Patrick Pegus
Mini Project 2
November 5, 2015
CMPSCI-689
Prof. Sridhar Mahadevan

1. (a) The state variables at time t are marginally independent because the observation at that time d-separates them. In other words, $P(S(t,1)|S(t,2)) = P(S(t,1))$ because $Y(t)$ is a collider node in their path. However, $P(S(t,1)|S(t,2)) \neq P(S(t,1))$ when $Y(t)$ then d-connects them. The state variables at time t are conditionally independent of the past history of state variables given the state variables at $t-1$ because those given variables d-separate them from past states.

   (b) To convert the factorial HMM to a regular HMM, collapse states $S(t,1), \ldots S(t,M)$ to a single state $S(t)$ that has K^M values, which is enough to represent all possible state combinations of the former states. Since the time complexity of the forward algorithm on an HMM is $O(L^2T)$ where L is the number of state values, the complexit of the converted HMM is $O\left(\left(K^M\right)^2 T\right) = O(K^{2M}T)$.

2. (a) **Derive Lagrangian**

$$L(w,\alpha) = \sum_{i=1}^{l} \left(y_i - \langle w, x_i \rangle\right)^2 - \alpha \|w\|^2$$

Lagrange dual

$$\max_{\alpha:\alpha\geq 0} L_D(\alpha) = \min_{w} L(w,\alpha)$$