Statistics Assignment

Question 1:

The quality assurance checks on the previous batches of drugs found that — it is 4 times more likely that a drug is able to produce a satisfactory result than not.

Given a small sample of 10 drugs, you are required to find the theoretical probability that at most, 3 drugs are not able to do a satisfactory job.

a) Propose the type of probability distribution that would accurately portray the above scenario, and list out the three conditions that this distribution follows.
b) Calculate the required probability.

Answer:

Let assume probability of drugs produce non satisfactory result is **X**, so probability of producing satisfactory result is **4X**, total Probability is always **1**.

Hence 4X + X = 1

5X = 1

X = 0.2

Probability of Non-Satisfactory Result **(X) = 0.2**

Probability of Satisfactory Result **(4X) = 4 * 0.2 = 0.8** (OR **1 – 0.2 = 0.8**)

Key Point on Distribution:

1. Here total number of trials are fixed as we perform quality check on sample of 10 drugs.
2. Each assurance check produces satisfactory or non-satisfactory result only two possible result so binary outcome.
3. Probability of satisfactory (0.8) & non-satisfactory (0.2) result is same for all trails.

Based on above conditions we can conclude that above sample distribution follows **Binomial Distribution.**

Probability of at most 3 drugs out of 10 produces not-satisfactory result (till 2 decimal) is:

$$\mathbf{P(X = r) = {}^{n}C_{r}\,(p)^{r}\,(1-p)^{n-r}}$$

$$P(X=r) = {}^{n}C_{r} (p)^{r} (1-p)^{n-r}$$

$P(X=0)$ For Sample Size = 10 & Probability of $(p) = 0.2$

So,

$$P(X=0) = {}^{10}C_{0} (0.2)^{0} (1-0.2)^{10-0}$$

$$= 1 (1) (0.8)^{10}$$

$$= \underline{0.1073}$$

Similarly we can calculate probability for random var $X = 1, 2, 3 \ldots$

X	P(X)	P(X) [Round to 2 Decimal]	Cumulative Probability P(X <= x)
0	0.1073	0.11	0.11
1	0.2684	0.27	0.38
2	0.3019	0.30	0.68
3	**0.2013**	**0.20**	**0.88**

So, the probability of almost 3 drugs produce non-satisfactory result is **0.88.**

Question 2:

For the effectiveness test, a sample of 100 drugs was taken. The mean time of effect was 207 seconds, with the standard deviation coming to 65 seconds. Using this information, you are required to estimate the range in which the population mean might lie — with a 95% confidence level.

a) Discuss the main methodology using which you will approach this problem. State all the properties of the required method. Limit your answer to 150 words.
b) Find the required range.

Answer:

I will use **Central Limit Theorem** to solve this question. Central limit theorem defines three rules:

1. Sampling Distribution mean = Population Mean
2. Sampling distribution standard error = **σ / Square Root(n)**
   where σ is Population Standard Deviation we can approximate Sample Standard Deviation(S) is equal to population standard deviation

so, Standard Error = **S / Square Root(n)**

3. Sampling distribution become normal distribution when we take sample size more than 30

But here we are not working with mean or standard deviation of sampling distribution but will use sample population and apply CLT properties to find margin error and confidence interval.

Margin Error: **Z*S / Square Root (n)**

Z-score associated with 95% confidence level is (-1.96 & +1.96)

$$\text{Confidence Interval} = \left(\bar{x} \pm \underbrace{\frac{Z^* S}{\sqrt{n}}}_{\text{Margin of Error}}\right)$$

$\bar{x}$ = Sample Mean = 207

n = Sample Size = 100

S = Standard Deviation = 65

Z^* = Z-Score associated with confidence level.

From Z-table we get ±1.96 for 95% Confidence level.

$$\text{Margin of Error} = \frac{Z^* S}{\sqrt{n}} = \frac{\pm 1.96 \times 65}{\sqrt{100}}$$

$$= \pm 12.74$$

$$\text{Confidence Interval} = (\bar{x} \pm \text{Margin of Error})$$

$$= (207 \pm 12.74)$$

$$= (194.26\ ,\ 219.74)$$

So, with **95%** confidence we can say population mean is lie between **194.26 & 219.74**

Question 3 (a):

The painkiller drug needs to have a time of effect of at most 200 seconds to be considered as having done a satisfactory job. Given the same sample data (size, mean, and standard deviation) of the previous question, test the claim that the newer batch produces a satisfactory result and passes the quality assurance test. Utilize 2 hypothesis testing methods to make your decision. Take the significance level at 5 %. Clearly specify the hypotheses, the calculated test statistics, and the final decision that should be made for each method.

Answer:

Null Hypothesis (**H_0**): $\mu <= 200$, as painkiller required at most 200 second to produce satisfactory result.

Alternate Hypothesis (**H_1**): $\mu > 200$, painkiller required more than 200 seconds to do satisfactory job.

In hypothesis test we either reject the null hypothesis or we fail to reject null hypothesis.

Sample Size: 100

Sample Mean: 207

Sample Standard Deviation: 65

Population Mean (Claim): 200

Significance Level: 5%

Critical Value Method:

Alternate hypothesis is mean > 200, it means critical region lie on right side of the curve, so we have to perform Upper Tailed Test, for that we have to calculate UCV (Upper Critical Value) and check the null hypothesis.

Cumulative Probability for 5% significance level on UCV = 1 – 0.05 = **0.95**

Z-Critical value **(Z_c)** for cumulative probability of **0.95 = 1.64** (average of two nearest z score)

Critical Value (UCV or LCV) = $\mu_{\bar{x}} + (Z_c \times \sigma_{\bar{x}})$

$\mu_{\bar{x}}$ = Sample Mean = 200

$\sigma_{\bar{x}}$ = Standard Error {unknown}

Z_c = Z-Critical value = 1.64

Standard Error ($\sigma_{\bar{x}}$) = $\frac{\sigma}{\sqrt{n}}$

σ = Standard Deviation = 65

n = Sample Size = 100

$$\sigma_{\bar{x}} = \frac{65}{\sqrt{100}} = 6.5$$

$$UCV = \mu_{\bar{x}} + (Z_c \times \sigma_{\bar{x}})$$

$$= 200 + (1.64 \times 6.5)$$

$$= \underline{\underline{210.66}}$$

So, our sample mean 207 is less than critical value **210.66**, which lie in acceptance region, so we failed to reject null hypothesis.

As per Critical value method hypothesis test, we can conclude that newer batch produced satisfactory result.

P Value Method:

Core concept of P value test is, lower p-value indicate higher chances of rejecting the null hypothesis.

First calculate Z-score and based on that calculate p-value from cumulative probability for given z-score.

$$\text{Z-Score } (Z) = \frac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}}$$

$\bar{x}$ = Sample Mean = 207

$\mu_{\bar{x}}$ = Population Mean = 200

$\sigma_{\bar{x}}$ = Standard Error = 6.5 (from above)

$$\text{Z-Score } (Z) = \frac{207 - 200}{6.5} = 1.076$$

Cumulative Probability from Z-table for 1.076 is 0.8577

$$\text{P-value} = 1 - 0.8577 = \underline{\underline{0.1423}}$$

Right tail test as Z-Score is positive.

P-value = 1 – 0.8577 = **0.1423 = 14 %**

Here P-value is greater than significance region or we can say our sample mean is outside of critical region (5%), which implies null hypothesis is true & we failed to reject null hypothesis.

As per P-value method hypothesis test, we can conclude that newer batch produced satisfactory result.

Question 3 (b):

You know that two types of errors can occur during hypothesis testing — namely Type-I and Type-II errors — whose probabilities are denoted by α and β respectively. For the current sample conditions (sample size, mean, and standard deviation), the value of α and β come out to be 0.05 and 0.45 respectively.

Now, a different sampling procedure (with different sample size, mean, and standard deviation) is proposed so that when the same hypothesis test is conducted, the values of α and β are controlled at 0.15 each. Explain under what conditions would either method be more preferred than the other, i.e. give an example of a situation where conducting a hypothesis test having α and β as 0.05 and 0.45 respectively would be preferred over having them both at 0.15. Similarly, give an example for the reverse scenario - a situation where conducting the hypothesis test with both α and β values fixed at 0.15 would be preferred over having them at 0.05 and 0.45 respectively. Also, provide suitable reasons for your choice (Assume that only the values of α and β as mentioned above are provided to you and no other information is available).

Answer:

Type-I & Type-II error probability which are denoted by α and β respectively are inversely related.

Type-I error occurred when we reject the null hypothesis but it actually true, this happens when we have very low α value (so high β value).

Type-II error occurred when we failed to reject the null hypothesis but it actually false, this happens when we have high α value ((so low β value).

For α = 0.05 & β = 0.45

H_0= At most 20% of drugs provide non satisfactory job (Mean <= 20)

H_1= More than 20% of drugs provide non satisfactory job (Mean > 20)

As this is the UCV test, assume after calculation we got

Sample Mean = 22

UCV = 21.5 for α = 0.05

UCV = 22.5 for α = 0.15

If we consider significance level to 0.15 then will fail to reject the null hypothesis (or reject alternative hypothesis and assumed null hypothesis is true) which might cause serious health issue to patients as we are allowing to sale less effective or faulty drugs in market.

On other hand if we use low alpha value say 0.05 then we have good evidence to reject null hypothesis and we prevent less effective/faulty drugs to be sold in market (or consumed by patients)

So, when we have serious impact to health or to the end user (consumer) or alpha value should be low

For α = 0.15 & β = 0.15

H_0= Painkiller took at most 200 seconds for effectiveness (Mean <= 200)

H_1= Painkiller took more than 200 seconds for effectiveness (Mean > 200)

As this is the UCV test, assume after calculation we got

Sample Mean = 207

UCV = 208 for α = 0.05

UCV = 206 for α = 0.15

Based on 0.05 significance level, we can reject the null hypothesis, where as if we increase alpha value to 0.15, we can say null hypothesis still hold truth.

As hypothesis is related to time required by the drugs for effectiveness we can still go with higher alpha value as we know drugs are effective but it might take more time if we fail to reject null hypothesis.

So, we can use higher alpha value 0.15 instead of taking 0.05 and rejecting the effective drugs based on effectiveness (As in this example we take time in seconds if effectiveness is in days than alpha value might be lesser than this)

Question 4:

Now, once the batch has passed all the quality tests and is ready to be launched in the market, the marketing team needs to plan an effective online ad campaign to attract new customers. Two taglines were proposed for the campaign, and the team is currently divided on which option to use.

Explain why and how A/B testing can be used to decide which option is more effective. Give a stepwise procedure for the test that needs to be conducted.

Answer:

A/B testing mainly used to compare UI related stuff like two different version of completely redesigned web pages or comparing specific UI section, in email ad campaign compare two email subject line / content for conversion ratio, site hit ratio on normal vs mobile friendly design.

A/B test normally define likeliness of end user of any app, web site or proportional offers/campaign, with A/B test analysis we can conclude which version of app/site works well for specific business and increase the profit in terms of site hit or sale.

A/B test follows below mentioned steps:

Assuming we doing online ad campaign on Google Ads

- First create two different Taglines for our ad campaign.
- Define Hypothesis based on tagline like Tagline-1 is as good as Tagline-2
- Analyze and target the tool/website where we want to post ad. (Assume google ads)
- Post two different ads on google with out two different taglines. So, half of the user will see add with Tagline-1 and another half will see tagline-2

- Whenever user interact with the ad captured the activity like user click on which ad (Tagline-1 or Tagline-2)
- Analyze this data using analytical tools.
- Check which tagline gets more hit and continue that tagline for rest of the ad campaign.