Linear Subspace

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Subspace of R^n $\Rightarrow$ if we say V is a subspace of R^n then V is a subset of R^n

Conditions for V to be a subspace of R^n, then this means 3 things

1) V contains the 0 vector ($\mathbf{0} = \begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix}$)

2) if $\vec{x}$ in V, then $c\vec{x}$ is also in V

- or if $\vec{x}$ is a subspace of V
- closure under multiplication

3) if $\vec{a}$ in V and $\vec{b}$ in V, then $\vec{a}+\vec{b}$ in V

- closure under addition

EXAMPLES

$V = \{\mathbf{0}\} = \left\{ \begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix} \right\}$

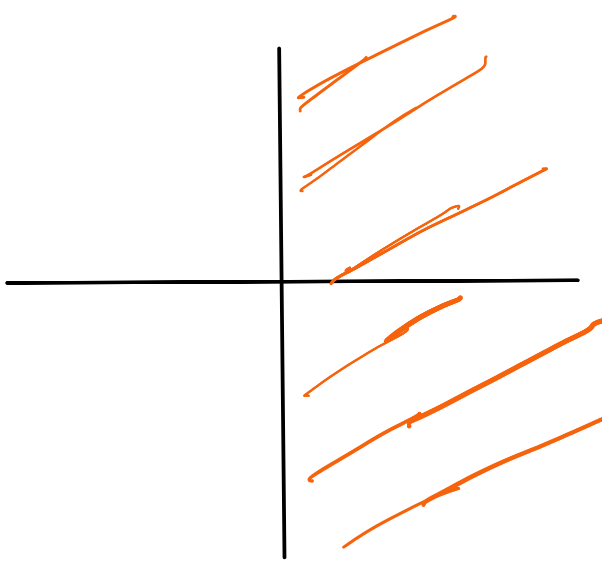
1) contains the zero vector

2) $c\begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix}$

3) $\begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix}$

So V is a subspace of R^3

2. $S = \left\{ \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \in \mathbb{R}^2 \;\middle|\; x_1 \geq 0 \right\}$



1) contains the zero vector

2) $c\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ if c is +ve we are good

but if c is <0 then we are outside $x_1 \geq 0$ so we are not in our subspace (S)

So, we are violating the condition of being a subspace

So its not a subspace of $\mathbb{R}^2$

3) $U = \text{span}(v_1, v_2, v_3)$ is U valid subspace of $\mathbb{R}^n$

: linear combinations of (v_1, v_2, v_3)

$\Rightarrow c_1 v_1 + c_2 v_2 + c_3 v_3$

1) if $c_1 = c_2 = c_3 = 0$, then span contains the zero vector

2) $a_1 c_1 v_1 + a_2 c_2 v_2 + a_3 c_3 v_3$

$\Rightarrow a_{11} v_1 + a_{22} v_2 + a_{33} v_3$ } still a linear combination of v_1, v_2, v_3 with new arbitrary constants

3) $c_1 v_1 + c_2 v_2 + c_3 v_3 + c_1 v_1 + c_2 v_2 + c_3 v_3$

$\Rightarrow c_{11} v_1 + c_{22} v_2 + c_{33} v_3$ }

$\Rightarrow$ So both 2), 3) are in U, so U is a subspace of $\mathbb{R}^n$

Basis of Subspace

If 1) $V = \text{span}(\vec{v_1}, \vec{v_2}, \dots \vec{v_n})$

2) All $\vec{v_1}, \vec{v_2} \dots \vec{v_n}$ are linearly independent

Then we can say : S is a basis of **V**

Basis = a "minimum" set of vectors that spans the subspace

So if you are able to remove a vector from a span and still able to create the subspace then its not a basis

There can be many basis for a space ($\mathbb{R}^2$)

Standard basis for $\mathbb{R}^2$ $\left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$

$\{ i, j \}$

And you can represent any vector in $\mathbb{R}^2$ by a combination of i & j

$\{v_1, v_2 \dots v_n\}$ = Basis for U

= $v_1, v_2 \dots$ are linearly independent

= $v_1, v_2 \dots$ span U

each vector in **U** is a unique combination of those vectors