Scheduling Mobiles with Multiple Antennas to Share a Slot in WiMAX Networks

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Abstract

The concept of sharing a slot across multiple users attained high attention these days: due to the availability of advanced signal processing techniques like Interference Alignment (IA) and Interference Nulling (IN); and multiple antennas at each user. As countably finite $(K/2)$ number of user pairs can share a slot in a K user decentralized channel, the same is not possible in a centralized network. While a protocol (based on 802.11n) has already been developed to optimally utilize the existing techniques in a decentralized network, no such work has been initiated on a centralized network. Surprisingly, the number of users sharing a slot in centralized network is always taken to be 2 (using Collaborative Spatial Multiplexing technique). This paper aims to study the maximum number of MSs that can share a slot in a centralized network based on state-of-the-art interference avoidance and reduction, and multiple antenna techniques.

Further, considering each MS to be moving with speed between $0 - 120$ Kmph, we investigate if those many number of MSs can be scheduled in a typical WiMAX network. In addition, we propose a scheduling algorithm that utilizes the properties of IA technique. Evaluating the proposed algorithm with simulations proves that performance of the network increases compared to other existing pairing techniques.

Keywords: MIMO, WiMAX, Signal to Interference Ratio

I. INTRODUCTION

Traditional wireless communication protocols eliminate interference due to multiple transmitters by allocating different frequencies and/or times for each transmission (Frequency or Time

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Division Multiple Access). With the advent of advanced antenna technologies, it is possible for several Mobile Stations (MSs) to share a slot. This concept -when used with Orthogonal Frequency Division Multiple Access (OFDMA) technology at the physical layer-is named as Collaborative Spatial Multiplexing in IEEE 802.16 networks [1]. And supports 2 MSs to share a slot for the uplink transmissions. The BS decides the MSs that can share a slot for the next time frame (of 5 ms duration), depending on the requests received from each MS, and the observed channel quality of each MS during the previous frame. This scheduling of MSs at the Base Station (BS) is termed as Pairing, considers only one antenna at each MS. However, the sudden emergence of smart-phones with multiple (2) antennas at each MS can toggle the performance of above protocols, due to increase in the values of interference in the network. Hence, the problem -to determine the maximum number of MSs that can share a same slot in a scenario where each MS: has mobility speeds upto 120 Kmph, uses 2 antenna for transmission all the time- becomes interesting.

The dependency of this problem on the Multiple Input Multiple Output (MIMO) technique applied at each MS (which is decided by the BS) is an important parameter to maximize performance of the network. Two contrasting MIMO techniques are Spatial Multiplexing (SM) and Spatial Diversity (SD). SM techniques improve the number of streams of data transmitted ($\leq$ min(N_t, N_r)) between a transmitter with N_t, and receiver with N_r number of antennas. Diversity techniques improve the reliability of the transmitted data, either by transmitting or receiving variants of the same data across several antennas. Few techniques that utilize the SM and SD techniques to data of different streams optimally in the same slot are also proposed in literature [2]. However, in this paper we consider only SM and SD techniques for evaluation, as other techniques can be generated by applying mentioned techniques repeatedly.

A. *Motivation*

Interference Alignment (IA) is a technique in which several interfering user pairs with single antenna transmit and receive data successfully, by aligning their transmissions along predetermined vectors [3]. It is proved that all user pairs (K) in a decentralized network can transmit data for half of the time in the network (independent of K). However, this technique can not be directly applied to a Point-to-Multipoint (PMP) network, due to the requirement of enormous computing power to generate IA vectors at the Base Station (centralized location).

While algorithms to generate IA vectors for each user-pair in a distributed network are available for different channels [4], these algorithms are not applicable in a Point-to-Multipoint (PMP) network, as all the receiving antennas are located at same location. A sample PMP network with BS with N_r antennas, and each MS equipped with N_t antennas can look similar to that of Fig. 1. IA technique, when complemented with Interference Nulling technique is utilized efficiently in [5] to share a slot across multiple MSs in an IEEE $802.11n$ network. The authors show with testbed results that an user-pair with more number of antennas than the current number of spatial streams transmitted in the network, can share the same slot for transmission. However, such a study is unavailable in a PMP network.

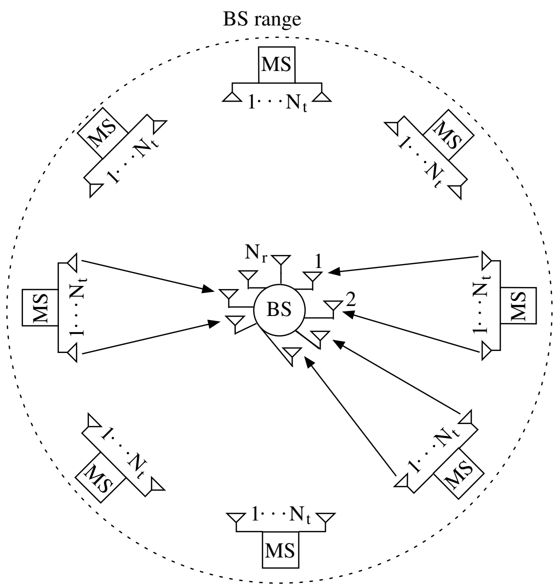

Fig. 1. Interference Alignment Technique as Applicable in a PMP Network.

Peters and Heath Jr. [6] study multiple antenna interference channels and propose a greedy algorithm to partition available users into several groups such that users in a group use IA to transmit data in a single slot, while users across groups use TDMA mechanism to transmit data across different slots. They model users pairs in the network as a connected graph, and partition the graph based on the position information of each transmitter and receiver in the network and provide fairness on rate requirements achieved among users. They model overhead of using IA in the network, and show that the achieved sum rate will become zero as more number of users share a slot.

While many studies [7], [8], [9], [10] propose algorithms to allocate a slot to several MSs (pairing schemes) such that the frame is utilized optimally, they do not question why only two MSs are shared in a WiMAX network. Also, these techniques assume that each MS is equipped with only one antenna.

B. Contributions

The main contributions of the paper are as follows:

1) We find the maximum number of MSs that can share an OFDM slot, when each MS is equipped with 2 antennas and uses a known MIMO technique for transmissions. We use SIR as a parameter to verify if a constant rate can be attained at each MS.
2) We propose an algorithm to schedule MSs in a typical WiMAX network, wherein each OFDM slot is optimally utilized by allocating to maximum number of MSs possible. Challenges and solutions to determine IA vectors with in the limited time are also discussed in detail.

Overall, we propose a working design to use IA technology in WiMAX networks. To the best of authors' knowledge, this is the first study towards utilizing IA technique in WiMAX networks. Relative to our previous work [11], in this paper, we analytically prove the maximum number of MSs sharing a slot as well as propose an algorithm to schedule MSs sharing a slot in a WiMAX network.

C. Organization

Section II provides the relevant background required to analyze the problem considered in this paper. Section III gives the system model considered, while Section IV studies about finding the maximum number of mobiles that can share a slot in a PMP network. In Section V, we study the challenges to utilize IA vectors in a typical wireless network. In Section VI, we propose a scheduling algorithm that schedules MSs based on the channel quality and MIMO technique of each MS participating in the share. In Section VII, we conclude the work and provide directions for possible works in the future.

II. Background

Interference: The chief factors contributing to the interference of a Point-to-Multipoint network can be classified in two ways:

a) Interference due to mobility of MS: The most common problems of frequency, time synchronization, Carrier and Doppler frequency offsets can be described in form of Inter Carrier Interference (ICI). Since OFDMA is used, Inter Symbol Interference (ISI) and interference due to signals taking multiple paths to reach the destination are almost eliminated by choosing the guard time appropriately. ICI can be eliminated in the network by transmitting same symbol redundantly along a set of adjacent subcarriers [12], [13], [14]. The disadvantage of these techniques is that the subcarriers are wasted. Conclude:

b) Interference due to multiple antennas: Several antennas using same slot for transmission will cause interference at the receiving antenna. Choosing or Scheduling different MSs such that interference can be mitigated at the receiving antennas is the responsibility of the BS. Some of the ways of reducing interference at the receiving antenna are beamforming, Interference Nulling (IN), and IA. Bemforming eliminates interference by transmitting data along a particular vector depending on the channel quality between the transmitting and receving antennas. IN enables a second transmitter to transmit data using the same slot without interfering with the ongoing transmission. Generating IN vectors is dependent only on channel quality between second transmitter and first receiver.

Interference Alignment: This technique is first brought into light through a seminal paper by Viveck et.al., [3], where they prove that all the K user pairs in a decentralized channel can transmit and receive data correctly for half of the time. Interestingly, it considers each user to be equipped with only one antenna. Data is transmitted at each antenna along a pre-determined vector. Antenna at the receiver is configured such that it receives data along only one vector (dependent on transmitting IA vector and channel coefficient between transmitting and receiving antennas). Generating IA vectors is dependent on the channel quality between **every** transmitter and the receiver. Ex: Two antennas of MS 'T' transmit data along two vectors $\overrightarrow{w_1}$ and $\overrightarrow{w_2}$, as shown in Fig. 2. Appropriate antennas receive data along vectors $\overrightarrow{w_1^H}$ and $\overrightarrow{w_2^H}$, where w_1^H, w_2^H are designed such that $w_1 \times w_1^H = 1$ and $w_1 \times w_2^H = 0$. Designing these IA vectors for different channels [4], [15], [16] has been a thoroughly studied topic. The streams received at antennas

$2T, 2T+1$ as shown in Fig. 2 can be taken as:

$$\begin{aligned} y_{2T} &= w_1 x_1 w_1^H + w_2 x_2 w_1^H \\ y_{2T+1} &= w_1 x_1 w_2^H + w_2 x_2 w_2^H \end{aligned} \tag{1}$$

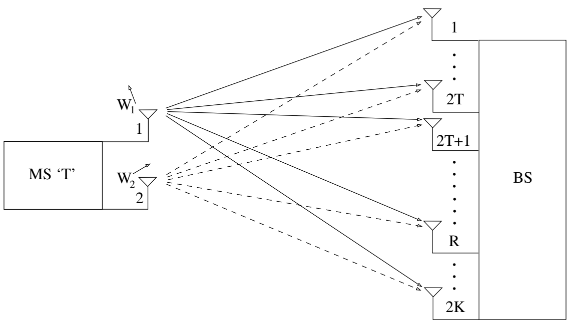

Fig. 2. System Model.

Signal to Interference Ratio: For a received signal to be interference free, the value of SIR should be above a threshold β, where β is dependent on the bit transmission rate. The SIR for a signal can be defined [17] as

$$SIR = \frac{S}{I} \tag{2}$$

where S is the power of the required signal at the receiver and I is the power of interfered signal at the receiver. β can be taken as $2^R - 1$, where R is the required rate of transmission in bps/Hz [18].

III. System Model

Let 'K' MSs share the same OFDM slot for transmitting to the BS. Each MS is equipped with 2 antennas and the BS is equipped with $M(\geq 2K)$ antennas. An antenna at each MS transmits the data that is aligned along a vector, beamed to a specific antenna at the BS. In other words, alignment vectors specific to each antenna of the MS are transmitted by the BS for each frame. The overhead of transmitting these beam vectors in each frame is presumed to be negligible when compared to the achievable data rates. The data received at antenna i of the BS, similar to Equation 6 in [19], can be written as

$$y_i(s) = \sum_{j=1}^{2K} d_{ij}^{-\rho} W_j H_{ij} x_j(s) W_i^H + n_{ij} \tag{3}$$

where W_j is the beam-formed (weighted) vector along which data is transmitted,
W_i^H is the inverse of the beam-formed matrix used at the BS,
$x_j(s)$ is the data transmitted across antenna j on subcarrier s,
H_{ij} is the channel coefficient between the transmitted antenna j and received antenna i,
n_{ij} is the Additive White Gaussian Noise associated with the channel,
d_{ij} is the distance between antenna i and antenna j,
and ρ is the path loss exponent of the channel. The ICI in a channel using OFDM is mainly due to adjacent subcarriers [19]. Also, assuming the channel to be frequency flat fading, Equation 3 can be modified as follows:

$$\begin{aligned} y_i(s) = & \sum_{j=1}^{2K} d_{ij}^{-\rho} W_j H_{ij} x_j(s) W_i^H + \sum_{j=1}^{2K} d_{ij}^{-\rho} W_j H_{ij} x_j(s-1) W_i^H \\ & + \sum_{j=1}^{2K} d_{ij}^{-\rho} W_j H_{ij} x_j(s+1) W_i^H + n_{ij} \end{aligned} \tag{4}$$

Equation 4 assumes that ICI occurs every time in the network. ICI in the network [20] is considered to occur at a probability equal to be a random number uniformly generated in the range of $[0, 0.1]$.

The SIR of i^{th} stream at antenna i on a OFDM slot using subcarrier s can be determined as

$$SIR = \frac{\left|W_i W_i^H H_{ii}(s)\right|^2}{\sum\limits_{l=s} \sum\limits_{\substack{j=1 \\ j \neq i}}^{2K} \left|W_j W_i^H H_{ij}(l)\right|^2 + \sum\limits_{\substack{l=s-1 \\ l=s+1}} \sum\limits_{j=1}^{2K} \left|W_j W_i^H H_{ij}(l)\right|^2} \tag{5}$$

From the concept of IA, it is obvious that

$$W_j W_i^H = \left\{ \begin{array}{ll} 1 & \text{if } i = j \\ 0 & \text{otherwise} \end{array} \right\}$$

Hence, $W_j W_i^H$ is always binary (i.e., 0 or 1). However, due to the factors such as imperfections in measured channel conditions and highly varying nature of the channel the IA vectors may not remain fixed even for $20\ ms$ (same as time duration of a typical super-frame in WiMAX Networks [1]) time frame, $W_l \times W_m^H$ will not be zero all the time and is considered to have minute errors in evaluations.

Different MIMO techniques can be achieved by varying the transmitted data $x_j(l)$ across different antennas with alignment vectors W_j. We consider both multiplexing and diversity techniques in this paper. A thorough study on the consequences of using different MIMO techniques is provided in the next section. For the numerical results, initially we place all MSs adjacent to each other as shown in Figure 2. Since each MS shares approximately the same channel to the BS, the SIR estimate remains same for several MSs (Note: Distance remains the same for all MSs). Later SIR value of each MS is made to be independent of other MSs, representing a more realistic scenario.

IV. System Analysis [For Maximizing Size of the Share]

We devote this section to determine the maximum number of MSs that can share an OFDM slot in a PMP network. For this section alone, we assume that IA vectors are computed and communicated to each MS participating in the share. This assumption is to ensure the focus remains on the number of MSs in the share and not on inherent complexities to use IA technique. In the subsequent section, we determine different ways to compute IA vectors correctly according to parameters of the considered network. Also, the SIR expressions for different MIMO techniques are specified and independent analysis is provided for two MIMO techniques: Spatial Multiplexing and Spatial Diversity (with Selection Combining receiver). However, this study can be extended to other receiver combining techniques also.

1) Spatial Multiplexing: When each MS utilizes Multiplexing technique, both antennas transmit different data along different IA vectors. Considering $2K$ antennas at the BS receive data from 2 antennas of the K MSs, the data received at any two antennas (l and m) of the BS from an associated MS can be written as

$$\begin{aligned} y_l &= \sum_{j=1}^{2K} d_{jl}^{-\rho} W_j H_{jl} x_j W_l^H + n_{jl} \\ y_m &= \sum_{j=1}^{2K} d_{jm}^{-\rho} W_j H_{jm} x_j W_m^H + n_{jm} \end{aligned} \tag{6}$$

The SIR of data received at antenna l, for the data transmitted at antenna i can be written as

$$SIR = \frac{\left|W_i H_{il} W_l^H\right|^2}{\sum\limits_{\substack{j=1 \\ j \neq i}}^{2K} \left|W_j H_{jl} W_l^H\right|^2} \tag{7}$$

We also know that ICI in the network is inevitable, and occurs randomly in the range of $[0, 0.1]$. Assuming the probability of occurrence of ICI in the network as α, SIR for the signal received at antenna 'l'', transmitted across an OFDMA slot using sub-carrier 's'' can be written similar to Equation 7 as in Equation 8.

$$SIR_l(s) = \frac{\left|W_i H_{il}(s) W_l^H\right|^2}{\sum\limits_{\substack{j=1 \\ r=s-1 \\ r=s+1}}^{j=2K} \alpha \left|W_j H_{jl}(r) W_l^H\right|^2 + \sum\limits_{\substack{j=1 \\ j \neq i}}^{j=2K} \left|W_j H_{jl}(s) W_l^H\right|^2} \tag{8}$$

where $\alpha \in [0, 0.1]$. Using ICI reduction techniques can further reduce α, and thus can improve SIR values of the required signal, at the cost of wasting the most precious bandwidth. Hence, we study assuming that the ICI occurs in the network with a uniform probability in the range of $[0, 0.1]$ [20].

To provide a theoretical insight, we verify if SIR(s) term in Equation 8 can fit in any of the existing, well-studied probability distributions. Study in that direction leads to the following conclusions:

- H_{ij} is a circularly symmetric Gaussian random variable. This is because the real and imaginary parts of the complex term to generate H_{ij} are generated from an identical and independent distribution [21] in the range of $[0, 1]$.
- Clearly, H_{ij}^2 follows a chi-square distribution with 1 degree of freedom [22].
- Similarly, $\sum\limits_{j=1, j \neq i}^{j=2K} H_{ij}^2$ follows a chi-square distribution with $2K - 1$ degrees of freedom.

It can be seen that both numerator and denominator of the SIR(s) term follows chi-square distribution with 1 and $6K - 1$ degrees of freedom. Hence, $(6K - 1 * SIR(s))$ follows F distribution with $(1, 6K - 1)$ degrees of freedom. However, this derivation does not take α and errors due to IA vectors into consideration.

The immediate aim in this section is to find the maximum number of MSs that can share

TABLE I
EACH MS USES MULTIPLEXING TECHNIQUE.

Minimum Rate Requirement (in bps/Hz)	β	Number of MSs (K)
2	3	$\Pr(SIR \geq 3) \geq 0.99$ $\Pr((6K-1)*SIR \geq 3*(6K-1))$ ≥ 0.99 $K \leq 1$
3	7	1
4	15	1
5	31	1

slot such that a minimum rate requirement is maintained at each MS. For a minimum rate requirement, the maximum number of MSs that can share a slot is computed using standard F distribution tables (with 99% significance), and is mentioned in Table I. While calculation steps for the rate requirement of $2\ bps/Hz$ are given in detail, values for other rate requirements are directly mentioned for reading elegance.

2) Spatial Diversity: When each MS utilizes Diversity technique for transmission, same data is transmitted across different antennas along same IA vectors. The SIR of data received at antennas l and $l+1$, for the data transmitted at antennas i and $i+1$ can be written as

$$\begin{aligned} SIR_l &= \frac{\left|W_i(H_{il}+H_{(i+1)l})W_l^H\right|^2}{\sum\limits_{\substack{j=1\\ j\neq i,i+1}}^{j=2K}\left|W_jH_{jl}W_l^H\right|^2}, \\ SIR_{l+1} &= \frac{\left|W_i(H_{i(l+1)}+H_{(i+1)l})W_l^H\right|^2}{\sum\limits_{\substack{j=1\\ j\neq i,i+1}}^{j=2K}\left|W_jH_{jl}W_l^H\right|^2} \end{aligned} \tag{9}$$

The above expressions utilize the fact that two antennas transmit data aligned on one vector W_i and receive across two antennas along same vector W_i^H.

SIR_{eff}:

In case of Receiver Diversity (RD) techniques, multiple antennas receive same signal transmitted across different antennas. Data is received at each antenna using a Zero Forcing receiver, and

TABLE II
EACH MS USES DIVERSITY TECHNIQUE.

Minimum Rate Requirement (in bps/Hz)	β	Number of MSs (K) SC receiver
2	3	3
3	7	2
4	15	1
5	31	1

SIR is computed at each antenna. Once SIR at each antenna is computed, SC receiver is used to determine SIR of the received data. For a SC receiver, SIR_{eff} can be expressed as:

$$\begin{aligned} SIR_{eff} = & SIR_l \;\text{ if } SIR_l \geq SIR_{l+1}; \\ & SIR_{l+1} \text{ otherwise} \end{aligned} \tag{10}$$

By proving that SIR_{eff} follows F distribution with $(1, 3K-1)$ degrees of freedom, number of MSs in the share can be obtained similar to that of Multiplexing case. The calculated values are shown in Table II.

Thus, it is proved that more than two MSs can share a slot for transmission (under favorable conditions), and that number is dependent on the MIMO technique employed at each MS in the share. Since a $2\ bps/hz$ is a reasonable rate requirement in next generation networks such as WiMAX networks, we prove that sharing a slot among multiple (≥ 2) MSs is indeed possible. Surprisingly, the analysis proved that only one MS can share a slot if each MS uses Multiplexing technique while three MSs can share using Diversity technique. We validate this analysis in the next section to evaluate the mentioned interesting observation. We verify the correctness of this analysis in the next section, and with experiments in Section IV-B.

A. Model Validation

In this section, we validate the analytical findings presented in IV-1 and IV-2 via extensive simulations. Using MATLAB, we simulate a MIMO channel in which four MSs share an OFDM slot. Each MS is equipped with two antennas each, and transmits data across both the antennas with the same power. Antenna correlations at each MS and correlation between different MSs is also modeled while modeling the MIMO channel. The channel quality of each MS is assumed to be different. The beam vectors corresponding to the channel quality of each MS are computed using $Max - SINR$ algorithm [23]. Different MIMO techniques are achieved by changing the alignment vectors at each antenna before transmission. For instance: Multiplexing is achieved when data at each antenna is aligned across different vectors and Diversity is achieved when two antennas of an MS transmit across same alignment vector. Also, SC receivers are employed at the BS to retrieve data without errors. We compute SIR values at different antennas of the BS, and plot a Quantile-Quantile (QQ) plot with SIR values generated at those antennas via simulations in Fig. 3, 4.

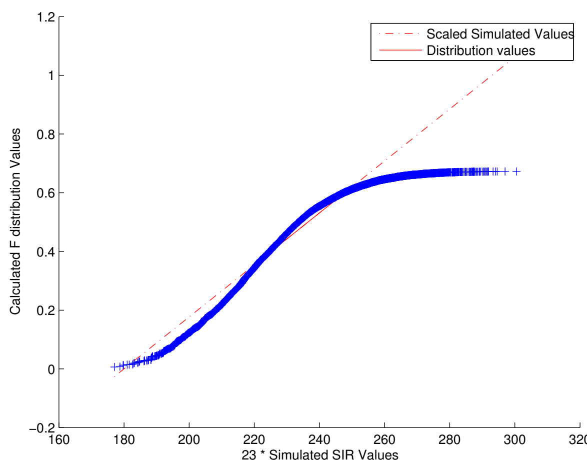

Fig. 3. Simulated Versus Analytical SIR Values in Multiplexing case.

An S-shaped curve in the plots (in Multiplexing and Diversity cases) denotes that the observed SIR values follow an F distribution with shorter tails. Hence, we conclude that the SIR values at each antenna can be approximated to the derived F distribution. The errors in this approximation are primarily due to the inherent randomness in the occurrence of ICI in the network and possible mis-alignment (frequent channel quality variations caused due to high mobility) of IA vectors between the transmitter and receiver. We do not consider modeling mobility in this analysis, due

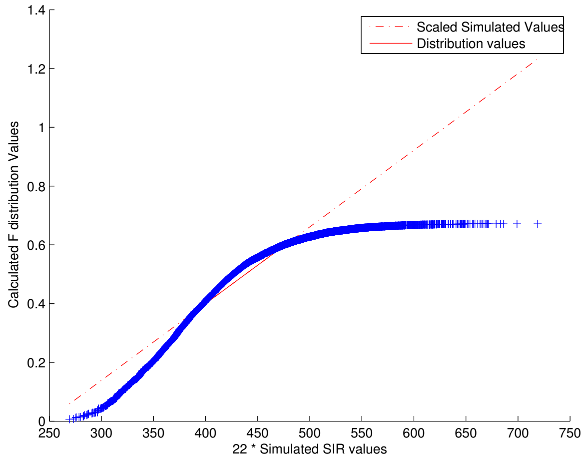

Fig. 4. Simulated Versus Analytical SIR Values in Diversity Technique with SC Receiver.

to the inherent complications. Hence, the model is only approximated and is not exact.

B. Experimental Validation

We consider only one slot is being used for transmission by all MSs in the network and increase the number of MSs in the network from 1 to 20. The authors in [24] propose a transmitter centric algorithm after realizing that the uplink interference in a multi-user system is highly fluctuant. Since each MS receives different SIR values for each transmission, we calculate SIR of required signal at each antenna of the BS, and plot the average SIR across all antennas observed at the BS. This average SIR is used to determine the achievable transmission rate (in bps/Hz) for each MS in the network. For the sake of numerical calculations, we assume the channel coefficients to be uniformly distributed Gaussian random variables with zero mean and unit variance while noise is circularly symmetric and uniformly distributed with zero mean and 0.1 variance.

Initially, we compute the maximum number of MSs that can share a slot when channel conditions are identical for each MS. Later, we compute the same with channel conditions being completely independent for each MS. Also, an SC receiver is used at the BS when Diversity techniques are employed.

Considering the total capacity of the network to be constant, the number of MSs sharing a slot is maximized when the requirement of each MS is minimized. We consider 2 bps/Hz as a norm in the analysis because it is highly visible in WiMAX networks, even at lower Modulation and Coding Schemes[Table 1.1 in [25]].

1) Numerical Results: When the number of MSs that share a slot is 1, the SIR value is maximum for all the four studied scenarios: Each MS with 1 (2) antennas and ICI can (not) be mitigated completely. As the number of MSs sharing a slot increases, the interference in the network also will increase, and a reduction in SIR value can be observed. The maximum MSs that can share a slot is limited to that number, where the average SIR falls below the threshold $\beta = 3$ (pertaining to R=2 bps/Hz). Also, note that β is calculated from Section III). The graphs are plotted with 95% confidence interval after performing $10,000$ iterations. Since β is dependent on rate requirement of each MS, number of MSs that share a slot depends on rate requirement of each participating MS.

When an MS with one antenna transmits only one stream of data, SIR value would be maximum as there is no interference in the network. Fig 5 provides the results for the scenario where all MSs have same channel quality while Fig 6 provides results when the channel quality of each MS is independent of other MS. Also, unless mentioned, the results are analyzed for a constant rate requirement of 2 bps/Hz and $\beta = 3$.

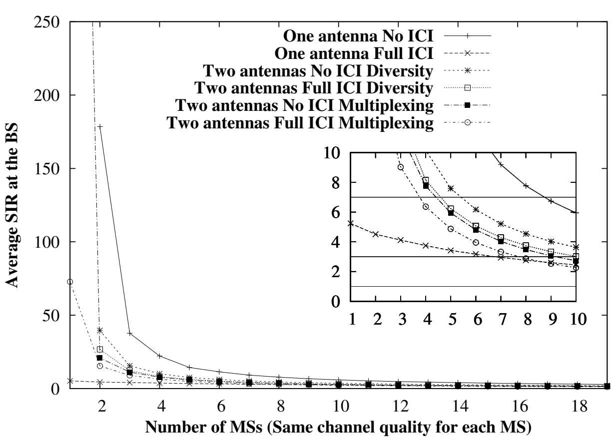

Fig. 5. Average SIR determined at different antennas of the receiver versus the number of MSs sharing a slot.

As can be observed from Fig. 5, more than 10 MSs can share a slot if each MS is equipped

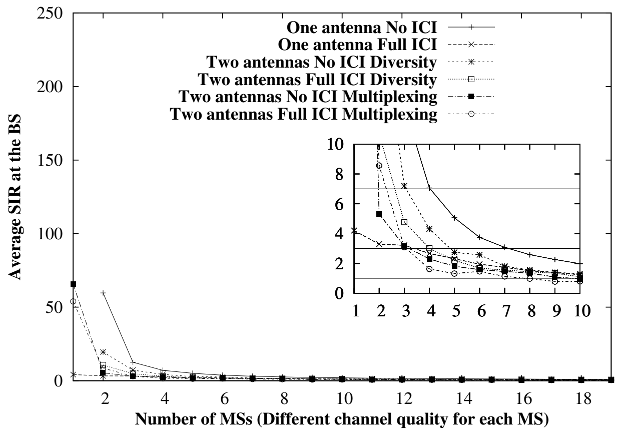

Fig. 6. Average SIR determined at different antennas of the receiver versus the number of MSs sharing a slot.

with only one antenna and ICI is mitigated completely. However, a maximum of 7 MSs can share a slot when channel quality of each MS is independent of the other, as can be observed from Fig 6. Hence, the number of MSs that share a slot depends on channel quality of each MS participating in the share.

Though multiplexing techniques multi-fold the achievable transmit rates depending on number of antennas used for transmission, prior work showed that using multiplexing techniques improves performance of the network only in high SIR regions [1]. Thus, it can be misleading to consider that every MS uses multiplexing techniques for transmission. We observed that diversity techniques attain more SIR values compared to that of multiplexing techniques reiterating this notion. Hence, it becomes noteworthy that in a real world scenario, the number of MSs that share a slot also depends on the MIMO technique employed by each MS sharing the slot.

In Fig. 5, we observed that more than 6 MSs can share a slot for transmission when ICI is not mitigated. However, this number reduces significantly as the channel conditions vary for each MS, as seen in Fig. 6. Also, it can be observed from Fig. 6 that the SIR values remain almost same when the number of MSs sharing a slot is above 8, irrespective of the number of antennas at each MS and ICI mitigation in the network. This confirms that the upper bound for the number of MSs that can share a slot is bounded to 8 if we plan to exploit MIMO techniques available at each MS.

We can also observe that when 2 MSs with one antenna share an OFDM slot for transmission, the SIR value remains close to the threshold β, conforming to the C-SM specifications in WiMAX standards [1]. However, we observed that more MSs can share a slot when each MS is equipped with multiple antennas. A maximum of 7 MSs can share an OFDM slot for a constant rate requirement of $2\ bps/Hz$ in the best possible settings. Consequently, a BS must need at least 14 antennas to receive data from each MS participating in the share. Thus, we provide a lower bound on the number of antennas required at the BS in a PMP network, such that rate requirements of each MS is satisfied. However, the number MSs sharing a slot is completely dependent on channel characteristics, rate requirements, and number of antennas at each MS participating in the share.

Though we showed that more than two MSs can share an OFDMA slot in a PMP network, several assumptions taken for granted in the above study needs to be revisited, before applying them directly in a standard network. Few such important assumptions are discussed in Section V, and we show that they are indeed applicable in a WiMAX network.

V. IA Vectors Computation

Section IV assumes that IA vectors are computed and communicated to each MS in the network. However, it is not trivial to calculate IA vectors in the network, as the available algorithms are computationally intensive and highly complicated in nature. For example, the distributed algorithms proposed in [4], [23] take enormous time (need approximately 10000 iterations) to converge as the number of MSs in the network increases. Hence, we devote this section to determine the IA vectors in a WiMAX network with in one frame duration.

Before we delve in to the challenges of finding IA vectors in a network, we perform an illustration to find out if we can ever schedule MSs satisfying the requirements mentioned in the above section.

Illustration

A total of $10-50$ MSs are deployed in the network randomly, moving at speeds of 120 Kmph. The Angle of Arrival (AoA) of the signals from each MS are noted at the BS for each super-frame. The distance of each MS is estimated at the BS in the initial connection establishment phase itself. Based on the speed of mobile, the relative difference in AoA and distance are calculated for each frame. This AoA, when considered based on the H values of each MS, can be considered as Correlative Angle between two users [8].

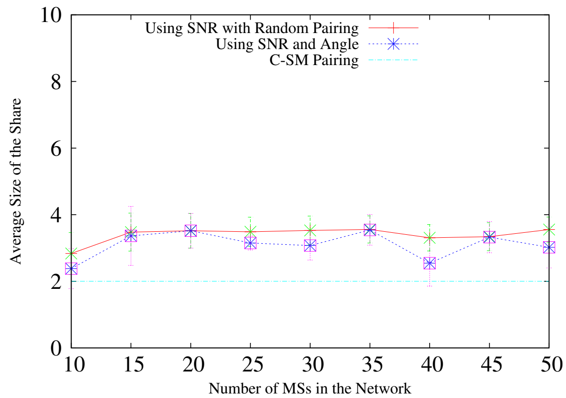

Fig. 7. Number of Mobiles Sharing a Slot

In this setting, we find out the maximum number of MSs that can share a slot in one OFDM slot, with each MS equipped with two antennas. SM and SD MIMO techniques are applied at each MS according to the SNR value of each MS. Each MS uses a MIMO technique randomly at each moment. Though this is not a idealistic measure, just for this illustration this assumption is considered. While a total of 3 MSs share a slot when each MS uses SM technique, a value of 4 is used when SD is used. A total of $100,000$ Monte-carlo iterations are performed and the number of MSs that can share is calculated in each iteration. The results are compared using random pairing with SNR technique. The number of MSs sharing a slot is observed to be 3 when SNR with random pairing is used. Similar observation is visible when SNR with AoA pairing is used. We used a minimum of 60 degree difference between the AoA values of two MSs participating in the share, similar to that of a typical sectored antenna system []. It is clearly visible from Fig. 7 that it is definitely possible to locate MSs with the required parameters. Also, we observed that this number is higher than the number (2) that C-SM technique provides in a typical WiMAX network. As this study is done for a single slot (time), extending this study to several OFDMA slots in a typical WiMAX frame is expected to provide more performance improvements and is further studied.

Challenges

Important challenges of using IA in a WiMAX network can be named as follows:

- Precise Channel Quality Estimation
- Average time and complexity to compute IA vectors
- Delayed IA vectors relevance

For the sake of computing IA vectors between BS and several MSs, we consider a distributed numerical approach (Max-SINR) algorithm proposed in [23], based on the principle, "The wireless channel exhibits the property of reciprocity alignment". In other words, the alignment vectors computed for a channel can simply be reciprocated when the transmitter and receiver are swapped, i.e., the alignment vectors are dependent on the channel values and not on the direction of the transmission. This assumption in a WiMAX network is acceptable, as the BS measures the channel quality of each MS once in each frame, only in the fast feedback channel [26], [1]. In a typical WiMAX network, the channel is estimated using techniques such as blind estimation, channel sounding [27], [28], etc., wherein the MS transmits a known signal to the BS, and the BS determines the error in the received signal to determine the quality of the channel. Since the channel quality is measured only in one direction and is used for determining the IA vectors across both the directions, the above assumption of reciprocity looks reasonable. Also, it has been proved that reciprocity of the channel is visible in a TDD frame structure [16], similar to that of a WiMAX network. Each step involved in generating IA vectors according to the Max-SINR algorithm can be summarized as follows:

Step 0: Generate initial precoding vectors randomly
Step 1: Find interference plus noise covariance matrix
Step 2: Find receiver combining vectors
Step 3: Reverse the link and use receiver combining vectors as precoding vectors
Step 4: Find interference plus noise covariance matrix
Step 5: Find receiver combining vectors
Step 6: Reverse the link and use receiver combining vectors as precoding vectors
Step 7: Goto Step 1 until precoding and receiver vectors converge

The IA vectors calculated are dependent only on H matrix, corresponding to the impact of each MS at several antennas of the BS. In a slow fading channel, the variations in the elements of H matrix across few super-frames will be minimal (as we do not consider sudden frequency specific disruptions in the network). In this scenario, we provide mechanisms to overcome the mentioned challenges in the following subsections.

A. Channel Estimation Precision

A typical MAC frame structure in a WiMAX network is as shown in Fig 8, with the duration of each frame being 5 ms. Each MS typically transmits the channel quality to the BS once in each frame in Frame Control Header (FCH) slot. IEEE 802.16m standard specifies that four frames will constitute a super frame (one in 20 ms), and use one FCH slot, thus utilizing the available OFDM slots for transmitting data optimally. The Modulation and Coding Schemes (MCS) used by each MS are updated according to the channel quality updated for each 5 ms frame. Also, we show later that IA vectors can be computed in 20 ms time. Hence, IA vectors generated from the channel estimated in the previous frame can be used for transmission in the current frame, similar to that of MCS schemes applicable. And precision of estimating the channel is sufficient to utilize IA vectors in a WiMAX network.

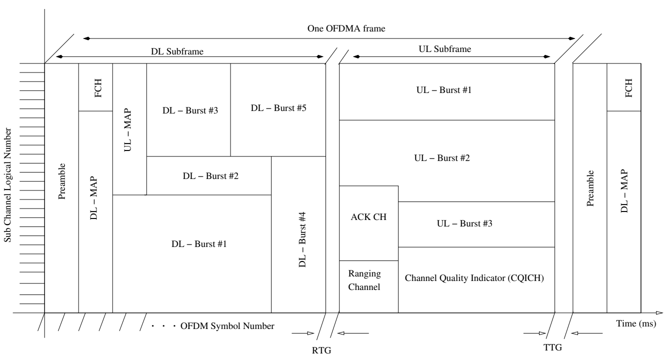

Fig. 8. MAC Frame Structure in WiMAX

B. Average Time to Compute IA Vectors

The numerical approach described in Max-SINR algorithm optimized to the current scenario as follows: The algorithm (Step 0) considers generating random IA vectors and updating them constantly such that the IA vectors converge after several iterations. This converging value is dependent on the channel gain vectors of each MS participating in the share. Given the fact that the channel quality of each MS does not vary drastically for adjacent super frames (of 20 ms duration due to slow fading channel), we use the IA vectors calculated in the previous frame as initial vectors to generate IA vectors for the next frame.

Corollary 5.1: The IA vectors converge early, when the initial random vectors are same as IA vectors generated for the previous frame.

since the channel qualities vary minimally in two adjacent frames (since we consider a slow fading channel), as long as the pairing set remains same, the IA vectors of the set also vary accordingly. Hence, it becomes logically appropriate to choose the initial vectors as IA vectors in the previous frame.

Validation:

We verified this proposition in a decentralized network with 10 users, with each user requiring 2 Degrees of Freedom. We found that by considering the IA vectors used in the previous frame, the average time consumed to generate IA vectors almost reduced to one third of the time taken when initial vectors are generated randomly.

As shown in Fig. 9, the optimization was able to reduce the overhead in the computation time and complexity of IA vectors.

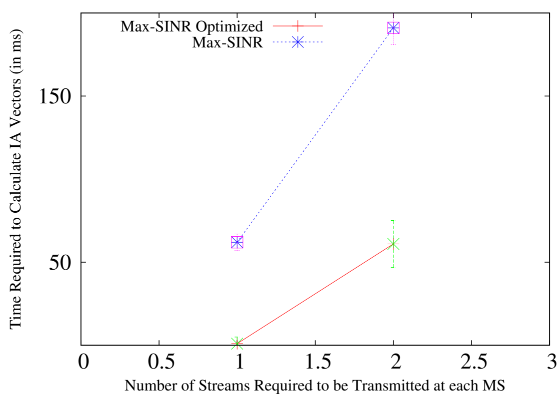

Fig. 9. Average Time Consumed to Generate IA Vectors

Example: Due to frequent channel quality exchange between each MS and the BS, the SNR values of each MS recorded at the BS vary continuously, and not in a step wise manner. Also, MSs are paired based on the SNR and values of H matrix. Hence, the IA vectors calculated for MSs participating in the share will vary differentially during the subsequent frame. Let MS_1, MS_2, MS_3 share a slot in time t_1. During time t_2, the SNR values of each MS varies differentially with few variations in H matrix[1]. If all the MSs participating in the share remain same at time t_2 also, we optimize the Max-SINR algorithm to use IA vectors used at time t_1 as initial random vectors. Evaluation showed that the computation time indeed reduced to almost one-third compared to the normal computation time.

C. Delayed IA Vectors Relevance

As mentioned in Section II, each MS transmits the channel quality to the BS once for each super frame [29]. This also means that the BS can transmit IA vectors to each MS participating in the share once in every 20 ms. Also, an average of $5-10$ ms time is consumed at the BS to generate IA vectors for each MS participating in the share, as in Fig. 9. However, the challenge remains if the IA vectors communicated will remain intact for the next 20 ms frame. A simple block diagram to denote the IA vectors is shown in Fig. 10.

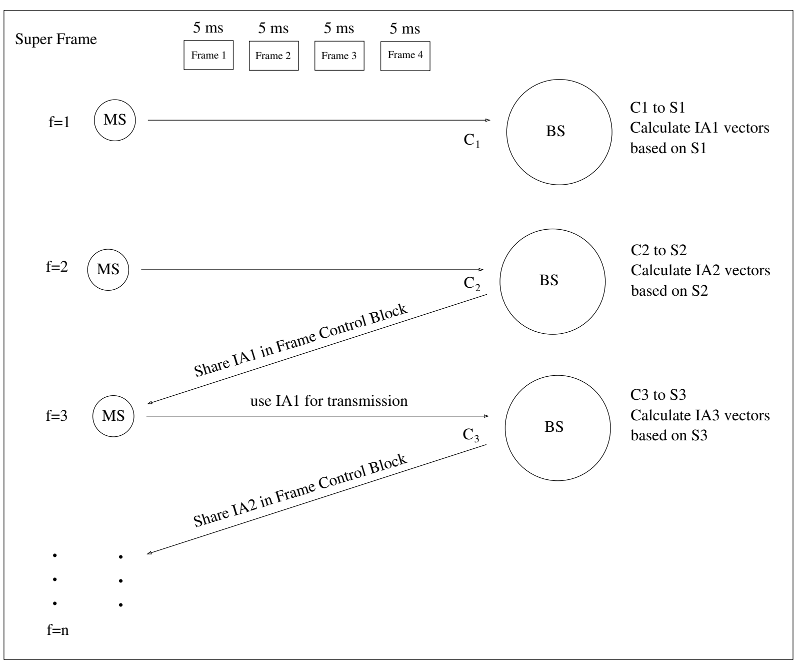

Fig. 10. Working Solution to find IA vectors

[1]generated by the mentioned three MSs sharing slot in previous frame

$C1$ to $S1$ at the BS denotes the instantaneous channel quality is converted to the channel quality measured statistically. This is done at the BS to ensure that the channel quality of each MS is always continuous and does not show high fluctuations in adjacent frames. Since, we consider a slow fading channel IA vectors will remain intact for the channel coherence time, which is 30 ms for the considered mobility of each MS. Hence, the IA vectors are relevant for the next frame duration.

VI. Algorithm to Schedule MSs

A. Related Work

In this subsection, we evaluate some of the basic algorithms that pair several (two) users in same slot.

Random Pairing: All MSs are ordered according to their SNR values [30]. An MS with highest SNR value is allocated the slot. Among the remaining MSs, one MS is randomly picked and allocated to the same slot. The process repeats for other OFDM slots at the BS, for each frame. Few disadvantages of this pairing scheme are:

1) The physical layer parameters of MSs, such as SNR, channel quality are not considered.

2) The rate constraints of each MS are not considered.

Double Proportional Fairness Pairing: This scheme assigns rank to each MS based on the ratio of required transmission rate to the average transmission rate achieved. Similar to the above, the first MS_i is the MS with highest SNR value. Before choosing the MS to be paired, we find the utility for each MS_k as

$$U_{ik} = \frac{R_i + R_k}{\overline{R_i} + \overline{R_k}} \tag{11}$$

where R_i, R_k is the rate required in the next frame, $\overline{R_i},\ \overline{R_k}$ is the average transmission rate achieved by MSs i, k. The MS that provides the maximum utility is paired with the Ms_i for the OFDM slot.

A notable point of D-PF [7] is that D-PF is proposed for each MS equipped with single antenna. The authors claim that the pairing algorithm can be extended to more than two MSs also. However, allocating MSs with multiple antennas will induce more interference, which needs to be considered in pairing. Towards this direction, we propose a scheduling algorithm as in Algorithm 1 in which we consider the presence of multiple antennas, interference and correlation angle between MSs sharing the slot.

B. Proposed Algorithm

Algorithm 1 Proposed Algorithm

Input: *SNR* of each MS
H Matrix: $(N * N_t) \times N_r$ size
N_t: Number of antennas at each MS
N_r: Number of antennas at BS
N: Number of MSs in the network
M_i: *MIMO* Technique at each MS
O: Set of *OFDM* slots to be allocated
Start:
Sort MSs based on *SNR* values
for each *OFDM* Slot $\emptyset$ **do**
    $allocated\_MS_{\emptyset}$ //Holds the MSs allocated same slot
    Find MS with largest *SNR* and is unallocated
    Allocate $allocated\_MS_{\emptyset}$ slot to MS_i with highest *SNR* value
    //Match MSs for this MS_i Starts
    Find C candidate MSs based on *CA-SUP* algorithm (2.3 in [8])
    Find IA vectors with C MSs as input using *Max-SINR-Optimized* algorithm
    Allocate $allocated\_MS_{\emptyset}$ to τ MSs determined from *Max-SINR-Optimized* algorithm
    //Match MSs for this MS_i Ends
end for
return $allocated\_MS$ //Set of $allocated\_MS_{\emptyset}$ for each *OFDM* slot $\emptyset$
End

Max-SINR-Optimized:
1) Step 1: $V_i^{[k]} = V_i^{[IA_{Prev-frame}]}$
2) Continue from Step 2 as mentioned in V.C in [23]
3) If values of τ MSs converge, exit
4) return IA vectors of converged MSs

The algorithm determines an MS with highest SNR value and assigns the slot to it. A set of C MSs that can share the same slot are generated using CA-SUP technique [8]. These C MSs are given as input to MAX-SINR-Optimized algorithm to determine τ MSs that can share the slot. This algorithm also determines the IA vectors to be used by each MS participating in the share. The value of τ is 3 if this MS uses a SM technique to transmit data, 4 when SD technique is used for transmission. In our scenario, the value of C depends on the total number of MSs that can share that slot. This number C should be large enough to provide a chance other MSs to share the slot, and small enough for $Max-SINR-Optimized$ algorithm to converge in a reasonable amount of time. For the purpose of simulations, we considered C to be 10. The C MSs are given as inputs to $Max-SINR-Optimized$ algorithm: With the initial values of IA vectors taken as the IA vectors used in the previous frame (BS has the sufficient data). We iterate until τ MSs converge to IA vectors. The same is represented in Algorithm 1.

C. Simulation Results

We use standard network simulator (ns-3) to evaluate performance of the proposed algorithm. A standard WiMAX network is simulated with each MS equipped with 2 antennas. However, only one antenna is used for transmission when D-PF and random pairing techniques are used. We do not consider using the best antenna for the transmissions, and consider each MS is equipped with only one antenna when using D-PF and random pairing techniques. Few important simulation parameters are mentioned in Table III.

TABLE III
SIMULATION PARAMETERS

Parameter	**Value**
Number of Antennas at MS	2
Carrier Frequency	2.4 GHz
Channel Bandwidth	10 MHz
OFDM Symbol Duration	102.86 μs
UGS Traffic	Constant Bit Rate - 64 Kbps
Confidence Interval	95%
Simulation Time	500 seconds

As we can observe from Fig. 11, there is substantial improvement in throughput of the network when the proposed scheme is used. This is because more number of MSs are sharing each OFDM slot using the proposed scheduling scheme. We also observed that D-PF and random pairing achieved similar throughput in the network. This is because we considered only UGS traffic in the network. The proposed algorithm assumes a constant rate requirement at each MS for transmissions. As other traffic classes such as Real Time Polling System (rTPS), non-real

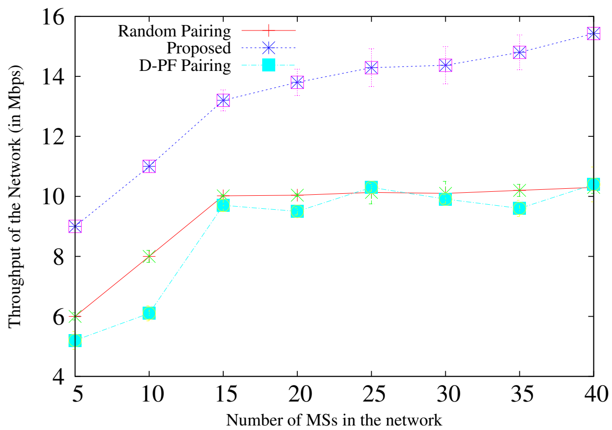

Fig. 11. Working Solution to find IA vectors

Time Polling System (n-rTPS), Best Effort (BE) transmit data at different rates in different frames, we do not consider them for the evaluations. Also, the MIMO technique applied at each MS is dependent on instantaneous SNR values. Hence, we do not observe the throughput scaling similar to the observation in Fig. 7.

VII. Conclusion

In this paper, we studied the effect of increasing the number of MSs that share a slot in a PMP network using both analysis and simulations. For a constant rate requirement of $2\ bps/Hz$, we observed (both in theory and simulations) that more than 2 MSs can share a slot when each MS is equipped with multiple antennas, which is in stark contrast to the current wireless network standards [1], which allow only 2 MSs to share a slot for transmission. The number of MSs sharing the slot is shown to vary for different rate requirements at each MS and different MIMO techniques applied at each participating MS. Once the number of MSs that can share a slot is determined, finding which MSs can share each slot in a typical WiMAX network is discussed and an algorithm is proposed to schedule MSs in each frame. We found that performance of the network is significantly improved by using the proposed scheduling algorithm. Studying the network for different rate requirements at each MS remains to be an open problem, and can be considered for future work.

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