# Matrix - vector products

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

Define $A\vec{x}$

$$\begin{bmatrix} a_{11} & \cdots & a_{1n} \\ a_{m1} & \cdots & a_{mn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

$\vec{v}_1 \quad \cdots \quad \vec{v}_n$

$$A = [\vec{v}_1 \; \vec{v}_2 \; \cdots \cdots \; \vec{v}_n]$$

$$A\vec{x} = \vec{v}_1 x_1 + \vec{v}_2 x_2 + \vec{v}_3 x_3 \cdots + \vec{v}_n x_n$$

$= \mathbb{R}^{m \times 1}$ vector

Interpretation of matrix multiplication
- = weighted combination of column vectors where $x$ is the weight
- = dot products of column vectors with $x$

## Introduction to the null space of a matrix

Subspace $S$

Recall for a subspace, the following should be true:

1) $\vec{0} \in S$
2) $\vec{v}_1, \vec{v}_2 \in S$, then $\vec{v}_1 + \vec{v}_2 \in S$
3) $\vec{v}_1 \in S$, $c \in \mathbb{R}$, then $c\vec{v}_1 \in S$

$A = m \times n$ matrix

$A\vec{x} = 0$ } homogenous equation

$N = \{ x \in \mathbb{R}^n \mid A\vec{x} = \vec{0} \}$ → for matrix multiplication to work, its $\mathbb{R}^n$

N is a set of all x's that satisfies $A\vec{x} = \vec{0}$

We want to know if N is a valid sub-space

1) Does it contain the zero vector

for it to contain the zero vector it should satisfy the

$$A\vec{x} = \vec{0}$$

is $A \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} = \vec{0}$

$$v_1 0 + v_2 0 + \dots + v_n 0 = 0$$

So $\vec{0}$ satisfies this equation. So N contains the zero vector

2) If $\vec{a_1}$ and $\vec{a_2}$ are members of N, is $\vec{a_1} + \vec{a_2}$ a member of N

$$\vec{a_1}, \vec{a_2} \in N$$

$$A\vec{v_1} = \vec{0}$$
$$A\vec{v_2} = \vec{0}$$

$$A(\vec{v_1} + \vec{v_2}) = 0$$

$$\underbrace{A\vec{v_1}}_{0} + \underbrace{A\vec{v_2}}_{0}$$

If $\vec{a_1}, \vec{a_2}$ are members of N, then $(\vec{a_1} + \vec{a_2})$ are also member of N

3) If $\vec{a_1} \in N$, then is $c\vec{a_1}$ a member of N?

$$A\vec{x} = 0$$

$$A\vec{a} = 0$$

$$A\,c\vec{a} = 0$$

$$c\,\underbrace{A\vec{a}}_{0} = 0$$

So, $c\,A\vec{a}$ also satisfies the $A\vec{x} = 0$ equation

→ We have showed that N is a valid subspace

→ Now this N has a special name : NULL SPACE of A

**For an arbitary matrix A , the null space is N OR N = N(A)**

**i.e , the set of all $\vec{x}$'s that give satisfy the equation $A\vec{x} = 0$**

## Relationship to linear independence

If column vectors of A are linearly independent then :

$$\boxed{\text{Null space of } A \qquad N(A) = \{\vec{0}\}}$$

that's because :

$x_1 v_1 + x_2 v_2 + \cdots x_n v_n = 0$, when

$v_1, v_2, \ldots$ are independent is when all

$x$'s are zero