Dimension of a column space or rank of matrix

Column space of A or $C(A)$

$A = [a_1, a_2 \dots a_n]$

$C(A) = \text{span}(\vec{a_1}, \vec{a_2}, \vec{a_3}, \vec{a_4}, \vec{a_5})$

Now basis of $C(\vec{A})$ = set of linearly independent vectors in $\text{span}(a_1, a_2 \dots a_5)$

$$\boxed{\dim(C(A)) = \#\text{ of vectors in basis of } C(A)}$$

So if $\{\vec{a_1}, \vec{a_2}, \vec{a_4}\}$ form basis of $C(A)$ then,

$$\dim(C(A)) = 3$$

$$\boxed{\dim(C(A)) \text{ is also called Rank}(A)}$$

Dimension of the null space or nullity

$B = \begin{bmatrix} 1 & 1 & 2 & -3 & 2 \\ 1 & 1 & 3 & 1 & 4 \end{bmatrix}$; B is a matrix

$N(B) = \{\vec{x} \in \mathbb{R}^5 \mid B\vec{x} = 0\}$: a set
: x is 5 dimensional

$N(B) = N(\text{rref}(B))$ reduced row echelon form of B

Let basis of N(B) be $\{\vec{v_1}, \vec{v_2}, \vec{v_3})$

Dimension of a subspace = # of elements in the basis of the subspace

or $\dim(N(B))$ = # of vectors/ elements

dimension of
null space of B

= # of vectors in basis of
null space of B

$$\boxed{\dim(N(B)) = \text{nullity of } B}$$