$

where the inertia is sum of the inertia of the propeller, gears and rotor of the motor (Fig.3.). For simplicity, we will assume that the propeller can be modeled with a flat plate and the other masses are lumped into it.

$$I_{pz} = \frac{m_{1mov}\left(b^2 + c^2\right)}{12} \tag{13}$$

As moments are free vectors the above torques can be transferred from acting around a point in the motor to the centre of the craft (intersection of the cross spars) without change. The above torques have to be calculated for each motor, as their speed may be different, and then summed to obtain the totally gyroscopic torque.

As two rotors rotate clockwise and two anticlockwise their gyroscopic torques should partially cancel. Only partially, because the maneuvers that give rise to gyroscopic torque (roll and pitch) are the result of commanded differences in rotor speeds.

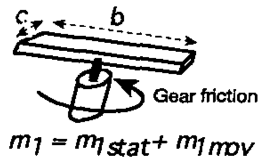

Figure 3. Model of motor/gear/rotor system

VIII. Coriolis Acceleration

The rotors spin within a plane parallel to the xy plane, so when the Draganflyer yaws the blades of the rotors experience coriolis acceleration. Coriolis acceleration represents the difference between the relative acceleration measured from non-rotating (instantaneous) axes and the relative acceleration measured from rotating (robot) axes.

The rotor (Fig.3.) spins around the z axis with an angular velocity, giving a point on it an instantaneous linear velocity in the xy plane. If the craft yaws, the blade of the propeller experiences a coriolis acceleration. The vector is in the xy plane perpendicular to the instantaneous linear velocity.

$$a_c = \omega_{pm} \times v_{lin} \tag{14}$$

This acceleration will result in the application of torque to the propeller and hence to the craft.

IX. Propeller Thrust

A propeller produces thrust by pushing air in a direction perpendicular to it's plane of rotation. Whether the airflow is in the direction of the angular velocity vector or opposite depends on the shape of the propeller. The airflow generates thrust to push the aircraft in reaction to the air drag on the blades. The thrust or lift force is:

$$F_{lift} = \frac{\rho C_l U^2 S}{2} = K_l \omega_{mp}^2 \tag{15}$$

where ρ is the fluid density, U is the flow velocity, C_l is the lift coefficient and S = span (b) * chord (c)

The drag force, which is parallel to the direction of blade motion is:

$$F_{drag} = \frac{\rho C_d U^2 S}{2} = K_d \omega_{mp}^2 \tag{16}$$

The Draganflyer rotors have a circular arc profile. With this profile the lift and drag forces increase as the angle of attack increases. While the rotor blades are fixed in place, they are quite flexible and probably change attack angle when they are accelerated.

As the blades rotate around axes that are away from the centre of the aircraft, the drag force produces a torque around the aerodynamic centre. As the two pairs of motors rotate in opposite directions these torques will balance to zero through the airframe when the motors are turning at the same velocity. When commanded to yaw, the controller decreases the velocity of one pair, to create a torque imbalance that causes the craft to turn around the z axis.

X. Mapping Control Space to Force Space

The controller gives commands in spherical coordinates which have to be mapped to motor voltage commands. This mapping is complicated by the highly coupled dynamics, the under actuated control, and the nonlinearity of the relationship between motor voltage and rotor thrust. Also, the aerodynamics change as the craft moves from one flight state to another (e.g. hover to forward translation).

In the hover state, the force balance is described by Equation 1. To move into another state requires the craft to roll, pitch or yaw. As a consequence of rolling or pitching, the rotor forces are no longer aligned with the z axis of the world frame (assuming stiff rotors) and the craft starts to fall. Equation 1 has to be modified to balance the z component of the forces with gravity. To cause the craft to roll

$$f_4 < f_2 \text{ and } \sum_{i=1}^{n} f_i * \cos(roll) = mg \qquad (17)$$

When moving from hover to roll, a force imbalance is created between the left and right rotors. This force imbalance has to overcome the inertial forces opposing rotation. Once it starts to roll, the gyroscopic forces cause it to pitch and this has to be corrected with imbalanced force from the front-back pair of rotors. Once it starts rolling, it will continue rolling because the inertial force is much larger than the air drag.

Consequently, the left-right pair of rotors has to apply opposing differential force to get it to roll back to the horizontal. By this time the roll forces have caused sideways translation (along the y axis), which has to be stopped by rolling in the other direction. And so the pilot has to continually apply corrections in an attempt to get the craft to hover again. A similar scenario occurs for every initial movement from the hover state.

XI. Alignment and Flexibility

The cross struts of the Draganflyer are carbon fibre tubes (Fig.4.). Each motor is press fitted into a bracket that attaches to the end of a cross strut. A gear on the shaft drives a larger gear supported by a bearing. The propeller is screwed onto the larger gear. As the struts are tubes, there is no simple way to align everything. However, we have noticed that the craft is easier to fly after careful alignment. Unfortunately, motor torques twist the motor bracket around the circular spar and crashes cause considerable misalignment.

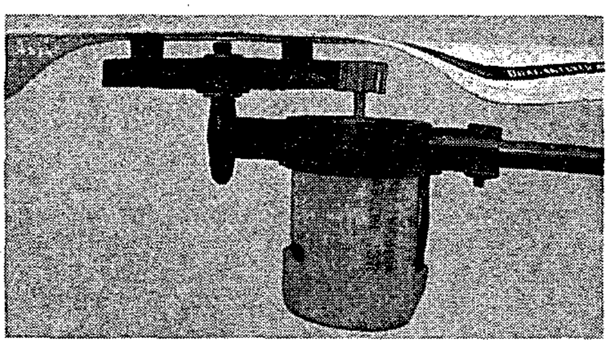

Figure 4. Motor, gear, rotor and blade assembly

When a rotor is distorted or unbalanced the angular momentum vector may not have the same direction as the angular velocity vector. Consequently, the propeller spins with a constant angular velocity but with a varying momentum. As a result the propeller tends to wobble and torques must be applied by the bearings to prevent this, but these cyclic torques can produce vibration and damage.

A propeller on the Draganflyer is fixed to a plastic gear with two screws (Fig. 4.). The gear turns around a single bearing, and the gear mechanism is quite flexible. So the propeller is free to wobble and any minor wobble is damped by the flexible propeller. However, when the propeller is badly out of alignment considerable vibration occurs.

XII. Flight Simulator

We developed a flight simulator (Fig.5.) to provide a test bed for evaluating models. The simulator is written in Java4GL and all the parameters for the models are stored in XML files. We have made initial measurements of all the parameters, but to get more accurate measurements we will have to build a number of test jigs.

When using the simulator, the first thing that is obvious is how difficult it is to get the simulated Draganflyer to stop rising or falling. To get it to hover at one height you have to adjust the throttle until both velocity and acceleration in the z direction are zero. Then as soon as you give any other command, it starts to rise or fall again.

XIII. Conclusion

The above discussion has assumed a stiff system. Yet it is well known that helicopter blades have hinges to enable them to be flexible. This flexibility increases the damping in the system making the control easier to stabilize. We suspect that the flexibility of the rotor blades achieves the same damping on the Draganflyer.

Our analysis has shown that the Draganflyer is a complex system. The under-actuated control and coupled dynamics explain why it is difficult for an inexperienced pilot to fly it. They also create a challenge for the computer control of such vehicles for indoor flight in constrained quarters. In this analysis we have developed a model of the Draganflyer. Our next step is to transform that model into a model for control.

Acknowledgment

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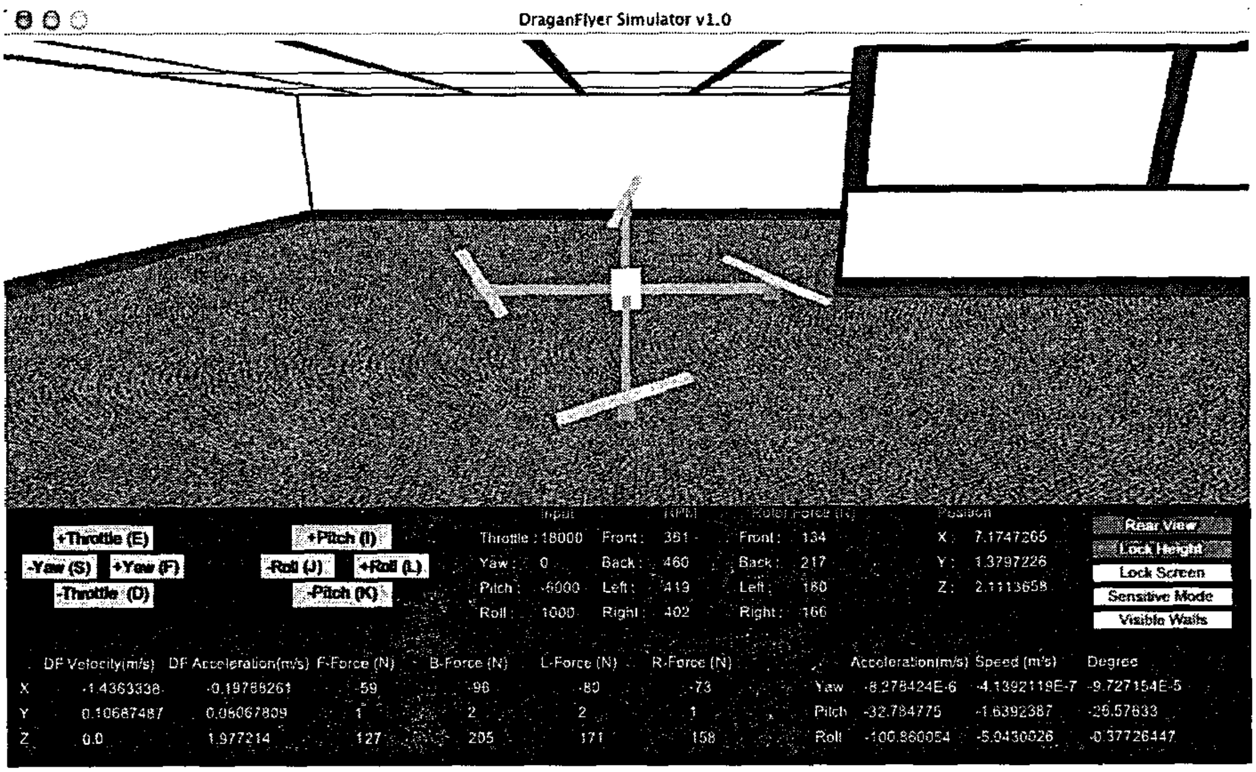

Fig. 5 Draganflyer flight simulator, top - environment, bottom – model parameters