# Stacks & Queues

- Understanding ADTs
- Stacks
- Queues
- Implementations of Stacks and Queues

# Part 1: Abstract Data Types (ADTs)

# ADT: Abstract Data Type

- abstraction of a data structure
- defines operations
- separate from implementation
- may have an “ideal” runtime for operations, but the actual runtimes will be determined by the implementation

# The Value of Defining a (narrowly defined) ADT

- HOW an operation is implemented matters.
- HOW an operation is implemented depends on how the data structure is implemented.
- Defining a data structure to do precisely what it needs to do and no more encourages discipline in programming, making code easier to understand.
- It forces a developer to think about which operations are ABSOLUTELY ESSENTIAL and which are not.
- Many algorithms depend on specific ADTs, so understanding the nature of an ADT can help with understanding the algorithm.

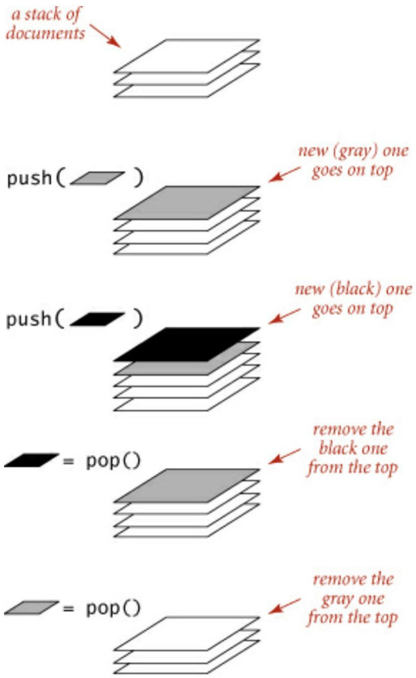
# Part 2: Stacks

Last in, first out (LIFO)

# Stacks


simple truth
Proud of what's NOT in our food.™
NEW
Light Refreshing Taste.
TOAST HOLIDAYS

# Typical Stack Operations



Operations on a pushdown stack

Picture from [1]

- *push*: adds an item to the top of the Stack
- *pop*: removes an item from the top of the Stack
- *isEmpty*: checks if the Stack is empty
- *size*: returns the number of elements in the Stack
- *peek* (or *top*): returns but does not remove the top element in the Stack

# How would you implement a Stack?

# Stacks: What are they good for?

# Stack Application:

## Dijkstra’s Two-Stack Algorithm for Expression Evaluation

EX:
(8 * ((7 + 3) - ((4+2)*(3 - 1))))

```
let S1 and S2 be empty stacks
for each character c in the expression do:
    if c is an operand
    then push it onto S1
    else if c is an operator
    then push it onto S2
    else if c is a right parenthesis
    then pop an operator o from S2
        pop the requisite number of operands from S1
        calculate the result of applying o to the operands and
        push the result onto S1


end for
return the last value on S1
```

# Stack Application:

## Memory Management

```
void foo(char *ptr) {
      char buf[16];
      strcpy(buf, ptr);
}

int main(int argh, char **argv) {
      foo(argv[1]);
      return 0;
}
```

# If a program uses a Stack, what does that tell us?

- We are dealing with data that needs to be stored and processed, and…
- we want to process it in LIFO order.

# Part 3: Queues

First in, first out...

# Typical Queue Operations

Picture from https://pngtree.com/freepng/people-png-elements_2835125.html

- *enqueue*: adds an item to the back of the Queue
- *dequeue*: removes an item from the front of the Queue
- *peek*: returns but does not remove the item from the front of the Queue
- *isEmpty*: determines if the Queue is empty or not
- *size*: returns the number of items in the Queue

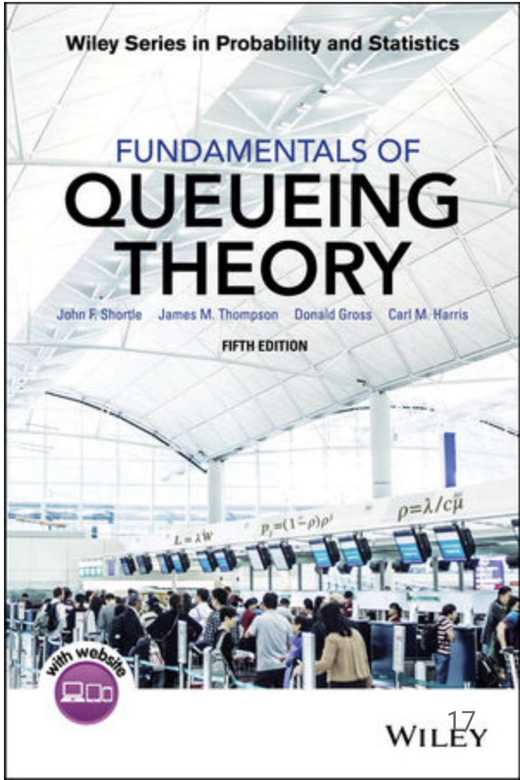
# How would you implement a Queue?

# Queues: What are they good for?

# Queue Application:

## Networks and Data Communication

- Sending
- Routing
- Receiving
- Processing



# If a program uses a Queue, what does that tell us?

- We are dealing with data that needs to be stored and processed, and…
- we want to process it in FIFO order.

# Part 4: Double-ended Queues (Deques)

Combines the functionality of a Stack and a Queue.

# Typical Deque Operations

- *addToFront*: (like *push*)
- *addToBack*: (like *enqueue*)
- *getFront*: (like *pop*)
- *getBack*: remove from the back
- *peekFront*
- *peekBack*
- *isEmpty*
- *size*

# Part 5: Implementations of Stacks and Queues

- Array-based Stack
- Array-based Queue
- Amortized Analysis
- Linked List Implementations

1. How much space should a Stack/Queue take if we have ***N*** elements?

2. How much time should it take to *push*/*pop* or *enqueue*/*dequeue* a single item?

# The Ideal Implementation...

- ...total memory required is proportional to the collection size N
- ...basic operations are independent of the collection size (i.e. can perform basic operations in O(1) time)

NOTE: Ideally, the size of a stack or queue should be dynamic—you shouldn’t need to specify the size when you create it.

# Arrays

- Built in to Java
- Size is fixed and specified when array is created
- So what can be done if you end up with more elements than you expected?

- If we wanted to implement a Stack with an array, what are some of the challenges and how might we handle them?

# Implementing a Stack with an array

- instance variables: array, integer for size
- handling errors: e.g. popping from an empty stack
- How do you implement:
  - *size*
  - *push*
  - *pop*
  - *isEmpty*
  - *peek*

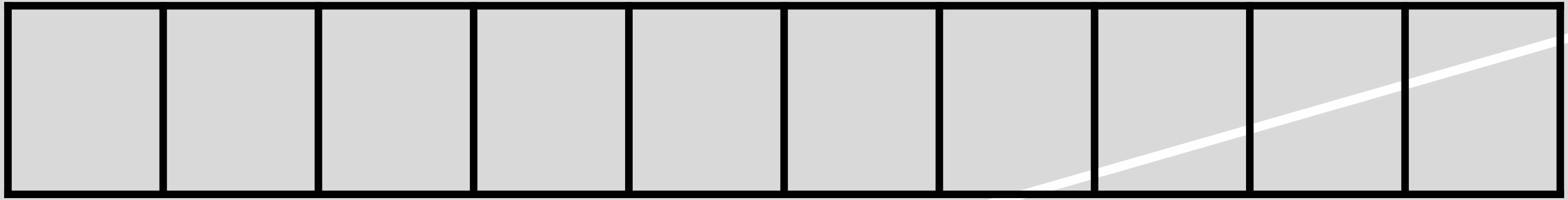
# Implementing a Stack with an array

- instance variables: array, integer for size
- handling errors: e.g. popping from an empty stack
- How do you implement:
  - *size*: keep track of it with a variable
  - *push*: use the size variable to indicate the next open index
  - *pop*: use the size variable to remove “top” element, reduce size
  - *isEmpty*: size is 0
  - *peek*: use the size variable to get the “top” element

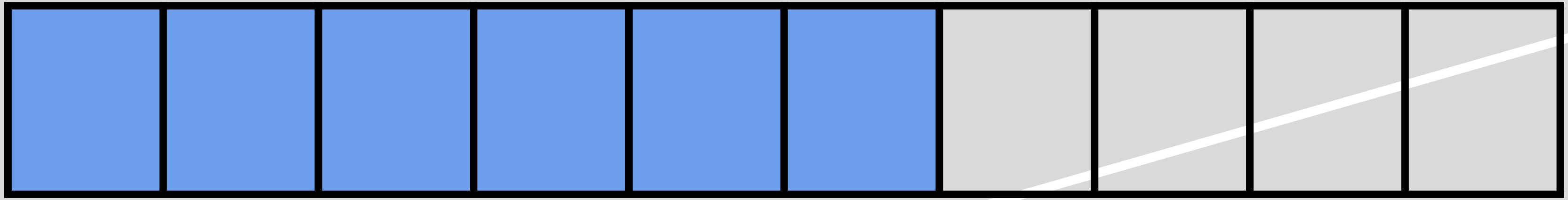
# Dynamic Resizing

- Start with an array of size X.
- When the stack is full, make a new array of size 2X and copy the items over.
- If the size of the stack falls below a threshold (around ¼ of X), make a new array of size X/2 and copy the items over (keeps the size of the stack between ½ full and full.)

Empty Stack
index 0 = the bottom of the stack
Capacity (N) = 10
Size = 0

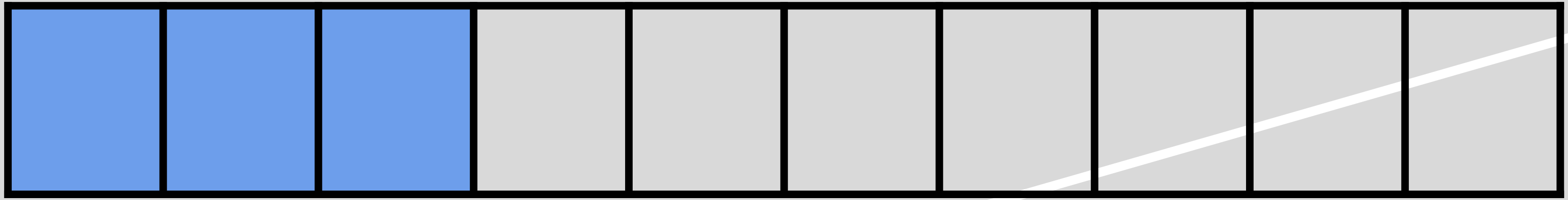


index 0 = the bottom of the stack
index 5 = the top of the stack
Capacity (N) = 10
Size = 6



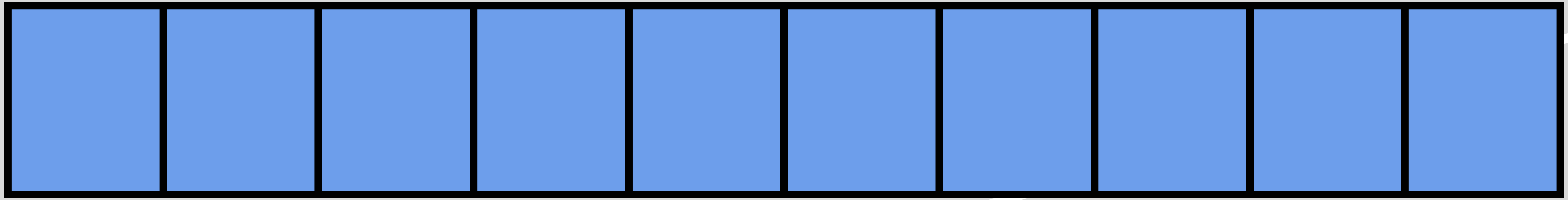
after *push*ing 6 items

index 0 = the bottom of the stack
index 2 = the top of the stack
Capacity (N) = 10
Size = 3



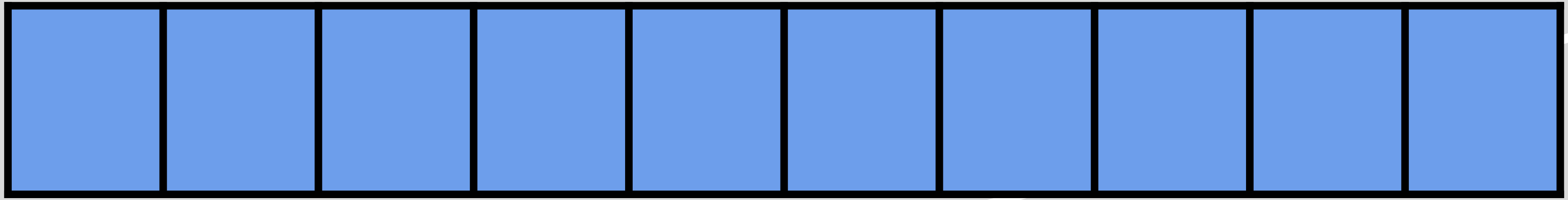
after *pop*ping 3 items

index 0 = the bottom of the stack
index 9 = the top of the stack
Capacity (N) = 10
Size = 10



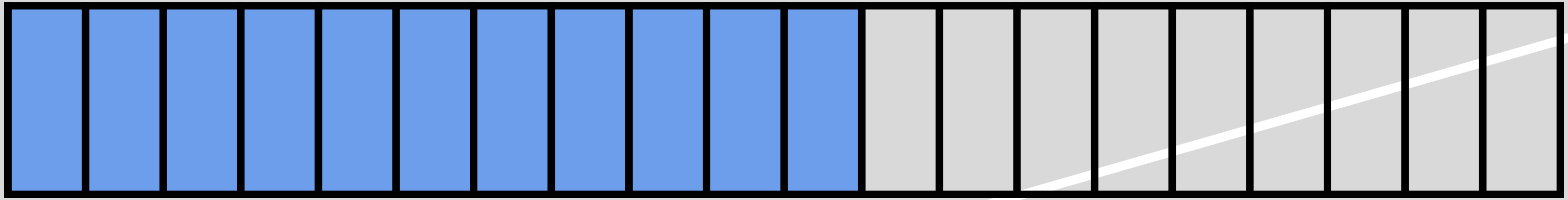
after pushing 7 items

index 0 = the bottom of the stack
index 9 = the top of the stack
Capacity (N) = 10
Size = 10



What do we do if we want to *push* one more item?

index 0 = the bottom of the stack
index 10 = the top of the stack
Capacity (N) = 20
Size = 11



Create a new array that is twice as large and copy the items over...but how much time does that take?

# Queue implementation with array

- Dynamically resize the same as the Stack
- Differences with Stack
    - Names of operations
    - Operations and indexes
        - Need to keep track of front and back
        - Need to allow for adding to back and removing from front
        - How to keep track of size
        - Wrap around to utilize full capacity

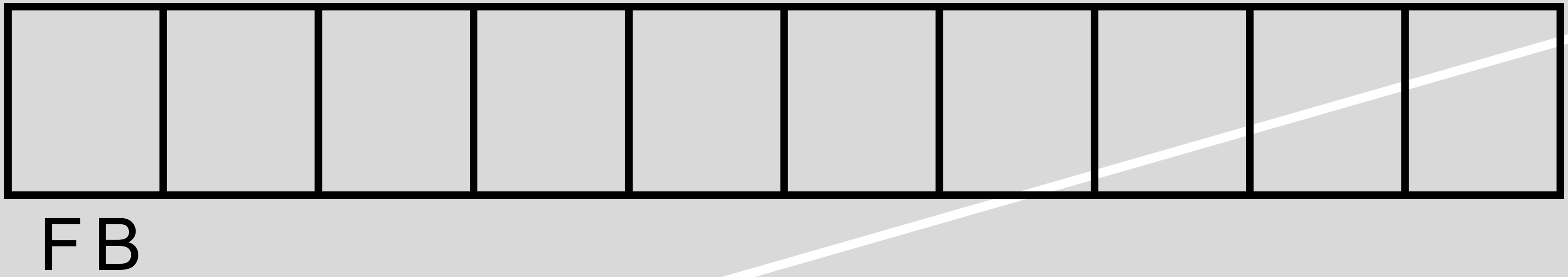
Empty Queue

F = index of the front

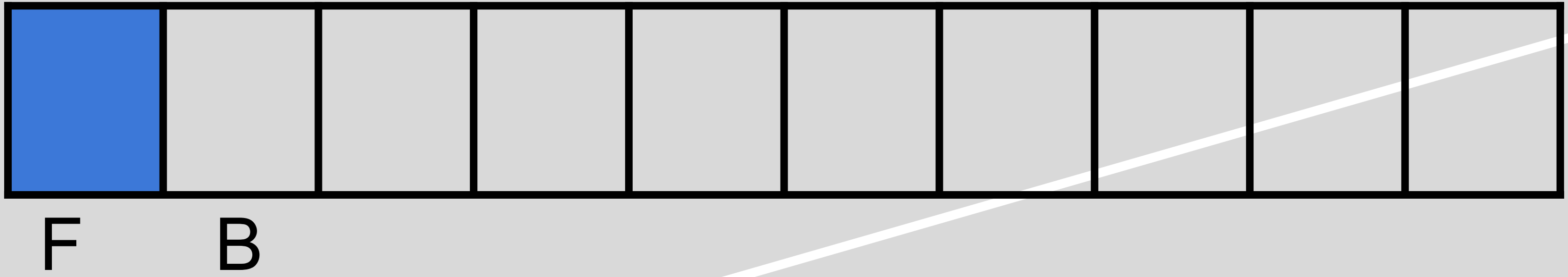
B = index of the back

Capacity (N) = 10

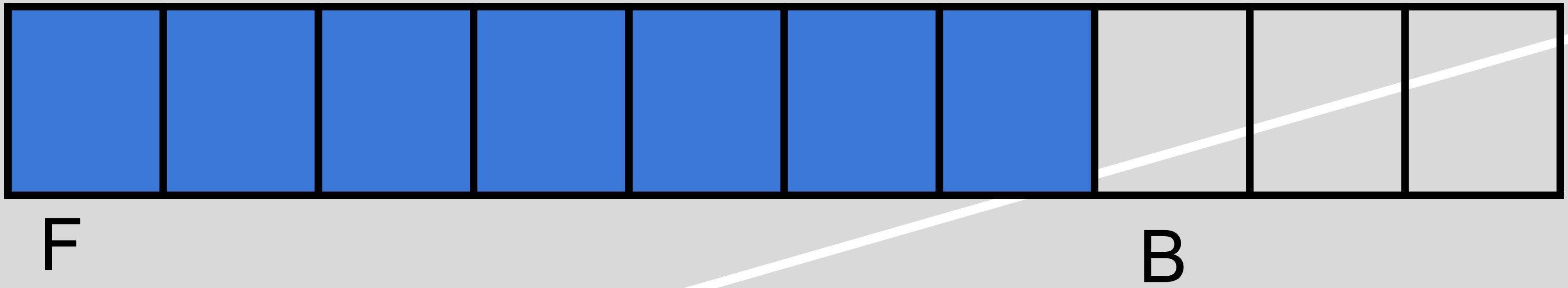
Size = 0



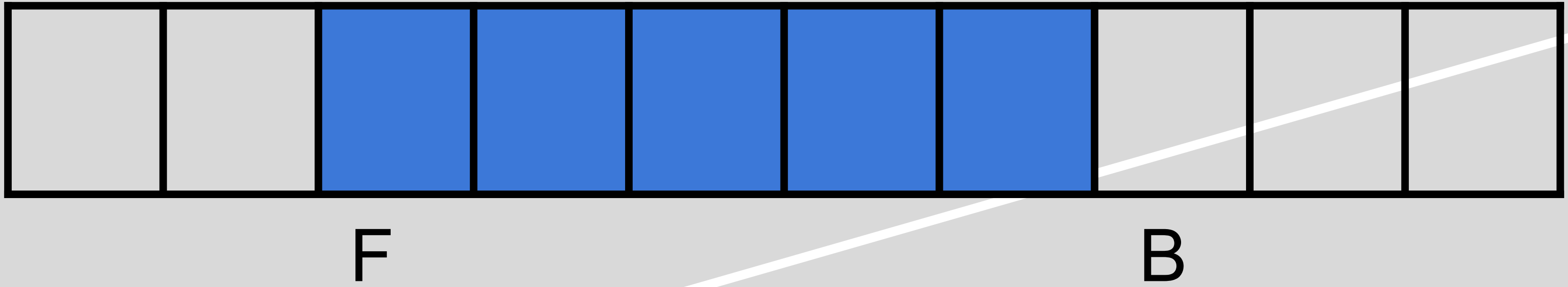
Enqueue 1 item
Size = 1



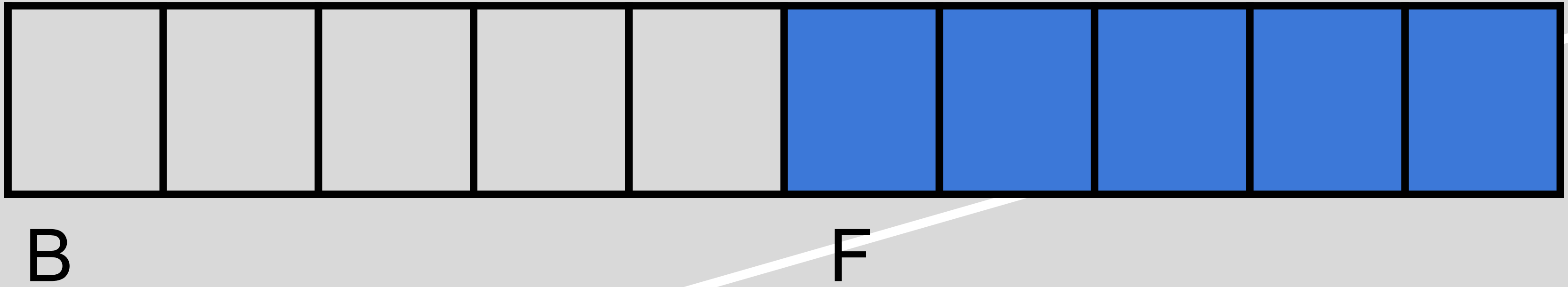
Enqueue 6 more items
Size = 7



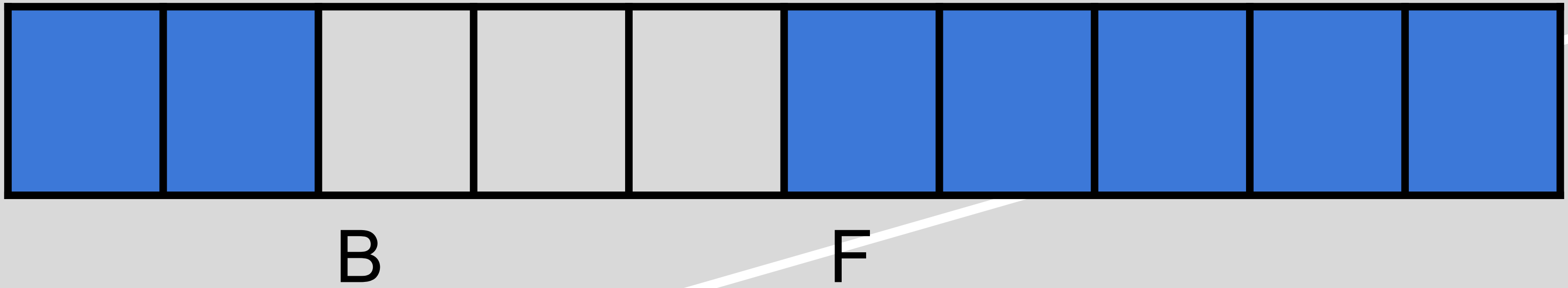
Dequeue 2 items
Size = 5



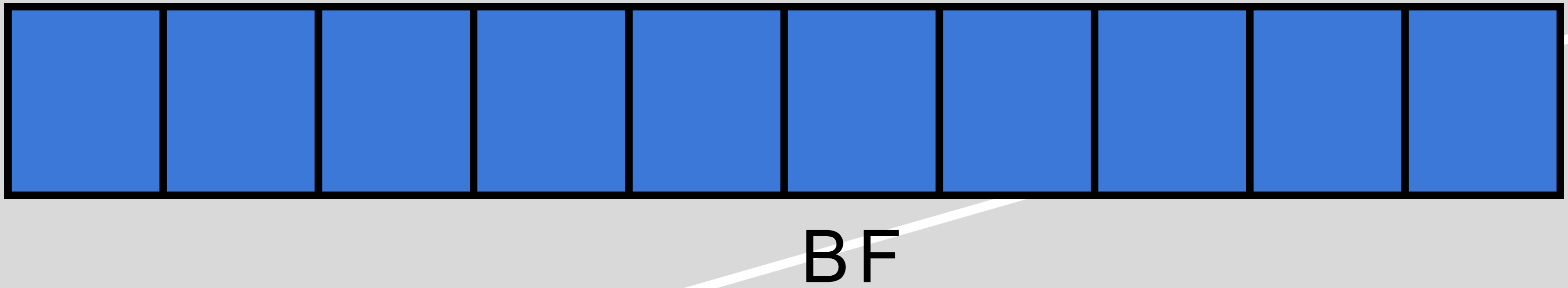
Dequeue 3 items and Enqueue 3 items
Size = 5
Index for Back wraps around



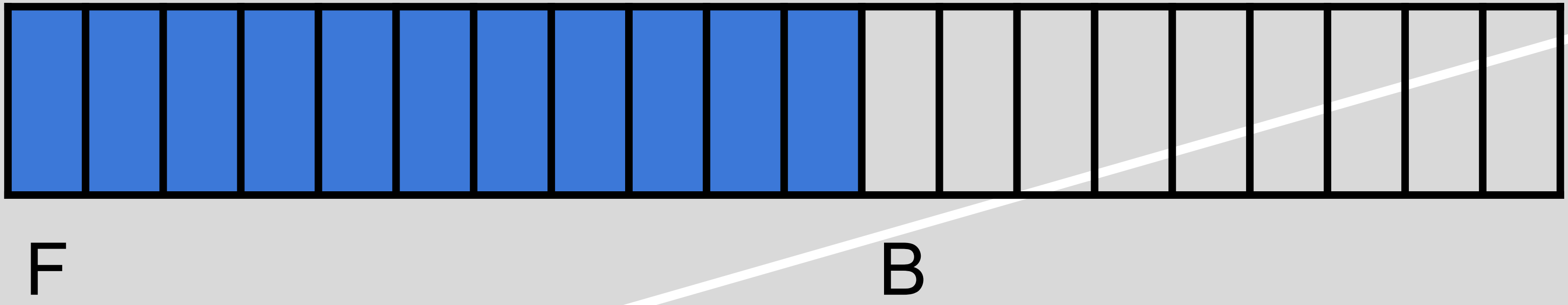
Enqueue 2 more items
Size = 7



Enqueue 3 more items
Size = 10



Enqueue 1 more item
Size = 11




Double the size of the array, copy the items over, and reset the pointers for front and back.

# Remember: The Ideal Implementation...

- ...total memory required is proportional to the collection size N
- ...basic operations are independent of the collection size (i.e. can perform basic operations in O(1) time)
- …the structure is dynamically resizable

# Analysis of Operations

Array-based Stack/Queue with dynamic size

- isEmpty
- size
- push/enqueue
- pop/dequeue

# Analysis of Operations

Array-based Stack/Queue with dynamic size

- isEmpty: O(1)
- size: O(1)
- push/enqueue: best-O(1); worst-O(N)
- pop/dequeue: best-O(1); worst-O(N)

This should look familiar, though, if you've started Project 2. The two things we need to make sure of in order to guarantee good *amortized* time are:

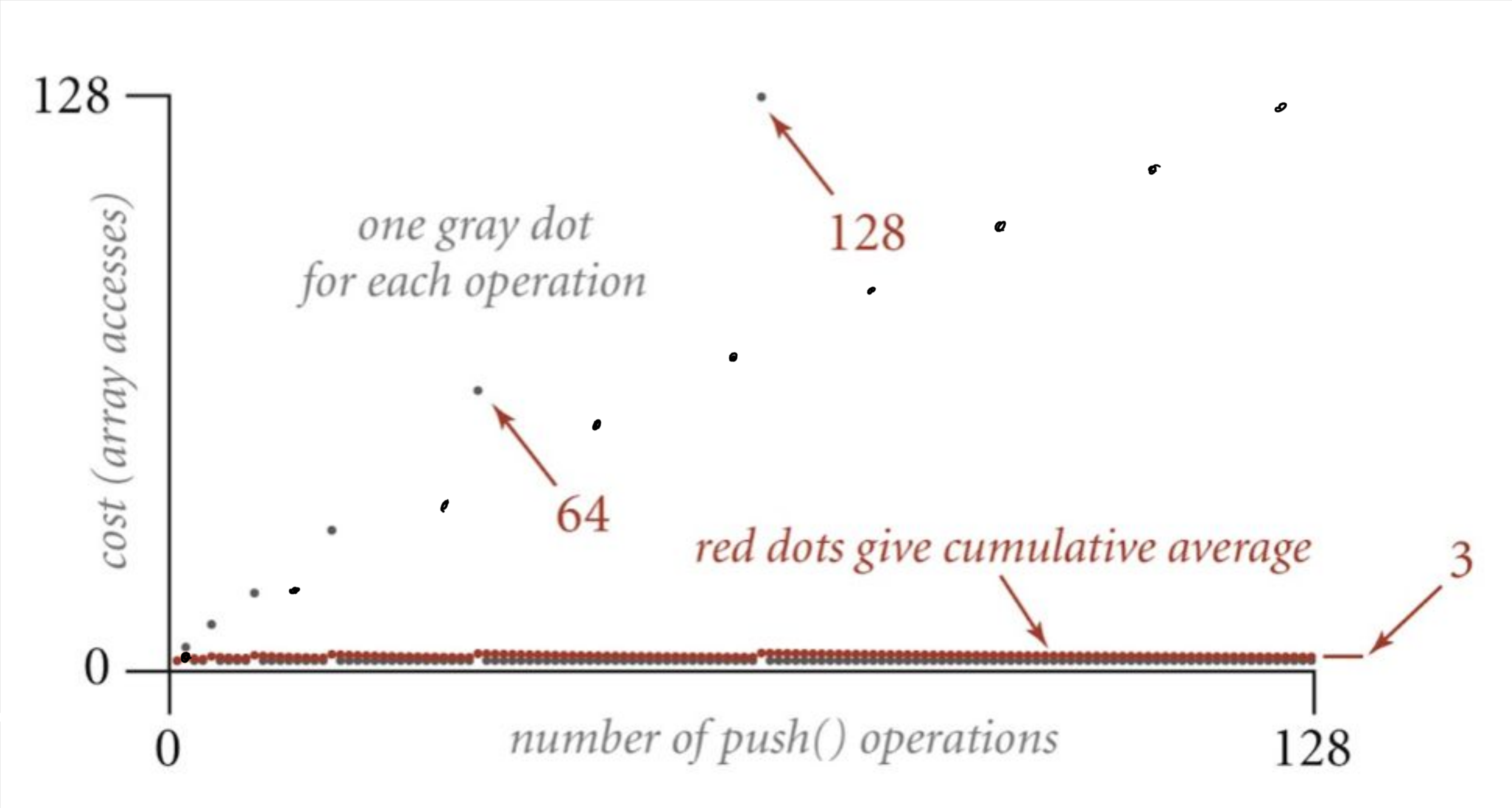
- the worst-case runtime should only happen when the array is resized
- the resizing should be done by doubling (or something similar to that)

# Amortized Runtime Analysis

- used when analyzing the runtime of a single operation on a data structure
- may be useful when the best-case and worst-case runtimes vary quite a bit *and* the worst-case doesn't happen very often
- calculated by:
  - considering a worst-case string of *N* operations
  - adding up the total cost
  - dividing by *N*

# Amortized Analysis for *push*

- A good cost model will be to count the *array accesses* (i.e. the number of times the array is accessed.)
- A worst-case scenario for *N* pushes is to just do *N* pushes in a row without any *pops*.
- Each one of these *pushes* will access the array 1 time.
- Plus, some of them will require resizing which will require *2M* array accesses, where *M* is the number of elements in the array when the array is resized.
- We will add up this total and then divide it by *N* to get the average cost of each *push*.




Picture from [1]

# Amortized cost of push where the cost model is the number of array accesses and we resize by doubling.

Assume: ① the array capacity starts at 1

② When we have N elements, the array is full.

$$\# \text{ of array accesses} = \underbrace{N}_{\text{1 for each push}} + \underbrace{(2 + 4 + 8 + \dots + N)}_{\text{\# for resizing}}$$

$$= N + 2(1 + 2 + 4 + \dots + N/2)$$

$$= N + 2\sum_{k=0}^{\log_2 N/2} 2^k = N + 2\left[\frac{2^{\log_2 N/2 + 1} - 1}{1}\right] = N + 2[N - 1]$$

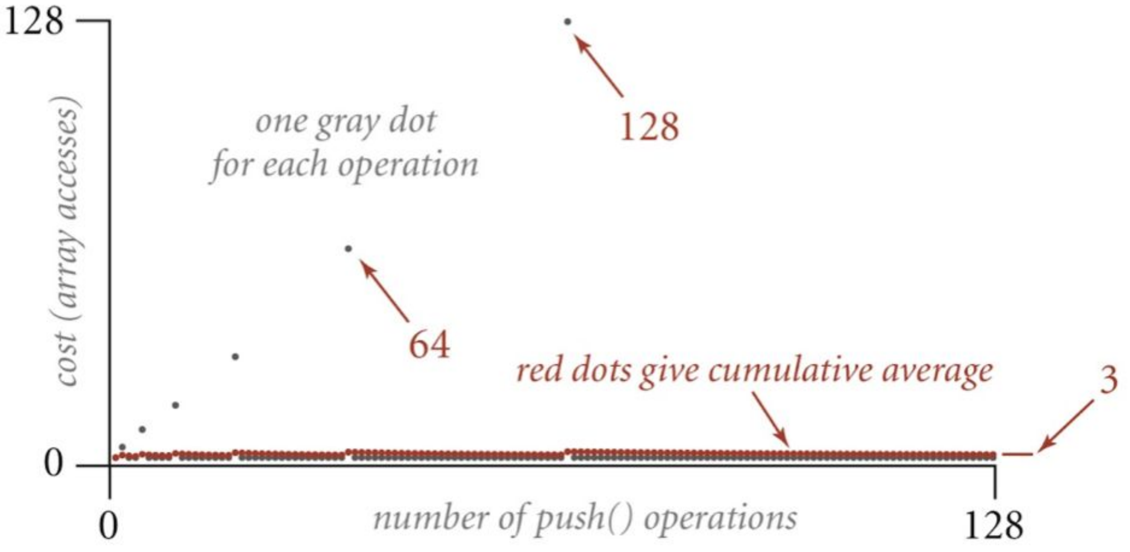
$$= N + 2N - 2 = 3N - 2$$

---

Dividing by N pushes: $\frac{3N-2}{N} = 3 - \cancel{\frac{2}{N}}^{\,0} \sim 3 \Rightarrow O(1)$

# Amortized Analysis for *push:* why the resizing strategy matters

- Notice how the dots for when the array is resized get further apart as N gets larger.
- This would not be the case if we resized by adding a constant amount to the array size.



# Amortized cost of push where the cost model is the number of array accesses and we resize by adding 100 to the array capacity.

Assume: array cap starts at 1 and that the array is full after N pushes

$$\#\text{ of array accesses} = N + 2(1 + 101 + 201 + \dots N-100)$$

$$= N + 2\sum_{k=0}^{\frac{N-101}{100}}(100k+1) = N + 2\left[100\sum_{k=0}^{\frac{N-101}{100}} k + \sum_{k=0}^{\frac{N-101}{100}} 1\right]$$

$$= N + 200\left[\underbrace{\frac{\left(\frac{N-101}{100}\right)\left(\frac{N-101}{100}+1\right)}{2}}_{O(N^2)}\right] + 2\left[\underbrace{\frac{N-101}{100}}_{O(N)}\right] + 2$$

amortized cost of push: $O(N)$

# Summary of Analysis for array-based Stacks and Queues

| Operation | Best Case | Worst Case | Amortized |
|---|---|---|---|
| push/enqueue | O(1) | O(N) | O(1) |
| pop/dequeue | O(1) | O(N) | O(1) |
| isEmpty | O(1) | O(1) | O(1) |
| peek | O(1) | O(1) | O(1) |
| size | O(1) | O(1) | O(1) |

# Linked List Implementation of Stacks and Queues

- As we discussed before, it takes O(1) time to add or remove elements from the front or the back of a linked list.
- This means that implementing a Stack or a Queue with a linked list is straightforward and will guarantee O(1) time for the basic operations.
- However, we should also consider the disadvantages of linked lists…
    - space requirements for the pointers
    - the fact that the nodes are not stored sequentially in memory, which affects the runtime

# What's better: implementing a Stack with an **array** or a **linked list**?

# References

[1] *Algorithms, Fourth Edition;* Robert Sedgewick and Kevin Wayne (and associated slides)