Constructing Initial Data for Black Hole/s System

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2 The 3+1 Decomposition

3 For Single BHs

4 For Binary BHs

5 BBHs in Orbits

6 Realistic BBHs Initial Data

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Introduction

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- In order to start any numerical relativity simulations we need to specify initial data that describe the initial state of the binary system. These initial data should be chosen as accurately and as realistically as possible.
- Main motive of the talk: How one can construct reliable initial data for binary black holes.

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Starting Point!

- The key idea behind NR simulations is split the four-dimensional manifold into "space+time" components and the covariant Einstein equations are converted into evolution equations for three-dimensional geometric field.

$$ds^2 = -\alpha^2 dt^2 + \gamma_{ij}(dx^i + \beta^i dt)(dx^j + \beta^j dt) \tag{1}$$

where, γ_{ij} is spatial three-dimensional induced metric.

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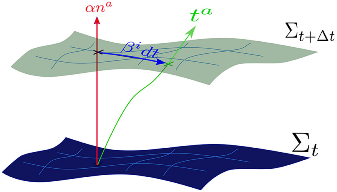
where, γ_{ij} is spatial three-dimensional induced metric.

- One introduces extrinsic curvature which defines the curvature for $t =$ constant hypersurface.

$$K_{ij} = -\frac{1}{2\alpha}(\partial_t \gamma_{ij} - D_i \beta_j - D_j \beta_i) \tag{2}$$

ADM Formalism



- Evolution Equations:

$$\partial_t \gamma_{ij} = -2\alpha K_{ij} + \mathcal{L}_\beta \gamma_{ij}$$

$$\partial_t K_{ij} = \alpha(R_{ij} - 2K_{il}K^l_j + KK_{ij}) - D_i D_j \alpha + \mathcal{L}_\beta K_{ij}$$
$$- 8\pi\alpha S_{ij} + 4\pi\alpha\gamma_{ij}(S - \rho)$$

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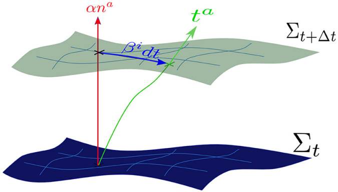

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- Constraint Equations:

$$R - K_{ij}K^{ij} + K^2 = 16\pi\rho$$

$$D_j(K^{ij} - \gamma^{ij}K) = 8\pi j^i$$

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- NO UNIQUENESS!

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- Introducing Conformal Decomposition!

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- Introducing Conformal Decomposition!
- We have two main approaches:
    - Conformal transverse traceless (CTT)
    - Conformal thin-sandwich (CTS)

Conformal Decomposition

- Introduce

$$\gamma_{ij} = \psi^4 \bar{\gamma}_{ij}; \quad R = \psi^{-4} \bar{R} - 8\psi^{-5} \bar{D}_k \bar{D}^k \psi$$

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- Further, define *conformal killing operator* L such that

$$(LW)^{ij} := D^i W^j + D^j W^i - \frac{2}{3} \gamma^{ij} D_k W^k$$

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- Next, the extrinsic curvature is split into its trace and its tracefree part

$$K^{ij} = \underbrace{A^{ij}}_{tracefree} + \underbrace{\frac{1}{3}\gamma^{ij} K;}_{trace} \quad A^{ij} = \psi^{-10} \bar{A}^{ij}$$

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- Finally, Hamiltonian constraint

$$8 \bar{D}_k \bar{D}^k \psi - \bar{R} \psi + \psi^{-7} \bar{A}^{ij} \bar{A}_{ij} - \frac{2}{3} \psi^5 K = -16\pi \psi \rho$$

Conformal Transverse Traceless

- We start by splitting $\bar{A}^{ij}$:

$$\bar{A}^{ij} = \underbrace{\bar{M}^{ij}}_{transverse-traceless} + \underbrace{\frac{1}{\bar{\sigma}}}_{weighting} (\bar{L}W)^{ij}$$

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- Solve Hamiltonian and Momentum constraints for the conformal factor ψ and killing vector W^i.

Conformal Thin-Sandwich

- Here we not only look for $\bar{\gamma}_{ij}$, but also for its time-derivatives

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- We already have Schwarzschild metric defining such BHs.

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$$ds^2 = -\left(\frac{1 - m/2r}{1 + m/2r}\right)^2 dt^2 + \psi^4[dr^2 + r^2(d\theta^2 + d\phi^2)]$$

where, $\psi = 1 + m/2r$

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- One can write initial data for this BHs as

$$\alpha = \left(\frac{1 - m/2r}{1 + m/2r}\right); \qquad \beta^i = 0; \qquad \gamma_{ij} = \psi^4 \delta_{ij}; \qquad K_{ij} = 0$$

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where,

$$H = \frac{mr}{r^2 + a^2(z/r)^2}$$

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- Initial data obtained is of form

$$\alpha = \frac{1}{\sqrt{1 + 2Hk_0 k_0}}; \qquad \beta^i = 2Hk_0 k_i;$$

$$\gamma_{ij} = \delta_{ij} + 2Hk_i k_j; \qquad K_{ij} = \frac{D_i\beta_j + D_j\beta_i}{2\alpha}$$

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BHs at rest

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- After applying boundary condition for conformal factor $\lim_{r\to\infty}\psi = 1$, we get

$$\psi_{BL} = 1 + \sum_{A=1}^{N} \frac{m_A}{2r_A}$$

Puncture Initial Data [1]

- Generalisation of Brill-Lindquist data with non-vanishing K_{ij}.

$$K = 0$$
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- Further, one uses CTT formalism with $\bar{\sigma} = 1$ and $\bar{M}_{ij} = 0$. Momentum constraint reduces to

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$$W^i = \frac{1}{4r}(7P^i + n^i n_j P^j) + \frac{1}{r^2}\epsilon_{ijk} n^j S^k$$

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- Conformal Bowen-York extrinsic curvature :

$$\bar{A}^{ij}_{BY} = \frac{3}{2r^2}\left[P^i n^j + P^j n^i - \left(\delta^{ij} - n^i n^j\right) P^k n_k\right] + \frac{3}{r^3}\left(n^i \epsilon^{jkl} + n^j \epsilon^{ikl}\right) S_k n_l$$

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- Next step is to solve Hamiltonian constraint.

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$$\psi = \psi_{BL} + u$$

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$$\psi = \psi_{BL} + u$$

- Inserting it in the constraint eqn;

$$\bar{D}^2 u + \frac{1}{8\psi_{BL}^7}\bar{A}_{ij}\bar{A}^{ij}\left(1 + \frac{u}{\psi_{BL}}\right)^{-7} = 0$$

Bowen-York Initial Data

- Bowen-York is an extension of Puncture initial data where we take isometry in account.

$$\begin{aligned}\bar{A}^{ij}_{\pm} =& \frac{3}{2r^2}\left[P^i n^j + P^j n^i - \left(\delta^{ij} - n^i n^j\right) P^k n_k\right] \\ & \pm \frac{3b^2}{2r^4}\left[P^i n^j + P^j n^i + \left(\delta^{ij} - 5n^i n^j\right) P^k n_k\right] \\ & + \frac{3}{r^3}\left(n^i \epsilon^{jkl} + n^j \epsilon^{ikl}\right) S_k n_l\end{aligned}$$

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- Because of isometry, we need boundary condition at each of the throats.

$$\partial_r \psi|_{r=a} = -\psi/2r|_{r=a}$$

r: coordinate distance to the center of the throat
a: radius

Remarks on BY extrinsic curvature

- We get Schwarzschild BHs from both puncture as well as Bowen-York initial data when

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- We get Schwarzschild BHs from both puncture as well as Bowen-York initial data when

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- Do we get Kerr BH if $P^i = 0$ and $S^i \neq 0$?

Remarks on BY extrinsic curvature

- We get Schwarzschild BHs from both puncture as well as Bowen-York initial data when

$$P^i = 0$$
$$S^i = 0$$

- No! It doesnt represent Kerr BH. Proved by Garat and Price! [2]

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- What does BY initial data represent?
- Bowen–York black hole is not stationary!

CTS based Initial Data

- Making most obvious choice for the free data

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- Impose the apparent horizon boundary conditions for the conformal factor and the shift.

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- We consider binary black holes in the quasi-steady stage.

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Quasi-Circular Orbits

- We consider binary black holes in the quasi-steady stage.
- Changing to a corotating coordinate system.
- $\partial_t g_{\mu\nu} = 0 \implies t^\mu$ is approximate symmetry of the spacetime.

Quasi-Circular Orbits

- We consider binary black holes in the quasi-steady stage.
- Changing to a corotating coordinate system.
- Implies the existence of an approximate helical Killing vector ξ^{μ} with

$$\mathcal{L}_{\xi} g_{\mu\nu} \approx 0$$

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- For lapse, $\partial_t K = 0$ gives me an elliptic equation.

$$\nabla^2 v = \frac{7}{8}(\alpha\psi)\psi^{-8}\bar{A}_{ij}\bar{A}^{ij}$$

where, we have punctured ansatz:

$$\alpha\psi = 1 - \left(\frac{c_1 m_1}{2r_1} + \frac{c_2 m_2}{2r_2}\right) + v$$

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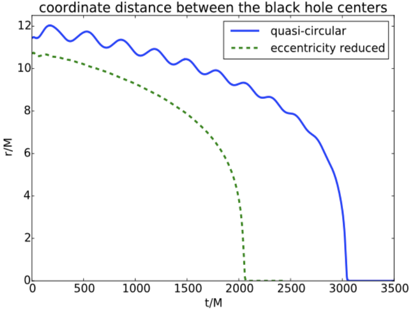
- For shift, $\partial_t \bar{\gamma}_{ij} = 0$

Quasi-Circular Orbits



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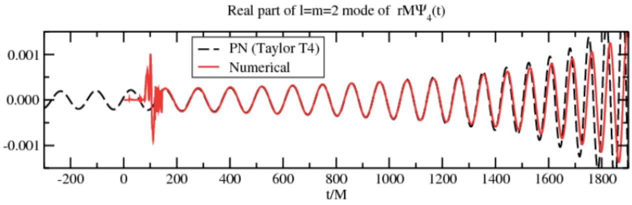
Post-Newtonian based Initial Data [4]

- While working with CTT or CTS, we concluded that the initial slice of our spacetime contains no gravitational radiation at all, even though we have two orbiting black holes.



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- PN initial data in ADMTT [3] gauge:

$$\gamma_{ij}^{PN} = \psi_{PN}^4 \delta_{ij} + h_{ij}^{TT}; \qquad K_{PN}^{ij} = \psi_{PN}^{-10} (\bar{A}_{BY}^{ij} + k_{TT}^{ij})$$

where,

$$\psi_{PN} = 1 + \sum_{A=1}^{2} \frac{E_A}{2r_A} + O(\epsilon^6); \quad E_A = \epsilon^2 m_A + \epsilon^4 \left(\frac{p_A^2}{2m_A} - \frac{m_1 m_2}{2r_{12}} \right)$$

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- Construct constraint satisfying initial data by using the CTT decomposition.

$$\bar{\gamma}_{ij} = \psi_{PN}^{-4} \gamma_{ij}^{PN}; \qquad \bar{M}^{ij} = \psi_{PN}^{10} \left(K_{PN}^{ij} - \frac{1}{3} \gamma_{PN}^{ij} K_{PN} \right); \qquad K = 0$$

Post-Newtonian based Initial Data [4]

- Inserting puncture ansatz $\psi = \psi_{PN} + u$ in CTT constraint equations for both Hamiltonian and Momentum.

Post-Newtonian based Initial Data [4]

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- Modified conformal factor leads to an increase in the ADM mass.

Post-Newtonian based Initial Data [4]

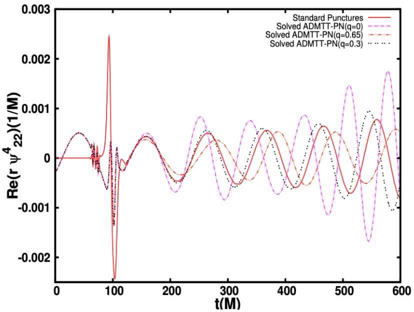
- Inserting puncture ansatz $\psi = \psi_{PN} + u$ in CTT constraint equations for both Hamiltonian and Momentum.
- Modified conformal factor leads to an increase in the ADM mass.
- Changing the conformal factor

$$\psi_{PN} \to \psi_{PN} - q\frac{m_1 m_2}{2r_{12}}\left(\frac{1}{2r_1} + \frac{1}{2r_2}\right)$$

Post-Newtonian based Initial Data [4]

- Inserting puncture ansatz $\psi = \psi_{PN} + u$ in CTT constraint equations for both Hamiltonian and Momentum.
- Modified conformal factor leads to an increase in the ADM mass.
- Changing the conformal factor

$$\psi_{PN} \rightarrow \psi_{PN} - q\frac{m_1 m_2}{2r_{12}}\left(\frac{1}{2r_1} + \frac{1}{2r_2}\right)$$



1 Introduction

2 The 3+1 Decomposition

3 For Single BHs

4 For Binary BHs

5 BBHs in Orbits

6 Realistic BBHs Initial Data

7 References

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Thanks!