

Matrix - vector products

$$A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix}$$

Define $A\vec{x}$

$$\begin{bmatrix} a_{11} & \dots & a_{1n} \\ a_{m1} & \dots & a_{mn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

$\vec{v}_1 \quad \dots \quad \vec{v}_n$

$$A = [\vec{v}_1 \quad \vec{v}_2 \quad \dots \quad \vec{v}_n]$$

$$\begin{aligned} A\vec{x} &= \vec{v}_1 x_1 + \vec{v}_2 x_2 + \vec{v}_3 x_3 + \dots + \vec{v}_n x_n \\ &= \mathbb{R}^{m \times 1} \text{ vector} \end{aligned}$$

Interpretation of matrix multiplication $\left\{ \begin{array}{l} = \text{weighted combination of column vectors where } x \text{ is the weight} \\ = \text{dot products of column vectors with } x \end{array} \right.$

Introduction to The null space of a matrix

Subspace S

Recall for a subspace, the following should be true:

- 1) $\vec{0} \in S$
- 2) $\vec{v}_1, \vec{v}_2 \in S$, then $\vec{v}_1 + \vec{v}_2 \in S$
- 3) $\vec{v}_1 \in S$, $c \in \mathbb{R}$, then $c\vec{v}_1 \in S$

$$A = m \times n \text{ matrix}$$

$$A \vec{x} = \vec{0}$$

} homogenous equation

$$N = \left\{ x \in \mathbb{R}^n \mid A \vec{x} = \vec{0} \right\} \rightarrow \text{for matrix multiplication to work, its } \mathbb{R}^n$$

N is a set of all x 's that satisfies $A \vec{x} = \vec{0}$

We want to know if N is a valid sub-space

1) Does it contain the zero vector

for it to contain the zero vector it should satisfy the

$$A \vec{x} = \vec{0}$$

$$\text{or } A \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} = \vec{0}$$

$$0_1 \cdot 0 + 0_2 \cdot 0 + \dots + 0_n \cdot 0 = 0$$

So $\vec{0}$ satisfies this equation. So N contains the zero vector

2) If \vec{a}_1 and \vec{a}_2 are members of N , is $\vec{a}_1 + \vec{a}_2$ a member of N

$$\vec{a}_1, \vec{a}_2 \in N$$

$$A \vec{v}_1 = \vec{0}$$

$$A \vec{v}_2 = \vec{0}$$

$$A (\vec{v}_1 + \vec{v}_2) = \vec{0}$$

$$A \vec{v}_1 + A \vec{v}_2$$

$$\downarrow \quad \downarrow$$

$$0 \quad 0$$

If \vec{a}_1, \vec{a}_2 are members of N , then $(\vec{a}_1 + \vec{a}_2)$ are also member of N

3) If $\vec{a}_1 \in N$, then is $c \vec{a}_1$ a member of N ?

$$A\vec{x} = 0$$

$$A\vec{a} = 0$$

$$A c\vec{a} = 0$$

$$c A\vec{a} = 0$$

So, $c A\vec{a}$ also satisfies the $A\vec{x} = 0$ equation

For an arbitrary matrix A , the null space is N OR $N = N(A)$ i.e., the set of all \vec{x} 's that give satisfy the equation $A\vec{x} = 0$

Relationship to linear independence

If column vectors of A are linearly independent then:

$$\text{Null space of } A \quad N(A) = \{ \vec{0} \}$$

that's because :

$x_1 v_1 + x_2 v_2 + \dots + x_n v_n = 0$, when

v_1, v_2, \dots are independent is when all
 x_i 's are zero