



DAMA 50

Written Assignment V

Submitted by:

Panagiotis Paltsokas

ID: std163861

[DAMA 50] Written Assignment 5

Panagiotis Paltsokas - std163861

Problem 7

Solve the following problem using `sagemath`:

A manufacturer says the Z-Phone smart phone has a mean consumer life of 42 months with a standard deviation of 8 months. Assuming a normal distribution, what is the probability a given random Z-Phone will last between 20 and 30 months?

```
In [1]: restore()
```

```
In [2]: %display latex
```

Define the necessary variables

```
In [3]: var('μ', 'σ', 'x1', 'x2')
```

```
Out[3]: (μ, σ, x1, x2)
```

Initialize the mean, standard deviation and x_1 and x_2 as defined in the problem

```
In [4]: μ=42
σ=8
x1=20
x2=30;
print(f"The consumer life of the Z-Phone follows a normal distribution with a mean of {μ} and standard deviation {σ}")
```

```
The consumer life of the Z-Phone follows a normal distribution with a mean of 42 and standard deviation 8
```

First we will calculate the z-scores z_1 and z_2 which are the standardized values for the months range, using the transformation $z = \frac{x-\mu}{\sigma}$

```
In [5]: z1 = n((x1-μ)/σ)
z2 = n((x2-μ)/σ);
z1, z2
```

```
Out[5]: (-2.750000000000000, -1.500000000000000)
```

Create a standard normal distribution and calculate the cumulative probability of the lifespan falling between x_1 and x_2 . Then convert that probability to percentage and print the result.

```
In [6]: G=RealDistribution('gaussian',1)
p=G.cum_distribution_function(z2)-G.cum_distribution_function(z1)
pct_p=n(100*p);
print(f"The probability that a given random Z-Phone will last between {x1} and {x2} months is {pct_p}%")
```

```
The probability that a given random Z-Phone will last between 20 and 30 months is 6.38274380338035%
```

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Problem 8

A casino proposes a lottery game, in which three numbers, are randomly drawn from the range $1, 2, \dots, 25$ each with an equal chance of being selected. If you bet \$1, you win \$10000 for guessing the three numbers in the correct order, \$1000 for guessing the three numbers but not in the correct order, and \$2 if the sum of your guessed numbers matches the sum of the numbers drawn. You have decided to bet \$1 on your lucky triplet of numbers $[1, 2, 3]$. Use a **sagemath** simulation to calculate your expected profit/loss including necessarily the following steps

- Create a sample of $n = 10^6$ triplets utilizing the function `ZZ.random_element(1,26)` and store them to a list named `w` (Attention, do not print them).
- Create the associated list of sums `ws` using `ws=list(map(sum,w))` (Attention, do not print them).
- Compute the number of triplets in the list `w` matching $[1, 2, 3]$ using `w.count([1,2,3])` and estimate the probability that you guessed the three numbers in the correct order.
- Following step (c) compute the number of triplets in `w` matching $[1, 3, 2], [2, 1, 3], [2, 3, 1], [3, 1, 2], [3, 2, 1]$ and estimate the probability that you guessed the three numbers not in the correct order.
- Calculate the number of triplets in the list `ws` matching the sum $1+2+3$ and estimate the probability of correctly guessing the sum without correctly guessing the three numbers.
- Using the results of steps (c),(d),(e) compute the expected profit/loss and comment on the result.

```
In [1]: restore()
```

```
In [2]: %display latex
```

(a)

Create a sample of $n = 10^6$ triplets utilizing the function `ZZ.random_element(1,26)` and store them to a list named `w`

```
In [3]: nos = 10^6
w = [[ZZ.random_element(1,26) for i in range (3)] for j in range (nos)]
```

(b)

Create the associated list of sums `ws` using `ws=list(map(sum,w))`

```
In [4]: ws=list(map(sum,w))
```

(c)

Let A be the event that the triplet is the lucky $[1, 2, 3]$ that we bet on.

First we count how many times event A is satisfied.

```
In [5]: nA = w.count([1,2,3]);nA
Out[5]: 62
```

The probability that event A is satisfied can be calculated with the classical probability $P(A) = \frac{||A||}{||w||}$

```
In [6]: pA = float(nA/nos);
print(f"The probability that we guessed the tree numbers in the correct order is {pA*100} %")
The probability that we guessed the tree numbers in the correct order is 0.006200000000000001 %
```

(d)

Let B be the event that the triplet contains 1,2 and 3 in the wrong order. That means the triplet is one of the following:

$[1, 3, 2], [2, 1, 3], [2, 3, 1], [3, 1, 2], [3, 2, 1]$

First we count how many times event B is satisfied.

```
In [7]: nB = w.count([1,3,2]) + w.count([2,1,3]) + w.count([2,3,1]) + w.count([3,1,2]) + w.count([3,2,1]);nB
Out[7]: 310
```

The probability that event B is satisfied can be calculated with the classical probability $P(B) = \frac{||B||}{||w||}$

```
In [8]: pB = float(nB/nos);
print(f"The probability that we guessed the three numbers not in the correct order is {pB*100}%")
The probability that we guessed the three numbers not in the correct order is 0.031%
```

(e)

Let C be the event that the sum of the three numbers is the same as ours, that is $1 + 2 + 3 = 6$, and at the same time the triplet does not contain the numbers 1, 2 or 3.

```
In [9]: pC = float(((ws.count(6)) - nA - nB)/nos)
print(f"There are {ws.count(6)} triplets whose sum is 6, and {nA+nB} triplets containing 1,2 or 3. Therefore there are {ws.count(6)-nA-nB} triplets with a sum of 6, that are do not contain the numbers 1,2 or 3.")
print(f"The probability of correctly guessing the sum without correctly guessing the three numbers is {pC*100}%")
There are 618 triplets whose sum is 6, and 372 triplets containing 1,2 or 3. Therefore there are 246 triplets with a sum of 6, that are do not contain the numbers 1,2 or 3.
The probability of correctly guessing the sum without correctly guessing the three numbers is 0.0246%
```

(f)

Our expected earnings per game are calculated by the probability of getting the correct triplet times 10.000\$ plus the probability of getting any other permutation of 1,2,3 times 1.000\$ plus the probability of getting a triplet with a sum of "6" times 2\$. Our participation costs 1\$ so that should be subtracted from our expected earnings to get the expected Profit/Loss amount per game.

```
In [10]: Expected_Earnings = pA*10000 + pB*1000 + pC*2
Game_cost = 1
Prof_Loss = Expected_Earnings - Game_cost
print(f"The expected profit/loss per game is {Prof_Loss}$")

The expected profit/loss per game is -0.06950800000000001$
```

It is obvious that we have a slight loss on average for each game played, which makes the game not favorable for the player as it will accumulate a big loss over multiple games.

[DAMA 50] Written Assignment 5

Panagiotis Paltsoakas - std163861

Problem 9

The probability density function (pdf) $f(x)$ of a continuous random variable X is

$$f(x) = \begin{cases} c(1+x^2) & -2 \leq x \leq 1 \\ 0 & \text{otherwise.} \end{cases}$$

- (i) Compute the value of c .
 - (ii) Find the cumulative distribution function (cdf) of the random variable X , by considering as c the value computed in question (i).
 - (iii) Compute the probability $P(X > -1)$ by expressing it in terms of the cdf.
-

(i)

The probability density function of a random variable X has the property that $\int_{-\infty}^{+\infty} f(t)dt = 1$.

$$\text{Therefore, } \int_{-2}^1 c \cdot (1+t^2)dt = 1 \Leftrightarrow c \cdot \left[t + \frac{t^3}{3} \right]_{-2}^1 = 1 \Leftrightarrow c \cdot \left(1 + \frac{1}{3} + 2 + \frac{8}{3} \right) = 1 \Leftrightarrow c \cdot \frac{18}{3} = 1 \Leftrightarrow c = \frac{1}{6}$$

(ii)

The probability density function of the random variable X is

$$f(x) = \begin{cases} \frac{1}{6} + \frac{x^2}{6}, & -2 \leq x \leq 1 \\ 0 & , \text{ Otherwise} \end{cases} \quad (9.1)$$

To find the cumulative distribution function (CDF) of X , we use $F_X(x) = \int_{-\infty}^x f_X(u)du$

- For $x < -2$, $F(x) = 0$
- For $x > 1$, $F(x) = 1$
- For $-2 \leq x \leq 1$,

$$F(x) = \int_{-2}^x \left(\frac{1}{6} + \frac{t^2}{6} \right) dt = \left[\frac{t}{6} + \frac{t^3}{18} \right]_{-2}^x = \frac{x}{6} + \frac{x^3}{18} + \frac{2}{6} + \frac{8}{18} = \frac{3x + x^3 + 14}{18} = \frac{1}{18}(x^3 + 3x + 14)$$

Finally, the (CDF) of X is :

$$F(x) = \begin{cases} 0 & , x < -2 \\ \frac{1}{18}(x^3 + 3x + 14) & , -2 \leq x \leq 1 \\ 1 & , x > 1 \end{cases} \quad (9.2)$$

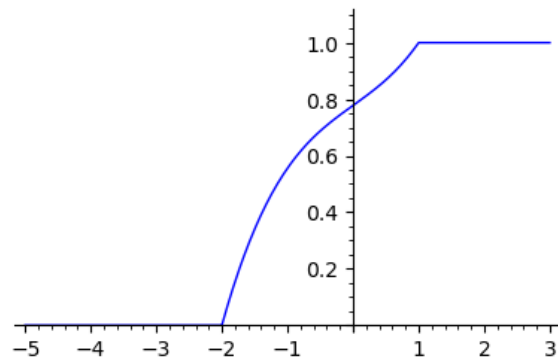


Fig. 1 The graph of the CDF of X

(iii)

We can find the $P(X > -1)$ by calculating the $P(X \leq -1)$ from the (CDF), since

$P(X \leq -1) = F(-1) = \frac{1}{18}(-1 - 3 + 14) = \frac{10}{18} = \frac{5}{9}$ and then $P(X > -1)$ will be the remaining probability.

Hence, $P(X > -1) = 1 - P(X \leq -1) = 1 - \frac{5}{9} = \frac{4}{9}$.

[DAMA 50] Written Assignment 5

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Problem 10

Solve the following problems using by hand calculation:

A. A certain grapefruit variety is grown in two regions in Greece. Both areas get infested from time to time with parasites that damage the crop. Let A be the event that region R1 is infected with parasites and B that region R2 is infected. Suppose $P(A) = 3/4$, $P(B) = 2/5$ and $P(A \cup B) = 4/5$. If the food inspection detects the parasite in a ship carrying grapefruits from R1, what is the probability region R2 is infected as well?

B. A fair coin is tossed independently twice. The random variable x counts the number of heads H that appeared. Compute the cumulative distribution $F(x)$ of x .

A.

Event A = Region R1 is infected with parasites, with $P(A) = 3/4$

Event B = Region R2 is infected with parasites, with $P(B) = 2/5$

Event $A \cup B$ = At least one of the two regions are infected with parasites, with $P(A \cup B) = 4/5$

We will examine the probability of region R2 fruits being infected as well, if we already know that the R1 fruits are infected with the parasite. We are therefore, looking for the conditional probability $P(B|A)$.

- From the *probabilistic inclusion-exclusion principle* we know that

$$\begin{aligned} P(A \cup B) &= P(A) + P(B) - P(A \cap B) \Leftrightarrow \frac{4}{5} = \frac{3}{4} + \frac{2}{5} - P(A \cap B) \Leftrightarrow P(A \cap B) = \frac{15 + 8 - 16}{20} \\ &\Leftrightarrow P(A \cap B) = \frac{7}{20} \end{aligned}$$

- Finally, from the *multiplication rule* we have

$$P(A \cap B) = P(B|A) \cdot P(A) \Leftrightarrow P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{7/20}{3/4} = \frac{7}{15}$$

B.

If X is the number of heads appearing in two independent tosses of a fair coin, then we can easily compute from our sample space $S = \{HH, HT, TH, TT\}$ that

- $P(X=0) = \frac{1}{4}$ that both tosses are tails
- $P(X=1) = \frac{1}{2}$ that one toss is heads and the other is tails, and
- $P(X=2) = \frac{1}{4}$ that both tosses are heads.

Now we will calculate the cumulative distribution function $F(x) = P(X \leq x)$

- For $x < 0$, $P(X \leq x) = 0$
- For $0 \leq x < 1$, $P(X \leq x) = \frac{1}{4}$
- For $1 \leq x < 2$, $P(X \leq x) = \frac{1}{4} + \frac{1}{2} = \frac{3}{4}$
- For $x \geq 2$, $P(X \leq x) = \frac{3}{4} + \frac{1}{4} = 1$

Finally we get :

$$F(x) = \begin{cases} 0, & x < 0 \\ 1/4, & 0 \leq x < 1 \\ 3/4, & 1 \leq x < 2 \\ 1, & x \geq 2 \end{cases} \quad (10.1)$$

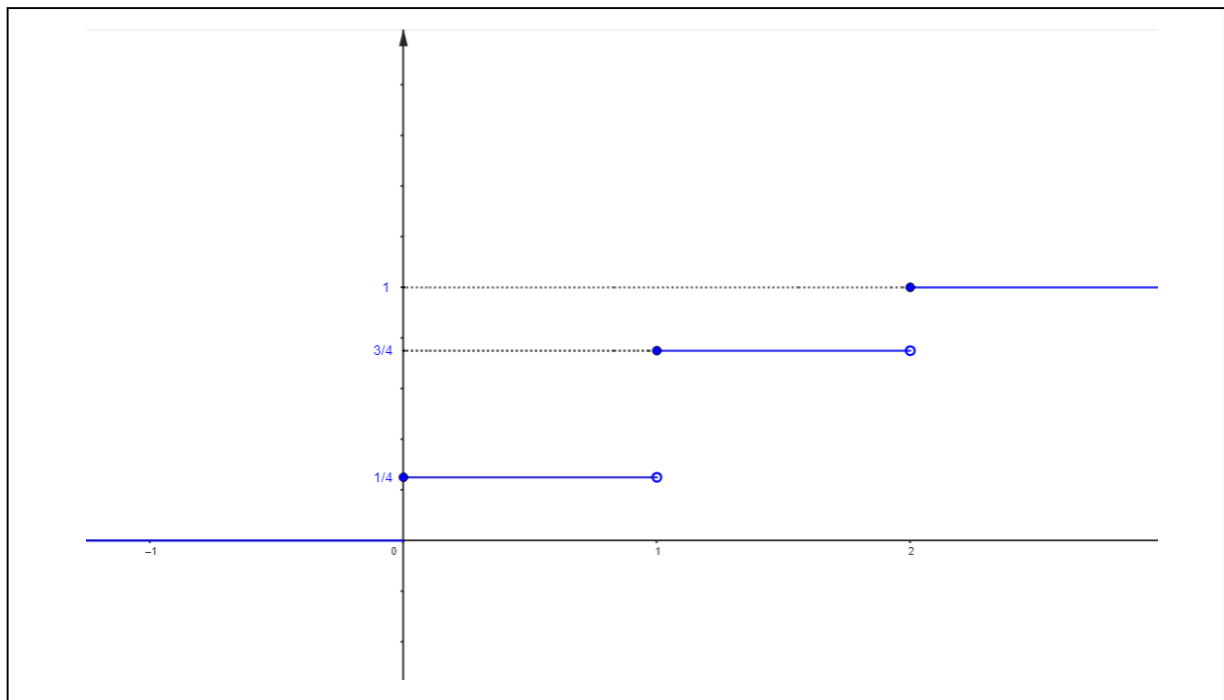


Fig. 2 Graph of the cumulative distribution function $F(x)$