Exponential distribution analysis

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Overview

The Central Limit Theorem (CLT) states that *"the distribution of averages of indipendent and identically distributed (iid) variables becomes that of a standard normal as the sample size increase."* so in this assignment it will be investigate the behaviour of the **exponential distribution** when looking at the distribution of averages of 40 exponentials (and more).

Simulations

Lets simulate an experiment with a random variable X with an **exponential distribution** with the following characteristics

- **rate parameter** $\lambda = 0.2$
  - **mean** $\mu = E[X_i] = 1/\lambda = 5$
  - **standard deviation** $\sigma = \sqrt{2xVar(X_i)} = 1/\lambda = 5$
  - **standard error** (SE) $\sigma/\sqrt{2n}$

For building the distribution of averages we will run thousands of simulation and for each simulation the following steps are executed:

- run the experiment n time where `n = 40, 80, 120, 160`,
  - be X_i the outcome for the experiment $i = 1..n$
- calculate the mean $\overline{X}_n$ using $X_1, X_2, .., X_n$
- calculate $\frac{\overline{X}_n - \mu}{\sigma/\sqrt{n}}$

The following the code snippet can be used to generate the simulation data.

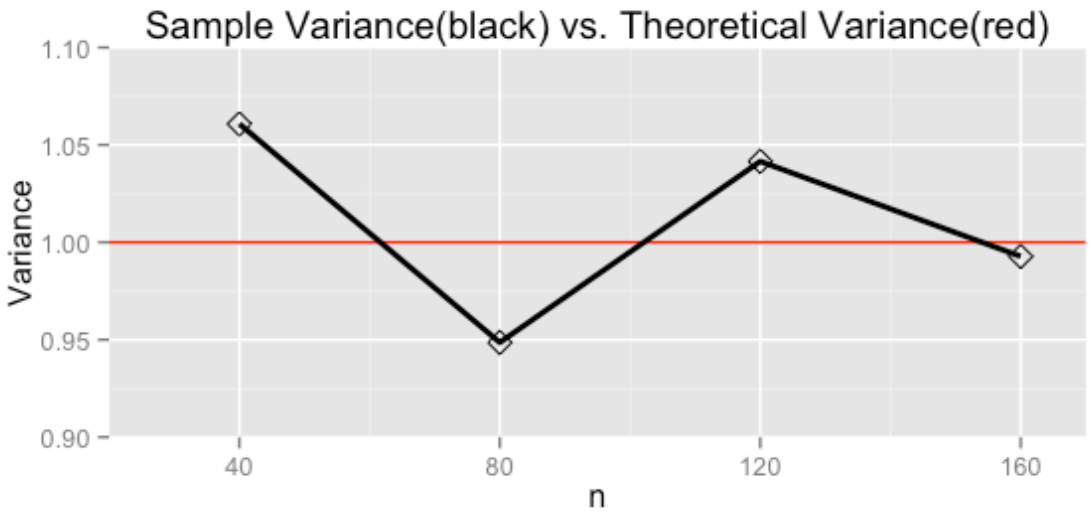
```
nosim = 1000
n = 40
clt_func <- function(x, n) sqrt(n) * (mean(x) - 5) / 5

dat <- data.frame(
  x = c(
    apply(matrix(rexp(n * nosim, 0.2), nrow = nosim), MARGIN = 1, clt_func, n),
    apply(matrix(rexp((2*n) * nosim, 0.2), nrow = nosim), MARGIN = 1, clt_func, (2*n)),
    apply(matrix(rexp((3*n) * nosim, 0.2), nrow = nosim), MARGIN = 1, clt_func, (3*n)),
    apply(matrix(rexp((4*n) * nosim, 0.2), nrow = nosim), MARGIN = 1, clt_func, (4*n))
    ),
  size = factor(rep(c(n, 2*n, 3*n, 4*n), rep(nosim, 4)))
  )
```

Sample Mean versus Theoretical Mean

Based on the **CLT** the **theoretical mean** is $\mu = 0$ and the value of the sample mean is **0.0506595** for n = 40. The **sample mean** is around the **theoretical mean** for `n = 40` and it converges more and more to it increasing the size of n (`n = 80, 120, 160`).

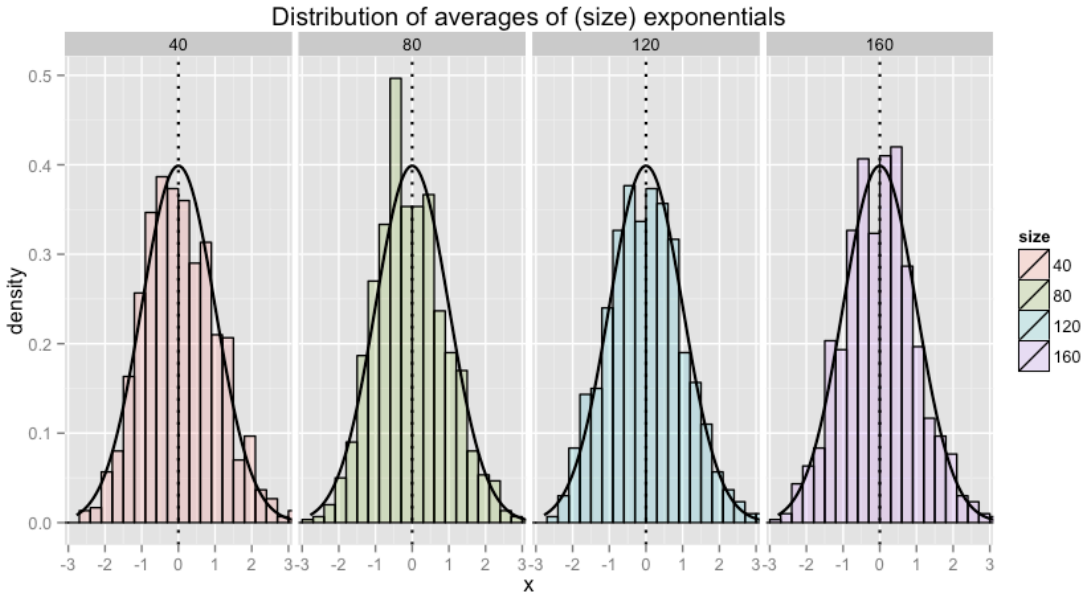
Sample Variance versus Theoretical Variance

Based on the **CLT** the **theoretical variance** is $\sigma^2 = 1$ and the value of the sample variance is **1.0607617** for n = 40. The **sample variance** is around the **theoretical variance** for `n = 40` and it converges more and more to it increasing the size of n (`n = 80, 120, 160`).



Distribution

Lets plot the distribution of the averages for `n = 40, 80, 120, 160` previously calculated over all of the simulations together with the standard normal distribution $N(0,1)$ (black bell curve). We can see from the plot below that the distribution of averages $(\frac{\overline{X}_n - \mu}{\sigma/\sqrt{n}})$ for `n = 40` is already a good approximation of the standard normal distribution $N(0,1)$ and gets better and better increasing the size of `n` as expected from the Central Limit Theorem.



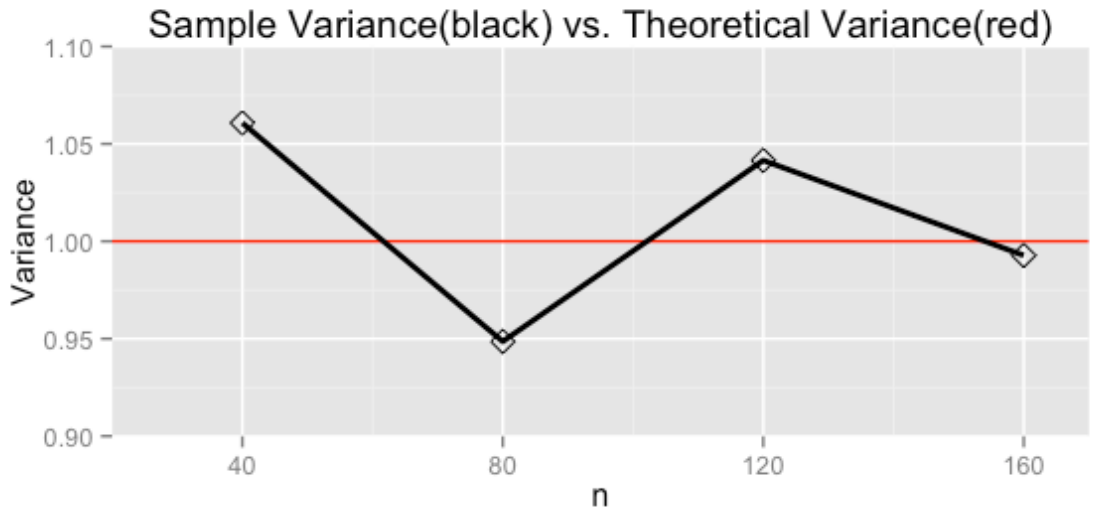
Appendix

Code chunk used to generate plot in **Sample Mean versus Theoretical Mean** section.

```
meanSims <- c(
    mean(dat[dat$size == n,]$x),
    mean(dat[dat$size == (2*n),]$x),
    mean(dat[dat$size == (3*n),]$x),
    mean(dat[dat$size == (4*n),]$x)
    )
g <- ggplot(data.frame(x = c(n, 2*n, 3*n, 4*n), y = meanSims), aes(x = x, y = y))
g <- g + geom_hline(yintercept = 0, color = "red") + geom_line(size = 1) + geom_point(shape = 5, s
ize = 3)
g <- g + labs(x = "n", y = "Mean", title ="Sample Mean(black) vs. Theoretical Mean(red)")
g <- g + coord_cartesian(xlim= c(20,170), ylim=c(-0.06,0.06))
g
```

Code chunk used to generate plot in **Sample Variance versus Theoretical Variance** section.

```
varianceSims <- c(
    sd(dat[dat$size == n,]$x)^2,
    sd(dat[dat$size == (2*n),]$x)^2,
    sd(dat[dat$size == (3*n),]$x)^2,
    sd(dat[dat$size == (4*n),]$x)^2
    )
g <- ggplot(data.frame(x = c(n, 2*n, 3*n, 4*n), y = varianceSims), aes(x = x, y = y))
g <- g + geom_hline(yintercept = 1, color = "red") + geom_line(size = 1) + geom_point(shape = 5, s
ize = 3)
g <- g + labs(x = "n", y = "Variance", title ="Sample Variance(black) vs. Theoretical Variance(red
)")
g <- g + coord_cartesian(xlim= c(20,170), ylim=c(0.9,1.1))
g
```



Code chunk used to generate plot in **Distribution** section.

```
g <- ggplot(dat, aes(x = x, fill = size)) + geom_histogram(alpha = .20, binwidth=.3, colour = "bla
ck", aes(y = ..density..))
g <- g + stat_function(fun = dnorm, size = 0.8)
g <- g + geom_vline(xintercept=c(0,0), color="black", linetype="dotted", size = 0.8)
g <- g + coord_cartesian(xlim= c(-3.1,3.1)) + labs(title = "Distribution of averages of (size) exp
onentials")
g + facet_grid(. ~ size)
```