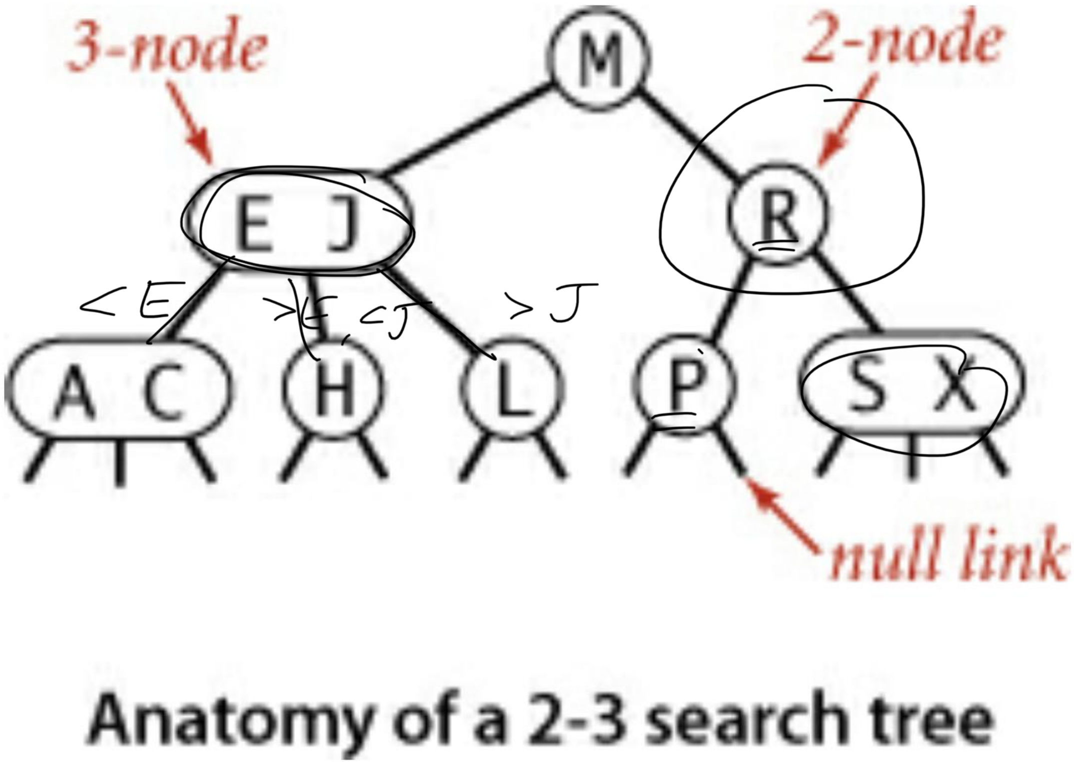
2-3 Trees

- An *ordered* symbol table
- A self-balancing tree

2-3 Trees

Key Idea: We need a tree that is more flexible than a BST so that it can more easily balance itself.

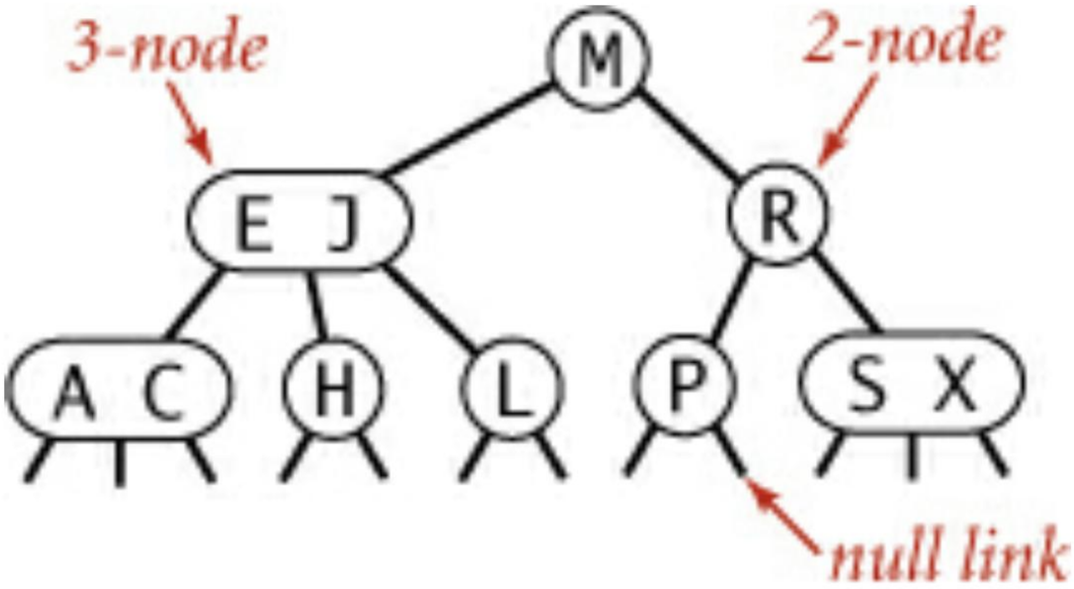


Picture from [1]

A 2-3 tree allows Nodes to have: (a) 1 key and two links or (b) 2 keys and three links

2-3 Trees

- Maintains Order
- Guarantees that every null link is the same distance from the root
- What is the height of the tree given N nodes?

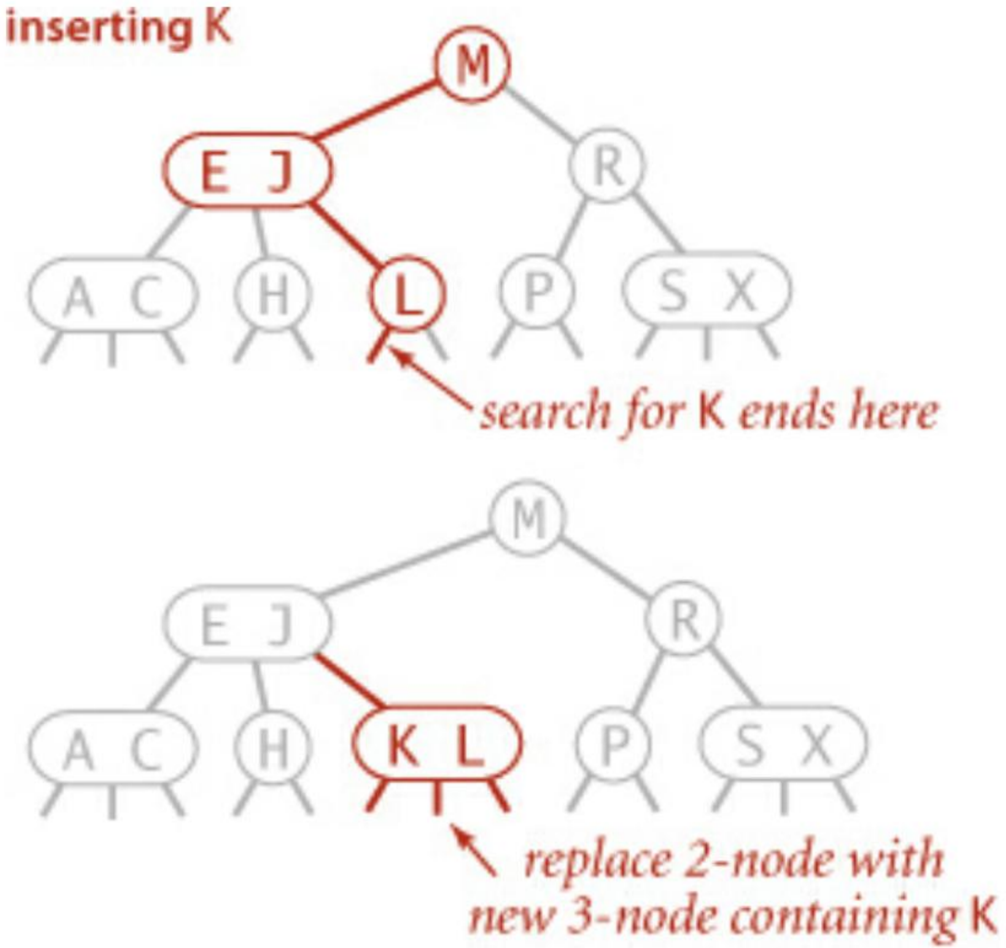


Anatomy of a 2-3 search tree

Picture from [1]

put(K):

adding to a 2-node

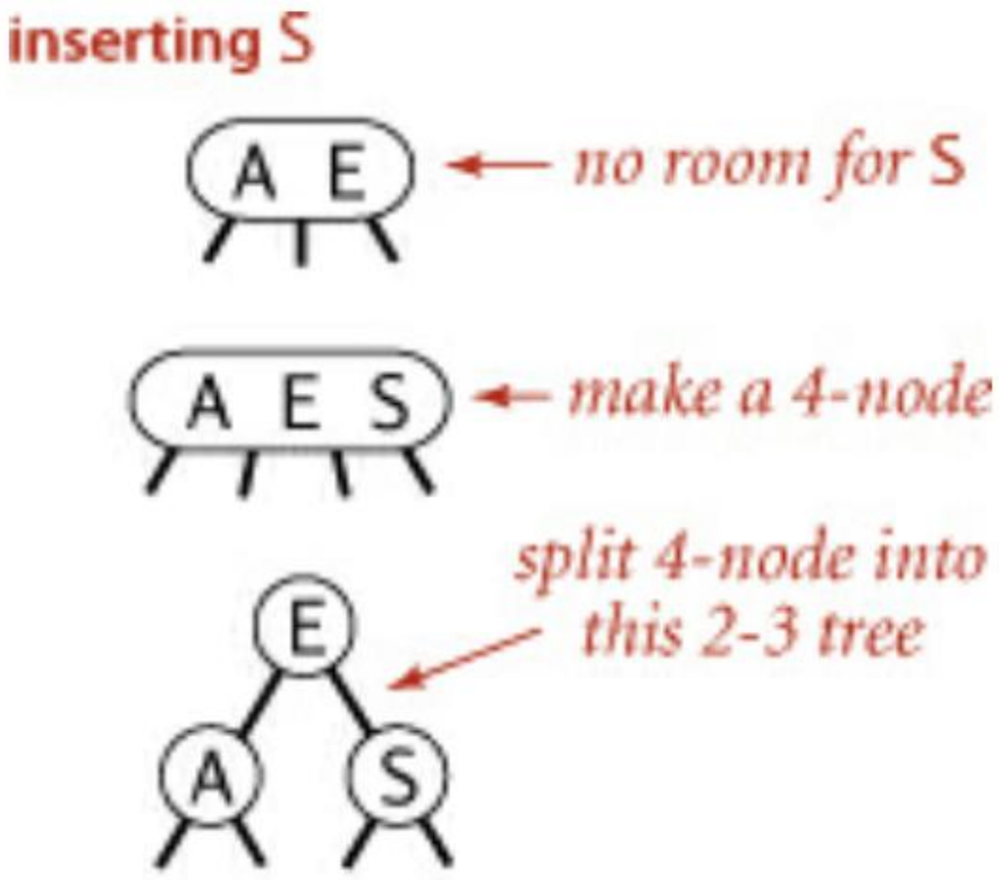


Picture from [1]

Insert into a 2-node

put(S):

adding to a single 3-node (the root of the tree)



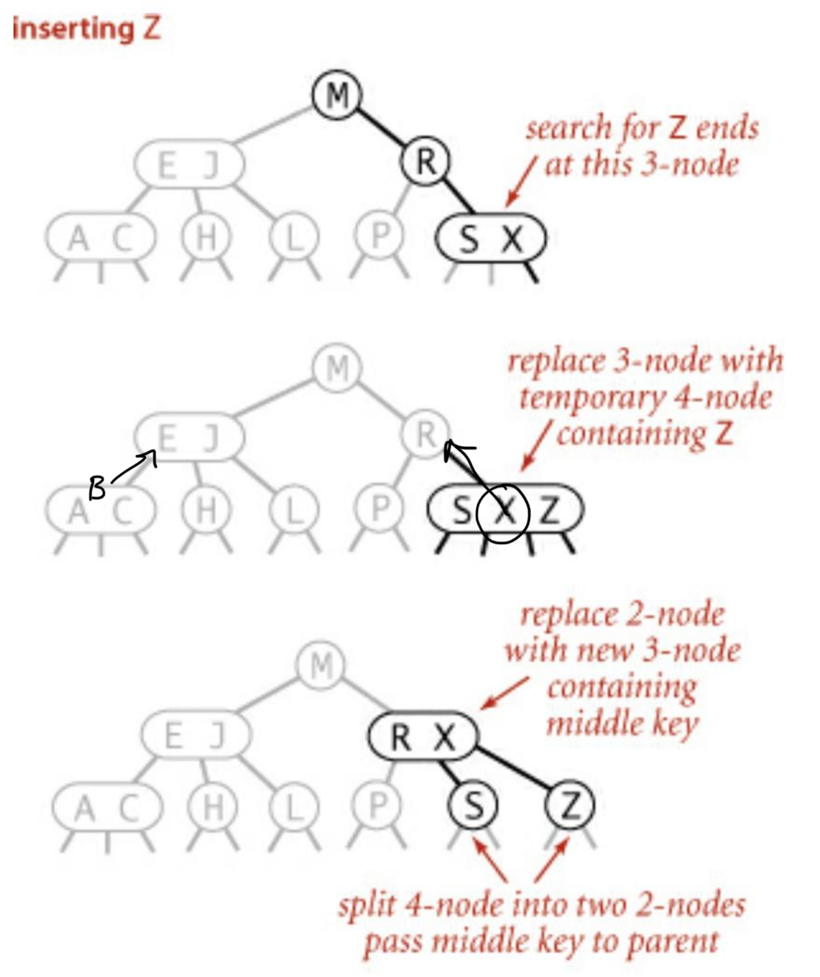
Insert into a single 3-node

Picture from [1]

put(Z):

adding to a 3-node

Picture from [1]

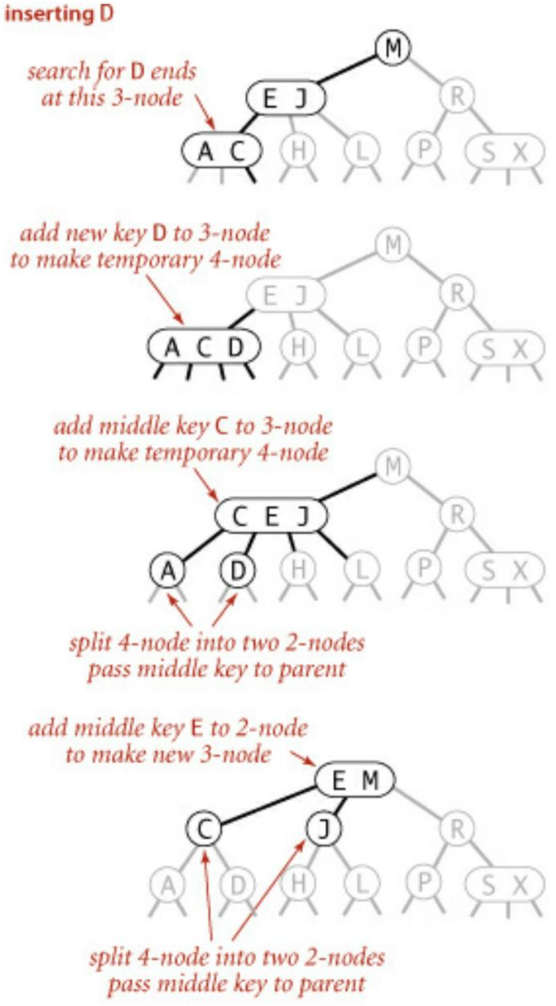


Insert into a 3-node whose parent is a 2-node

put(D):

adding to a 3-node

Picture from [1]

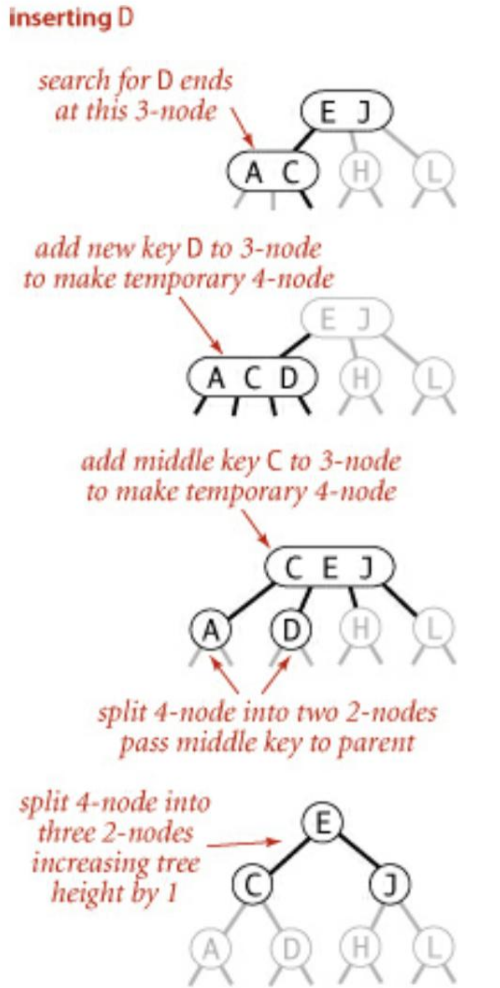


Insert into a 3-node whose parent is a 3-node

put(D):

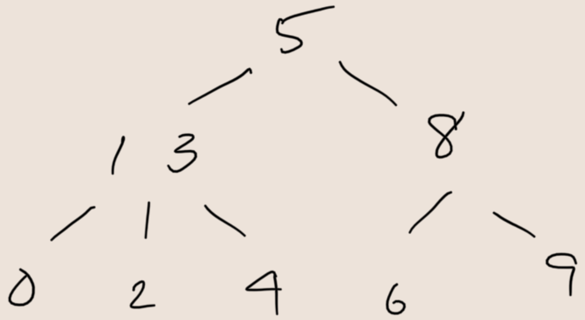
adding to a 3-node

Picture from [1]

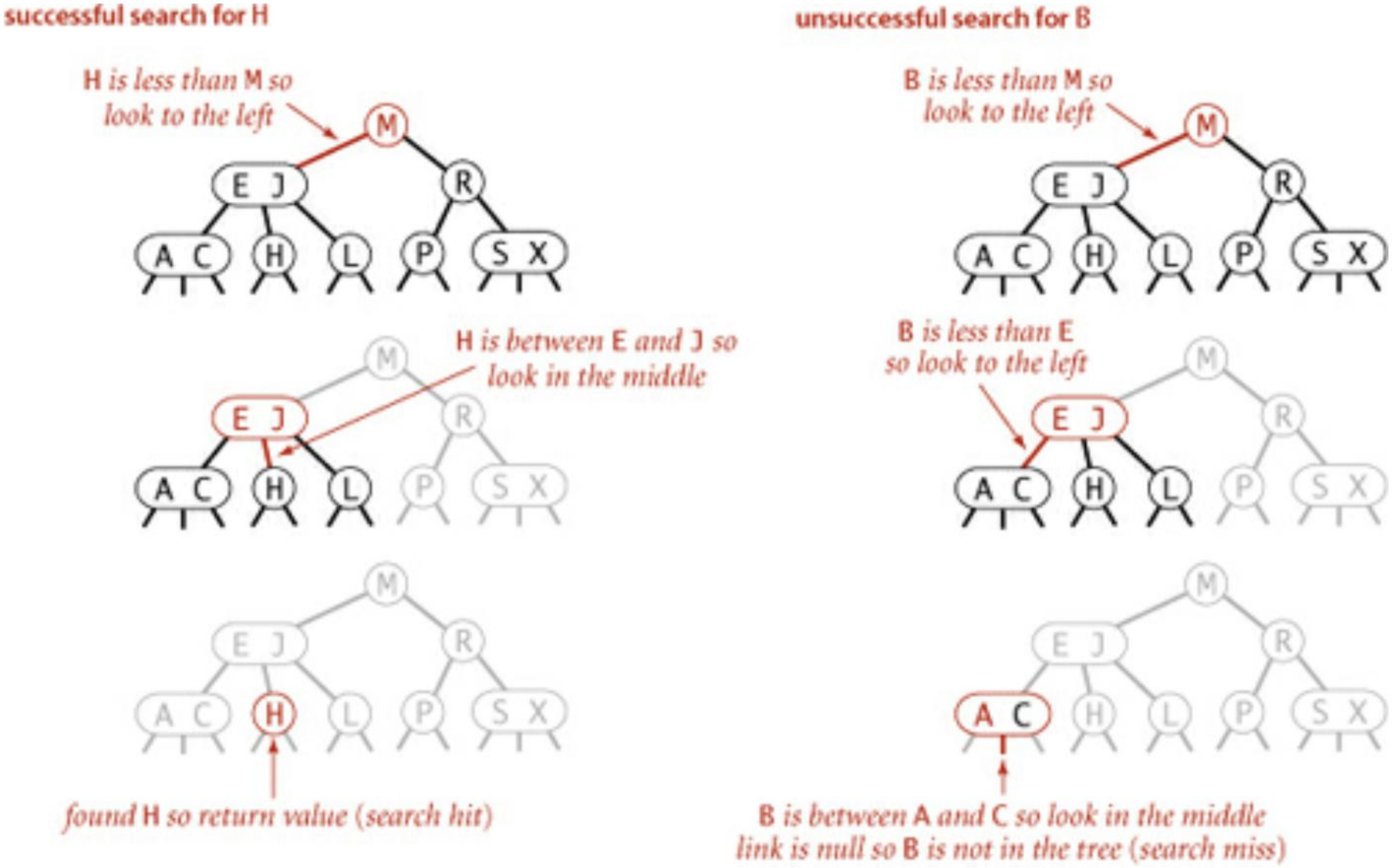


Splitting the root

Example: ~~insert 8~~, ~~insert 2~~, ~~insert 3~~, ~~insert 4~~, ~~insert 0~~, ~~insert 9~~, ~~insert 6~~, insert 5, insert 1

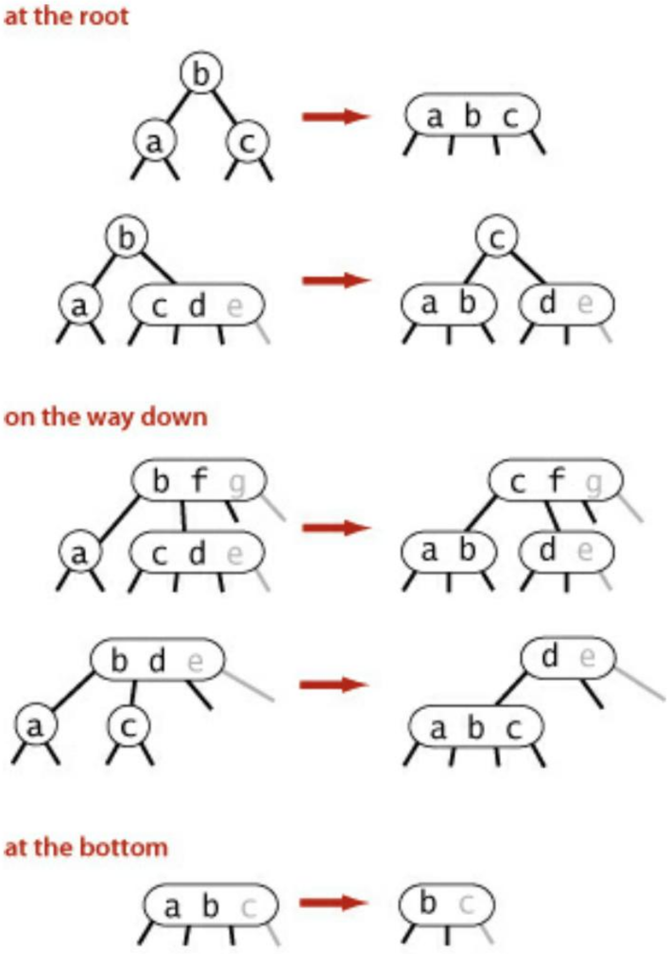


get



Picture from [1]

Search hit (left) and search miss (right) in a 2-3 tree



Transformations for delete the minimum

Big Idea for Deletion:

It is easy to delete a key from a 3-node (or a 4-node) at the bottom of the tree. The tricky part is deleting a 2-node. (WHY?)

So we can transform the tree on the way down to ensure that the current node is not a 2-node. (HOW?)

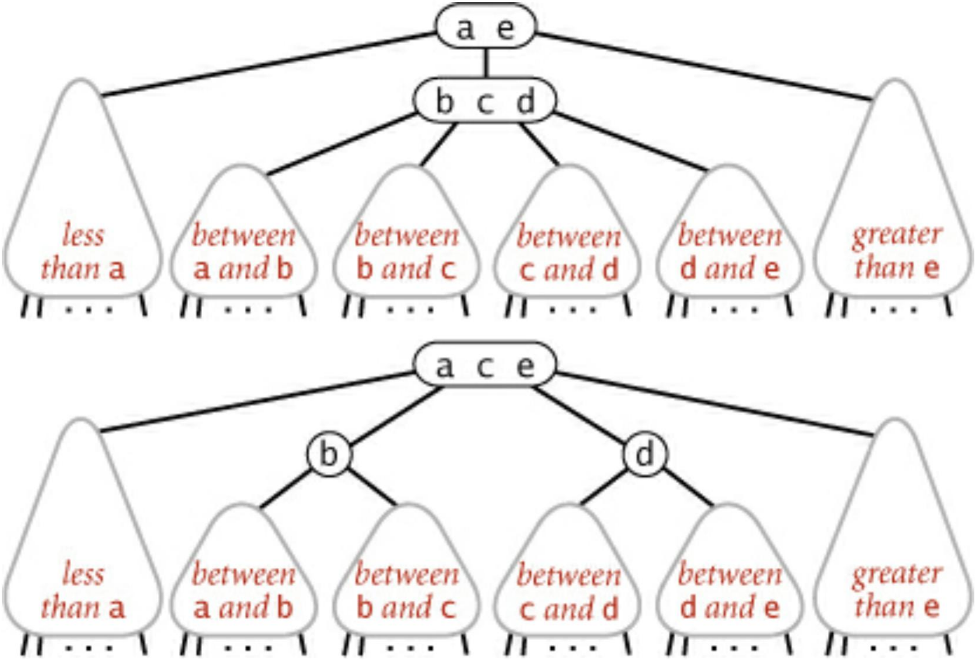
Picture from [1]

Example: delete 3, delete 9, delete 5, delete 0, delete 1, delete 8, delete 4, delete 6, delete 7, delete 2

Analysis of **put** algorithm

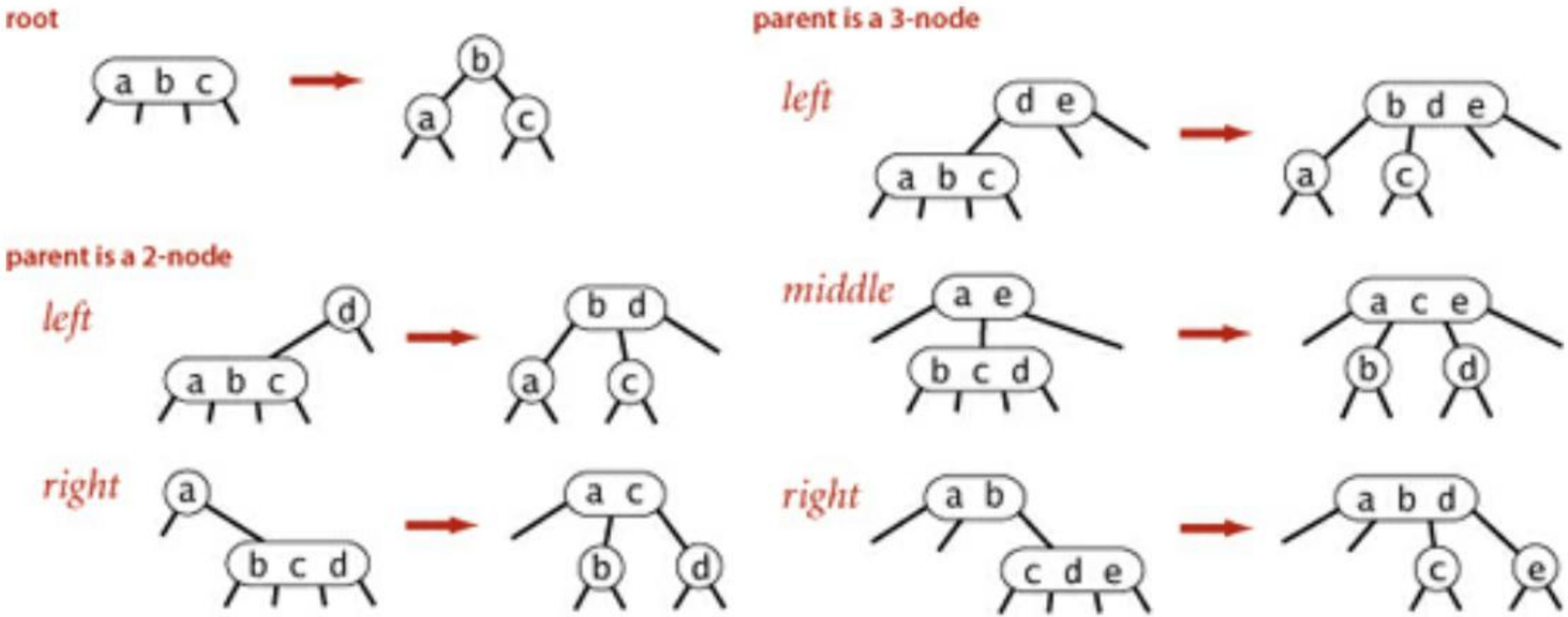
- ***local*** transformations: only the specified nodes need to be examined—number of links changed is bounded by a small constant
- ***global*** properties of the tree are preserved (order, balance, height*)

*height is increased by 1 when the root splits (all null nodes still have equal depth)



Splitting a 4-node is a local transformation that preserves order and perfect balance

Picture from [1]



Splitting a temporary 4-node in a 2-3 tree (summary)

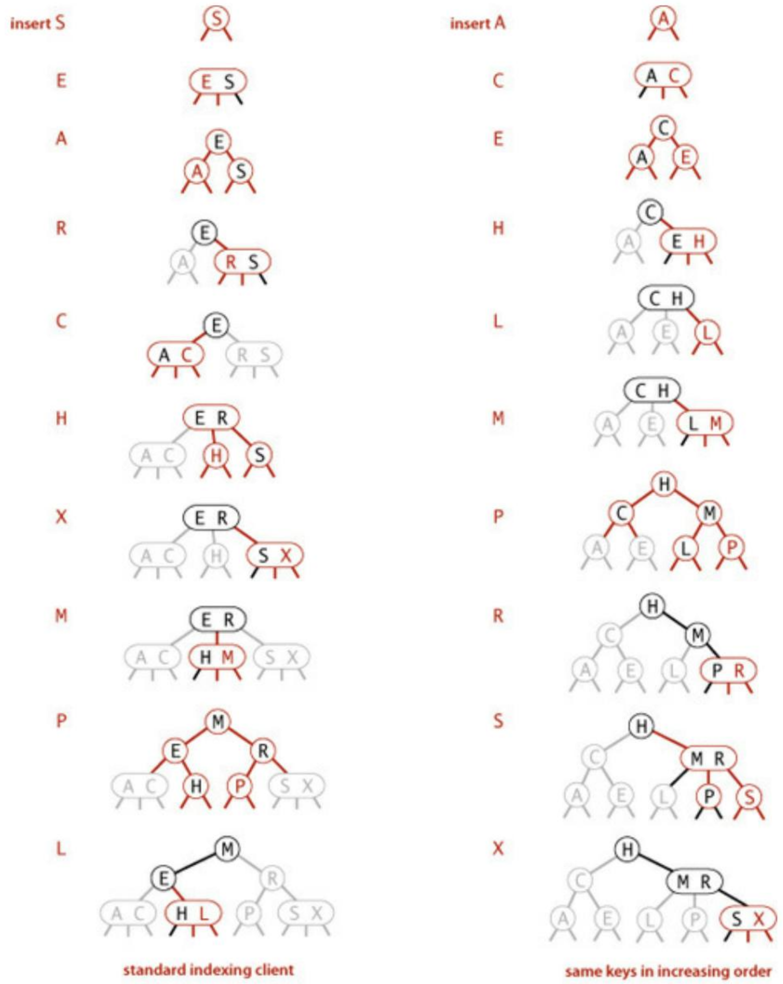
Picture from [1]

Proposition F

Search and insert operations in a 2-3 tree with *N* keys are guaranteed to visit at most *lgN* nodes.

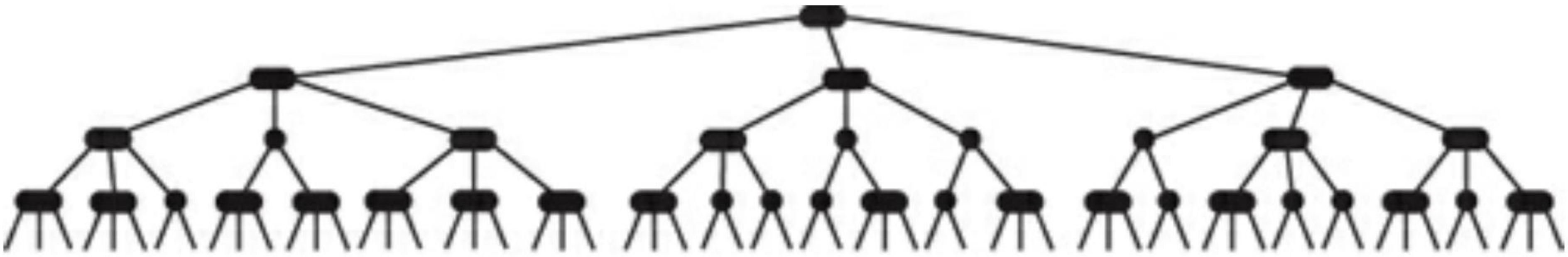
- Consider the two "extremes": the tree is made up of all 3 nodes or the tree is made up of all 2 nodes
- All transformations (local) take constant time
- Each operation touches nodes on a single path

$\log_3 N \leq h \leq \log_2 N$



2-3 construction traces

Picture from [1]



Picture from [1]

Typical 2-3 tree built from random keys

How might this be different if the keys are in decreasing order?

2-3 Trees

Pros

- Good **guaranteed** worst-case performance for basic operations

Cons

- Not “standard” trees—include two kinds of nodes
- Difficult to implement
- Implementation overhead could make it even worse to use than regular BST

References

[1] *Algorithms, Fourth Edition;* Robert Sedgewick and Kevin Wayne (and associated slides)
[2] Slides from https://www.cs.princeton.edu/~rs/talks/LLRB/RedBlack.pdf