University of Zurich[UZH]

Master Thesis Biostatistics

Recruitment rate stochasticity at the design stage of a clinical trial

Supervision by Malgorzata Roos

Pilar Pastor

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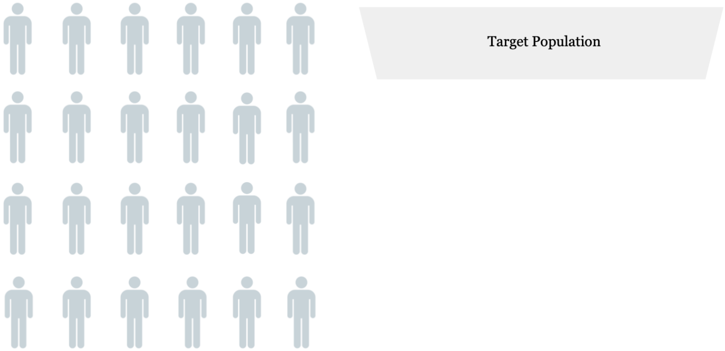
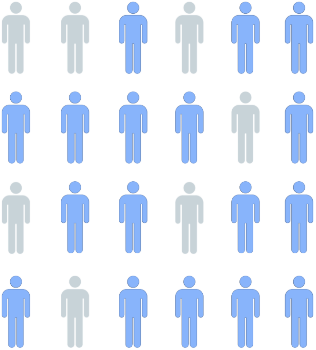
Why recruitment rates?

- Timely recruitment vital to the success of a clinical trial
- Inadequate number of subjects $\rightarrow$ lack of power
- Recruitment period too long $\rightarrow$ competing treatments
- Recruitment of patients varies at each stage
- Accrual = Cumulative Recruitment
- Carter (2004)

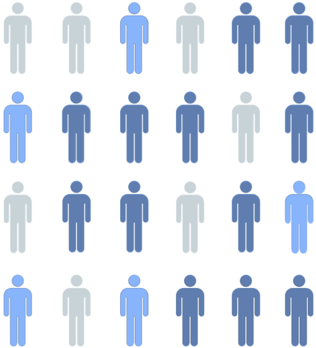
Target Population

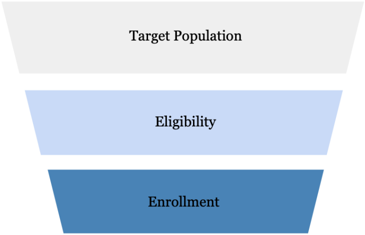


Eligibility

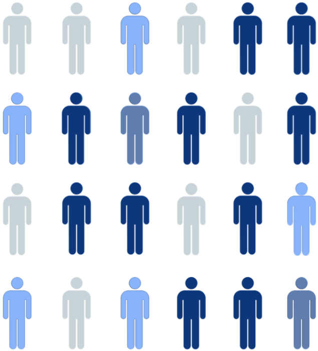


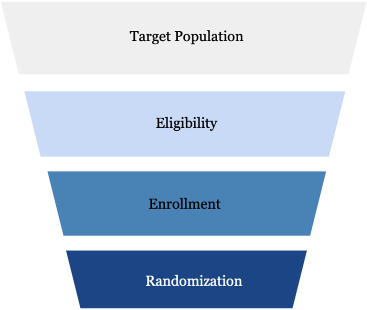
Enrollment



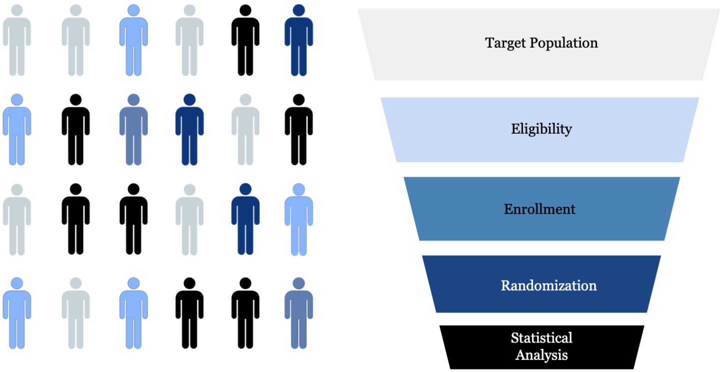


Randomization

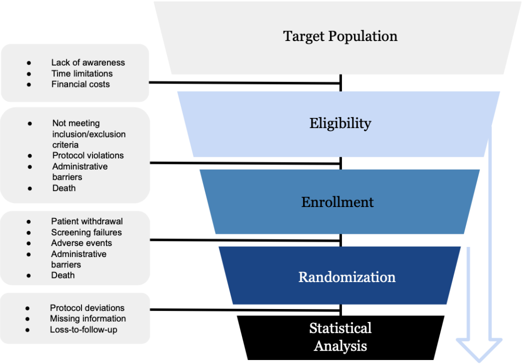




Statistical Analysis



Patient Attrition



Uncertainty

- **Aleatory**: randomness inherent and unpredictable
- **Epistemic**: arises from limited knowledge about parameters

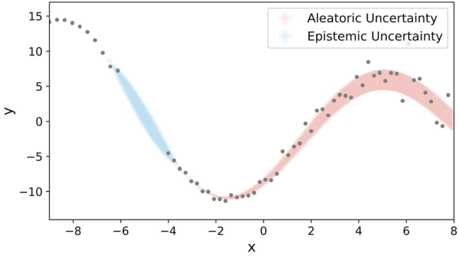


Figure: Visualization of two types of uncertainty (Yang and Li, 2023)

Models for Counts

Methods	Counts	Expectation	Variance	Aleatory	Epistemic
Expectation	$C(t) = \lambda t$	λt	0	No	No
Poisson	$C(t) \sim \mathrm{Po}(\lambda \mathrm{t})$	λt	λt	Yes	No
Negative Binomial	$C(t) \sim \mathrm{Po}(\tilde{}\mathrm{t}); \Lambda \sim \mathrm{G}(\alpha, \beta)$	$\frac{\alpha}{\beta}$	$\frac{\alpha(\beta+1)}{\beta^2}$	Yes	Yes

Table: Aleatory and epistemic uncertainty in accrual shown by different models for counts.

Study

- Time $t = 550$ days
- Recruitment Rate $\lambda = \frac{\textit{Counts}}{\textit{Time}} = 0.591$

Study

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- Recruitment Rate $\lambda = \frac{Counts}{Time} = 0.591$
- Models for Counts:
  - Expectation: $\mathrm{EC(t)} = \lambda \mathrm{t} = 0.591 \cdot 550 = 325$
  - Poisson: $C(t) \sim \mathrm{Po}(\lambda \mathrm{t})$
  - Negative Binomial: $C(t) \sim \mathrm{Po}(\tilde{\ }\mathrm{t}); \Lambda \sim \mathrm{G}(\alpha, \beta)$

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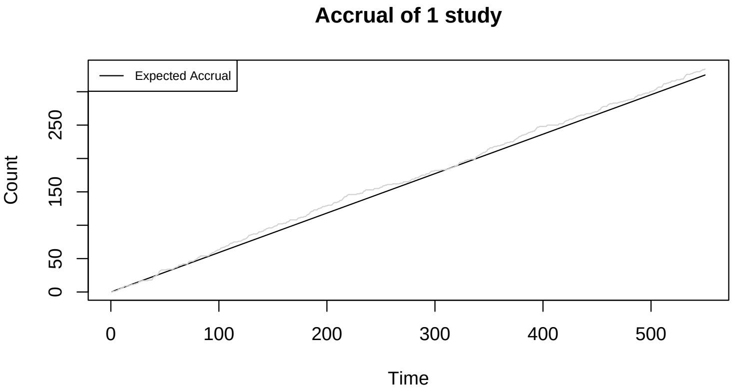
  - Negative Binomial: $C(t) \sim \mathrm{Po}(\Lambda t)$; $\Lambda \sim \mathrm{G}(\alpha, \beta)$

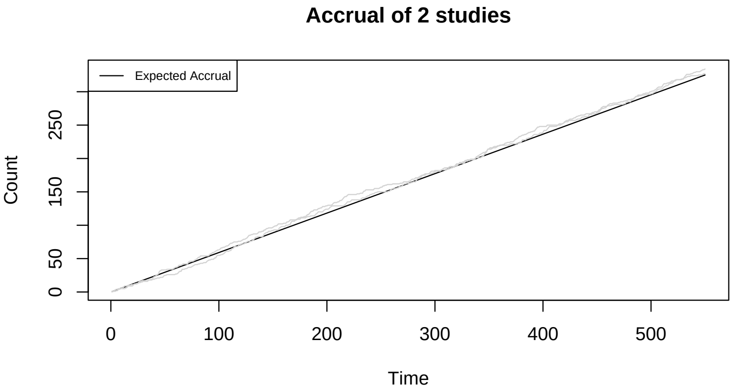
Accrual at time point t

- **Expectation**: $\mathrm{E}C(t) = \underbrace{\mathrm{E}C + \ldots + C}_{t \text{ times}} = t\mathrm{E}C = \lambda t$
- **Poisson**: $\underbrace{\mathrm{Po}(\lambda) + \ldots + \mathrm{Po}(\lambda)}_{t \text{ times}} = \mathrm{Po}(\lambda t)$

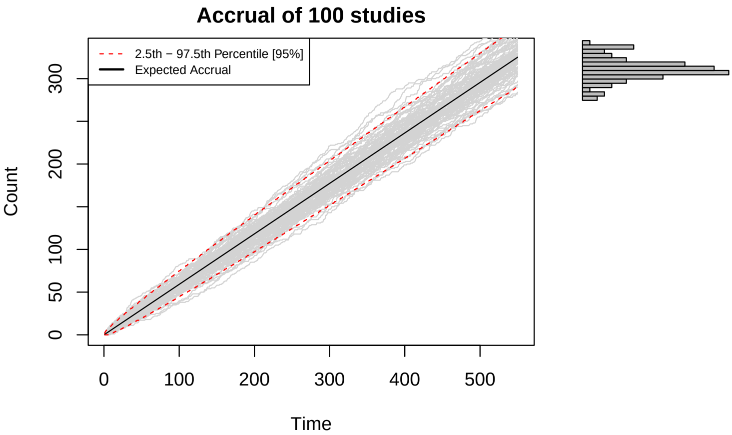
Accrual of 1 study



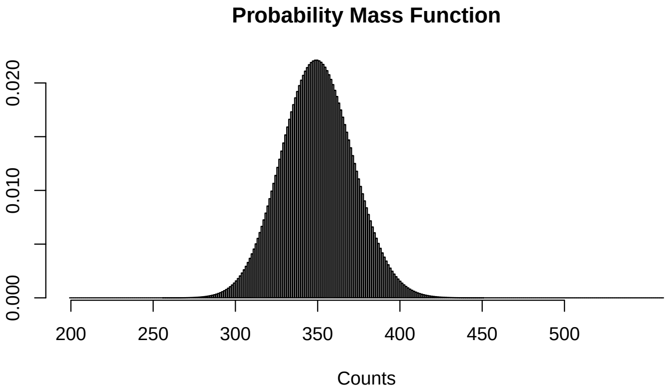
Accrual of 2 studies



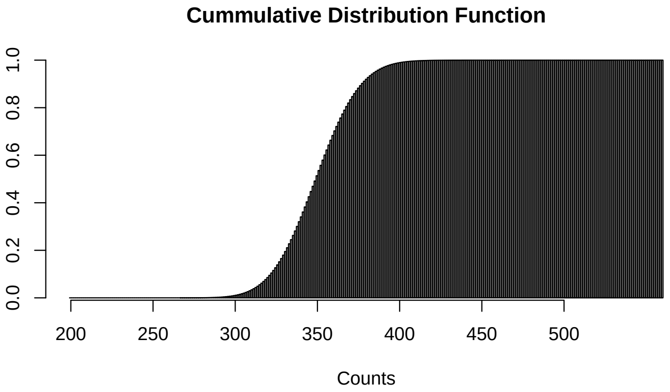
Accrual of 100 studies



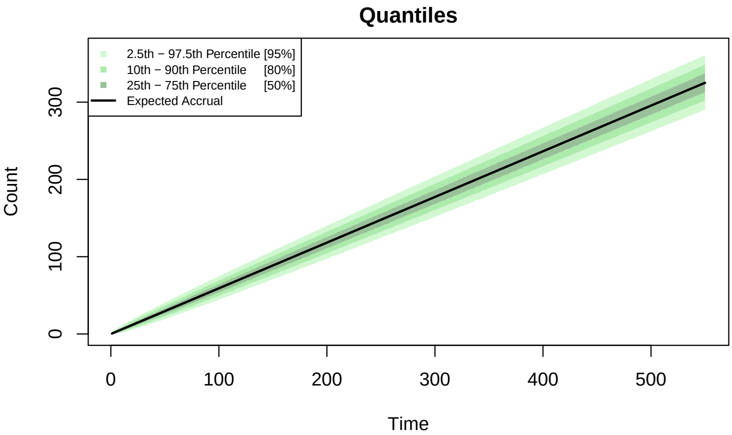
Theoretical PMF



Theoretical CDF



Uncertainty bands



Negative binomial derived from Poisson-Gamma model

Recruitment in one unit of time ($t = 1$)

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- Models for Counts:
  - Expectation: $\mathrm{E}C(t) = \lambda t = 0.591 \cdot 550 = 325$
  - **Poisson**: $C(t) \sim \mathrm{Po}(\lambda t)$
  - **Negative Binomial**: $C(t) \sim \mathrm{Po}(\Lambda t); \Lambda \sim \mathrm{G}(\alpha, \beta)$
    - $\alpha = 325$
    - $\beta = 1.5 \cdot 365$
    - $\mathrm{E}\Lambda = \frac{\alpha}{\beta} = 0.591 = \lambda$

Comparison between Poisson and Negative Binomial

Comparison between Poisson and Negative Binomial

Summary

- Models for **counts**
- Extended Carter's simulation to exact distributions

Next steps

- Application to simulation on Carter (2004)
- Models for **time**
  - Theoretical
  - Application on Carter (2004)
- Shiny App
- Predictions using theoretical models developed on Daniore Nittas dataset of rates (cite?)

References

Carter, R. E. (2004). Application of stochastic processes to participant recruitment in clinical trials. *Controlled clinical trials*, 25(5):429–436.

Liu, J., Jiang, Y., Wu, C., Simon, S., Mayo, M. S., Raghavan, R., and Gajewski, B. J. (2023). *accrual: Bayesian Accrual Prediction*. R package version 1.4.

Spiegelhalter, D., Pearson, M., and Short, I. (2011). Visualizing uncertainty about the future. *Science*, 333(6048):1393–1400.

Yang, C.-I. and Li, Y.-P. (2023). Explainable uncertainty quantifications for deep learning-based molecular property prediction. *Journal of Cheminformatics*, 15(1):13.