2.(a) Amortized cost of push where the cost model is the # of array accesses and we resize by multiplying the size by C.

Assume: ① array capacity starts at 1

② when we have N elements, the array is full

$$\#\text{ of array accesses} = \underbrace{N}_{1 \text{ for each push}} + \underbrace{\left(2 + 2c + 2c^2 + 2c^3 + \cdots + 2\frac{N}{c}\right)}_{\text{while resizing}}$$

$$= N + 2\left(1 + c + c^2 + \cdots + N/c\right)$$

$$= N + 2 \cdot \sum_{k=0}^{\log \frac{N}{c}} c^k$$

$$= N + 2\left(\frac{c^{\log \frac{N}{c} + 1} - 1}{c - 1}\right) = N + 2\left(\frac{N-1}{c-1}\right)$$

$$= N + \frac{2N}{c-1} - \frac{2}{c-1}$$

for average runtime

dividing by N pushes: $1 + \dfrac{2}{c-1} - \underbrace{\dfrac{2}{N(c-1)}}_{\to 0}$

$$\sim 1 + \frac{2}{c-1} \Rightarrow O(1)$$

2(b) Amortised cost of push where the cost model is the # of accesses and we resize by adding c to the array capacity

Assume: ① array capacity starts at 1

② when we have N elements, the array is full

# of array accesses $= N + 2(1 + (1+c) + (1+2c) + \dots + N-c)$

$$= N + 2 \sum_{k=0}^{\frac{N-c-1}{c}} (1 + kc)$$

$kc + 1 = N - c$

$k = \frac{N-c-1}{c}$

$$= N + 2\left[c \sum_{k=0}^{\frac{N-c-1}{c}} k + \sum_{k=0}^{\frac{N-c-1}{c}} 1 \right]$$

$$= N + 2c\underbrace{\left[\frac{\left(\frac{N-c-1}{c}\right)\left(\frac{N-c-1}{c} + 1\right)}{2} \right]}_{O(N^2)} + 2\underbrace{\left[\frac{N-c-1}{c}\right]}_{O(N)} + 2$$

for average runtime divide by $N \Rightarrow O(N)$