YONSEI UNIVERSITY
Department of Computer Science

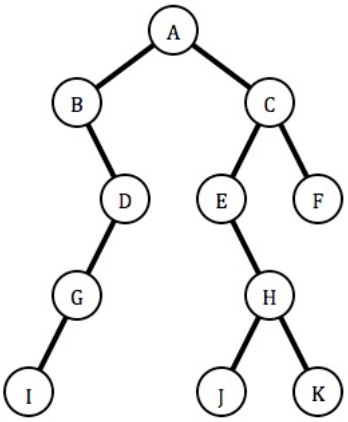
CSI2103, Data Structures, FALL 2016

Homework No.3

DUE: 2016.11.09 23:59:59pm

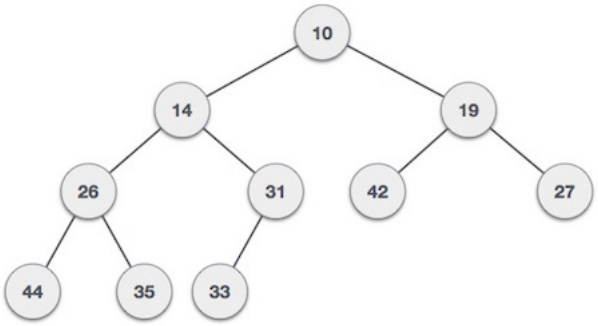
- Please don't forget to write down your student ID and name.
- All answers must be in **English**.
- You should submit a PDF file generated by MS Word, HWP or LaTeX via YSCEC
  A scanned-handwritten answer is **NOT** acceptable!
- By submitting your homework, you pledge on your honor that
  - You have completed all answers in the homework on your own
  - You have not used any unauthorized materials while completing this homework
  - You have not given anyone else access to your homework
- Unless specifically asked for, do not write code as a solution. You should write the general process, not just code, for solving the problem.

1. (2pt) Give the pre-order and post-order traversals of the following tree, rooted at A.



2. (2pt) Let T be an ordered tree with more than one node. Is it possible that the pre-order traversal of T visits the nodes in the same order as the post-order traversal of T? If so, give an example; otherwise, explain why this cannot occur. Likewise, is it possible that the pre-order traversal of T visits the nodes in the reverse order of the post-order traversal of T? If so, give an example; otherwise, explain why this cannot occur. If you need to give an example, it is acceptable to draw it on a sheet of paper and insert a picture of the drawing into your submission (not as a separate file). Please draw **very** neatly.

3. (2pt) Consider the following Min-Heap. Show the steps to insert an element with value 3 and the final resulting heap. Next, with the updated heap, show the steps to delete the element with value 10 as well as the resulting heap. For this problem, it is acceptable to draw the steps out on a sheet of paper and include a picture of the drawing in your submission (not as a separate file). Please draw **very** neatly and clearly show all steps and labels.



4. (5pt) We will define a hash function that maps from an integer to a value in a range by a 3-tuple, (a, b, N). So, we have $h : \mathbb{Z} \to \{0, ..., N-1\}$, and $h(x) = ax+b \pmod N$, where $0 < a < N$ and $0 \leq b < N$. A desirable property for a hash function is that for any value, c, and element, d, in the range $\{0, ..., N-1\}$, the probability $\Pr(h(c) = d) = \frac{1}{N}$. In other words, given a random integer, we want each element in the range $\{0, ..., N-1\}$ to be equally likely to be the value computed by the hash function. To do this, people often choose N to be a prime number. However, this is not the only way to achieve this probability.

   a) Give a hash function, (a, b, N), such that $N \geq 5$ is not prime but $\Pr(h(c) = d) = \frac{1}{N}$ holds.

   b) Give an example where $N \geq 5$ is not prime and the probability does not hold.

5. (9pt) A bloom filter is a data structure that uses hash functions. A bloom filter is defined by k hash functions, $h_1, ..., h_k$, that all map an integer to the range $\{0, ..., N-1\}$, and an array of size N initially filled with 0's. There are two operations on bloom filters, $Insert(x)$ and $Contains(x)$. To insert x into the bloom filter, apply each of the hash functions to x and set the index that is returned by the hash function to 1. To check containment, apply each of the hash functions to the x and return true if *all* of the indices returned by the hash functions are set to 1. If *any* are 0, return false. Unfortunately, there may be cases where $Contains(x)$ returns true even if x had not been inserted into the bloom filter. Consider the following bloom filter:

   - $k = 3, N = 11$
   - $h_1(x) = 3x + 1 \pmod{11}$
   - $h_2(x) = 6x + 5 \pmod{11}$
   - $h_3(x) = 10x + 1 \pmod{11}$

   a) Perform the following insertions and show the state of the bloom filter array after each one: $15, 1000, 22, 5$.

b) Using the described bloom filter, give an example of a series of insertions and a different value, a, such that after the insertions are completed the bloom filter returns true for $Contains(a)$ even though a was not inserted.

c) Using the described bloom filter, give an example of a series of insertions such that the bloom filter returns true when asked if it contains any positive integer.