Introduction to Computational and Algorithmic Thinking

LECTURE 5 – ITERATION, LISTS, AND ALGORITHM DESIGN

Announcements

This lecture: Iteration, Lists, and Algorithm Design

Reading: Read Chapter 3 of Conery

Acknowledgement: Some of the lecture slides are based on CSE 101 lecture notes by Prof. Kevin McDonald at SBU and the textbook by John Conery.

The Sieve of Eratosthenes

- As a motivating example of studying (i) **iteration** (code that repeats a list of steps), (ii) **lists**, and (iii) the thought process for **designing algorithms**, we will look at an ancient algorithm for finding prime numbers called **the Sieve of Eratosthenes**
- A **prime** is a natural number greater than 1 that has no divisors other than 1 and itself
- In modern times, prime numbers play an important role in encrypting data, including Internet traffic
- Non-prime numbers are called **composite** numbers
  - Example primes: 5, 11, 73, 9967, . . .
  - Example composites: 10 (2x5), 99 (3x3x11)

The Sieve of Eratosthenes

- The basic idea of the algorithm is simple. Below, it is briefly described in **pseudocode**:

  make a list of numbers, starting with 2

  repeat the following steps until done:

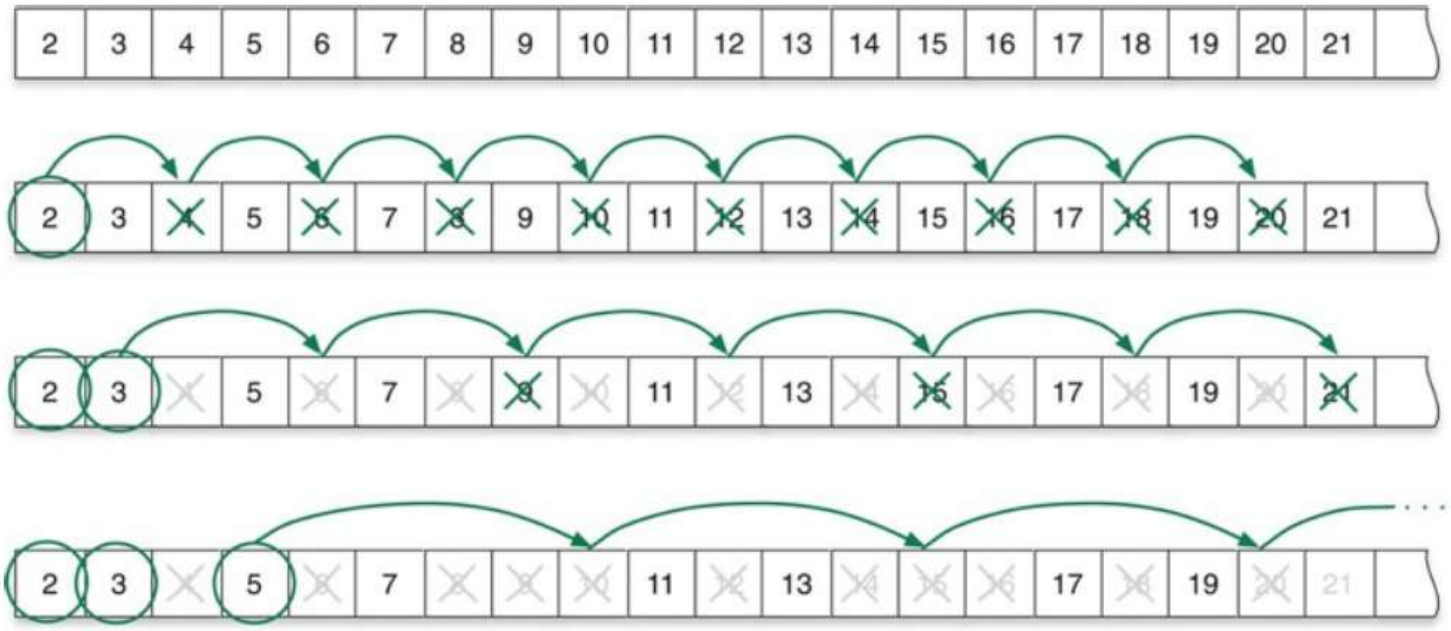
  the first unmarked number in the list is prime

  cross off multiples of the most recent prime

- So, first we cross off multiples of 2.
- Then, we cross off multiples of 3 that were not crossed off in the first round (e.g., 6 is a multiple of 2 and 3, so it was crossed off in the first round).
- Next, we cross off multiples of 5 that were not crossed off in the first two rounds. Note that because 4 is a multiple of 2, all multiples of 4 were crossed off in the first round.

The Sieve of Eratosthenes

- The algorithm continues in this fashion until there are no more numbers to cross off
- We will explore the stopping condition in more detail later



Devising an algorithm

- The method depicted in the previous slide works well for short lists

- But what if you want to find prime numbers between 2 and 100? 1000?
  - It's a tedious process to write out a list of 100 numbers
  - It takes a lot of paper to make a list of 1000 numbers
  - Chances are you will make a few arithmetic mistakes (this is a boring job!)

- Can we turn this method into a computation?
- Yes, but we need to be more precise about the steps

Devising an algorithm

- A detailed specification of the starting condition is there in the pseudocode (e.g., "make a list")
- What about the other steps? "Cross off" and "next number" need to be clearly defined if we're going to use Python
- The stopping condition is not so clear just yet
  - When do we stop the process? Perhaps when all the numbers are crossed off?
- As you've probably guessed by now, we will write a program to implement the Sieve of Eratosthenes algorithm
- We will need to explore a few new ideas in Python first, however

Collections

- In everyday life we often encounter collections of things
  - Course catalog: a collection of course descriptions
  - Parking lot: a collection of vehicles
- Mathematicians also work with collections
  - Matrix (a table of numbers)
  - Sequence (e.g., 1, 1, 2, 3, 5, 8, ...)
- In computer science we make a collection by defining a **data structure** that includes references to **objects**
- The term **object** simply means *generic piece of data*
  - Objects include numbers, strings, dates, and others
- Using programming terminology, a **container** is an object that contains other objects

Lists

- The simplest kind of container in Python is called a **list**
- One way to make a list is to enclose a set of objects in square brackets:

  ages = [61, 32, 19, 37, 42, 39]
- The above statement is an assignment statement, actually
- Python **allocates** space in its **object store**, which is a fancy term for a particular section in the memory of the computer
- Python creates an object to represent the list and associates the name **ages** with the new object
- The **len** function tells us how many elements are in a list:

  len(ages) # returns the value 6

Lists of strings

- Any kind of object can be stored in a list
- This statement defines a list with three strings:

  breakfast = ['green eggs', 'ham', 'toast']
- Note what happens when we ask Python how many objects are in this list:

  len(breakfast) # returns the value 3
- Python did not count the individual letters with a list
  - cf. **len('apple')** returns 5 # with a string, it counts the individual letters
- The list contains three string objects, so the return value of the call to **len** is 3

Empty lists

- We can also make a list with no objects:

  cars = []

- The value on the right side of that expression is a valid list
- An empty list is still a list, even though it contains no objects
  - A bag with nothing in it is still a bag, even though it contains nothing in it.
- The length of an empty list is 0

  len(cars) # returns the value 0

- It may seem strange to create a list with nothing in it, but usually we do so because we need to wait until later to fill in the contents

Iteration

- After building a container, we often want to do something with each item in it
- The idea is to "step through" the container to "visit" each object
- This type of operation is called **iteration**
  - From the Latin word *iter*, for "path" or "road"
- For example, to find the largest item in an (unsorted) list, an algorithm would need to visit every item during its search
- We'll look at this algorithm a little later

for-loops

- The simplest way to “visit” every item in a list is to use a **for**-loop
- This example prints every item in the list **cars** :

```
for car in cars: # "for each car in cars"
    print(car)
```

- Note that the statements inside a for-loop – the **body** of the loop – must be **indented**
- Python assigns **car** to be the first item in the list and then executes the indented statement(s)
- Then it gets the next item, assigns it to **car**, and executes the indented statement(s) again
- It repeats until all the items in list have been processed

for-loops

- Suppose we had this code:

```
cars = ['Kia', 'Honda', 'Toyota', 'Ford']
for car in cars:
    print(car + ' ' + str(len(car)))
```

- The for-loop would output this:

```
Kia 3
Honda 5
Toyota 6
Ford 4
```

- Note that **len(car)** gives the length of each car string in the list as we "visit" that car
  - **len(cars)** would give what?

Example: sum()

- Consider a function that computes the sum of the numbers in a list
- Such a function exists in Python (it's called **sum()**), but let's write our own so we can understand for-loops better
- First, we'll initialize a variable **total** to zero
- Then, a for-loop will add each number in the list to **total**
  - The statement **total += num** means "add **num** to the value of **total**"
  - An alternative way of writing this would be

    total = total + num
- After all items have been added, the loop will terminate, and the function returns the final value of **total**

Example: sum()

```
def sum(nums):
    total = 0
    for num in nums:
        total += num
    return total
```

Initialize a variable to store the running total

- Example:

```
t = sum([3, 5, 1]) # t will equal 9
```

- See sum_tests.py

Example: sum()

```
def sum(nums):
    total = 0
    for num in nums:
        total += num
    return total
```

Visit each number in the list of numbers

- Example:

```
t = sum([3, 5, 1]) # t will equal 9
```

Example: sum()

```
def sum(nums):
    total = 0
    for num in nums:
        total += num
    return total
```

Add each number to the running total

- Example:

```
t = sum([3, 5, 1]) # t will equal 9
```

Example: sum()

```
def sum(nums):
    total = 0
    for num in nums:
        total += num
    return total
```

Return the final total

- Example:

```
t = sum([3, 5, 1]) # t will equal 9
```

Example: sum()

- Let's *trace the execution* of this code to understand it better
- A red arrow will indicate the current line of code we are executing
- A table of values will show how the variables change value over time

Trace execution: sum()

```
def sum(nums):
    total = 0
    for num in nums:
        total += num
    return total
```

Variable	Value
total	0

- Example:

```
t = sum([3, 5, 1]) # t will equal 9
```

Trace execution: sum()

```
def sum(nums):
    total = 0
    for num in nums:
        total += num
    return total
```

Variable	Value
total	0
num	3

- Example:

```
t = sum([3, 5, 1]) # t will equal 9
```

Trace execution: sum()

```
def sum(nums):
    total = 0
    for num in nums:
        total += num
    return total
```

Variable	Value
total	3
num	3

- Example:

```
t = sum([3, 5, 1]) # t will equal 9
```

Trace execution: sum()

```
def sum(nums):
    total = 0
    for num in nums:
        total += num
    return total
```

Variable	Value
total	3
num	5

- Example:

```
t = sum([3, 5, 1]) # t will equal 9
```

Trace execution: sum()

```
def sum(nums):
    total = 0
    for num in nums:
        total += num
    return total
```

Variable	Value
total	8
num	3

- Example:

```
t = sum([3, 5, 1]) # t will equal 9
```

Trace execution: sum()

```
def sum(nums):
    total = 0
    for num in nums:
        total += num
    return total
```

Variable	Value
total	8
num	1

- Example:

t = sum([3, 5, 1]) # t will equal 9

Trace execution: sum()

```
def sum(nums):
    total = 0
    for num in nums:
        total += num
    return total
```

Variable	Value
total	9
num	1

- Example:

```
t = sum([3, 5, 1]) # t will equal 9
```

Trace execution: sum()

```
def sum(nums):
    total = 0
    for num in nums:
        total += num
    return total
```

Variable	Value
total	9
num	1

- Example:

```
t = sum([3, 5, 1]) # t will equal 9
```

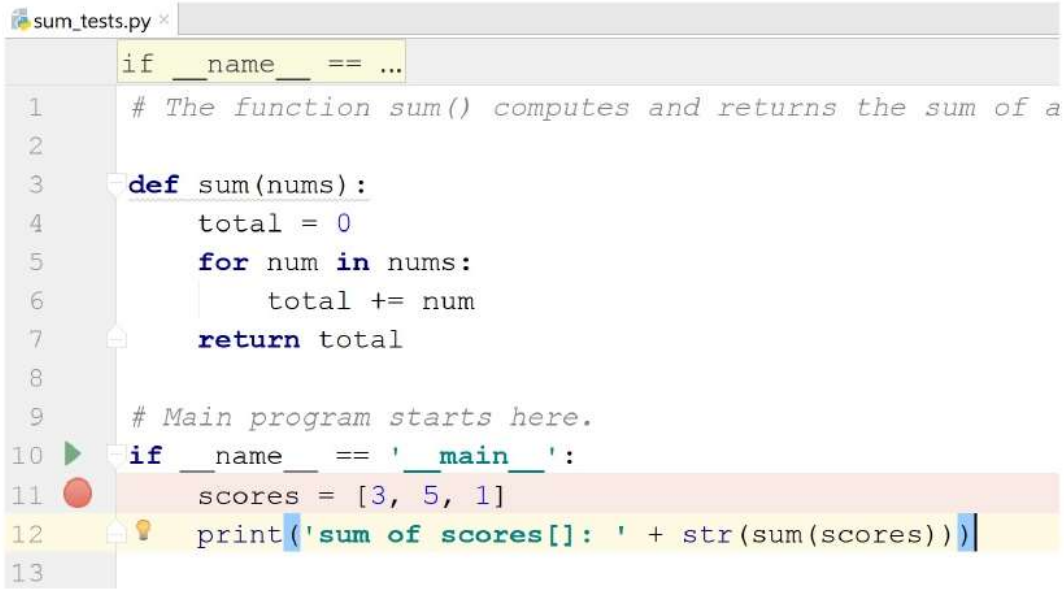
Trace execution in PyCharm

- PyCharm features a powerful tool called a **debugger** which can help you trace the execution of your program
  - Usually we use a debugger to help find bugs

- First we will set a ***breakpoint*** by clicking the mouse to the left of the line where we want the computer to pause execution

- In sum_tests.py let's put a breakpoint on line 11

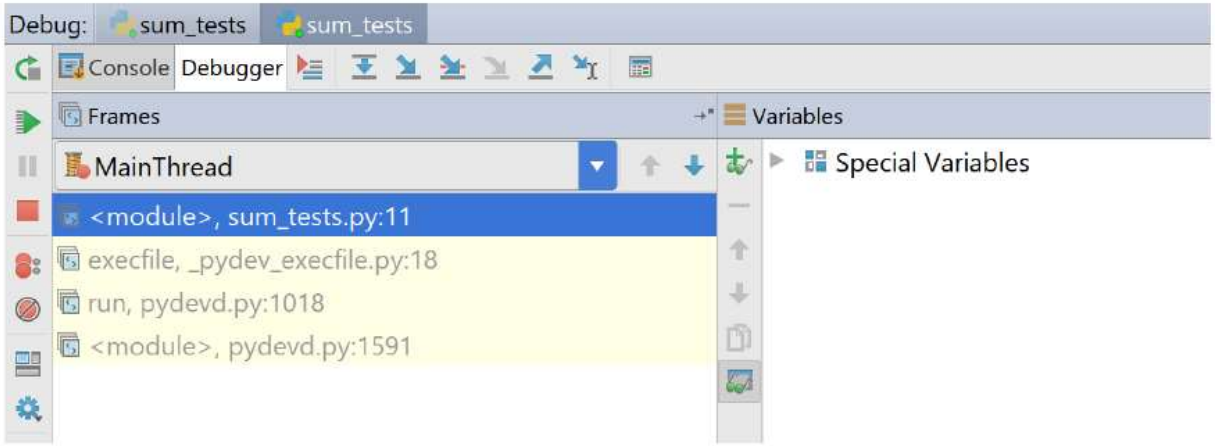
Trace execution in PyCharm



- When we tell the computer to run the program, it will stop at that line and not execute it until we tell it to
- When you first try the debugger, PyCharm may ask you to install some updates. Do them.

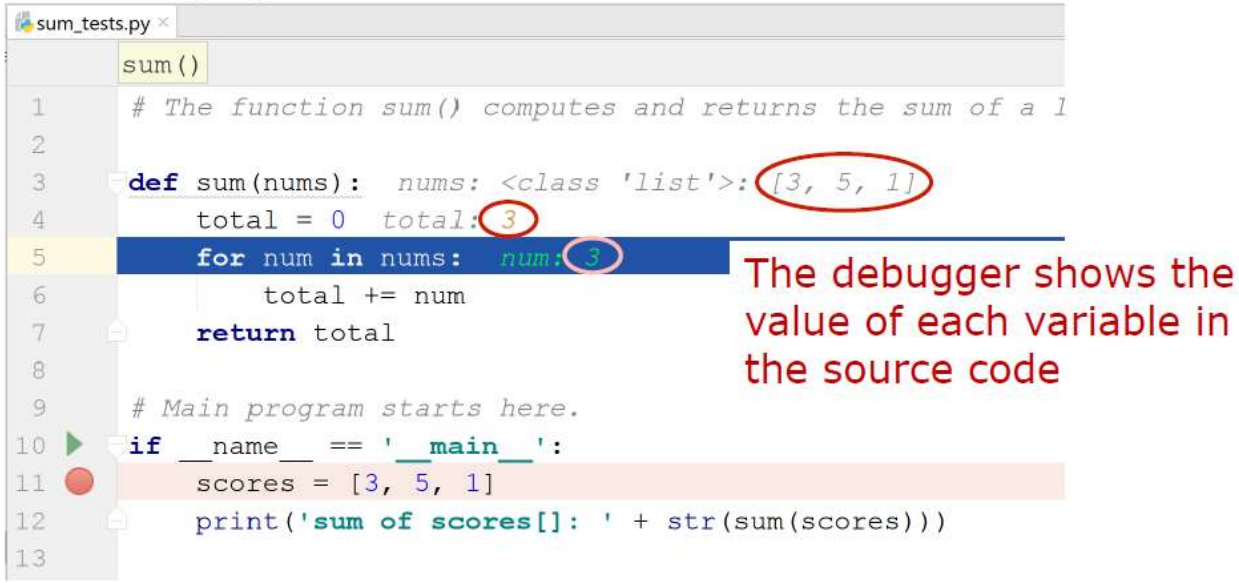
Trace execution in PyCharm

- To begin execution, right-click on sum_tests.py and pick "Debug 'sum_tests'". The computer stops at line 11.
- A "Debugger" panel opens
  - On the right we see a sub-panel named "Variables" that will show the values of variables as our program runs



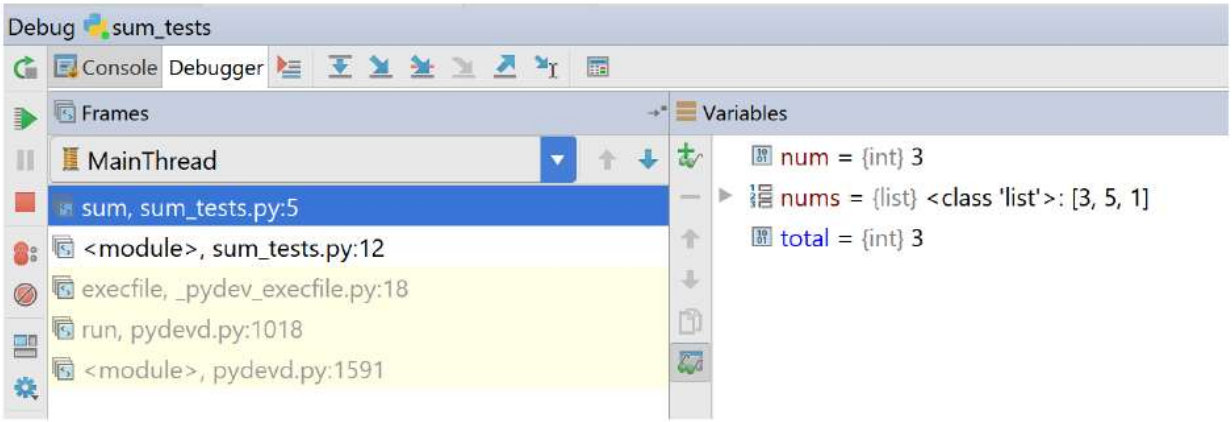
Trace execution in PyCharm

- Every time we hit the green arrow (Resume Program button) PyCharm will execute another line of code
- PyCharm highlights in blue what line it will execute *next*



Trace execution in PyCharm

Here's the state of the program after hitting the green arrow several times:



. In lab you will practice using the debugger – getting familiar with this tool will save you hours of headaches later on

List indexes

- We often need an item in the middle of a list
- If a list has n item, the locations in the list are numbered from 0 to n-1 (not 1 through n)
- The notation **a[i]** stands for “the item at location I in list a”
- **a[i]** is said aloud as “a sub I”
- In programming, we use the word **index** to refer to the numerical position of an element in a list
- Example: **scores = [89, 78, 92, 63, 92]**
- **scores[0]** is 89
- **scores[2]** is 92
- **scores[5]** gives an “index out of range” error (why?)

List indexes

- The **index** method will tell us the position of an element in a list
- If the requested element is not in the list, the Python interpreter will generate an error
- Example: **scores = [89, 78, 92, 63, 92]**
  - **scores.index(92)** is 2, the index of the first occurrence of 92 in the **scores** list
  - **scores.index(99)** generates this error: "ValueError: 99 is not in list"

List indexes

- If your program needs the index of a value, and you're not sure if the value is in the list, use an if-statement in conjunction with the **in** operator to first make sure the item is actually in the list
- Example:

```
vowels = ['a', 'e', 'i', 'o', 'u']
letter = 'e'
if letter in vowels:
    print('That letter is at index ' +
    str(vowels.index(letter)) + '.')
else:
    print('That letter is not in the list.')
```

- Output for this example: **That letter is at index 1.**

Iteration using list indexes

- A common programming "idiom" uses a for-loop based on a list index:

```
for i in range(n):
    # do something with i
```

- **range(n)** means "the sequence of integers starting from zero and ranging up to, but not including, **n**"
- Python executes the body of the loop **n** times
- **i** is set to every value between 0 and **n-1** (i.e., **n** is not included)
- For example, the **partial_total** function on the next slide computes and returns the sum of the first **k** values in a list (see sieve.py)

Iteration using list indexes

This function computes and returns the sum of the first **k** values in a list

```
def partial_total(nums, k):
    total = 0
    for i in range(k):
        total += nums[i]
    return total
```

Initialize the variable to store the running total

- Example:
  - **a = [4, 2, 8, 3, 1]**
  - **partial_total(a, 3) # returns the value 14**
  - **partial_total(a, 1) # returns the value 4**
  - **partial_total(a, 6) # error**

Iteration using list indexes

This function computes and returns the sum of the first **k** values in a list

```
def partial_total(nums, k):
    total = 0
    for i in range(k):
        total += nums[i]
    return total
```

Generate indexes 0 through *k*-1

- Example:
  - **a = [4, 2, 8, 3, 1]**
  - **partial_total(a, 3) # returns the value 14**
  - **partial_total(a, 1) # returns the value 4**
  - **partial_total(a, 6) # error**

Iteration using list indexes

This function computes and returns the sum of the first **k** values in a list

```
def partial_total(nums, k):
    total = 0
    for i in range(k):
        total += nums[i]
    return total
```

Add each number to the running total

- Example:
  - **a = [4, 2, 8, 3, 1]**
  - **partial_total(a, 3) # returns the value 14**
  - **partial_total(a, 1) # returns the value 4**
  - **partial_total(a, 6) # error**

Iteration using list indexes

This function computes and returns the sum of the first **k** values in a list

```
def partial_total(nums, k):
    total = 0
    for i in range(k):
        total += nums[i]
    return total
```

- Example:
  - **a = [4, 2, 8, 3, 1]**
  - **partial_total(a, 3) # returns the value 14**
  - **partial_total(a, 1) # returns the value 4**
  - **partial_total(a, 6) # error**

Iteration using list indexes

- Let's trace the execution of this function for one example

```
def partial_total(nums, k):
    total = 0
    for i in range(k):
        total += nums[i]
    return total
```

Variable	Value
total	0

- Example:
- **a = [4, 2, 8, 3, 1]**
- **partial_total(a, 3) # returns the value 14**

Iteration using list indexes

- Let's trace the execution of this function for one example

```
def partial_total(nums, k):
    total = 0
    for i in range(k):
        total += nums[i]
    return total
```

Variable	Value
total	0
i	0

- Example:
- **a = [4, 2, 8, 3, 1]**
- **partial_total(a, 3) # returns the value 14**

Iteration using list indexes

- Let's trace the execution of this function for one example

```
def partial_total(nums, k):
    total = 0
    for i in range(k):
        total += nums[i]
    return total
```

Variable	Value
total	4
i	0
nums[i]	4

- Example:
- **a = [4, 2, 8, 3, 1]**
- **partial_total(a, 3) # returns the value 14**

Iteration using list indexes

- Let's trace the execution of this function for one example

```
def partial_total(nums, k):
    total = 0
    for i in range(k):
        total += nums[i]
    return total
```

Variable	Value
total	4
i	1
nums[i]	4

- Example:
- **a = [4, 2, 8, 3, 1]**
- **partial_total(a, 3) # returns the value 14**

Iteration using list indexes

- Let's trace the execution of this function for one example

```
def partial_total(nums, k):
    total = 0
    for i in range(k):
        total += nums[i]
    return total
```

Variable	Value
total	6
i	1
nums[i]	2

- Example:
- **a = [4, 2, 8, 3, 1]**
- **partial_total(a, 3) # returns the value 14**

Iteration using list indexes

- Let's trace the execution of this function for one example

```
def partial_total(nums, k):
    total = 0
    for i in range(k):
        total += nums[i]
    return total
```

Variable	Value
total	6
i	2
nums[i]	2

- Example:
- **a = [4, 2, 8, 3, 1]**
- **partial_total(a, 3) # returns the value 14**

Iteration using list indexes

- Let's trace the execution of this function for one example

```
def partial_total(nums, k):
    total = 0
    for i in range(k):
        total += nums[i]
    return total
```

Variable	Value
total	14
i	2
nums[i]	8

- Example:
- **a = [4, 2, 8, 3, 1]**
- **partial_total(a, 3) # returns the value 14**

Iteration using list indexes

- Let's trace the execution of this function for one example

```
def partial_total(nums, k):
    total = 0
    for i in range(k):
        total += nums[i]
    return total
```

Variable	Value
total	14
i	2
nums[i]	8

- Example:
- **a = [4, 2, 8, 3, 1]**
- **partial_total(a, 3) # returns the value 14**

String indexes

- Strings and lists have much in common, including indexing
- Notation like **name[i]** would give us the character at index **i** of the string **name**, just as **nums[i]** gives us the element at index **i** of the list **nums**
- Examples:

```
title = 'Lord of the Rings'
print(title[0]) # prints L
print(title[6]) # prints f
j = 10
print(title[j]) # prints e
```

Making lists of numbers

- The **range** function can be used to make a list of integers
- This example makes a list of the numbers from 0 to 9:

```
nums = list(range(10))
```

- Note that **list** is the name of a **class** in Python
- A class describes what kinds of data an object can store
- In general, if we use a class name as a function, Python will create an object of that class
- These functions are called **constructors** because they construct new objects
- More on this topic later in the course

Back to the Sieve algorithm

- We now have all the pieces we need to make a list of prime numbers
- We will use a Python **list** object to represent a "worksheet" of numbers that we will progressively cross off
- It will initially have all the integers from 2 to *n* (the upper limit)
- We will use for-loops to iterate over the list to cross off composite numbers
- If we pass *two* values to **range**, it uses one as the lower limit and the other as the upper limit (minus 1)
- For example, to make a list of numbers between 2 and 99 we would type **list(range(2, 100))**

Back to the Sieve algorithm

- The steps of the algorithm are easier to explain if we add two "placeholder" values at the front of the list to represent 0 and 1 (neither of which is a prime number)
- Python has a special value called **None** that stands for "no object"
- Since the expression **a + b** means "concatenate **a** and **b**" where **a** and **b** are lists, the statement below creates the initial worksheet:

```
worksheet = [None, None] + list(range(2,100))
```

- With the two placeholders at the front, we now know any number **I** will be at **worksheet[i]**
  - For example, the number 5 will be at **worksheet[5]** instead of **worksheet[3]**

PythonLabs

- PythonLabs is a set of Python modules developed for the course textbook
- PythonLabs homepage: http://ix.cs.uoregon.edu/~conery/eic/python/index.html
- Installation instructions: http://ix.cs.uoregon.edu/~conery/eic/python/installation.html

SieveLab

- The module for the Sieve algorithm is named SieveLab
- SieveLab has:
  - A complete implementation of a **sieve** function for finding prime numbers
  - Functions that use algorithm animation to generate graphical displays to show how the algorithm works
- In PyCharm, right-click on the Examples folder and select the menu option “Mark Directory as” and choose “Sources Root”

SieveLab

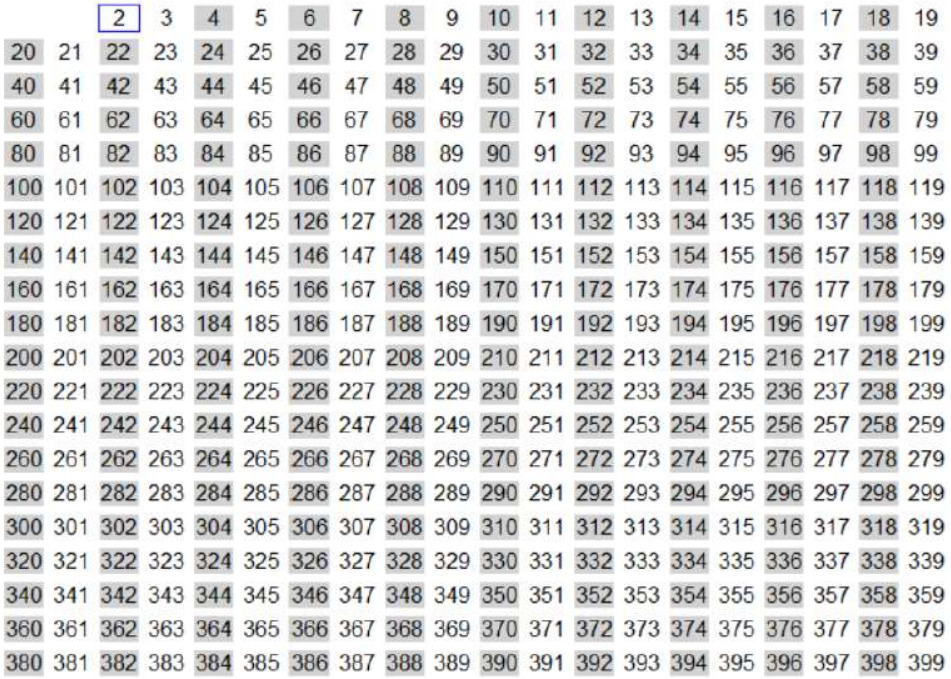
- Below you can see an example of how to use the SieveLab module

```
import PythonLabs.SieveLab
worksheet = [None, None] + list(range(2, 400))
PythonLabs.SieveLab.view_sieve(worksheet)
```

- We can call a SieveLab function named **mark_multiples** to see how the algorithm removes multiples of a specified value
- The two arguments to **mark_multiples** are a number **k** and the **worksheet** list
- The screen will be updated to show that **k** is prime (indicated by a blue square)
- Gray boxes will be drawn over all the multiples of **k**

SieveLab

PythonLabs.SieveLab.mark_multiples(2, worksheet):



	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79
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300	301	302	303	304	305	306	307	308	309	310	311	312	313	314	315	316	317	318	319
320	321	322	323	324	325	326	327	328	329	330	331	332	333	334	335	336	337	338	339
340	341	342	343	344	345	346	347	348	349	350	351	352	353	354	355	356	357	358	359
360	361	362	363	364	365	366	367	368	369	370	371	372	373	374	375	376	377	378	379
380	381	382	383	384	385	386	387	388	389	390	391	392	393	394	395	396	397	398	399

SieveLab

We can call SieveLab's **erase_multiples** function to erase the marked numbers

Let's erase the multiples of 2 using this function

SieveLab

PythonLabs.SieveLab.erase_multiples(2, worksheet):

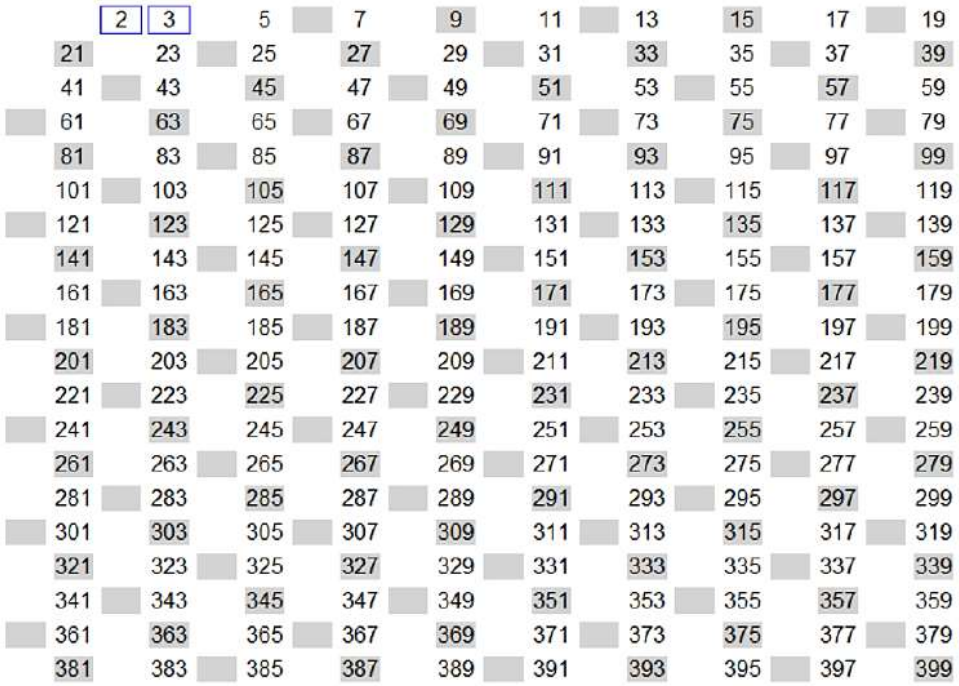
	2	3	5	7	9	11	13	15	17	19
21		23	25	27	29	31	33	35	37	39
41		43	45	47	49	51	53	55	57	59
61		63	65	67	69	71	73	75	77	79
81		83	85	87	89	91	93	95	97	99
101		103	105	107	109	111	113	115	117	119
121		123	125	127	129	131	133	135	137	139
141		143	145	147	149	151	153	155	157	159
161		163	165	167	169	171	173	175	177	179
181		183	185	187	189	191	193	195	197	199
201		203	205	207	209	211	213	215	217	219
221		223	225	227	229	231	233	235	237	239
241		243	245	247	249	251	253	255	257	259
261		263	265	267	269	271	273	275	277	279
281		283	285	287	289	291	293	295	297	299
301		303	305	307	309	311	313	315	317	319
321		323	325	327	329	331	333	335	337	339
341		343	345	347	349	351	353	355	357	359
361		363	365	367	369	371	373	375	377	379
381		383	385	387	389	391	393	395	397	399

SieveLab

- After erasing multiples of 2, the lowest unmarked number is 3, so on the next round we will remove multiples of 3
- We repeat the “marking” and “erasing” steps until only prime numbers are left
- Let’s see what this process looks like for marking and erasing multiples of 3, 5 and 7

SieveLab

PythonLabs.SieveLab.mark_multiples(3, worksheet):



	2	3	5	7	9	11	13	15	17	19		
21	23	25	27	29	31	33	35	37	39			
41	43	45	47	49	51	53	55	57	59			
61	63	65	67	69	71	73	75	77	79			
81	83	85	87	89	91	93	95	97	99			
101	103	105	107	109	111	113	115	117	119			
121	123	125	127	129	131	133	135	137	139			
141	143	145	147	149	151	153	155	157	159			
161	163	165	167	169	171	173	175	177	179			
181	183	185	187	189	191	193	195	197	199			
201	203	205	207	209	211	213	215	217	219			
221	223	225	227	229	231	233	235	237	239			
241	243	245	247	249	251	253	255	257	259			
261	263	265	267	269	271	273	275	277	279			
281	283	285	287	289	291	293	295	297	299			
301	303	305	307	309	311	313	315	317	319			
321	323	325	327	329	331	333	335	337	339			
341	343	345	347	349	351	353	355	357	359			
361	363	365	367	369	371	373	375	377	379			
381	383	385	387	389	391	393	395	397	399			

SieveLab

PythonLabs.SieveLab.erase_multiples(3, worksheet):

	2	3	5	7		11	13		17	19
		23	25		29	31		35	37	
41		43		47	49		53	55		59
61			65	67		71	73		77	79
		83	85		89	91		95	97	
101		103		107	109		113	115		119
121			125	127		131	133		137	139
		143	145		149	151		155	157	
161		163		167	169		173	175		179
181			185	187		191	193		197	199
		203	205		209	211		215	217	
221		223		227	229		233	235		239
241			245	247		251	253		257	259
		263	265		269	271		275	277	
281		283		287	289		293	295		299
301			305	307		311	313		317	319
		323	325		329	331		335	337	
341		343		347	349		353	355		359
361			365	367		371	373		377	379
		383	385		389	391		395	397	

SieveLab

PythonLabs.SieveLab.mark_multiples(5, worksheet):

		2	3	5	7		11	13		17	19
			23	25		29	31		35	37	
	41		43		47	49		53	55		59
	61			65	67		71	73		77	79
			83	85		89	91		95	97	
	101		103		107	109		113	115		119
	121			125	127		131	133		137	139
			143	145		149	151		155	157	
	161		163		167	169		173	175		179
	181			185	187		191	193		197	199
			203	205		209	211		215	217	
	221		223		227	229		233	235		239
	241			245	247		251	253		257	259
			263	265		269	271		275	277	
	281		283		287	289		293	295		299
	301			305	307		311	313		317	319
			323	325		329	331		335	337	
	341		343		347	349		353	355		359
	361			365	367		371	373		377	379
			383	385		389	391		395	397	

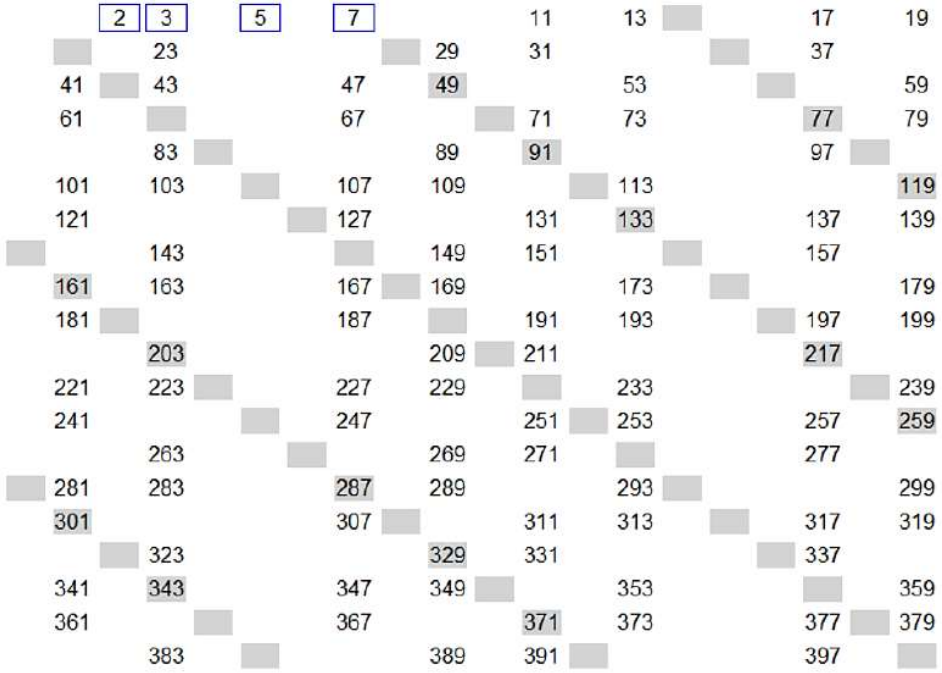
SieveLab

PythonLabs.SieveLab.erase_multiples(5, worksheet):

	2	3	5	7		11	13		17	19
		23			29	31			37	
41		43		47	49		53			59
61				67		71	73		77	79
		83			89	91			97	
101		103		107	109		113			119
121				127		131	133		137	139
		143			149	151			157	
161		163		167	169		173			179
181				187		191	193		197	199
		203			209	211			217	
221		223		227	229		233			239
241				247		251	253		257	259
		263			269	271			277	
281		283		287	289		293			299
301				307		311	313		317	319
		323			329	331			337	
341		343		347	349		353			359
361				367		371	373		377	379
		383			389	391			397	

SieveLab

PythonLabs.SieveLab.mark_multiples(7, worksheet):



SieveLab

PythonLabs.SieveLab.erase_multiples(7, worksheet):

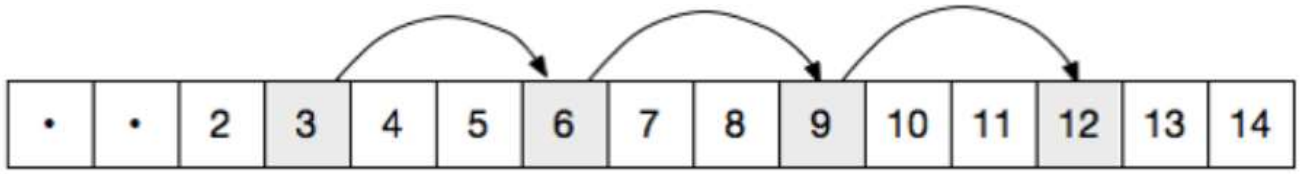
	2	3	5	7		11	13	17	19
		23			29	31		37	
41		43		47			53		59
61				67		71	73		79
		83			89			97	
101		103		107	109		113		
121				127		131		137	139
		143			149	151		157	
		163		167	169		173		179
181				187		191	193	197	199
					209	211			
221		223		227	229		233		239
241				247		251	253	257	
		263			269	271		277	
281		283			289		293		299
				307		311	313	317	319
		323				331		337	
341				347	349		353		359
361				367			373	377	379
		383			389	391		397	

Sieve algorithm: a helper function

- An important step toward implementing the Sieve algorithm is to write a function that solves a small part of the problem
- The function **sift** will make a single pass through the worksheet
- Pass it a number **k**, and **sift** will find and remove multiples of **k**
- For example, to sift out multiples of 5 from the list called **worksheet** we would type this: **sift(5, worksheet)**
- **sift** has a very specific purpose, and it is unlikely to be used except as part of an implementation of the Sieve algorithm
- Programmers call special-purpose functions like this **helper Functions**

Stepping through the worksheet

- On each call to sift we want to find multiples of k
- The first one is 2xk
- Notice that the remaining multiples (3xk, 4xk, etc) are all k steps apart:



- We can use a for-loop with a **range** expression to walk through the list:

```
for i in range (2*k, len(a), k):
```

- Note this **range** expression has three arguments: the starting point, the ending point, and the **step size (k)**

Stepping through the worksheet

- If we want to remove a number from the worksheet, we could use the Python **del** statement, which deletes an item from a list
- But this would shorten the list and make it harder to walk through on future iterations
- Our solution: replace the items with placeholders (**None** objects)
- The complete implementation of the **sift** function:

```
def sift(k, a):
    for i in range(2*k, len(a), k):
        a[i] = None
```

Stepping through the worksheet

```
def sift(k, a):
    for i in range(2*k, len(a), k):
        a[i] = None
```

- An example of **sift** in action:

```
worksheet = [None, None] + list(range(2, 16))
```

- **worksheet** is now:

```
[None, None, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15]
```

- Now call **sift(2, worksheet)**
- **worksheet** becomes this:

```
[None, None, 2, 3, None, 5, None, 7, None, 9, None, 11, None, 13, None, 15]
```

The sieve() function

- Now that we have a helper function to do the hard work, writing the **sieve** function is straightforward
- When a program has helpers, a function like **sieve** (which is called to solve the complete problem) is known as a **top-level function**
- We have to write a loop that starts by sifting multiples of 2 and keep calling **sift** until all composite numbers are removed
- This loop can stop when the next number to send to **sift** is greater than the square root of **n**
- The for-loop that controls the loop should set **k** to every value from 2 up to the square root (why?) of **n**:

```
for k in range(2, sqrt(n))
```

The sieve() function

for k in range(2, sqrt(n))

- There is a problem with this code: we cannot pass a floating-point value to **range**
- If we "round up" the square root, we'll have what we want: an integer that is greater than the highest possible prime factor of **n**
- A function named **ceil** in Python's math library does this operation
- **ceil** is short for "ceiling"
- A corresponding function named **floor** rounds a floating-point value down to the nearest integer

sieve()'s main loop

- One important detail: before sifting out multiples of a number, we make sure we haven't already removed it
- For example, we don't sift multiples of 4 because 4 was already removed when sifting multiples of 2
  - **sift** would still work, but our program would be less efficient
- The main loop looks like this:

```
for k in range(2, ceil(sqrt(n))):
    if worksheet[k] is not None:
        sift(k, worksheet)
```

- Note that the expression **x is not None** is the preferred way of testing to see if **x** is a reference to the **None** object

Sieve: remove the placeholders

- There is just one last step: to make the final list we have to remove the **None** objects from the worksheet
- A new helper function called **non_nulls** returns a copy of the worksheet, but without any **None** objects
- It makes an initial empty list named **res** (for “result”)
- Then it uses a for loop to look at every item in the input list
- If an item is not **None**, the item is appended to **res** using the **append** method for lists
- When the iteration is complete, **res** is returned as the result of the
- function call

Sieve: remove the placeholders

```
def non_nulls(a):
    res = []
    for x in a:
        if x is not None:
            res.append(x)
    return res
```

Initialize `res[]` to be the empty list

- Example:

```
worksheet = [None, None, 2, 3, None, 5, None, 7, None, None,
                    None, 11, None, 13, None, None]
worksheet = non_nulls(worksheet)
```

- **worksheet** is now: **[2, 3, 5, 7, 11, 13]**

Sieve: remove the placeholders

```
def non_nulls(a):
    res = []
    for x in a:
        if x is not None:
            res.append(x)
    return res
```

Visit each element in the list `a[]`

- Example:

```
worksheet = [None, None, 2, 3, None, 5, None, 7, None, None,
                    None, 11, None, 13, None, None]
worksheet = non_nulls(worksheet)
```

- **worksheet** is now: **[2, 3, 5, 7, 11, 13]**

Sieve: remove the placeholders

```
def non_nulls(a):
    res = []
    for x in a:
        if x is not None:
            res.append(x)
    return res
```

See if **x** is actually a number

- Example:

```
worksheet = [None, None, 2, 3, None, 5, None, 7, None, None,
                    None, 11, None, 13, None, None]
worksheet = non_nulls(worksheet)
```

- **worksheet** is now: **[2, 3, 5, 7, 11, 13]**

Sieve: remove the placeholders

```
def non_nulls(a):
    res = []
    for x in a:
        if x is not None:
            res.append(x)
    return res
```

If `x` is a number, append it to `res[]`

- Example:

```
worksheet = [None, None, 2, 3, None, 5, None, 7, None, None,
                    None, 11, None, 13, None, None]
worksheet = non_nulls(worksheet)
```

- **worksheet** is now: **[2, 3, 5, 7, 11, 13]**

Aside: appending to a List

- **+=** can be used to concatenate one string to the end of another
- This syntax can also be used to append one list to another
- Example:

fruits = ['apple', 'orange']

fruits += ['banana', 'mango', 'pear']

- **fruits** is now: **['apple', 'orange',**
  'banana', 'mango', 'pear']

fruits += ['pineapple']

- **fruits** is now: **['apple', 'orange',**
  'banana', 'mango', 'pear',
  'pineapple']

The Sieve algorithm: completed!

- We can now put all the pieces together
- Import the **math** library to get access to **sqrt** and **ceil**
- In the body of the **sieve** function we need to:
  - Create the **worksheet** with two initial **None** objects and all integers from 2 to **n**
  - Add the for-loop that calls **sift**
  - Call **non_nulls** to remove the **None** objects from the **worksheet**
- See sieve.py and the next slide for the code
- See PythonLabs/SieveLab.py: lines 12–28 for the textbook's implementation of the **sieve** function
- Run sieve_visualization.py to see it in action

Completed sieve() function

```
from math import
*
def sift(k, a):
   ... # see earlier slides

def non_nulls(a):
   ... # see earlier slides

def sieve(n):
   worksheet = [None, None] + list(range(2, n))
   for k in range(2, ceil(sqrt(n))):
      if worksheet[k] is not None:
         sift(k, worksheet)
   return non_nulls(worksheet)

primes = sieve(100)
print(primes)
```

See sieve.py

Abstraction

- Now that we have a function for making lists of prime numbers we can save it for later use
- We can use it to answer questions about primes
  - How many primes are less than *n*?
  - What is the largest gap between successive primes?
  - What are some twin primes (two prime numbers that differ only by 2, like 17 and 19)?
  - Many other questions are possible.
- This is a good example of **abstraction**: we have a nice, neat package that we can save and **reuse**
- In the future, we don't have to worry about the implementation details of **sieve**: we can just use it!
- We (and people who use it) just need to know that **sieve(n)** makes a list of prime numbers from 2 to **n**

Additional examples

- We'll now take a look at some additional examples of how to use for-loops and lists to solve problems in Python

Example: find the maximum

- Even though there already exists a function in Python that finds the maximum value in a list (it's called **max**), we will write our own algorithm for performing this task
- The basic idea is to *iterate* over the list and keep track of the largest value we have seen up to that point
- We begin by taking the value at index 0 as the maximum
- We continue with the remainder of the list, comparing the next value with the current maximum and updating the maximum if and when we find a value larger than the current maximum

Example: find_max.py

```
def find_max(nums):
    maximum = nums[0]
    for i in range(1, len(nums)):
        if nums[i] > maximum:
            maximum = nums[i]
    return maximum

ages = [20, 16, 22, 30, 17, 24]
max_age = find_max(ages) # max_age will be 30
print('Maximum age: ' + str(max_age))
```

Trace execution: find_max.py

```
def find_max(nums):
    maximum = nums[0]
    for i in range(1, len(nums)):
        if nums[i] > maximum:
            maximum = nums[i]
    return maximum

ages = [20, 16, 22, 30, 17, 24]
max_age = find_max(ages) # max_age will be 30
print('Maximum age: ' + str(max_age))
```

Variable	Value
maximum	20

Trace execution: find_max.py

```
def find_max(nums):
    maximum = nums[0]
    for i in range(1, len(nums)):
        if nums[i] > maximum:
            maximum = nums[i]
    return maximum

ages = [20, 16, 22, 30, 17, 24]
max_age = find_max(ages) # max_age will be 30
print('Maximum age: ' + str(max_age))
```

Variable	Value
maximum	20
i	1

Trace execution: find_max.py

```
def find_max(nums):
    maximum = nums[0]
    for i in range(1, len(nums)):
        if nums[i] > maximum:
            maximum = nums[i]
    return maximum

ages = [20, 16, 22, 30, 17, 24]
max_age = find_max(ages) # max_age will be 30
print('Maximum age: ' + str(max_age))
```

Variable	Value
maximum	20
i	1
nums[i]	16

Trace execution: find_max.py

```
def find_max(nums):
    maximum = nums[0]
    for i in range(1, len(nums)):
        if nums[i] > maximum:  # False
            maximum = nums[i]
    return maximum

ages = [20, 16, 22, 30, 17, 24]
max_age = find_max(ages) # max_age will be 30
print('Maximum age: ' + str(max_age))
```

Variable	Value
maximum	20
i	1
nums[i]	16

Trace execution: find_max.py

```
def find_max(nums):
    maximum = nums[0]
    for i in range(1, len(nums)):
        if nums[i] > maximum:
            maximum = nums[i]
    return maximum

ages = [20, 16, 22, 30, 17, 24]
max_age = find_max(ages) # max_age will be 30
print('Maximum age: ' + str(max_age))
```

Variable	Value
maximum	20
i	2
nums[i]	22

Trace execution: find_max.py

```
def find_max(nums):
    maximum = nums[0]
    for i in range(1, len(nums)):
        if nums[i] > maximum:
            maximum = nums[i]
    return maximum

ages = [20, 16, 22, 30, 17, 24]
max_age = find_max(ages) # max_age will be 30
print('Maximum age: ' + str(max_age))
```

Variable	Value
maximum	20
i	2
nums[i]	22

Trace execution: find_max.py

```
def find_max(nums):
    maximum = nums[0]
    for i in range(1, len(nums)):
        if nums[i] > maximum:    # True
            maximum = nums[i]
    return maximum

ages = [20, 16, 22, 30, 17, 24]
max_age = find_max(ages) # max_age will be 30
print('Maximum age: ' + str(max_age))
```

Variable	Value
maximum	20
i	2
nums[i]	22

Trace execution: find_max.py

```
def find_max(nums):
    maximum = nums[0]
    for i in range(1, len(nums)):
        if nums[i] > maximum:
            maximum = nums[i]
    return maximum

ages = [20, 16, 22, 30, 17, 24]
max_age = find_max(ages) # max_age will be 30
print('Maximum age: ' + str(max_age))
```

Variable	Value
maximum	22
i	2
nums[i]	22

Trace execution: find_max.py

```
def find_max(nums):
    maximum = nums[0]
    for i in range(1, len(nums)):
        if nums[i] > maximum:
            maximum = nums[i]
    return maximum

ages = [20, 16, 22, 30, 17, 24]
max_age = find_max(ages) # max_age will be 30
print('Maximum age: ' + str(max_age))
```

Variable	Value
maximum	22
i	3
nums[i]	30

Trace execution: find_max.py

```
def find_max(nums):
    maximum = nums[0]
    for i in range(1, len(nums)):
        if nums[i] > maximum:
            maximum = nums[i]
    return maximum

ages = [20, 16, 22, 30, 17, 24]
max_age = find_max(ages) # max_age will be 30
print('Maximum age: ' + str(max_age))
```

Variable	Value
maximum	22
i	3
nums[i]	30

Trace execution: find_max.py

```
def find_max(nums):
    maximum = nums[0]
    for i in range(1, len(nums)):
        if nums[i] > maximum:        # True
            maximum = nums[i]
    return maximum

ages = [20, 16, 22, 30, 17, 24]
max_age = find_max(ages) # max_age will be 30
print('Maximum age: ' + str(max_age))
```

Variable	Value
maximum	22
i	3
nums[i]	30

Trace execution: find_max.py

```
def find_max(nums):
    maximum = nums[0]
    for i in range(1, len(nums)):
        if nums[i] > maximum:
            maximum = nums[i]
    return maximum

ages = [20, 16, 22, 30, 17, 24]
max_age = find_max(ages) # max_age will be 30
print('Maximum age: ' + str(max_age))
```

Variable	Value
maximum	30
i	3
nums[i]	30

Trace execution: find_max.py

```
def find_max(nums):
    maximum = nums[0]
    for i in range(1, len(nums)):
        if nums[i] > maximum:
            maximum = nums[i]
    return maximum

ages = [20, 16, 22, 30, 17, 24]
max_age = find_max(ages) # max_age will be 30
print('Maximum age: ' + str(max_age))
```

Variable	Value
maximum	30
i	4
nums[i]	17

Trace execution: find_max.py

```
def find_max(nums):
    maximum = nums[0]
    for i in range(1, len(nums)):
        if nums[i] > maximum:
            maximum = nums[i]
    return maximum

ages = [20, 16, 22, 30, 17, 24]
max_age = find_max(ages) # max_age will be 30
print('Maximum age: ' + str(max_age))
```

Variable	Value
maximum	30
i	4
nums[i]	17

Trace execution: find_max.py

```
def find_max(nums):
    maximum = nums[0]
    for i in range(1, len(nums)):
        if nums[i] > maximum:    # False
            maximum = nums[i]
    return maximum

ages = [20, 16, 22, 30, 17, 24]
max_age = find_max(ages) # max_age will be 30
print('Maximum age: ' + str(max_age))
```

Variable	Value
maximum	30
i	4
nums[i]	17

Trace execution: find_max.py

```
def find_max(nums):
    maximum = nums[0]
    for i in range(1, len(nums)):
        if nums[i] > maximum:
            maximum = nums[i]
    return maximum

ages = [20, 16, 22, 30, 17, 24]
max_age = find_max(ages) # max_age will be 30
print('Maximum age: ' + str(max_age))
```

Variable	Value
maximum	30
i	5
nums[i]	24

Trace execution: find_max.py

```
def find_max(nums):
    maximum = nums[0]
    for i in range(1, len(nums)):
        if nums[i] > maximum:
            maximum = nums[i]
    return maximum

ages = [20, 16, 22, 30, 17, 24]
max_age = find_max(ages) # max_age will be 30
print('Maximum age: ' + str(max_age))
```

Variable	Value
maximum	30
i	5
nums[i]	24

Trace execution: find_max.py

```
def find_max(nums):
    maximum = nums[0]
    for i in range(1, len(nums)):
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ages = [20, 16, 22, 30, 17, 24]
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print('Maximum age: ' + str(max_age))
```

Variable	Value
maximum	30
i	5
nums[i]	24

Trace execution: find_max.py

```
def find_max(nums):
    maximum = nums[0]
    for i in range(1, len(nums)):
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            maximum = nums[i]
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ages = [20, 16, 22, 30, 17, 24]
max_age = find_max(ages) # max_age will be 30
print('Maximum age: ' + str(max_age))
```

Variable	Value
maximum	30
i	5
nums[i]	24

Example: count the vowels

- A for-loop can be used to iterate over the characters of a string
- To see how this works, let's look a function called **count_vowels** that counts the number of vowels (lowercase or uppercase) in a word
- To make this problem a little easier to solve, we will call the **lower()** method for strings, which makes a copy of a given string and changes all the uppercase letters to lowercase (**upper()** makes all letters uppercase)
- Strings are **immutable** (unchangeable) quantities
- If we want to convert a string into lowercase, all we can really do is make a lowercase copy of it and then replace the original string with the new one

Example: vowels.py

```
def count_vowels(word):
    vowels = 'aeiou'
    num_vowels = 0
    for letter in word.lower(): # search through a
        if letter in vowels:        # lowercase copy of
            num_vowels += 1         # the original word
    return num_vowels

word = 'Cider'
print('The number of vowels in ' + word + ' is ' +
str(count_vowels(word))) # will print 2
```

Example: vowels.py

```
def count_vowels(word):
    vowels = 'aeiou'
    num_vowels = 0
    for letter in word.lower():
        if letter in vowels:
            num_vowels += 1
    return num_vowels

word = 'Cider'
print('The number of vowels in ' + word + ' is ' +
str(count_vowels(word))) # will print 2
```

Variable	Value

Example: vowels.py

```
def count_vowels(word):
    vowels = 'aeiou'
    num_vowels = 0
    for letter in word.lower():
        if letter in vowels:
            num_vowels += 1
    return num_vowels

word = 'Cider'
print('The number of vowels in ' + word + ' is ' +
str(count_vowels(word))) # will print 2
```

Variable	Value
num_vowels	0

Example: vowels.py

```
def count_vowels(word):
    vowels = 'aeiou'
    num_vowels = 0
    for letter in word.lower():
        if letter in vowels:
            num_vowels += 1
    return num_vowels

word = 'Cider'
print('The number of vowels in ' + word + ' is ' +
str(count_vowels(word))) # will print 2
```

Variable	Value
num_vowels	0
letter	c

Example: vowels.py

```
def count_vowels(word):
    vowels = 'aeiou'
    num_vowels = 0
    for letter in word.lower():
        if letter in vowels:
            num_vowels += 1
    return num_vowels

word = 'Cider'
print('The number of vowels in ' + word + ' is ' +
str(count_vowels(word))) # will print 2
```

Variable	Value
num_vowels	0
letter	c

Example: vowels.py

```
def count_vowels(word):
    vowels = 'aeiou'
    num_vowels = 0
    for letter in word.lower():
        if letter in vowels:        # False
            num_vowels += 1
    return num_vowels

word = 'Cider'
print('The number of vowels in ' + word + ' is ' +
str(count_vowels(word))) # will print 2
```

Variable	Value
num_vowels	0
letter	c

Example: vowels.py

```
def count_vowels(word):
    vowels = 'aeiou'
    num_vowels = 0
    for letter in word.lower():
        if letter in vowels:
            num_vowels += 1
    return num_vowels

word = 'Cider'
print('The number of vowels in ' + word + ' is ' +
str(count_vowels(word))) # will print 2
```

Variable	Value
num_vowels	0
letter	i

Example: vowels.py

```
def count_vowels(word):
    vowels = 'aeiou'
    num_vowels = 0
    for letter in word.lower():
        if letter in vowels:
            num_vowels += 1
    return num_vowels

word = 'Cider'
print('The number of vowels in ' + word + ' is ' +
str(count_vowels(word))) # will print 2
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Variable	Value
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letter	i

Example: vowels.py

```
def count_vowels(word):
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        if letter in vowels:        # True
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word = 'Cider'
print('The number of vowels in ' + word + ' is ' +
str(count_vowels(word))) # will print 2
```

Variable	Value
num_vowels	0
letter	i

Example: vowels.py

```
def count_vowels(word):
    vowels = 'aeiou'
    num_vowels = 0
    for letter in word.lower():
        if letter in vowels:
            num_vowels += 1
    return num_vowels

word = 'Cider'
print('The number of vowels in ' + word + ' is ' +
str(count_vowels(word))) # will print 2
```

Variable	Value
num_vowels	1
letter	i

Example: vowels.py

```
def count_vowels(word):
    vowels = 'aeiou'
    num_vowels = 0
    for letter in word.lower():
        if letter in vowels:
            num_vowels += 1
    return num_vowels

word = 'Cider'
print('The number of vowels in ' + word + ' is ' +
str(count_vowels(word))) # will print 2
```

Variable	Value
num_vowels	1
letter	d

Example: vowels.py

```
def count_vowels(word):
    vowels = 'aeiou'
    num_vowels = 0
    for letter in word.lower():
        if letter in vowels:
            num_vowels += 1
    return num_vowels

word = 'Cider'
print('The number of vowels in ' + word + ' is ' +
str(count_vowels(word))) # will print 2
```

Variable	Value
num_vowels	1
letter	d

Example: vowels.py

```
def count_vowels(word):
    vowels = 'aeiou'
    num_vowels = 0
    for letter in word.lower():
        if letter in vowels:        # False
            num_vowels += 1
    return num_vowels

word = 'Cider'
print('The number of vowels in ' + word + ' is ' +
str(count_vowels(word))) # will print 2
```

Variable	Value
num_vowels	1
letter	d

Example: vowels.py

```
def count_vowels(word):
    vowels = 'aeiou'
    num_vowels = 0
    for letter in word.lower():
        if letter in vowels:
            num_vowels += 1
    return num_vowels

word = 'Cider'
print('The number of vowels in ' + word + ' is ' +
str(count_vowels(word))) # will print 2
```

Variable	Value
num_vowels	1
letter	e

Example: vowels.py

```
def count_vowels(word):
    vowels = 'aeiou'
    num_vowels = 0
    for letter in word.lower():
        if letter in vowels:
            num_vowels += 1
    return num_vowels

word = 'Cider'
print('The number of vowels in ' + word + ' is ' +
str(count_vowels(word))) # will print 2
```

Variable	Value
num_vowels	1
letter	e

Example: vowels.py

```
def count_vowels(word):
    vowels = 'aeiou'
    num_vowels = 0
    for letter in word.lower():
        if letter in vowels:          # True
            num_vowels += 1
    return num_vowels

word = 'Cider'
print('The number of vowels in ' + word + ' is ' +
str(count_vowels(word))) # will print 2
```

Variable	Value
num_vowels	1
letter	e

Example: vowels.py

```
def count_vowels(word):
    vowels = 'aeiou'
    num_vowels = 0
    for letter in word.lower():
        if letter in vowels:
            num_vowels += 1
    return num_vowels

word = 'Cider'
print('The number of vowels in ' + word + ' is ' +
str(count_vowels(word))) # will print 2
```

Variable	Value
num_vowels	2
letter	e

Example: vowels.py

```
def count_vowels(word):
    vowels = 'aeiou'
    num_vowels = 0
    for letter in word.lower():
        if letter in vowels:
            num_vowels += 1
    return num_vowels

word = 'Cider'
print('The number of vowels in ' + word + ' is ' +
str(count_vowels(word))) # will print 2
```

Variable	Value
num_vowels	2
letter	r

Example: vowels.py

```
def count_vowels(word):
    vowels = 'aeiou'
    num_vowels = 0
    for letter in word.lower():
        if letter in vowels:
            num_vowels += 1
    return num_vowels

word = 'Cider'
print('The number of vowels in ' + word + ' is ' +
str(count_vowels(word))) # will print 2
```

Variable	Value
num_vowels	2
letter	r

Example: vowels.py

```
def count_vowels(word):
    vowels = 'aeiou'
    num_vowels = 0
    for letter in word.lower():
        if letter in vowels:          # False
            num_vowels += 1
    return num_vowels

word = 'Cider'
print('The number of vowels in ' + word + ' is ' +
str(count_vowels(word))) # will print 2
```

Variable	Value
num_vowels	2
letter	r

Example: vowels.py

```
def count_vowels(word):
    vowels = 'aeiou'
    num_vowels = 0
    for letter in word.lower():
        if letter in vowels:
            num_vowels += 1
    return num_vowels

word = 'Cider'
print('The number of vowels in ' + word + ' is ' +
str(count_vowels(word))) # will print 2
```

Variable	Value
num_vowels	2
letter	r

A list of lists

- In Python, a list can contain objects of any type
- A list is an object. Therefore, a list can contain other lists!
- Imagine that we have a group of 4 students, and for each student we have 3 exam scores:
- **scores = [[89, 85, 90], [78, 85, 72],**
  [99, 86, 92], [82, 84, 79]]
- To access a particular score, we need to give two indexes: first, which student we are interested in (0 through 3)
- Second, which score of that student we are interested in (0 through 2)
- Example: **scores[3][1]** is student 3's score on exam 1 (which is 84)

Example: compute averages (v1)

- Let's write some code that will compute the average score the students earned on each exam
- We will write more than one version of the program, but let's start things off simply
- In the first version we will "hard-code" several values (the number of students and the number of scores) in the program
- Then we will generalize things a bit and use variables for these values

Example: averages_v1.py

```
scores = [[89, 85, 90], [78, 85, 72],
          [99, 86, 92], [82, 84, 79]]

averages = [0, 0, 0]

for student in scores:
    averages[0] += student[0]
    averages[1] += student[1]
    averages[2] += student[2]

for i in range(3):
    averages[i] /= 4

print(averages)
```

Example: averages_v1.py

```
scores = [[89, 85, 90], [78, 85, 72],
          [99, 86, 92], [82, 84, 79]]

averages = [0, 0, 0]

for student in scores:
    averages[0] += student[0]
    averages[1] += student[1]
    averages[2] += student[2]

for i in range(3):
    averages[i] /= 4

print(averages)
```

When this loop ends, `averages[0]` will contain the sum 89+78+99+82

Example: averages_v1.py

```
scores = [[89, 85, 90], [78, 85, 72],
          [99, 86, 92], [82, 84, 79]]

averages = [0, 0, 0]

for student in scores:
    averages[0] += student[0]
    averages[1] += student[1]
    averages[2] += student[2]

for i in range(3):
    averages[i] /= 4

print(averages)
```

When this loop ends, `averages[0]` will contain the sum 89+78+99+82

Example: averages_v1.py

```
scores = [[89, 85, 90], [78, 85, 72],
          [99, 86, 92], [82, 84, 79]]

averages = [0, 0, 0]

for student in scores:
    averages[0] += student[0]
    averages[1] += student[1]
    averages[2] += student[2]

for i in range(3):
    averages[i] /= 4

print(averages)
```

When this loop ends, `averages[1]` will contain the sum 85+85+86+84

Example: averages_v1.py

```
scores = [[89, 85, 90], [78, 85, 72],
          [99, 86, 92], [82, 84, 79]]

averages = [0, 0, 0]

for student in scores:
    averages[0] += student[0]
    averages[1] += student[1]
    averages[2] += student[2]

for i in range(3):
    averages[i] /= 4

print(averages)
```

When this loop ends, `averages[2]` will contain the sum 90+72+92+79

Example: compute averages (v2)

- The first version of our code has a major negative: the algorithm will work only for a class of four students who took three exams
- Suppose we had a larger or smaller class? Or suppose the students took more or fewer exams?
- We'll develop a better (but more complicated) version of the algorithm that can adapt to larger/smaller class sizes and more/fewer exams
- Our approach will rely on **nested loops**, which means we will have one loop inside of another
- Nested loops will become increasingly important as we progress through the course

Example: averages_v2.py

- One other thing before we look at the program
- Recall that syntax like **'Hi'*3** will create a new string by repeating a given string a desired number of times
  - For instance, **'Hi'*3** equals **'HiHiHi'**
- In a similar manner, **[0]*3** would create a list containing 3 zeroes, namely, **[0, 0, 0]**
- As we can see, the * notation with strings and lists is essentially a form of concatenation

Example: averages_v2.py

```
scores = [[89, 85, 90], [78, 85, 72], [99, 86, 92],
          [82, 84, 79]]
num_students = len(scores)
num_exams = len(scores[0]) # each student took the
averages = [0] * num_exams # same number of exams

for student in scores:
   for i in range(0, num_exams): # nested loops
      averages[i] += student[i]

for i in range(0, num_exams):
   averages[i] /= num_students

print(averages)
```

Example: compute averages (v3)

- In a third and final version of our exam average calculator, we will *encapsulate* (enclose or wrap) the computations inside of a function **compute_averages(students)**
- The function will take the list of scores as its argument
- After computing the exam averages, the function will return a list of the average scores
- So we'll now see that Python functions can return many values at once (via a list), not just a single number or string

Example: averages_v3.py

```
scores = [[89, 85, 90], [78, 85, 72], [99, 86, 92],
          [82, 84, 79]]
def compute_averages(students):
    num_students = len(students)
    num_exams = len(students[0])
    avgs = [0] * num_exams

    for student in students:
        for i in range(0, num_exams):
            avgs[i] += student[i]

    for i in range(0, num_exams):
        avgs[i] /= num_students

    return avgs

averages = compute_averages(scores)
print(averages)
```

Example: bottles of beer/milk

- We'll conclude the examples on a lighter note by looking at a program that prints the lyrics of the song "99 Bottles of Beer on the Wall"
- In this song, the singer needs to count from 99 down to 0
- The **range** command lets us count up, but it also can count down if we give a negative number for the step size
- For example, **range(10,-1,-1)** will count down from 10 to 0 by 1s
- So **list(range(10,-1,-1))** would generate the list **[10, 9, 8, 7, 6, 5, 4, 3, 2, 1, 0]**
- The code on the next slide asks the user for the starting number so that we can start from a value other than 99

Example: bottles.py

```
age = int(input('How old are you? '))

if age < 21:
   drink_type = 'milk'
else:
   drink_type = 'beer';

num_bottles = int(input('How many bottles of ' +
                        drink_type + ' do you have? '))

for bottle in range(num_bottles, -1, -1):
    if bottle > 1:
      print(str(bottle) + ' bottles of ' + drink_type +
            ' on the wall!')
   elif bottle == 1:
      print('1 bottle of ' + drink_type + ' on the wall!')
   else:
      print('No bottles of ' + drink_type + ' on the wall!')
```

Questions?