Assignment of ET 4389
Complex Networks - from Nature to Man-made Networks

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1 Topological properties of real-world networks

In the attachments, a real-world network is assigned to each group. All the networks in the attachments are given in the following format: each row "a b" denotes a link from a to b. By default, we assume each network is undirected, thus, if a is connected to b, b is also connected to a. Let G be the real-world network you get.

1.1) What is the number of nodes N, the number of links L, the link density p, the average degree $E[D]$ and the degree variance $Var[D]$? Plot the degree distribution. Does the degree distribution follow a power law distribution $\Pr[D = k] \sim ck^{\gamma}$? If so, what is the power exponent γ (plot the fitting curve)?

1.2) What is the degree correlation (assortativity) ρ_D? What is its physical meaning?

1.3) What is the clustering coefficient C?

1.4) What is the average hopcount $E[H]$ of the shortest paths between all node pairs? What is the diameter $H_{\max}$?

1.5) What is the largest eigenvalue (spectral radius) λ_1 of the adjacency matrix?

1.6) What is the second smallest eigenvalue μ_{N-1} of the Laplacian matrix (algebraic connectivity)?

2 Effect of degree correlation on network properties

2.1) By applying degree-preserving rewirings to your real-world network G, you can obtain a network with degree correlation $\rho \in [0.3, 0.31]$ without changing the degree of each node. Generate 100 such network instances each with degree correlation $\rho \in [0.3, 0.31]$. Compute the average of each network metric asked in Question 1 (excluding those mentioned in 1.1) over these 100 network instances.

2.2) By applying degree-preserving rewirings to your real-world network G, you can obtain a network with degree correlation ranging over $[-0.31, -0.3]$ without changing the degree of each node. Generate 100 such network instances with degree correlation $\rho \in [-0.31, -0.3]$. Compute the average of each network metric asked in Question 1 (excluding those mentioned in 1.1) over these 100 network instances.

2.3) Compare the results of Question 1(excluding 1.1), Question 2.1 and 2.2 in one table. Discuss the effect of degree correlation on all the other network metrics (excluding those mentioned in 1.1). Assume that your real network G and the two classes of networks of 2.1 and 2.2 are the three possible designs for a communication network. Discuss each metric ($C, E[H], H_{\max}, \lambda_1, \mu_{N-1}$) individually: what is its physical meaning with respect to a communication network, which design is the best and why. Taking all the metrics into account, which design will you recommend? (In case any of your network is disconnected, modify the metric "average hopcount $E[H]$ of the shortest paths" so that the average distance property of these three classes of networks are comparable.

3 Network Models

3.1) Generate 100 instances of Erdős-Rényi (ER) random graphs with the same number N of nodes and the same link density p as your real-world network G. Plot the degree distribution of these 100 network instances and the theoretical degree distribution.

3.2) Generate 100 instances of Barabasi-Albert (BA) scale-free networks with $m = 3$ and with the same number N of nodes as your real-world network G. Plot the degree distribution of these 100 network realizations and using curve-fitting to determine its power exponent γ.

4 Percolation and Network Robustness

4.1) Plot the relative size S_1 of the largest cluster as a function of the percentage f of nodes that are randomly removed from the network for ER random networks generated in 3.1. Tips: the plot should be the average over the 100 iterations. Within each iteration: firstly, generate an ER network described in 3.1), secondly, at each step, remove randomly $\lfloor \frac{N}{100} \rfloor$ nodes; compute the relative size of the largest cluster S_1, the total percentage f of nodes that have been removed from G; these steps are repeated until all nodes are removed or until $S_1 = 0$.

4.2) Similar to 4.1) plot the relative size S_1 of the largest cluster as a function of the percentage f of nodes that are randomly removed from the network for BA networks generated in 3.2.

4.3) Similar to 4.1) plot the relative size S_1 of the largest cluster as a function of the percentage f of nodes that are randomly removed from your real-world network. Note that in this case, the same real-world network topology is used in each of the 100 iterations.

4.4) Compare all the results to determine which network (model) is more/less robust and explain why. Are your results in 4.1 and 4.2 consistent with the theoretical percolation threshold discussed during the lecture? If not, what are the possible reasons?

4.5) Plot the relative size S_1 of the largest cluster as a function of the percentage f of nodes that are randomly removed from the network for the networks described in 2.1) and 2.2) respectively. Compare these results with 4.3. Which class of networks is more/less robust? In this case, are assortative or disassortative networks more robust? Why?

5 Opinion interactions

Simulate (Non-consensus Opinion) NCO model on three classes of networks: a) ER random graph with $N = 10000$ and average degree 4 ; b) network obtained by applying degree-preserving rewirings to a) so that the degree correlation $\rho \in [0.3, 0.31]$ and the degree of each node remains the same as in a); c) network obtained by applying degree-preserving rewirings to a) so that the degree correlation $\rho \in [-0.31, -0.3]$ and the degree of each node remains the same as in a);

Plot the relative size S_1 of the largest cluster of nodes with positive opinion in the steady state as a function of the percentage f of nodes initially with positive opinion. For each of these three network classes, the plot should be obtained as the average over 100 (1000 if possible) iterations. $f = i * 0.01$ where $i \in [0, 100]$. Within each iteration, a network from a given class is generated, upon which, the NCO dynamic is simulated once for each possible value f to determine the relative size S_1 of the largest cluster of nodes with positive opinion in the steady state.

Compare the results for these three classes of networks. In which class of networks the minority opinion is more likely to survive? Why?

6 Extra Information

1. A report is required to include your results, observations, conclusions and interpretations, as asked in the assignment. However, a LONG report is NOT recommended.

2. In order to justify your observations or conclusions, feel free to improve the simulations mentioned above, such as increasing the number of iterations or computing extra measures etc.

3. For this assignment, we can as well generate an ER network with N nodes and average degree k (or link density p) by randomly assigning the $Nk/2$ (or $\frac{N(N-1)}{2}p$) links to the $\frac{N(N-1)}{2}$ possible positions (node pairs). This differs from the definition of an ER network where the number of links is a random variable instead of a constant.