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Theory Question 1

The axioms of inner product are:

$$\begin{aligned}
\textbf{Non-negativity:}\ & f(\vec{x},\vec{x}) \geq 0 \text{ if } 0,\ \vec{x} \text{ must be } \vec{0} \\
\textbf{Linearity:}\ & f(\vec{x}+\vec{y},\vec{z}) = f(\vec{x},\vec{z}) + f(\vec{y},\vec{z}) \\
\textbf{Scalar multiple:}\ & f(\alpha\vec{x},\vec{y}) = \alpha \cdot f(\vec{x},\vec{y}) \\
\textbf{Symmetry:}\ & f(\vec{x},\vec{y}) = f(\vec{y},\vec{x})
\end{aligned}$$

Non-negativity

This axiom is not satisfied by δ. Let $\omega_i = \vec{0}$. Then $\delta(\omega_i, \omega_i) = \frac{0}{0}$, which is undefined.

Linearity

This axiom is not satisfied by δ. Let $\|\omega_i\|_2 \neq 1$ or $\|\omega_j\|_2 \neq 1$, for example let $\omega_i = [1, 2], \omega_j = [1, 0], \omega_k = [0, 1]$. Then $\delta(\omega_i + \omega_j, \omega_k) \approx 0.7$ and $\delta(\omega_i, \omega_k) + \delta(\omega_j, \omega_k) \approx 0.9$.

Scalar multiple

This axiom is not satisfied by δ, since it normalizes vectors. For example, let $\alpha = 2$ and $\omega_i = [1, 0]$. Then $\delta(\alpha\omega_i, \omega_i) = 1$ and $\alpha \cdot \delta(\omega_i, \omega_i) = 2$.