

6. (a) Basis step:

When T is 1 node, $i(T) = 0$

Inductive Step:

Let T_1, T_2, T_3 be full binary trees and

$$T_4 = T_1 \cdot T_2 \cdot T_3$$

$$\text{Then, } i(T_4) = i(T_1) + i(T_2) + i(T_3) + 1$$

(b) Basis Step: When T is 1 node, $e(T) = 1$

Inductive Step: Let T_1, T_2, T_3 be full ternary trees and $T_4 = T_1 \cdot T_2 \cdot T_3$

$$e(T_4) = e(T_1) + e(T_2) + e(T_3)$$

(c) Conjecture: $e(T) = 2i(T) + 1$

Basis Step: When T is just the root, $e(T) = 1$ and $i(T) = 0$

$$1 = 2 \times 0 + 1 \quad \checkmark$$

Inductive step: Let T_1, T_2, T_3 be full ternary trees and $T_4 = T_1 \cdot T_2 \cdot T_3$

Assume as the IH that $e(T_1) = 2i(T_1) + 1$,

$e(T_2) = 2i(T_2) + 1$ and $e(T_3) = 2i(T_3) + 1$

$$e(T_4) = e(T_1) + e(T_2) + e(T_3)$$

$$= 2i(T_1) + 1 + 2i(T_2) + 1 + 2i(T_3) + 1$$

$$= 2(i(T_1) + i(T_2) + i(T_3) + 1) + 1$$

$$= 2i(T_4) + 1 \quad (\text{by definition from (a)})$$