Time Series Analysis - Home Project # 2

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Abstract

In this project I estimated the Unconditional Standard Deviation and Value-at-Risk (VaR) of portfolio consisting four financial instruments with Models of the General Autoregressive Conditional Heteroscedacticity (GARCH) Family (Engele, 2001). The portfolio consists of the follwoing three indexes: Standard and Poor's 500 (SP500), the Deutsche Aktion Index (DAX) and the Warszawski Indeks Giełdowy (WIG20), as well as one cryptocurrency, Bitcoin (BTC). The choice of the Models from the GARCH Family was made on the basis of an comparison analysis. After the comparison and further analysis I choosed an AR(1)-GARCH-t(2,1) and GARCH(5,5). The long-term annulaized unconditional standard deviation is 24.08% (AR(1)-GARCH-t(2,1)) and 12.17 (GARCH(5,5)) for in-sample data. For the out-sample data the number of days were losses are higher than assued VaR are 0.004 (AR(1)-GARCH-t(2,1)) and 0.008 for(GARCH(5,5)).

Time Series Analysis - Home Project #2 Results

Estimation Garch-Models with in-sample data

Packages and data

```
library(tidyverse)
library(xts) # e.g. xts objects
library(fBasics) # e.g. basicStats()
library(tseries)# e.g. jarque.bera.test()
library(car) # e.g. durbinWatsonTest()
library(FinTS) # e.g. ArchTest()
library(fGarch) # e.g. garchFit()
library(quantmod) # e.g. getSymbol()
library(rugarch) # e.g. ugarchfit()
```

Functions

```
setwd("/Users/philipp/Library/Mobile Documents/com~apple~CloudDocs/Uni/Time Series Analysis/Project")

source("function_compareICs.GARCH.R")
source("function_compare.ICs.ugarchfit.R")
```

Choice of cryptocurrency

```
x <- c(9,1,2,7,9,7)

sum(x)%%5+1
```

```
## [1] 1
```

The result is 1, hence the choice of cryptocurrency is Bitcoin (BTC)

Analysis of Portfolio

```
SP500 <-
  getSymbols(Symbols = "^GSPC",
             from = "2018-01-02",
             to = "2021-06-01",
             auto.assign = FALSE)

DAX <-
  getSymbols(Symbols = "^GDAXI",
             from = "2018-01-02",
             to = "2021-06-01",
             auto.assign = FALSE)

WIG20 <-
  getSymbols(Symbols = "^WIG20",
             from = "2018-01-02",
             to = "2021-06-01",
             auto.assign = FALSE)

BTC <-
  getSymbols(Symbols = "BTC-USD",
             from = "2018-01-02",
             to = "2021-06-01",
             auto.assign = FALSE)
```

```
SP500 <- SP500[, 6]
names(SP500) <- "SP500"

DAX <- DAX[, 6]
names(DAX) <- "DAX"

BTC <- BTC[, 6]
names(BTC) <- "BTC"

WIG20 <- WIG20[, 6]
names(WIG20) <- "WIG20"
```

```
SP500$SP500.r <- diff.xts(log(SP500$SP500))

DAX$DAX.r <- diff.xts(log(DAX$DAX))

BTC$BTC.r <- diff.xts(log(BTC$BTC))

WIG20$WIG20.r <- diff.xts(log(WIG20$WIG20))
```

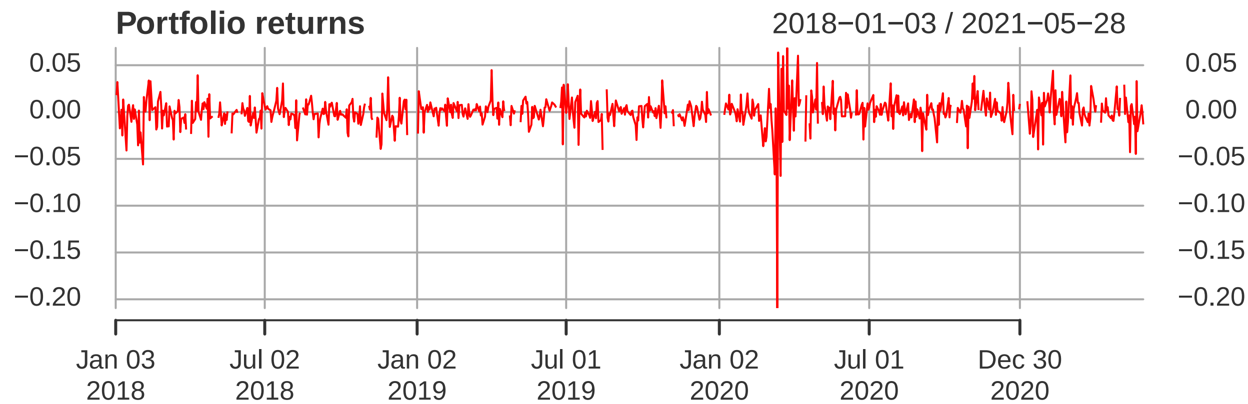
The gathered quotes of stock indices and cryptocurrencies are put togehter in a portfolio with equal shares.

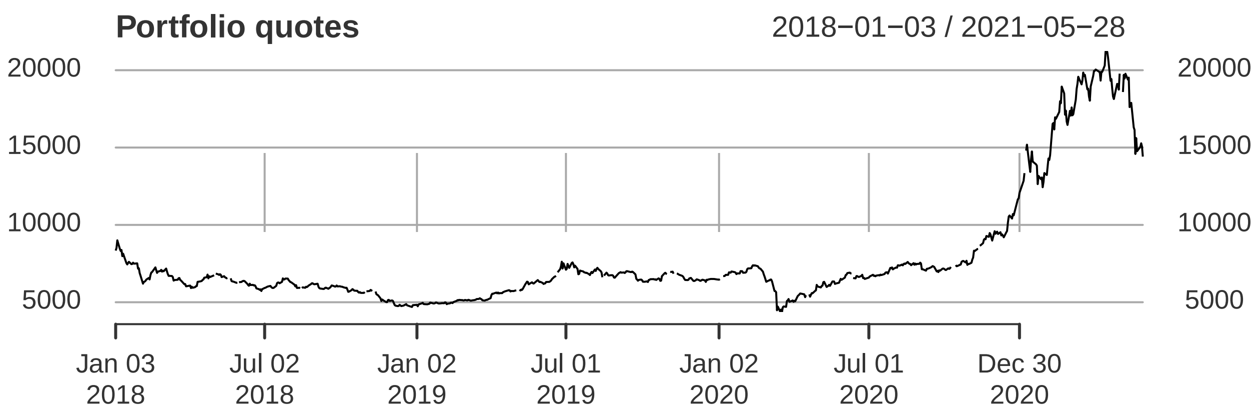
```
quotes <- 0.25*SP500$SP500.r+0.25*DAX$DAX.r+0.25*BTC$BTC.r+0.25*WIG20$WIG20.r
names(quotes) <- "quotes.r"
quotes$quotes <- 0.25*SP500$SP500+0.25*DAX$DAX+0.25*BTC$BTC+0.25*WIG20$WIG20

quotes$obs <- 1:length(quotes$quotes)
```

Plot of quotations and returns.

```
par(mfrow = c(2, 1))
plot(quotes$quotes.r,
     type = "l", col = "red", lwd = 1,
     main = "Portfolio returns")
plot(quotes$quotes,
     type = "l", col = "black", lwd = 1,
     main = "Portfolio quotes")
```

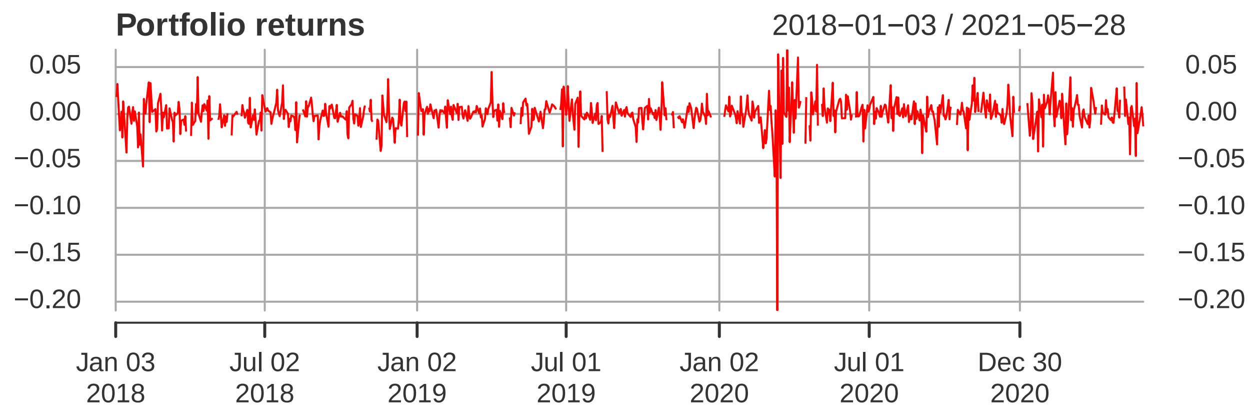


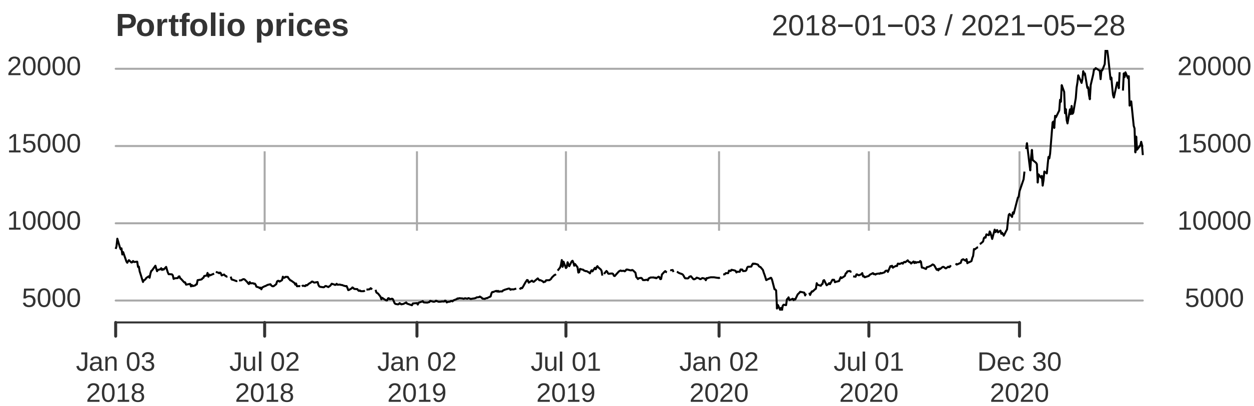


Examining the returns of the portfolio we see changes between 0.05 and -0.05 except in Febuary 2018 there is a return of -0.06 and in March 2020 there are rapid changes in return from approx. 0.07 to approx -0.21. The COVID-19 pandemic and its uncertainty for the financial market could be a reason for the big differences in returns of our portfolio in March 2020.

Plot of quotations and returns

```
par(mfrow = c(2, 1))
plot(quotes$quotes.r,
     type = "l", col = "red", lwd = 1,
     main = "Portfolio returns")
plot(quotes$quotes,
     type = "l", col = "black", lwd = 1,
     main = "Portfolio prices")
```



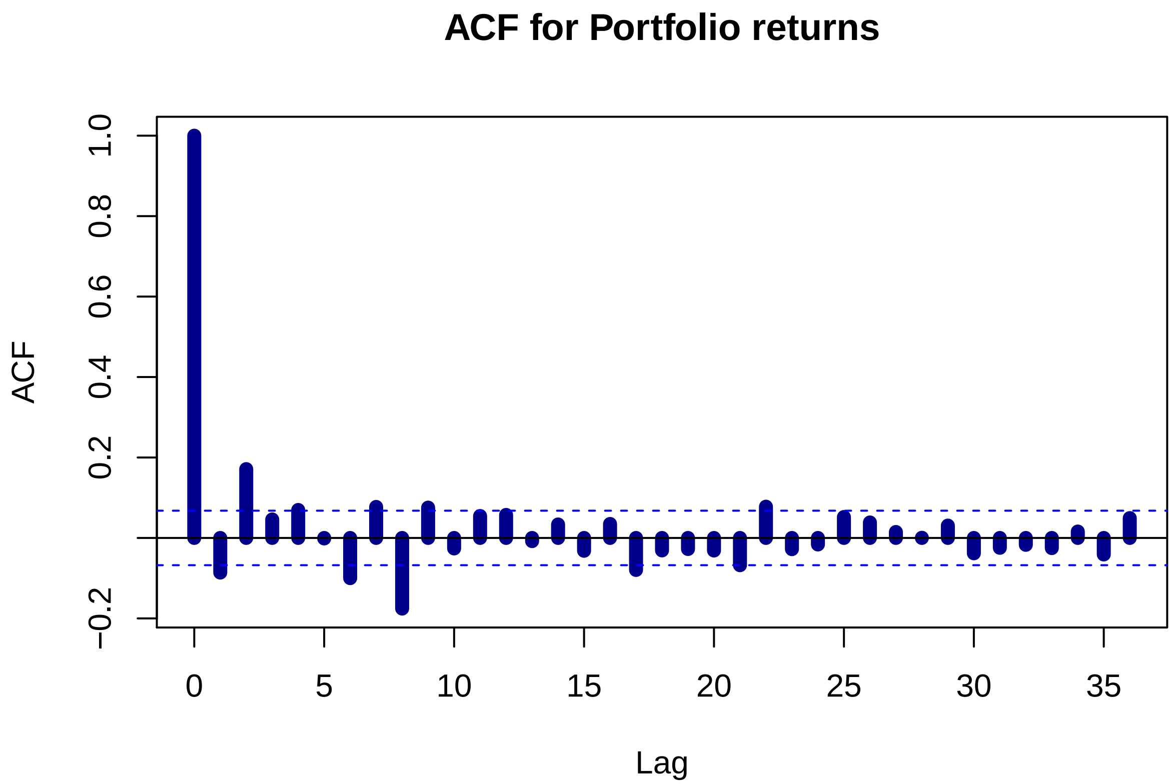



```
par(mfrow = c(1, 1))
```

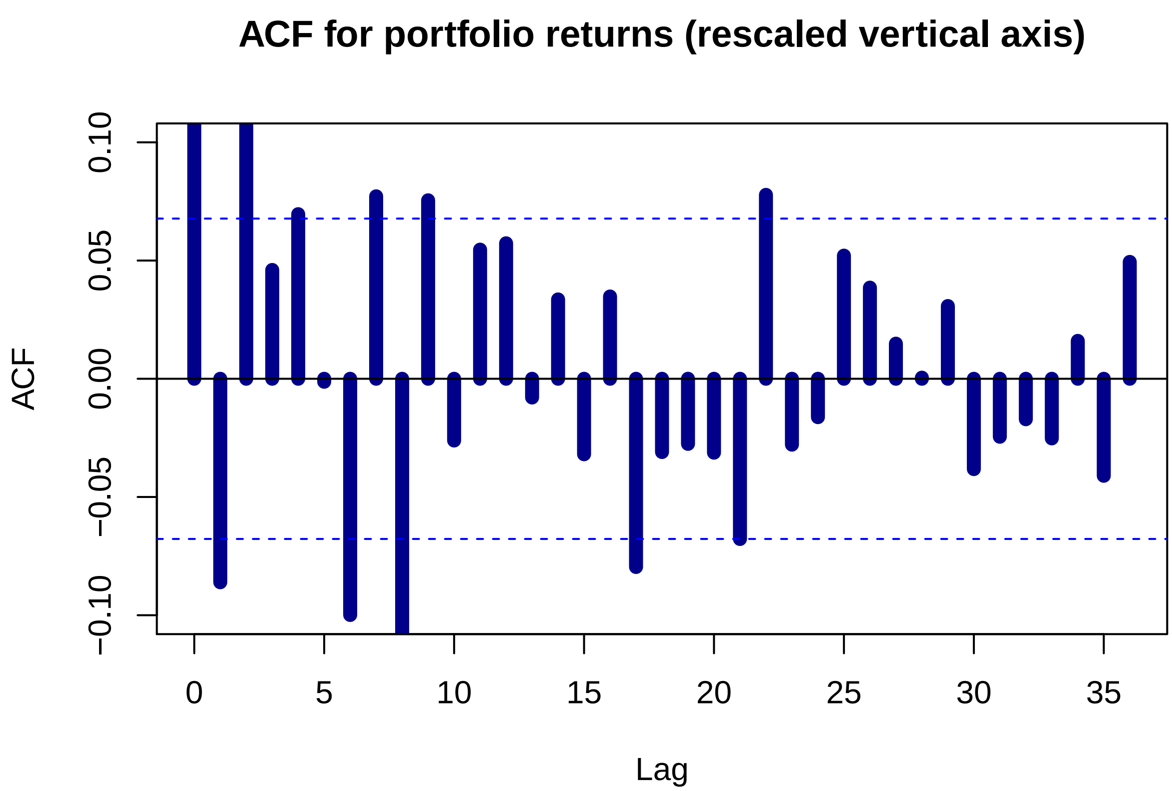
The behavior of our portfolio prices supports this assumption. The price drops in March where the COVID-19 pandemic and restriction occur in Europe and USA. Nevertheless, the prices recover fast and increase even more. The fast and high increase as well as the drop of prices from December can be cause by Bitcoin which has a similar behavior during this time.

Plot of ACF for returns

```
acf(quotes$quotes.r, lag.max = 36, na.action = na.pass,
    col = "darkblue", lwd = 7,
    main = "ACF for Portfolio returns")
```



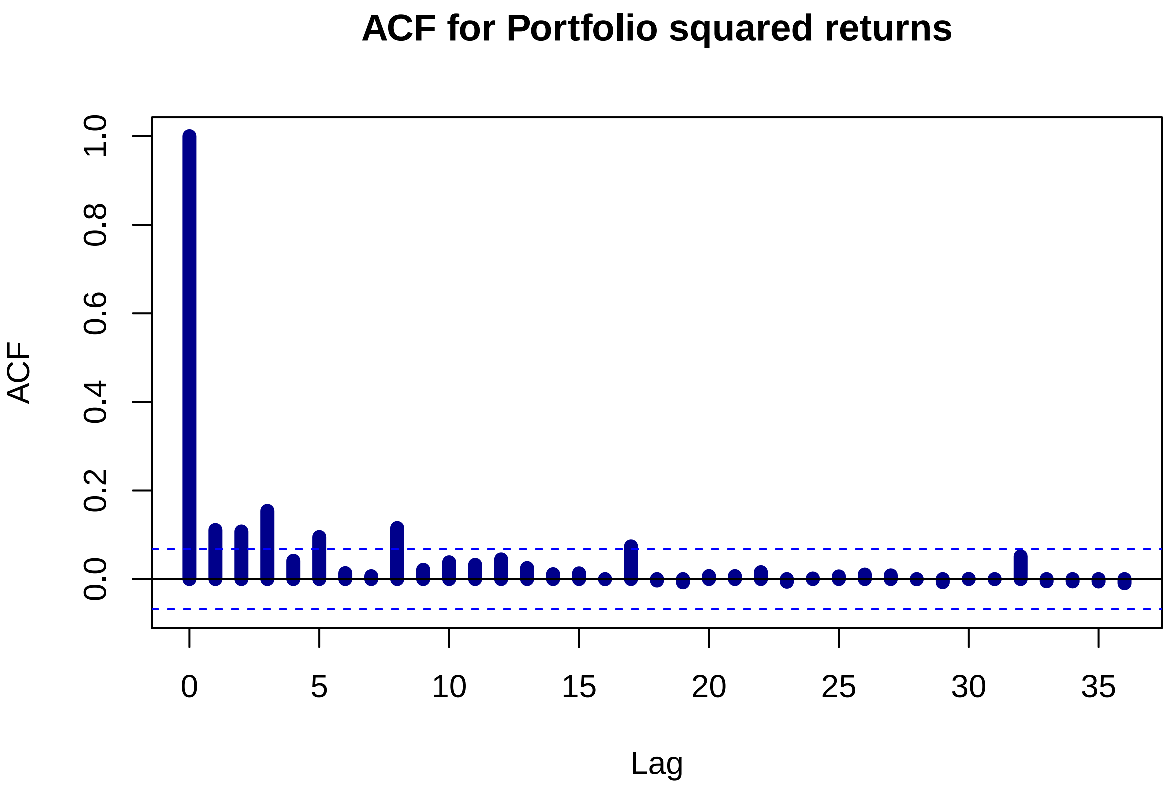

```r
acf(quotes$quotes.r, lag.max = 36, na.action = na.pass,
    ylim = c(-0.1, 0.1),
    col = "darkblue", lwd = 7,
    main = "ACF for portfolio returns (rescaled vertical axis)")
```



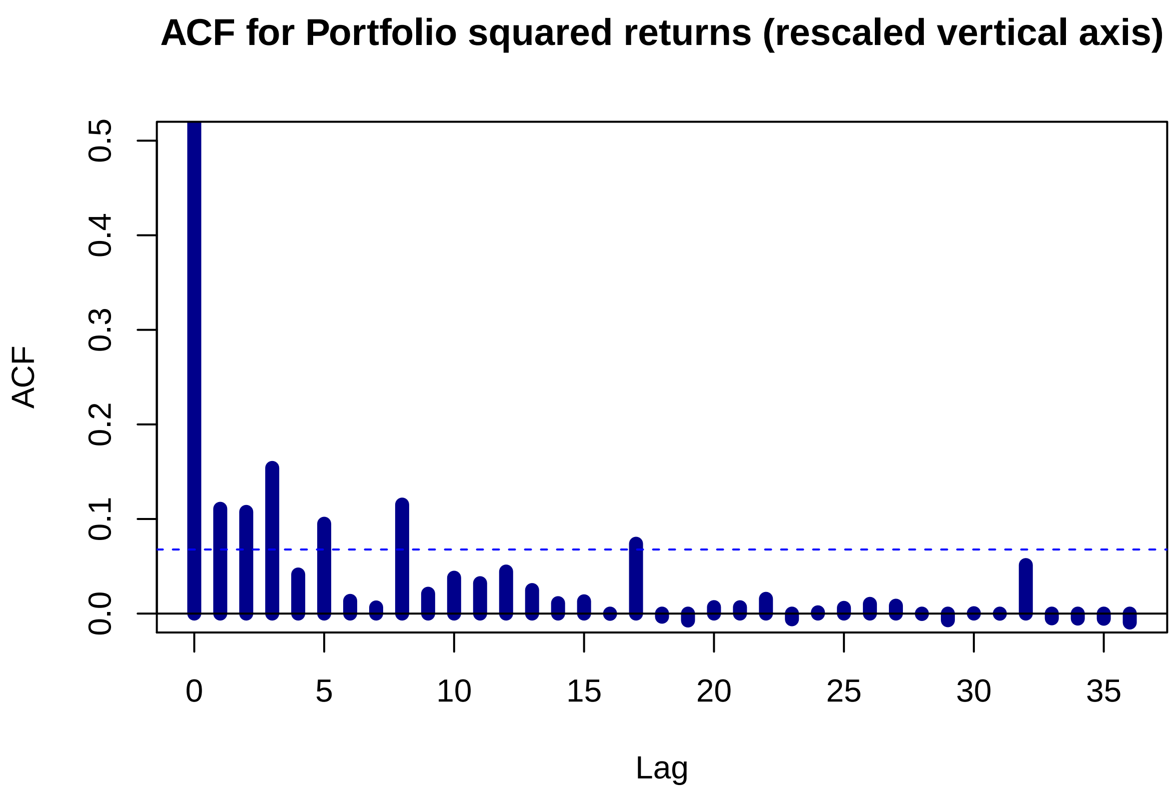
The plot of the ACF for my portfolio returns shows to some extent an autoregressive relationship. This can be used to build an ARIMA Model

Plot of ACF for squared returns

```
acf(quotes$quotes.r^2, lag.max = 36, na.action = na.pass,
    col = "darkblue", lwd = 7,
    main = "ACF for Portfolio squared returns")
```




```
acf(quotes$quotes.r^2, lag.max = 36, na.action = na.pass,
    ylim = c(0, 0.5),
    col = "darkblue", lwd = 7,
    main = "ACF for Portfolio squared returns (rescaled vertical axis)")
```

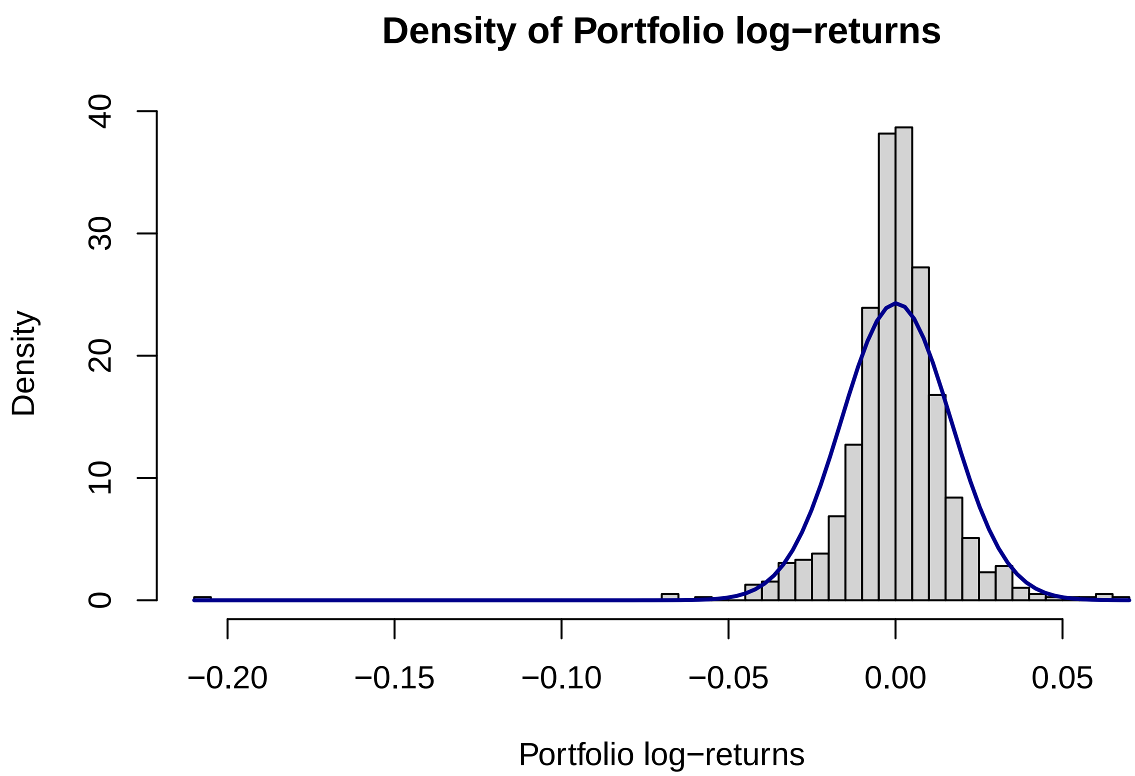


The plot of the ACF of squares of my portfolio shows a autoregressive relationship between each other. Thus we can use it to build a GARCH Model.

```
basicStats(quotes$quotes.r)
```

```
##                quotes.r
## nobs         837.000000
## NAs           51.000000
## Minimum       -0.209420
## Maximum        0.068545
## 1. Quartile   -0.006169
## 3. Quartile    0.007769
## Mean           0.000196
## Median         0.000754
## Sum            0.153698
## SE Mean        0.000586
## LCL Mean      -0.000954
## UCL Mean       0.001345
## Variance       0.000270
## Stdev          0.016418
## Skewness      -2.621073
## Kurtosis      34.809267
```

```
hist(quotes$quotes.r, prob = T, breaks = 40, main = "Density of Portfolio log-returns", xlab = "Portfoli
curve(dnorm(x, mean = mean(quotes$quotes.r, na.rm = T),
            sd  = sd(quotes$quotes.r, na.rm = T)),
      col = "darkblue", lwd = 2, add = TRUE)
```



The comparison of the distribution of our portfolio returns with a normal distribution shows a highly leptokurtic distribution.

```
basicStats(quotes$quotes.r)
```

```
##                  quotes.r
## nobs           837.000000
## NAs             51.000000
## Minimum         -0.209420
## Maximum          0.068545
## 1. Quartile     -0.006169
## 3. Quartile      0.007769
## Mean             0.000196
## Median           0.000754
## Sum              0.153698
## SE Mean          0.000586
## LCL Mean        -0.000954
## UCL Mean         0.001345
## Variance         0.000270
## Stdev            0.016418
## Skewness        -2.621073
## Kurtosis        34.809267
```

The basic Statistics show among others a Skewness of -2.62, which says, that the left tail is thicker than the right tail of the distribution. The Kurtosis which is the excess Kurtosis of 34.81 suggest a thicker thail than the normal distribution. The Kurtosis would be at approx 39.

```
jarque.bera.test(na.omit(quotes$quotes.r))
```

```
##
##  Jarque Bera Test
```

```
##
## data:  na.omit(quotes$quotes.r)
## X-squared = 40806, df = 2, p-value < 2.2e-16
```

Additionally, the p-value of the Jarque-Bera test is approx 0.00000000000000002. Hence, we can reject the Null-Hypothesis about normality to a confidence level of 99%.

```
ArchTest(quotes$quotes.r,
         lags = 5)
```

```
##
##  ARCH LM-test; Null hypothesis: no ARCH effects
##
## data:  quotes$quotes.r
## Chi-squared = 25.901, df = 5, p-value = 0.00009326
```

```
ArchTest(quotes$quotes.r,
         lags = 50)
```

```
##
##  ARCH LM-test; Null hypothesis: no ARCH effects
##
## data:  quotes$quotes.r
## Chi-squared = 81.724, df = 50, p-value = 0.00308
```

The result of the Arch Test has a p-value of 0.000009 for 5 lags and 0.003 for 50 lags. Thus we can reject the Null-Hypothesis for both to a confidence level of 99% which says that the residuals exhibits no conditional heteroscedasticity (Arch Effects). This result suggest the application of an ARCH Model.

```
quotes$quotes.r[is.na(quotes$quotes.r)] <- 0
```

Replace NA quotes with 0, since the ugarfit() function does not accept missing values.

Estimation GARCH Models

Garch(5,5)

```
spec <- ugarchspec(
  variance.model = list(model = "sGARCH",
                        garchOrder = c(5, 5)),
  mean.model = list(armaOrder = c(0, 0),
                    include.mean = TRUE),
  distribution.model = "norm")

quotes$quotes.r[is.na(quotes$quotes.r)] <- 0


k.garch55 <- ugarchfit(spec = spec,
                       data = quotes$quotes.r)

k.garch55
```

```
##
## *---------------------------------*
## *          GARCH Model Fit        *
## *---------------------------------*
##
## Conditional Variance Dynamics
```

```
## ------------------------------------
## GARCH Model  : sGARCH(5,5)
## Mean Model   : ARFIMA(0,0,0)
## Distribution : norm
##
## Optimal Parameters
## ------------------------------------
##         Estimate  Std. Error  t value Pr(>|t|)
## mu      0.000280    0.000421 0.665181 0.505935
## omega   0.000053    0.000017 3.218187 0.001290
## alpha1  0.092345    0.044200 2.089283 0.036682
## alpha2  0.063682    0.032689 1.948122 0.051400
## alpha3  0.183507    0.038460 4.771339 0.000002
## alpha4  0.000000    0.023488 0.000008 0.999993
## alpha5  0.135089    0.037753 3.578252 0.000346
## beta1   0.000000    0.225669 0.000002 0.999999
## beta2   0.141559    0.086261 1.641045 0.100788
## beta3   0.000000    0.170162 0.000001 0.999999
## beta4   0.000000    0.087795 0.000003 0.999997
## beta5   0.157484    0.055333 2.846119 0.004426
##
## Robust Standard Errors:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu      0.000280    0.000481 0.582310 0.560358
## omega   0.000053    0.000017 3.156940 0.001594
## alpha1  0.092345    0.062710 1.472584 0.140863
## alpha2  0.063682    0.060658 1.049845 0.293789
## alpha3  0.183507    0.119588 1.534496 0.124908
## alpha4  0.000000    0.052592 0.000004 0.999997
## alpha5  0.135089    0.049279 2.741293 0.006120
## beta1   0.000000    0.285939 0.000001 0.999999
## beta2   0.141559    0.293544 0.482240 0.629636
## beta3   0.000000    0.222007 0.000001 0.999999
## beta4   0.000000    0.238770 0.000001 0.999999
## beta5   0.157484    0.175423 0.897738 0.369325
##
## LogLikelihood : 2425.439
##
## Information Criteria
## ------------------------------------
##
## Akaike       -5.7669
## Bayes        -5.6991
## Shibata      -5.7673
## Hannan-Quinn -5.7409
##
## Weighted Ljung-Box Test on Standardized Residuals
## ------------------------------------
##                         statistic p-value
## Lag[1]                     0.8015 0.37066
## Lag[2*(p+q)+(p+q)-1][2]    3.8399 0.08326
## Lag[4*(p+q)+(p+q)-1][5]    7.3814 0.04187
## d.o.f=0
## H0 : No serial correlation
```

```
##
## Weighted Ljung-Box Test on Standardized Squared Residuals
## ------------------------------------
##                          statistic p-value
## Lag[1]                     0.08565  0.7698
## Lag[2*(p+q)+(p+q)-1][29]   8.14939  0.9588
## Lag[4*(p+q)+(p+q)-1][49]  14.58656  0.9798
## d.o.f=10
##
## Weighted ARCH LM Tests
## ------------------------------------
##              Statistic Shape Scale P-Value
## ARCH Lag[11]   0.01208 0.500 2.000  0.9125
## ARCH Lag[13]   3.56115 1.493 1.855  0.2773
## ARCH Lag[15]   4.06170 2.471 1.754  0.4546
##
## Nyblom stability test
## ------------------------------------
## Joint Statistic:  4.1372
## Individual Statistics:
## mu     0.68067
## omega  0.18037
## alpha1 0.03064
## alpha2 0.17831
## alpha3 0.16749
## alpha4 0.20621
## alpha5 0.11389
## beta1  0.21639
## beta2  0.21698
## beta3  0.21611
## beta4  0.22692
## beta5  0.16426
##
## Asymptotic Critical Values (10% 5% 1%)
## Joint Statistic:          2.69 2.96 3.51
## Individual Statistic:     0.35 0.47 0.75
##
## Sign Bias Test
## ------------------------------------
##                    t-value   prob sig
## Sign Bias           0.0867 0.9309
## Negative Sign Bias  1.2208 0.2225
## Positive Sign Bias  0.8614 0.3893
## Joint Effect        3.3670 0.3384
##
##
## Adjusted Pearson Goodness-of-Fit Test:
## ------------------------------------
##   group statistic p-value(g-1)
## 1    20     115.8    6.847e-16
## 2    30     161.2    2.827e-20
## 3    40     205.2    1.913e-24
## 4    50     266.6    1.040e-31
##
```

```
##
## Elapsed time : 0.2598331
```

The AR(5)-EGARCH(2,1) model

```
spec <- ugarchspec(
  variance.model = list(model = "eGARCH",
                        garchOrder = c(2, 1)),
  mean.model = list(armaOrder = c(5, 0),
                    include.mean = TRUE),
  distribution.model = "norm")


k.ar5egarch21 <- ugarchfit(spec = spec,
                           data = quotes$quotes.r)

k.ar5egarch21
```

```
##
## *---------------------------------*
## *          GARCH Model Fit        *
## *---------------------------------*
##
## Conditional Variance Dynamics
## -----------------------------------
## GARCH Model  : eGARCH(2,1)
## Mean Model   : ARFIMA(5,0,0)
## Distribution : norm
##
## Optimal Parameters
## ------------------------------------
##         Estimate  Std. Error  t value Pr(>|t|)
## mu     -0.000271    0.001234 -0.21945 0.826302
## ar1     0.016457    0.054013  0.30469 0.760602
## ar2     0.069401    0.032463  2.13786 0.032528
## ar3     0.050267    0.039448  1.27426 0.202573
## ar4     0.045652    0.022224  2.05416 0.039960
## ar5     0.051431    0.039931  1.28800 0.197746
## omega  -0.577304    0.321517 -1.79556 0.072564
## alpha1 -0.079757    0.049469 -1.61225 0.106906
## alpha2 -0.061273    0.061764 -0.99205 0.321174
## beta1   0.930335    0.036764 25.30527 0.000000
## gamma1  0.191909    0.070811  2.71016 0.006725
## gamma2  0.050988    0.077174  0.66069 0.508811
##
## Robust Standard Errors:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu     -0.000271    0.003721 -0.072752 0.942004
## ar1     0.016457    0.143923  0.114347 0.908963
## ar2     0.069401    0.030757  2.256394 0.024046
## ar3     0.050267    0.067586  0.743747 0.457030
## ar4     0.045652    0.014588  3.129430 0.001751
## ar5     0.051431    0.069907  0.735703 0.461912
## omega  -0.577304    1.002232 -0.576019 0.564603
```

```
## alpha1 -0.079757    0.083103 -0.959738 0.337187
## alpha2 -0.061273    0.146441 -0.418415 0.675643
## beta1   0.930335    0.113949  8.164458 0.000000
## gamma1  0.191909    0.119082  1.611570 0.107056
## gamma2  0.050988    0.146666  0.347648 0.728104
##
## LogLikelihood : 2428.843
##
## Information Criteria
## ------------------------------------
##
## Akaike       -5.7750
## Bayes        -5.7072
## Shibata      -5.7754
## Hannan-Quinn -5.7490
##
## Weighted Ljung-Box Test on Standardized Residuals
## ------------------------------------
##                          statistic p-value
## Lag[1]                     0.00488  0.9443
## Lag[2*(p+q)+(p+q)-1][14]   7.28282  0.6332
## Lag[4*(p+q)+(p+q)-1][24]  11.95589  0.5547
## d.o.f=5
## H0 : No serial correlation
##
## Weighted Ljung-Box Test on Standardized Squared Residuals
## ------------------------------------
##                          statistic p-value
## Lag[1]                     0.00355  0.9525
## Lag[2*(p+q)+(p+q)-1][8]    4.81846  0.3781
## Lag[4*(p+q)+(p+q)-1][14]   6.60497  0.5576
## d.o.f=3
##
## Weighted ARCH LM Tests
## ------------------------------------
##             Statistic Shape Scale P-Value
## ARCH Lag[4]    0.6819 0.500 2.000  0.4089
## ARCH Lag[6]    1.6968 1.461 1.711  0.5611
## ARCH Lag[8]    2.1424 2.368 1.583  0.7127
##
## Nyblom stability test
## ------------------------------------
## Joint Statistic:  3.7743
## Individual Statistics:
## mu     0.92689
## ar1    0.10338
## ar2    0.15229
## ar3    0.03226
## ar4    0.11665
## ar5    0.16799
## omega  0.36282
## alpha1 0.21763
## alpha2 0.11200
## beta1  0.34526
```

```
## gamma1 0.15083
## gamma2 0.03697
##
## Asymptotic Critical Values (10% 5% 1%)
## Joint Statistic:          2.69 2.96 3.51
## Individual Statistic:     0.35 0.47 0.75
##
## Sign Bias Test
## ------------------------------------
##                    t-value   prob sig
## Sign Bias           0.2996 0.7645
## Negative Sign Bias  0.4726 0.6366
## Positive Sign Bias  0.5980 0.5500
## Joint Effect        1.6134 0.6564
##
##
## Adjusted Pearson Goodness-of-Fit Test:
## ------------------------------------
##   group statistic    p-value(g-1)
## 1    20     84.65 0.0000000002901
## 2    30     98.02 0.0000000020256
## 3    40    114.35 0.0000000025087
## 4    50    126.62 0.0000000086020
##
##
## Elapsed time : 0.3643219
```

AR(1)-EGARCH(2,1)

```
spec <- ugarchspec(
  variance.model = list(model = "eGARCH",
                        garchOrder = c(2, 1)),
  mean.model = list(armaOrder = c(1, 0),
                    include.mean = TRUE),
  distribution.model = "norm")



k.ar1egarch21 <- ugarchfit(spec = spec,
                           data = quotes$quotes.r)

k.ar1egarch21
```

```
##
## *---------------------------------*
## *          GARCH Model Fit        *
## *---------------------------------*
##
## Conditional Variance Dynamics
## -----------------------------------
## GARCH Model  : eGARCH(2,1)
## Mean Model   : ARFIMA(1,0,0)
## Distribution : norm
##
```

```
## Optimal Parameters
## ------------------------------------
##         Estimate  Std. Error   t value Pr(>|t|)
## mu      0.000076    0.000451  0.169408 0.865475
## ar1     0.017616    0.039576  0.445120 0.656233
## omega  -0.531918    0.165693 -3.210267 0.001326
## alpha1 -0.089378    0.051176 -1.746495 0.080725
## alpha2 -0.022758    0.057082 -0.398684 0.690126
## beta1   0.935414    0.019485 48.007114 0.000000
## gamma1  0.236447    0.075268  3.141376 0.001682
## gamma2  0.007433    0.078565  0.094614 0.924622
##
## Robust Standard Errors:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu      0.000076    0.000500  0.15282 0.878536
## ar1     0.017616    0.044916  0.39220 0.694910
## omega  -0.531918    0.244318 -2.17715 0.029469
## alpha1 -0.089378    0.068439 -1.30595 0.191571
## alpha2 -0.022758    0.083841 -0.27144 0.786056
## beta1   0.935414    0.028419 32.91564 0.000000
## gamma1  0.236447    0.133726  1.76814 0.077038
## gamma2  0.007433    0.123765  0.06006 0.952108
##
## LogLikelihood : 2424.028
##
## Information Criteria
## ------------------------------------
##
## Akaike       -5.7731
## Bayes        -5.7279
## Shibata      -5.7732
## Hannan-Quinn -5.7557
##
## Weighted Ljung-Box Test on Standardized Residuals
## ------------------------------------
##                         statistic p-value
## Lag[1]                   0.009799 0.92115
## Lag[2*(p+q)+(p+q)-1][2]  2.568169 0.08030
## Lag[4*(p+q)+(p+q)-1][5]  5.228029 0.08857
## d.o.f=1
## H0 : No serial correlation
##
## Weighted Ljung-Box Test on Standardized Squared Residuals
## ------------------------------------
##                          statistic p-value
## Lag[1]                     0.03832  0.8448
## Lag[2*(p+q)+(p+q)-1][8]    3.60462  0.5753
## Lag[4*(p+q)+(p+q)-1][14]   5.40370  0.7181
## d.o.f=3
##
## Weighted ARCH LM Tests
## ------------------------------------
##             Statistic Shape Scale P-Value
## ARCH Lag[4]    0.7261 0.500 2.000  0.3941
```

```
## ARCH Lag[6]    1.3851 1.461 1.711  0.6408
## ARCH Lag[8]    1.6165 2.368 1.583  0.8176
##
## Nyblom stability test
## ------------------------------------
## Joint Statistic:  3.3586
## Individual Statistics:
## mu     1.37279
## ar1    0.15761
## omega  0.38419
## alpha1 0.18982
## alpha2 0.10081
## beta1  0.37017
## gamma1 0.14579
## gamma2 0.03502
##
## Asymptotic Critical Values (10% 5% 1%)
## Joint Statistic:          1.89 2.11 2.59
## Individual Statistic:     0.35 0.47 0.75
##
## Sign Bias Test
## ------------------------------------
##                    t-value   prob sig
## Sign Bias           0.2104 0.8334
## Negative Sign Bias  0.5432 0.5871
## Positive Sign Bias  0.5822 0.5606
## Joint Effect        1.4992 0.6824
##
##
## Adjusted Pearson Goodness-of-Fit Test:
## ------------------------------------
##   group statistic p-value(g-1)
## 1    20     107.0    2.821e-14
## 2    30     144.8    2.416e-17
## 3    40     157.6    3.488e-16
## 4    50     186.7    5.989e-18
##
##
## Elapsed time : 0.172174
```

AR(1)-EGARCH(2,1)-mean

```
spec <- ugarchspec(
  variance.model = list(model = "sGARCH",
                        garchOrder = c(2, 1)),
  mean.model = list(armaOrder = c(1, 0),
                    include.mean = TRUE,
                    archm = TRUE, archpow = 1),
  distribution.model = "norm")


k.ar1garchm21 <- ugarchfit(spec = spec,
                           data = quotes$quotes.r)
```

```
k.ar1garchm21
```

```
##
## *---------------------------------*
## *          GARCH Model Fit        *
## *---------------------------------*
##
## Conditional Variance Dynamics
## -----------------------------------
## GARCH Model  : sGARCH(2,1)
## Mean Model   : ARFIMA(1,0,0)
## Distribution : norm
##
## Optimal Parameters
## ------------------------------------
##         Estimate  Std. Error  t value Pr(>|t|)
## mu     -0.003192    0.001856 -1.71997 0.085438
## ar1     0.007089    0.041107  0.17244 0.863090
## archm   0.295462    0.144170  2.04940 0.040423
## omega   0.000015    0.000002  5.95416 0.000000
## alpha1  0.127347    0.047739  2.66755 0.007641
## alpha2  0.035319    0.058378  0.60501 0.545169
## beta1   0.782570    0.019397 40.34531 0.000000
##
## Robust Standard Errors:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu     -0.003192    0.002567 -1.24365  0.21363
## ar1     0.007089    0.046022  0.15403  0.87759
## archm   0.295462    0.220588  1.33943  0.18043
## omega   0.000015    0.000011  1.36077  0.17359
## alpha1  0.127347    0.115977  1.09804  0.27219
## alpha2  0.035319    0.107723  0.32787  0.74301
## beta1   0.782570    0.078840  9.92608  0.00000
##
## LogLikelihood : 2416.739
##
## Information Criteria
## ------------------------------------
##
## Akaike       -5.7580
## Bayes        -5.7185
## Shibata      -5.7582
## Hannan-Quinn -5.7429
##
## Weighted Ljung-Box Test on Standardized Residuals
## ------------------------------------
##                         statistic  p-value
## Lag[1]                     0.9565 0.328063
## Lag[2*(p+q)+(p+q)-1][2]    3.6602 0.009003
## Lag[4*(p+q)+(p+q)-1][5]    7.3697 0.013770
## d.o.f=1
## H0 : No serial correlation
##
## Weighted Ljung-Box Test on Standardized Squared Residuals
```

```
## ------------------------------------
##                          statistic p-value
## Lag[1]                      0.2227  0.6370
## Lag[2*(p+q)+(p+q)-1][8]     6.0817  0.2270
## Lag[4*(p+q)+(p+q)-1][14]    8.6202  0.3177
## d.o.f=3
##
## Weighted ARCH LM Tests
## ------------------------------------
##              Statistic Shape Scale P-Value
## ARCH Lag[4]    0.2412 0.500 2.000  0.6233
## ARCH Lag[6]    0.5119 1.461 1.711  0.8887
## ARCH Lag[8]    1.1244 2.368 1.583  0.9052
##
## Nyblom stability test
## ------------------------------------
## Joint Statistic:  10.7372
## Individual Statistics:
## mu     0.8624
## ar1    0.4726
## archm  0.9219
## omega  0.7555
## alpha1 0.1496
## alpha2 0.2140
## beta1  0.3179
##
## Asymptotic Critical Values (10% 5% 1%)
## Joint Statistic:          1.69 1.9 2.35
## Individual Statistic:     0.35 0.47 0.75
##
## Sign Bias Test
## ------------------------------------
##                     t-value   prob sig
## Sign Bias            0.1616 0.8717
## Negative Sign Bias  1.0184 0.3088
## Positive Sign Bias  0.7891 0.4303
## Joint Effect        3.2571 0.3537
##
##
## Adjusted Pearson Goodness-of-Fit Test:
## ------------------------------------
##   group statistic       p-value(g-1)
## 1    20      99.8 0.0000000000005826
## 2    30     114.4 0.0000000000043662
## 3    40     131.9 0.0000000000050668
## 4    50     150.5 0.0000000000029761
##
##
## Elapsed time : 0.361223
```

AR(1)-GARCH-t(2,1)

```
spec <- ugarchspec(
  variance.model = list(model = "sGARCH",
```

```r
                        garchOrder = c(2, 1)),
  mean.model = list(armaOrder = c(1, 0),
                    include.mean = TRUE),
  distribution.model = "std")

k.ar1garcht21 <- ugarchfit(spec = spec,
                           data = quotes$quotes.r)

k.ar1garcht21
```

```
##
## *---------------------------------*
## *          GARCH Model Fit        *
## *---------------------------------*
##
## Conditional Variance Dynamics
## -----------------------------------
## GARCH Model  : sGARCH(2,1)
## Mean Model   : ARFIMA(1,0,0)
## Distribution : std
##
## Optimal Parameters
## ------------------------------------
##         Estimate  Std. Error  t value Pr(>|t|)
## mu      0.000514    0.000341  1.50865  0.13139
## ar1    -0.013639    0.031409 -0.43425  0.66411
## omega   0.000012    0.000001  9.19920  0.00000
## alpha1  0.090869    0.055662  1.63252  0.10257
## alpha2  0.015245    0.059917  0.25444  0.79916
## beta1   0.858586    0.023784 36.09914  0.00000
## shape   3.223449    0.310635 10.37698  0.00000
##
## Robust Standard Errors:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu      0.000514    0.000360  1.42893  0.15302
## ar1    -0.013639    0.034226 -0.39851  0.69026
## omega   0.000012    0.000002  7.48736  0.00000
## alpha1  0.090869    0.061196  1.48489  0.13757
## alpha2  0.015245    0.061787  0.24674  0.80511
## beta1   0.858586    0.019649 43.69669  0.00000
## shape   3.223449    0.286106 11.26663  0.00000
##
## LogLikelihood : 2490.048
##
## Information Criteria
## ------------------------------------
##
## Akaike       -5.9332
## Bayes        -5.8937
## Shibata      -5.9333
## Hannan-Quinn -5.9180
##
## Weighted Ljung-Box Test on Standardized Residuals
## ------------------------------------
```

```
##                          statistic  p-value
## Lag[1]                       1.586 0.207958
## Lag[2*(p+q)+(p+q)-1][2]      4.530 0.001307
## Lag[4*(p+q)+(p+q)-1][5]      8.329 0.005607
## d.o.f=1
## H0 : No serial correlation
##
## Weighted Ljung-Box Test on Standardized Squared Residuals
## ------------------------------------
##                          statistic p-value
## Lag[1]                    0.000746  0.9782
## Lag[2*(p+q)+(p+q)-1][8]   8.032364  0.0932
## Lag[4*(p+q)+(p+q)-1][14] 10.489137  0.1667
## d.o.f=3
##
## Weighted ARCH LM Tests
## ------------------------------------
##             Statistic Shape Scale P-Value
## ARCH Lag[4]   0.05304 0.500 2.000  0.8179
## ARCH Lag[6]   0.20729 1.461 1.711  0.9671
## ARCH Lag[8]   0.49167 2.368 1.583  0.9825
##
## Nyblom stability test
## ------------------------------------
## Joint Statistic:  23.1208
## Individual Statistics:
## mu     0.6825
## ar1    0.4858
## omega  2.4857
## alpha1 0.1231
## alpha2 0.1850
## beta1  0.1442
## shape  0.1154
##
## Asymptotic Critical Values (10% 5% 1%)
## Joint Statistic:          1.69 1.9 2.35
## Individual Statistic:     0.35 0.47 0.75
##
## Sign Bias Test
## ------------------------------------
##                    t-value   prob sig
## Sign Bias          0.08929 0.9289
## Negative Sign Bias 1.61358 0.1070
## Positive Sign Bias 0.63274 0.5271
## Joint Effect       4.76154 0.1901
##
##
## Adjusted Pearson Goodness-of-Fit Test:
## ------------------------------------
##   group statistic p-value(g-1)
## 1    20     53.25 0.0000428985
## 2    30     81.24 0.0000007607
## 3    40     66.08 0.0043372904
## 4    50     96.63 0.0000585436
```

```
##
##
## Elapsed time : 0.3926601
```

AR(1)-EGARCH-t(2,1)-mean

```
spec <- ugarchspec(
  variance.model = list(model = "eGARCH",
                        garchOrder = c(2, 1)),
  mean.model = list(armaOrder = c(1, 0),
                    include.mean = TRUE,
                    archm = TRUE, archpow = 1),
  distribution.model = "std")


k.ar1egarchmt21 <- ugarchfit(spec = spec,
                             data = quotes$quotes.r)

k.ar1egarchmt21
```

```
##
## *---------------------------------*
## *          GARCH Model Fit        *
## *---------------------------------*
##
## Conditional Variance Dynamics
## -----------------------------------
## GARCH Model  : eGARCH(2,1)
## Mean Model   : ARFIMA(1,0,0)
## Distribution : std
##
## Optimal Parameters
## ------------------------------------
##         Estimate  Std. Error   t value Pr(>|t|)
## mu     -0.002115    0.001276  -1.65738 0.097442
## ar1     0.006024    0.020728   0.29059 0.771361
## archm   0.183963    0.094910   1.93828 0.052589
## omega  -0.436974    0.054270  -8.05186 0.000000
## alpha1 -0.166655    0.073445  -2.26911 0.023262
## alpha2  0.099795    0.072076   1.38458 0.166180
## beta1   0.948533    0.006259 151.54913 0.000000
## gamma1  0.150051    0.075772   1.98030 0.047670
## gamma2  0.052588    0.070603   0.74484 0.456371
## shape   3.222518    0.392955   8.20072 0.000000
##
## Robust Standard Errors:
##         Estimate  Std. Error   t value Pr(>|t|)
## mu     -0.002115    0.001070  -1.97637 0.048113
## ar1     0.006024    0.013951   0.43176 0.665915
## archm   0.183963    0.081376   2.26065 0.023781
## omega  -0.436974    0.030310 -14.41677 0.000000
## alpha1 -0.166655    0.079469  -2.09711 0.035984
## alpha2  0.099795    0.073620   1.35554 0.175245
## beta1   0.948533    0.003621 261.95826 0.000000
```

```
## gamma1  0.150051    0.066514   2.25593 0.024075
## gamma2  0.052588    0.063015   0.83452 0.403986
## shape   3.222518    0.323671   9.95616 0.000000
##
## LogLikelihood : 2494.335
##
## Information Criteria
## ------------------------------------
##
## Akaike       -5.9363
## Bayes        -5.8798
## Shibata      -5.9366
## Hannan-Quinn -5.9146
##
## Weighted Ljung-Box Test on Standardized Residuals
## ------------------------------------
##                         statistic  p-value
## Lag[1]                      1.164 0.280737
## Lag[2*(p+q)+(p+q)-1][2]     4.640 0.001015
## Lag[4*(p+q)+(p+q)-1][5]     8.249 0.006052
## d.o.f=1
## H0 : No serial correlation
##
## Weighted Ljung-Box Test on Standardized Squared Residuals
## ------------------------------------
##                          statistic p-value
## Lag[1]                      0.1516  0.6971
## Lag[2*(p+q)+(p+q)-1][8]     3.3332  0.6244
## Lag[4*(p+q)+(p+q)-1][14]    4.8303  0.7902
## d.o.f=3
##
## Weighted ARCH LM Tests
## ------------------------------------
##             Statistic Shape Scale P-Value
## ARCH Lag[4]    0.1459 0.500 2.000  0.7025
## ARCH Lag[6]    0.4414 1.461 1.711  0.9082
## ARCH Lag[8]    0.6475 2.368 1.583  0.9686
##
## Nyblom stability test
## ------------------------------------
## Joint Statistic:  2.966
## Individual Statistics:
## mu     0.78512
## ar1    0.37543
## archm  1.03533
## omega  0.38398
## alpha1 0.24933
## alpha2 0.08042
## beta1  0.37799
## gamma1 0.08598
## gamma2 0.25998
## shape  0.09143
##
## Asymptotic Critical Values (10% 5% 1%)
```

```
## Joint Statistic:          2.29 2.54 3.05
## Individual Statistic:     0.35 0.47 0.75
##
## Sign Bias Test
## ------------------------------------
##                    t-value   prob sig
## Sign Bias          0.02261 0.9820
## Negative Sign Bias 1.03928 0.2990
## Positive Sign Bias 0.15518 0.8767
## Joint Effect       1.23309 0.7451
##
##
## Adjusted Pearson Goodness-of-Fit Test:
## ------------------------------------
##   group statistic p-value(g-1)
## 1    20     31.32      0.03727
## 2    30     36.87      0.14955
## 3    40     39.13      0.46408
## 4    50     58.52      0.16553
##
##
## Elapsed time : 0.698092
```

AR(1)-EGARCH-t(2,1)

```
spec <-
  ugarchspec(variance.model = list(model = "eGARCH", garchOrder = c(2, 1)),
             mean.model = list(armaOrder = c(1, 0), include.mean = F),
             distribution.model = "std")


k.ar1egarcht21 <- ugarchfit(spec = spec,
                            data = quotes$quotes.r)

k.ar1egarcht21
```

```
##
## *---------------------------------*
## *          GARCH Model Fit        *
## *---------------------------------*
##
## Conditional Variance Dynamics
## -----------------------------------
## GARCH Model  : eGARCH(2,1)
## Mean Model   : ARFIMA(1,0,0)
## Distribution : std
##
## Optimal Parameters
## ------------------------------------
##         Estimate  Std. Error   t value Pr(>|t|)
## ar1    -0.001778    0.030026 -0.059205  0.95279
## omega  -0.407620    0.061665 -6.610278  0.00000
## alpha1 -0.179853    0.071289 -2.522865  0.01164
## alpha2  0.100520    0.069725  1.441651  0.14940
```

```
## beta1   0.951766    0.007064 134.728519  0.00000
## gamma1  0.146814    0.096391   1.523112  0.12773
## gamma2  0.058615    0.091395   0.641336  0.52131
## shape   3.269719    0.414907   7.880613  0.00000
##
## Robust Standard Errors:
##         Estimate  Std. Error    t value Pr(>|t|)
## ar1    -0.001778    0.032640  -0.054464 0.956566
## omega  -0.407620    0.028054 -14.530021 0.000000
## alpha1 -0.179853    0.071913  -2.500980 0.012385
## alpha2  0.100520    0.065163   1.542582 0.122932
## beta1   0.951766    0.003102 306.792846 0.000000
## gamma1  0.146814    0.103514   1.418302 0.156103
## gamma2  0.058615    0.100890   0.580978 0.561255
## shape   3.269719    0.356548   9.170490 0.000000
##
## LogLikelihood : 2492.898
##
## Information Criteria
## ------------------------------------
##
## Akaike       -5.9376
## Bayes        -5.8924
## Shibata      -5.9378
## Hannan-Quinn -5.9203
##
## Weighted Ljung-Box Test on Standardized Residuals
## ------------------------------------
##                         statistic  p-value
## Lag[1]                     0.7281 0.393506
## Lag[2*(p+q)+(p+q)-1][2]    3.6755 0.008713
## Lag[4*(p+q)+(p+q)-1][5]    6.5172 0.029698
## d.o.f=1
## H0 : No serial correlation
##
## Weighted Ljung-Box Test on Standardized Squared Residuals
## ------------------------------------
##                          statistic p-value
## Lag[1]                      0.2159  0.6422
## Lag[2*(p+q)+(p+q)-1][8]     3.0650  0.6736
## Lag[4*(p+q)+(p+q)-1][14]    4.5331  0.8248
## d.o.f=3
##
## Weighted ARCH LM Tests
## ------------------------------------
##             Statistic Shape Scale P-Value
## ARCH Lag[4]    0.2770 0.500 2.000  0.5987
## ARCH Lag[6]    0.6090 1.461 1.711  0.8611
## ARCH Lag[8]    0.8038 2.368 1.583  0.9510
##
## Nyblom stability test
## ------------------------------------
## Joint Statistic:  1.9771
## Individual Statistics:
```

```
## ar1    0.29176
## omega  0.27047
## alpha1 0.20518
## alpha2 0.07237
## beta1  0.26834
## gamma1 0.06342
## gamma2 0.22801
## shape  0.16340
##
## Asymptotic Critical Values (10% 5% 1%)
## Joint Statistic:          1.89 2.11 2.59
## Individual Statistic:     0.35 0.47 0.75
##
## Sign Bias Test
## ------------------------------------
##                     t-value   prob sig
## Sign Bias          0.185679 0.8527
## Negative Sign Bias 0.892624 0.3723
## Positive Sign Bias 0.002815 0.9978
## Joint Effect       0.846108 0.8384
##
##
## Adjusted Pearson Goodness-of-Fit Test:
## ------------------------------------
##   group statistic p-value(g-1)
## 1    20     37.10   0.00771406
## 2    30     47.19   0.01779131
## 3    40     76.21   0.00033667
## 4    50    103.44   0.00000913
##
##
## Elapsed time : 0.2299399
```

Comparison

```
compare.ICs.ugarchfit(c("k.garch55",
                        "k.ar5egarch21",
                        "k.ar1egarch21",
                        "k.ar1garchm21",
                        "k.ar1garcht21",
                        "k.ar1egarchmt21",
                        "k.ar1egarcht21"))
```

```
## $ICs
##      Akaike     Bayes   Shibata Hannan.Quinn           model
## 1 -5.766879 -5.699068 -5.767282    -5.740885       k.garch55
## 2 -5.775013 -5.707201 -5.775416    -5.749018   k.ar5egarch21
## 3 -5.773066 -5.727859 -5.773247    -5.755737   k.ar1egarch21
## 4 -5.758037 -5.718481 -5.758175    -5.742874   k.ar1garchm21
## 5 -5.933210 -5.893654 -5.933349    -5.918047   k.ar1garcht21
## 6 -5.936283 -5.879774 -5.936564    -5.914621 k.ar1egarchmt21
## 7 -5.937629 -5.892422 -5.937809    -5.920300  k.ar1egarcht21
##
## $which.min
```

```
##       Akaike        Bayes       Shibata Hannan.Quinn
##            7            5            7            7
```

The comparison considers four different criteria (Akaike, Bayes, Shibata and Hannan.Quinn). Two of the criteria (Akaike and Shibata) suggest an AR(1)-EGARCH-t(2,1)-mean model and the other two (Bayes and Hannan.Quinn) suggest a AR(1)-GARCH-t(2,1) model.

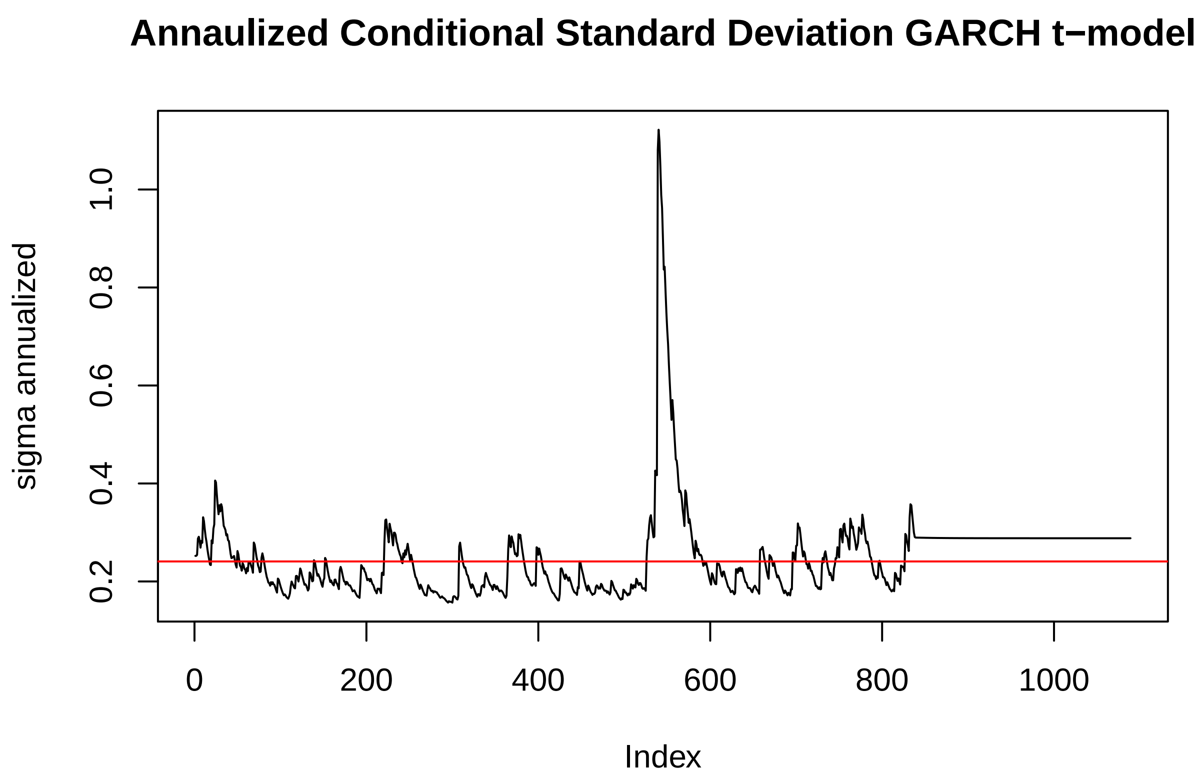
Results

(a) estimates of annualized conditional standard deviation in the in-sample period produced by the two models

Annualized conditional standard deviation AR(1)-GARCH-t(2,1):

```
sigma.forecast.longrun1 <- ugarchforecast(k.ar1garcht21, n.ahead = 252)
unconditional_sigma1 <-
  sqrt(
    k.ar1garcht21@model$pars["omega", 1] /
      (1 -
         k.ar1garcht21@model$pars["alpha1", 1] -
         k.ar1garcht21@model$pars["beta1", 1]))

plot(
  c(as.numeric(k.ar1garcht21@fit$sigma * sqrt(252)),
    as.numeric(sigma.forecast.longrun1@forecast$sigmaFor * sqrt(252))),
  main = "Annaulized Conditional Standard Deviation GARCH t-model",
  type = "l",
  ylab = "sigma annualized")
abline(h = unconditional_sigma1 * sqrt(252), col = "red")
```



```
max(k.ar1garcht21@fit$sigma) * sqrt(252)*100
```

```
## [1] 112.2047
```

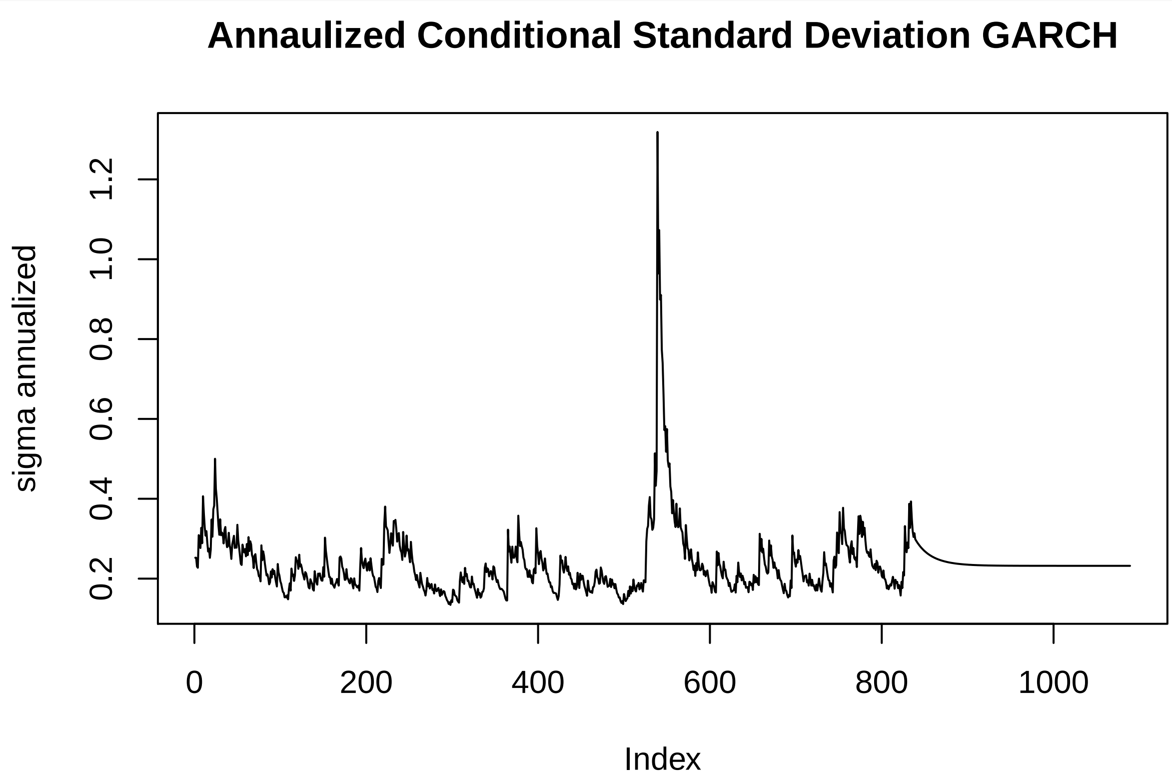
```
unconditional_sigma1 * sqrt(252)*100
```

```
## [1] 24.08314
```

In the plot we see the annualized conditional standard deviation of AR(1)-GARCH-t(2,1). The conditional sigma should converge to to the long run estimate of sigma, which is not the case. This is in contrast to the conventional literature. The deviation could have many different reasons, finding these would exceed the limit of this paper. The maximum value of estimated conditional standard deviation (sigma) is 112.2% and Long-term unconditional level is 24.08% annualized.

Annualized conditional standard deviation AR(1)-Garch-t(2,1)

```
sigma.forecast.longrun2 <- ugarchforecast(k.ar1egarcht21, n.ahead = 252)
unconditional_sigma2 <-
  sqrt(
    k.ar1egarcht21@model$pars["omega", 1]/
      (1 -
         k.ar1egarcht21@model$pars["alpha1", 1] -
         k.ar1egarcht21@model$pars["beta1", 1]))
plot(
  c(as.numeric(k.ar1egarcht21@fit$sigma * sqrt(252)),
    as.numeric(sigma.forecast.longrun2@forecast$sigmaFor * sqrt(252))),
  main = "Annaulized Conditional Standard Deviation GARCH",
  type = "l",
  ylab = "sigma annualized")
abline(h = unconditional_sigma2 * sqrt(252), col = "red")
```



```r
#omega is negative!

max(k.ar1egarcht21@fit$sigma) * sqrt(252)*100
```

```
## [1] 131.8686
```

```r
unconditional_sigma2 * sqrt(252)*100
```

```
## [1] NaN
```

In the plot we see the annualized conditional standard deviation of AR(1)-EGARCH-t(2,1). The conditional sigma cannot be estimated because the omega is negative an thus the square-root of the formular for the unconditional is not applicable. A reason for that could be the AR(1) in the model, hence I will try it without.

AR(0)-EGARCH-t(2,1)

```r
spec <-
  ugarchspec(variance.model = list(model = "eGARCH", garchOrder = c(2, 1)),
             mean.model = list(armaOrder = c(0, 0), include.mean = F),
             distribution.model = "std")


k.ar0egarcht21 <- ugarchfit(spec = spec,
                            data = quotes$quotes.r)

k.ar0egarcht21
```

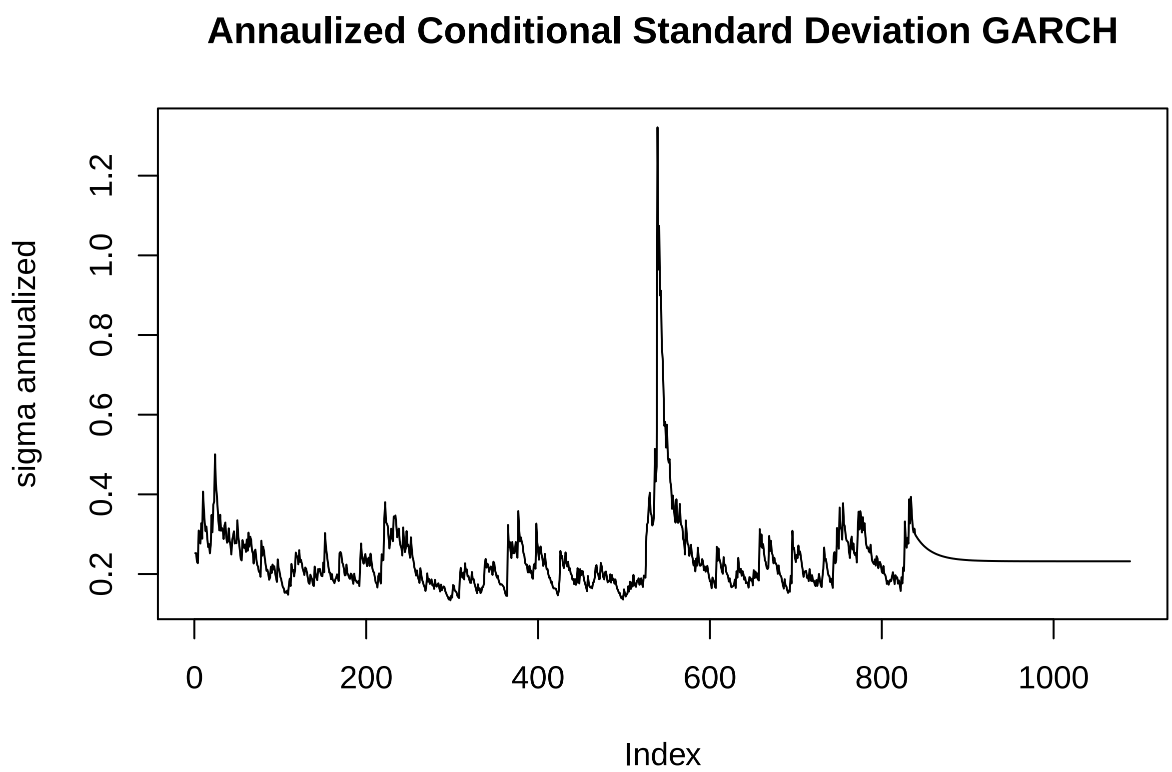
```
##
## *---------------------------------*
## *          GARCH Model Fit        *
## *---------------------------------*
##
## Conditional Variance Dynamics
## -----------------------------------
## GARCH Model  : eGARCH(2,1)
## Mean Model   : ARFIMA(0,0,0)
## Distribution : std
##
## Optimal Parameters
## ------------------------------------
##         Estimate  Std. Error   t value Pr(>|t|)
## omega  -0.407756    0.061653  -6.61367 0.000000
## alpha1 -0.180124    0.071055  -2.53498 0.011245
## alpha2  0.100662    0.069636   1.44553 0.148308
## beta1   0.951752    0.007063 134.75288 0.000000
## gamma1  0.147289    0.095909   1.53571 0.124611
## gamma2  0.058249    0.091063   0.63966 0.522395
## shape   3.270761    0.414692   7.88720 0.000000
##
## Robust Standard Errors:
##         Estimate  Std. Error   t value Pr(>|t|)
## omega  -0.407756    0.027880 -14.62565 0.000000
## alpha1 -0.180124    0.071874  -2.50609 0.012207
## alpha2  0.100662    0.065241   1.54293 0.122847
## beta1   0.951752    0.003089 308.13663 0.000000
## gamma1  0.147289    0.101500   1.45112 0.146747
```

```
## gamma2  0.058249    0.098776   0.58971 0.555385
## shape   3.270761    0.352813   9.27051 0.000000
##
## LogLikelihood : 2492.896
##
## Information Criteria
## ------------------------------------
##
## Akaike       -5.9400
## Bayes        -5.9005
## Shibata      -5.9402
## Hannan-Quinn -5.9249
##
## Weighted Ljung-Box Test on Standardized Residuals
## ------------------------------------
##                         statistic p-value
## Lag[1]                     0.6484 0.42069
## Lag[2*(p+q)+(p+q)-1][2]    3.5873 0.09737
## Lag[4*(p+q)+(p+q)-1][5]    6.4165 0.07166
## d.o.f=0
## H0 : No serial correlation
##
## Weighted Ljung-Box Test on Standardized Squared Residuals
## ------------------------------------
##                          statistic p-value
## Lag[1]                      0.2148  0.6431
## Lag[2*(p+q)+(p+q)-1][8]     3.0614  0.6742
## Lag[4*(p+q)+(p+q)-1][14]    4.5286  0.8253
## d.o.f=3
##
## Weighted ARCH LM Tests
## ------------------------------------
##             Statistic Shape Scale P-Value
## ARCH Lag[4]    0.2782 0.500 2.000  0.5979
## ARCH Lag[6]    0.6120 1.461 1.711  0.8602
## ARCH Lag[8]    0.8058 2.368 1.583  0.9507
##
## Nyblom stability test
## ------------------------------------
## Joint Statistic:  1.7359
## Individual Statistics:
## omega  0.27043
## alpha1 0.20684
## alpha2 0.07269
## beta1  0.26833
## gamma1 0.06374
## gamma2 0.22816
## shape  0.16371
##
## Asymptotic Critical Values (10% 5% 1%)
## Joint Statistic:          1.69 1.9 2.35
## Individual Statistic:     0.35 0.47 0.75
##
## Sign Bias Test
```

```
## -----------------------------------
##                     t-value   prob sig
## Sign Bias           0.03532 0.9718
## Negative Sign Bias  0.79354 0.4277
## Positive Sign Bias  0.10950 0.9128
## Joint Effect        0.80849 0.8474
##
##
## Adjusted Pearson Goodness-of-Fit Test:
## -----------------------------------
##   group statistic p-value(g-1)
## 1    20     63.91    9.177e-07
## 2    30     84.40    2.584e-07
## 3    40    118.84    5.325e-10
## 4    50    172.26    1.276e-15
##
##
## Elapsed time : 0.1612699
```

Annualized conditional standard deviation AR(0)-EGarch-t(2,1)

```
sigma.forecast.longrun3 <- ugarchforecast(k.ar0egarcht21, n.ahead = 252)
unconditional_sigma3 <-
  sqrt(
    k.ar0egarcht21@model$pars["omega", 1]/
      (1 -
         k.ar0egarcht21@model$pars["alpha1", 1] -
         k.ar0egarcht21@model$pars["beta1", 1]))
plot(
  c(as.numeric(k.ar0egarcht21@fit$sigma * sqrt(252)),
    as.numeric(sigma.forecast.longrun3@forecast$sigmaFor * sqrt(252))),
  main = "Annaulized Conditional Standard Deviation GARCH",
  type = "l",
  ylab = "sigma annualized")
abline(h = unconditional_sigma3 * sqrt(252), col = "red")
```




```
#omega is negative!

max(k.ar0egarcht21@fit$sigma) * sqrt(252)*100
```

```
## [1] 132.1283
```

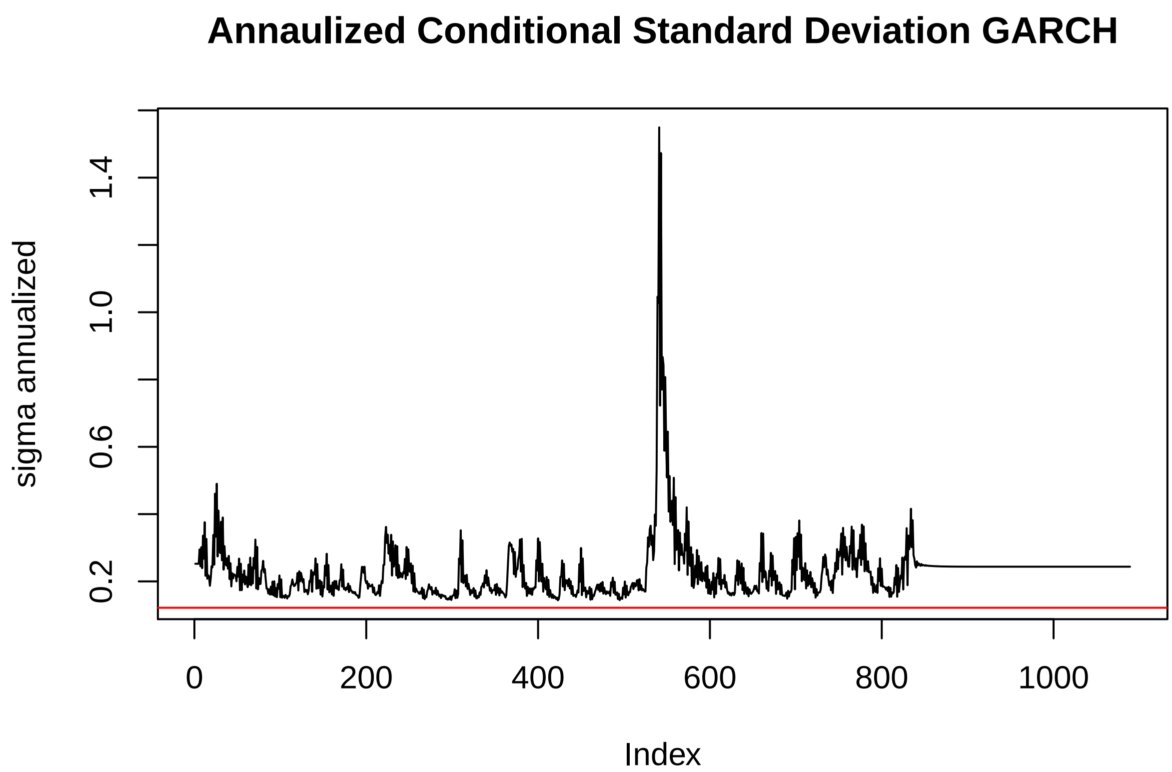
```
unconditional_sigma3 * sqrt(252)*100
```

```
## [1] NaN
```

Even without AR() the result of unconditional sigma is not applicable. Thus, I decided to choose a different model for the following estimations.

Annualized conditional standard deviation Garch(5,5)

```
sigma.forecast.longrun4 <- ugarchforecast(k.garch55, n.ahead = 252)
unconditional_sigma4 <-
  sqrt(
    k.garch55@model$pars["omega", 1]/
      (1 -
         k.garch55@model$pars["alpha1", 1] -
         k.garch55@model$pars["beta1", 1]))
plot(
  c(as.numeric(k.garch55@fit$sigma * sqrt(252)),
    as.numeric(sigma.forecast.longrun4@forecast$sigmaFor * sqrt(252))),
  main = "Annaulized Conditional Standard Deviation GARCH",
  type = "l",
  ylab = "sigma annualized")
abline(h = unconditional_sigma4 * sqrt(252), col = "red")
```




```
max(k.garch55@fit$sigma) * sqrt(252)*100
```

```
## [1] 154.9474
```

```
unconditional_sigma4 * sqrt(252)*100
```

```
## [1] 12.16885
```

In this case the unconditional sigma is applicable. However, it still does not converge to its long run estimation. The maximum value of estimated conditional standard deviation (sigma) is 154.95% and Long-term unconditional level is 12.17% annualized.

Estimation Garch-Models with out-sample data

(b) estimates of the Value-at-Risk produced by the two models in the out-of-sample period

Choice of the out-of-the sample data.

```
k <- sum(x)%%6+1
k
```

```
## [1] 6
```

The result is k=6 hence, the out-of-sample date starts 2020-06-01.

```
quotes2 <- quotes["2020-05-30/2021-06-01"]
quotes2$quotes.r[is.na(quotes$quotes.r)] <- 0
```

AR(1)-Garch-t(2,1)

```
spec <- ugarchspec(# variance equation
  variance.model = list(model = "sGARCH",
```

```r
                        garchOrder = c(2, 1)),
  mean.model = list(armaOrder = c(1, 0),
                    include.mean = TRUE),
  distribution.model = "std")

ar1garcht21 <- ugarchfit(spec = spec,
                         data = quotes2$quotes.r)

ar1garcht21
```

```
##
## *---------------------------------*
## *          GARCH Model Fit        *
## *---------------------------------*
##
## Conditional Variance Dynamics
## -----------------------------------
## GARCH Model  : sGARCH(2,1)
## Mean Model   : ARFIMA(1,0,0)
## Distribution : std
##
## Optimal Parameters
## ------------------------------------
##         Estimate  Std. Error  t value Pr(>|t|)
## mu      0.001881    0.000624  3.01640 0.002558
## ar1    -0.101344    0.050498 -2.00687 0.044764
## omega   0.000028    0.000029  0.99712 0.318706
## alpha1  0.000000    0.126433  0.00000 1.000000
## alpha2  0.149519    0.136789  1.09306 0.274366
## beta1   0.758030    0.176555  4.29344 0.000018
## shape   3.266229    0.839680  3.88985 0.000100
##
## Robust Standard Errors:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu      0.001881    0.000609  3.08897 0.002009
## ar1    -0.101344    0.051710 -1.95984 0.050014
## omega   0.000028    0.000025  1.12852 0.259100
## alpha1  0.000000    0.210389  0.00000 1.000000
## alpha2  0.149519    0.174477  0.85695 0.391470
## beta1   0.758030    0.145422  5.21264 0.000000
## shape   3.266229    0.750276  4.35337 0.000013
##
## LogLikelihood : 728.1499
##
## Information Criteria
## ------------------------------------
##
## Akaike       -5.8393
## Bayes        -5.7398
## Shibata      -5.8408
## Hannan-Quinn -5.7992
##
## Weighted Ljung-Box Test on Standardized Residuals
## ------------------------------------
```

```
##                          statistic p-value
## Lag[1]                      0.1728  0.6776
## Lag[2*(p+q)+(p+q)-1][2]     0.9455  0.7722
## Lag[4*(p+q)+(p+q)-1][5]     1.7868  0.7717
## d.o.f=1
## H0 : No serial correlation
##
## Weighted Ljung-Box Test on Standardized Squared Residuals
## ------------------------------------
##                          statistic p-value
## Lag[1]                      0.7262  0.3941
## Lag[2*(p+q)+(p+q)-1][8]     1.6552  0.9080
## Lag[4*(p+q)+(p+q)-1][14]    2.8640  0.9626
## d.o.f=3
##
## Weighted ARCH LM Tests
## ------------------------------------
##             Statistic Shape Scale P-Value
## ARCH Lag[4]    0.1224 0.500 2.000  0.7265
## ARCH Lag[6]    0.8196 1.461 1.711  0.8000
## ARCH Lag[8]    0.9726 2.368 1.583  0.9283
##
## Nyblom stability test
## ------------------------------------
## Joint Statistic:  1.2264
## Individual Statistics:
## mu     0.1008
## ar1    0.1153
## omega  0.3519
## alpha1 0.6324
## alpha2 0.3539
## beta1  0.4439
## shape  0.3998
##
## Asymptotic Critical Values (10% 5% 1%)
## Joint Statistic:          1.69 1.9 2.35
## Individual Statistic:     0.35 0.47 0.75
##
## Sign Bias Test
## ------------------------------------
##                    t-value   prob sig
## Sign Bias           0.1256 0.9002
## Negative Sign Bias  0.1657 0.8686
## Positive Sign Bias  0.6758 0.4998
## Joint Effect        0.8926 0.8272
##
##
## Adjusted Pearson Goodness-of-Fit Test:
## ------------------------------------
##   group statistic p-value(g-1)
## 1    20     15.91       0.6629
## 2    30     21.79       0.8290
## 3    40     30.09       0.8465
## 4    50     51.58       0.3731
```

```
##
##
## Elapsed time : 0.134002
```

Garch(55)

```r
spec <- ugarchspec(
  variance.model = list(model = "sGARCH",
                        garchOrder = c(5, 5)),
  mean.model = list(armaOrder = c(0, 0),
                    include.mean = TRUE),
  distribution.model = "norm")


garch55 <- ugarchfit(spec = spec,
                     data = quotes2$quotes.r)

garch55
```

```
##
## *---------------------------------*
## *          GARCH Model Fit        *
## *---------------------------------*
##
## Conditional Variance Dynamics
## -----------------------------------
## GARCH Model  : sGARCH(5,5)
## Mean Model   : ARFIMA(0,0,0)
## Distribution : norm
##
## Optimal Parameters
## ------------------------------------
##         Estimate  Std. Error  t value Pr(>|t|)
## mu      0.001734    0.000848 2.044691 0.040885
## omega   0.000077    0.000015 5.274909 0.000000
## alpha1  0.000000    0.111203 0.000000 1.000000
## alpha2  0.080861    0.069830 1.157972 0.246875
## alpha3  0.046525    0.091702 0.507349 0.611910
## alpha4  0.047289    0.028157 1.679449 0.093065
## alpha5  0.092642    0.056711 1.633580 0.102347
## beta1   0.000001    0.897305 0.000001 0.999999
## beta2   0.000000    0.646002 0.000000 1.000000
## beta3   0.332723    0.878650 0.378675 0.704929
## beta4   0.000000    0.679187 0.000000 1.000000
## beta5   0.000000    1.024117 0.000000 1.000000
##
## Robust Standard Errors:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu      0.001734    0.001037 1.672603 0.094405
## omega   0.000077    0.000238 0.324856 0.745290
## alpha1  0.000000    0.806142 0.000000 1.000000
## alpha2  0.080861    0.105426 0.766993 0.443086
## alpha3  0.046525    0.725110 0.064162 0.948841
## alpha4  0.047289    0.246848 0.191571 0.848078
```

```
## alpha5  0.092642    0.576168 0.160789 0.872259
## beta1   0.000001    5.523470 0.000000 1.000000
## beta2   0.000000    1.626800 0.000000 1.000000
## beta3   0.332723    5.222893 0.063705 0.949205
## beta4   0.000000    3.875267 0.000000 1.000000
## beta5   0.000000    6.179852 0.000000 1.000000
##
## LogLikelihood : 714.0781
##
## Information Criteria
## ------------------------------------
##
## Akaike       -5.6848
## Bayes        -5.5143
## Shibata      -5.6893
## Hannan-Quinn -5.6162
##
## Weighted Ljung-Box Test on Standardized Residuals
## ------------------------------------
##                         statistic p-value
## Lag[1]                      2.087  0.1485
## Lag[2*(p+q)+(p+q)-1][2]     3.048  0.1362
## Lag[4*(p+q)+(p+q)-1][5]     3.988  0.2556
## d.o.f=0
## H0 : No serial correlation
##
## Weighted Ljung-Box Test on Standardized Squared Residuals
## ------------------------------------
##                          statistic p-value
## Lag[1]                      0.7803  0.3771
## Lag[2*(p+q)+(p+q)-1][29]    5.7917  0.9957
## Lag[4*(p+q)+(p+q)-1][49]   13.2518  0.9916
## d.o.f=10
##
## Weighted ARCH LM Tests
## ------------------------------------
##              Statistic Shape Scale P-Value
## ARCH Lag[11]     1.466 0.500 2.000  0.2260
## ARCH Lag[13]     1.912 1.493 1.855  0.5571
## ARCH Lag[15]     2.052 2.471 1.754  0.7942
##
## Nyblom stability test
## ------------------------------------
## Joint Statistic:  3.4737
## Individual Statistics:
## mu     0.04875
## omega  0.12306
## alpha1 0.60394
## alpha2 0.09362
## alpha3 0.16081
## alpha4 0.28011
## alpha5 0.26688
## beta1  0.18232
## beta2  0.22830
```

```
## beta3  0.15760
## beta4  0.21610
## beta5  0.27874
##
## Asymptotic Critical Values (10% 5% 1%)
## Joint Statistic:          2.69 2.96 3.51
## Individual Statistic:     0.35 0.47 0.75
##
## Sign Bias Test
## ------------------------------------
##                    t-value   prob sig
## Sign Bias           0.2258 0.8215
## Negative Sign Bias  0.1815 0.8561
## Positive Sign Bias  0.7807 0.4357
## Joint Effect        0.8437 0.8390
##
##
## Adjusted Pearson Goodness-of-Fit Test:
## ------------------------------------
##   group statistic p-value(g-1)
## 1    20     34.70    0.0151728
## 2    30     59.68    0.0006768
## 3    40     78.34    0.0001891
## 4    50     89.64    0.0003533
##
##
## Elapsed time : 0.234272
```

Standardization of returns.

```
quotes2$rstd <- (quotes2$quotes.r - mean(quotes2$quotes.r, na.rm = T)) /
  sd(quotes2$quotes.r ,na.rm = T)
```

1% empirical quantile

```
q01 <- quantile(quotes2$rstd, 0.01, na.rm = T)
q01
```

```
##        1%
## -3.105542
```

Calculation the value-at-risk (VaR)

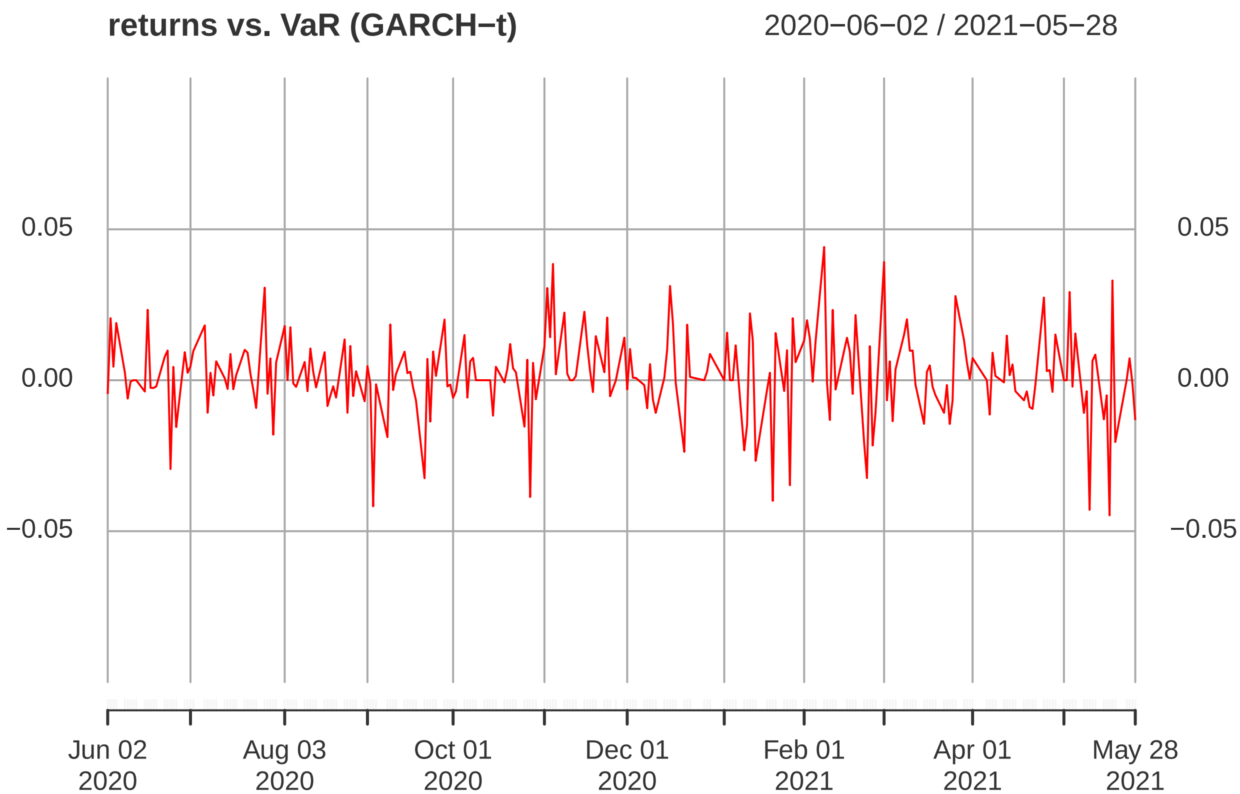
```
quotes2$VaR1 <- q01 * ar1garcht21@fit$sigma

quotes2$VaR2 <- q01 * garch55@fit$sigma
```

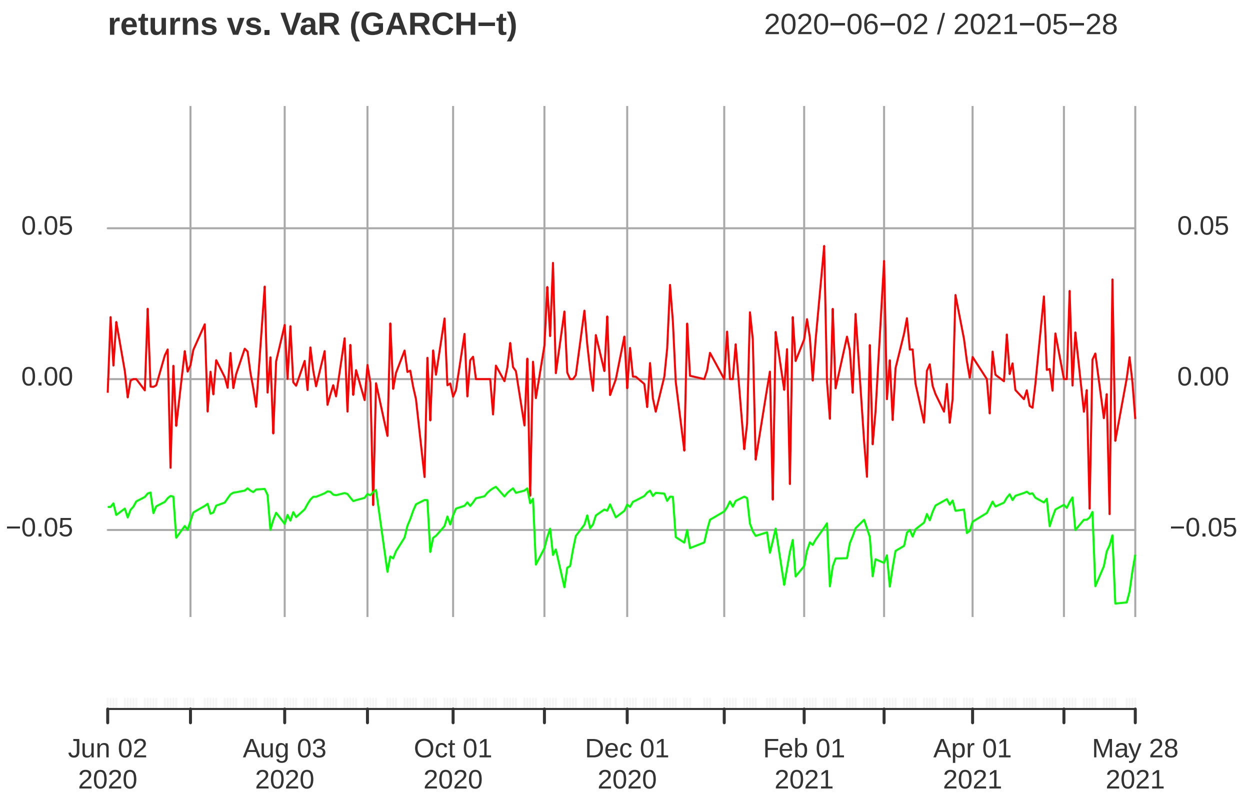
Plot of returns vs VaR

```
plot(quotes2$quotes.r,
     main = "returns vs. VaR (GARCH-t)",
     col = "red", lwd = 1, type = 'l',
```

```r
     ylim = c(-0.1, 0.1))
abline(h = 0, lty = 2)
```

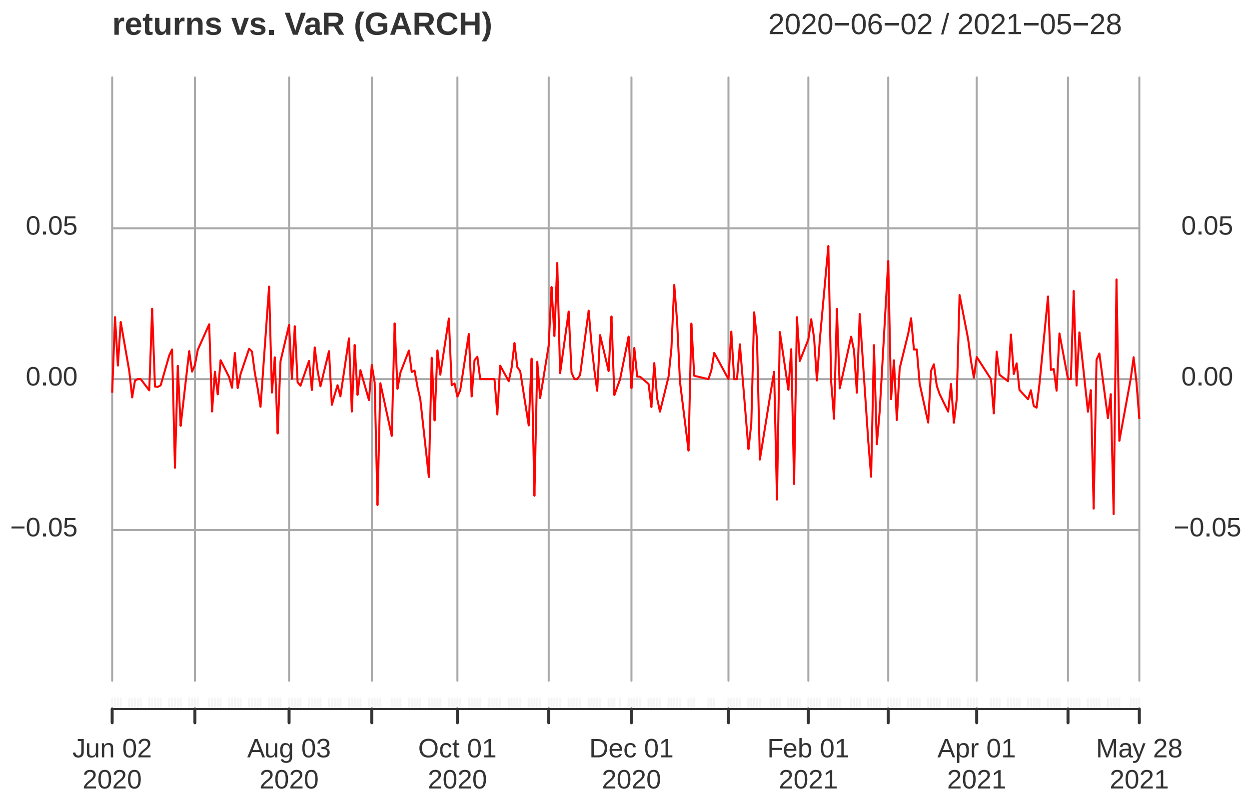



```r
lines(quotes2$VaR1, type = 'l', col = "green")
```

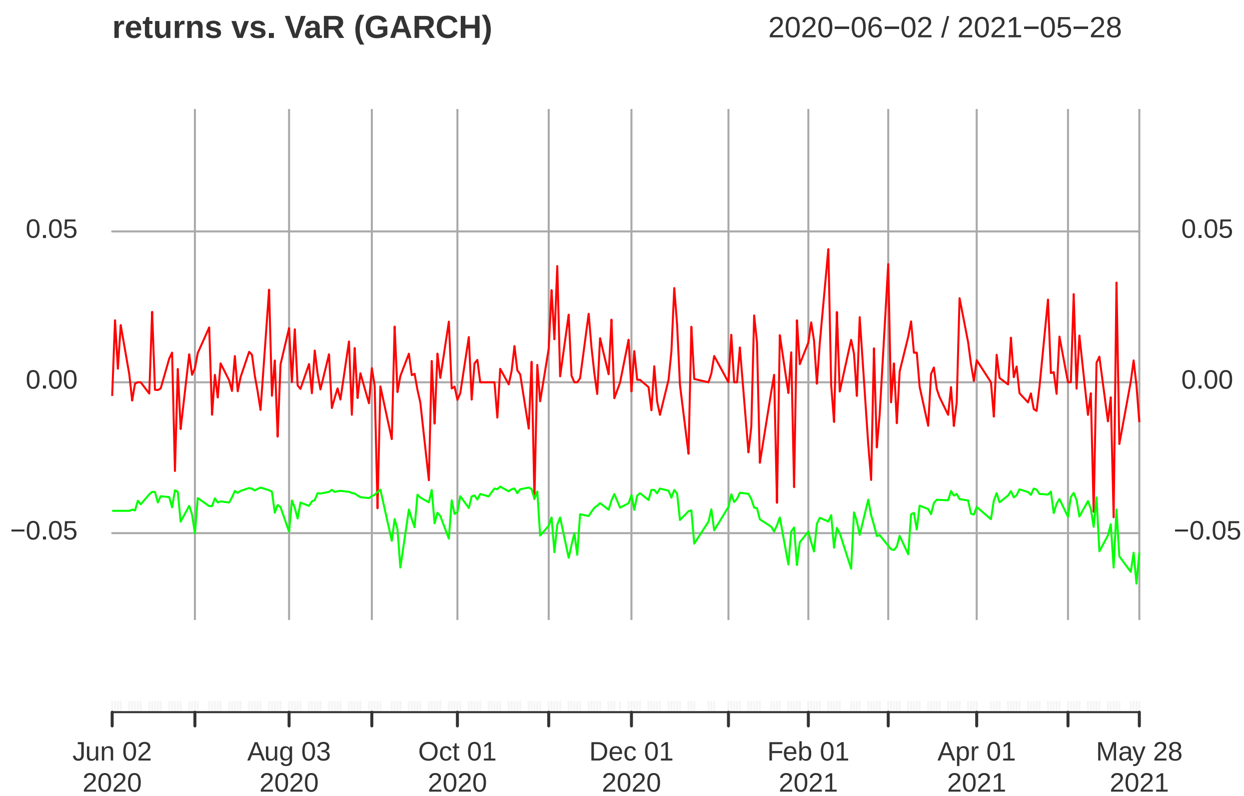


In the plot of returns and VaR of AR(1)-GARCH-t(2,1) has been crossed 2 times.

```
plot(quotes2$quotes.r,
     main = "returns vs. VaR (GARCH)",
     col = "red", lwd = 1, type = 'l',
     ylim = c(-0.1, 0.1))
abline(h = 0, lty = 2)
```




```
lines(quotes2$VaR2, type = 'l', col = "green")
```



In the plot of returns and VaR of GARCH(5,5) has been crossed 3 times.

In how many days losses were higher the assumed VaR?

```
sum(quotes2$quotes.r < quotes2$VaR1) / length(quotes2$VaR1)
```

```
## [1] 0.004048583
```

The number of days were losses are higher than assumed VaR for (AR(1)-GARCH-t(2,1)) is 0.004.

```
sum(quotes2$quotes.r < quotes2$VaR2) / length(quotes2$VaR2)
```

```
## [1] 0.008097166
```

The number of days were losses are higher than assumed VaR for GARCH(5,5) is 0.008.