Analysis of OCT decays

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1.1.1   Estimate noise

A fit of the residuals of smoothing splines provides the parameters for a model of the random noise.

```
##
##  Noise fit parameters:
## a_1 :  8.53
## a_2 :  224
```

The smoothing curve, residuals and noise model can be seen in Fig. 1. The noise presents a notable exponential decay.

1.1.2   Mono-exponential fit

A mono-exponential fit is attempted using the noise model.

The Birge ratio analysis rejects this model

```
##
##  MonoExp decay parameters:
```
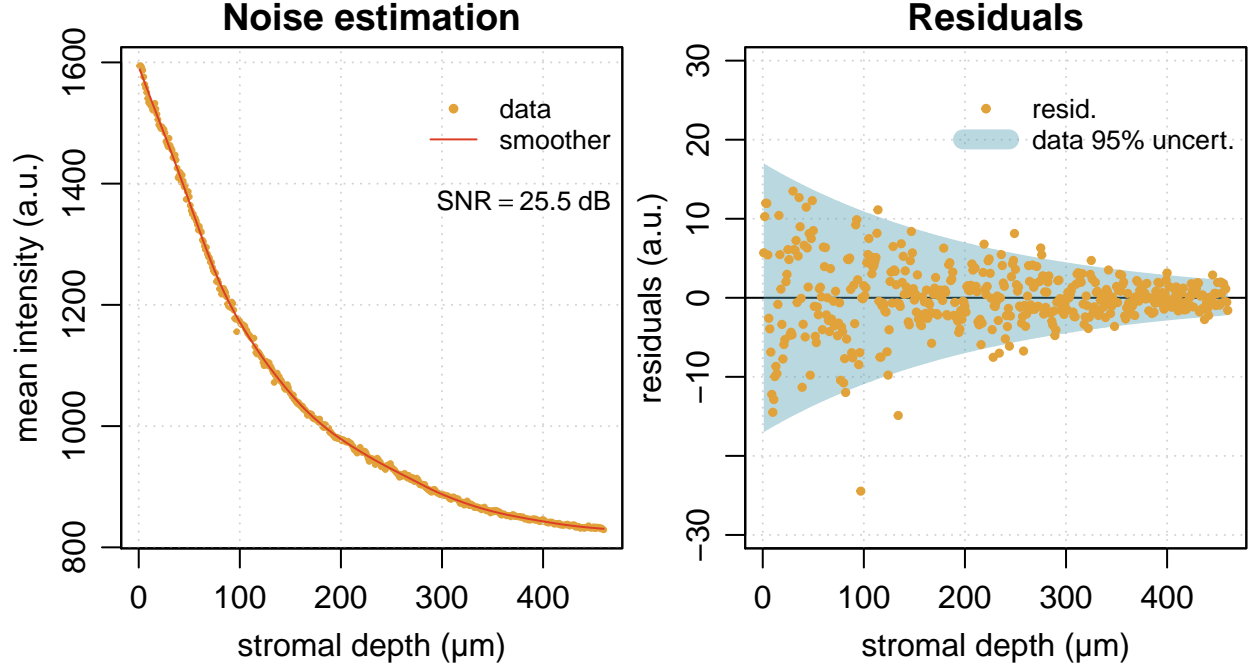
Figure 1: Splines smoothing and noise estimation

```
## b_1 :   807
## b_2 :   808
## b_3 :   259
##
##
## ndf        : 457
## br         : 2.5
## CI95(br)   : 0.87-1.1
## !!! WARNING: br out of interval !!!
```

It can be seen in Fig. 2 that the residuals present notable serial correlation.

1.1.3 Modulated-exponential fit

Considering that the mono-exponential is deemed inadequate, the model with mean-depts modulation is fit to the data. The default parameters are used, notably with a number of control points $M = 10$.

1.1.3.1 Prior sampling

The moments-matching prior is used for the exponential parameters, and one can check the dispersion of the exponential curves generated from this prior (prior predictive sampling) in
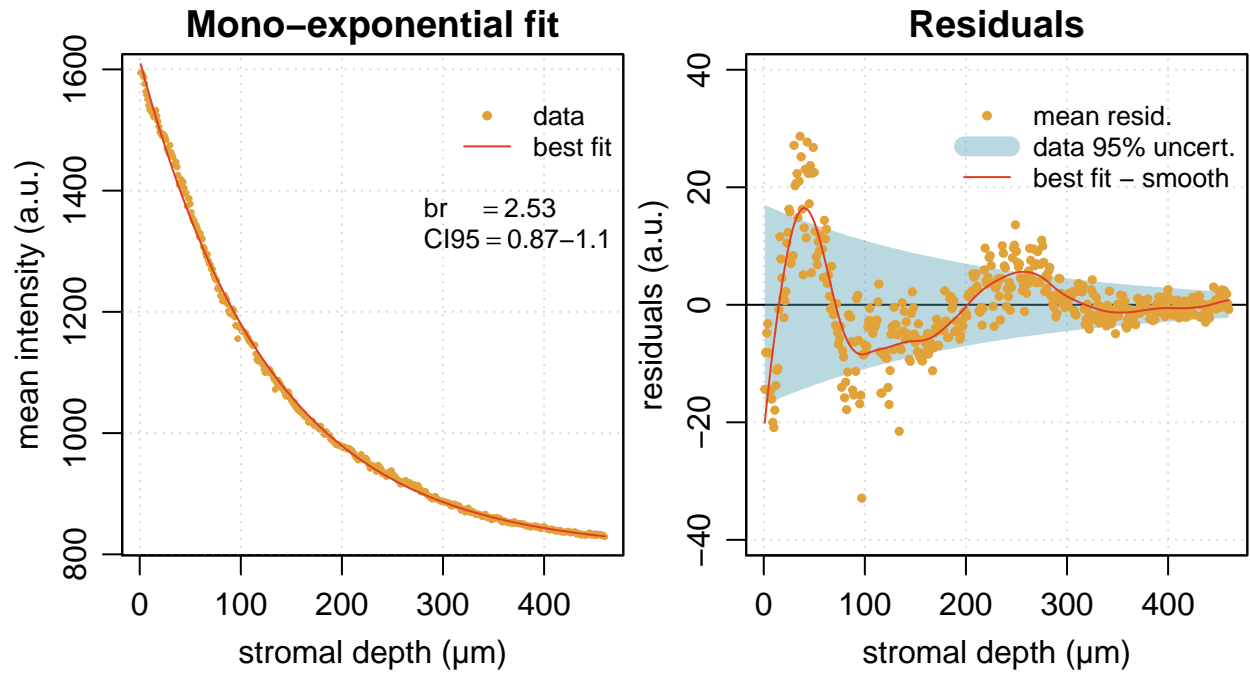
Figure 2: Mono-exponential fit and residuals.

Fig. 3. Prior probability intervals for the control values are also shown in this figure (right panel).

1.1.3.2 Posterior sampling

The priors pdf seems correct, and one can now draw samples from the posterior pdf.

The Birge ratio analysis validates this model.

```
## Active pts.: 4 / 10
## ndf        : 451
## br         : 1
## CI95(br)   : 0.87-1.1
##
```

As can be seen in Fig. 4, the residuals have been notably improved from the mono-exponential model.
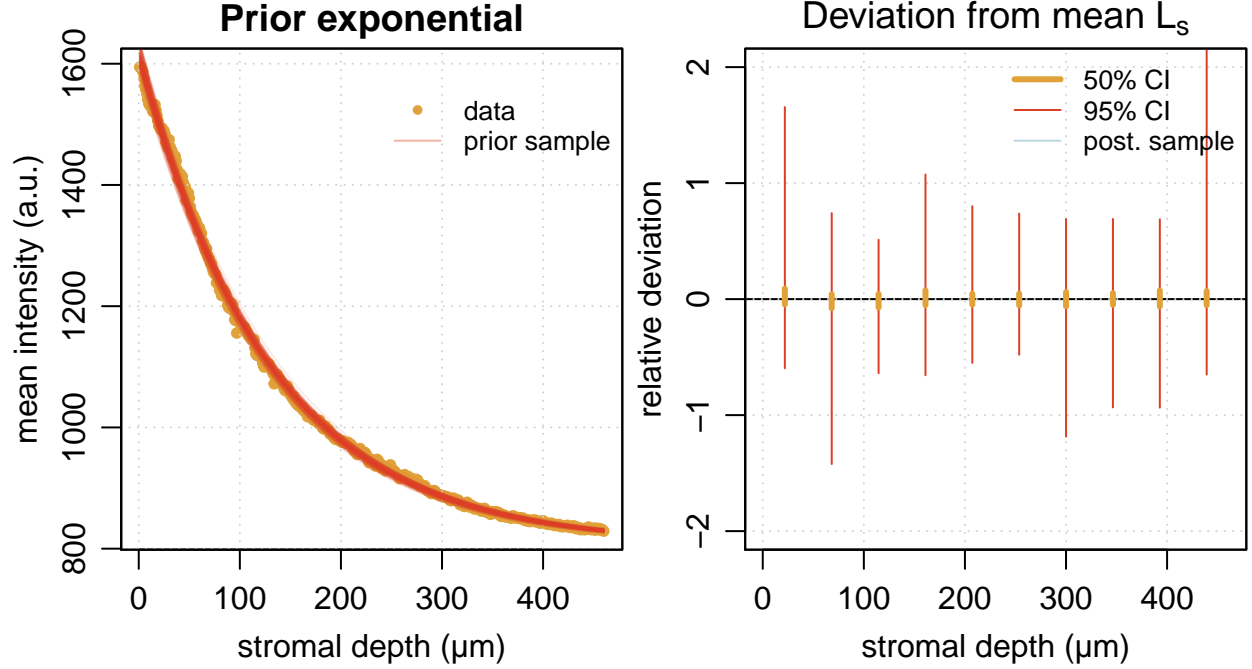
Figure 3: Samples from prior pdf.

1.2 In-vivo signal

1.2.1 Estimate noise

A fit of the residuals of smoothing splines provides the parameters for a model of the random noise.

```
##
##  Noise fit parameters:
## a_1 :  0.0092
## a_2 :  10000
```

The smoothing curve, residuals and noise model can be seen in Fig. 5. In thsi example, the noise is homogeneous, and the exponential model tends to a constant.

1.2.2 Mono-exponential fit

A mono-exponential fit is attempted using the noise model.

The Birge ratio analysis rejects this model

```
##
##  MonoExp decay parameters:
```
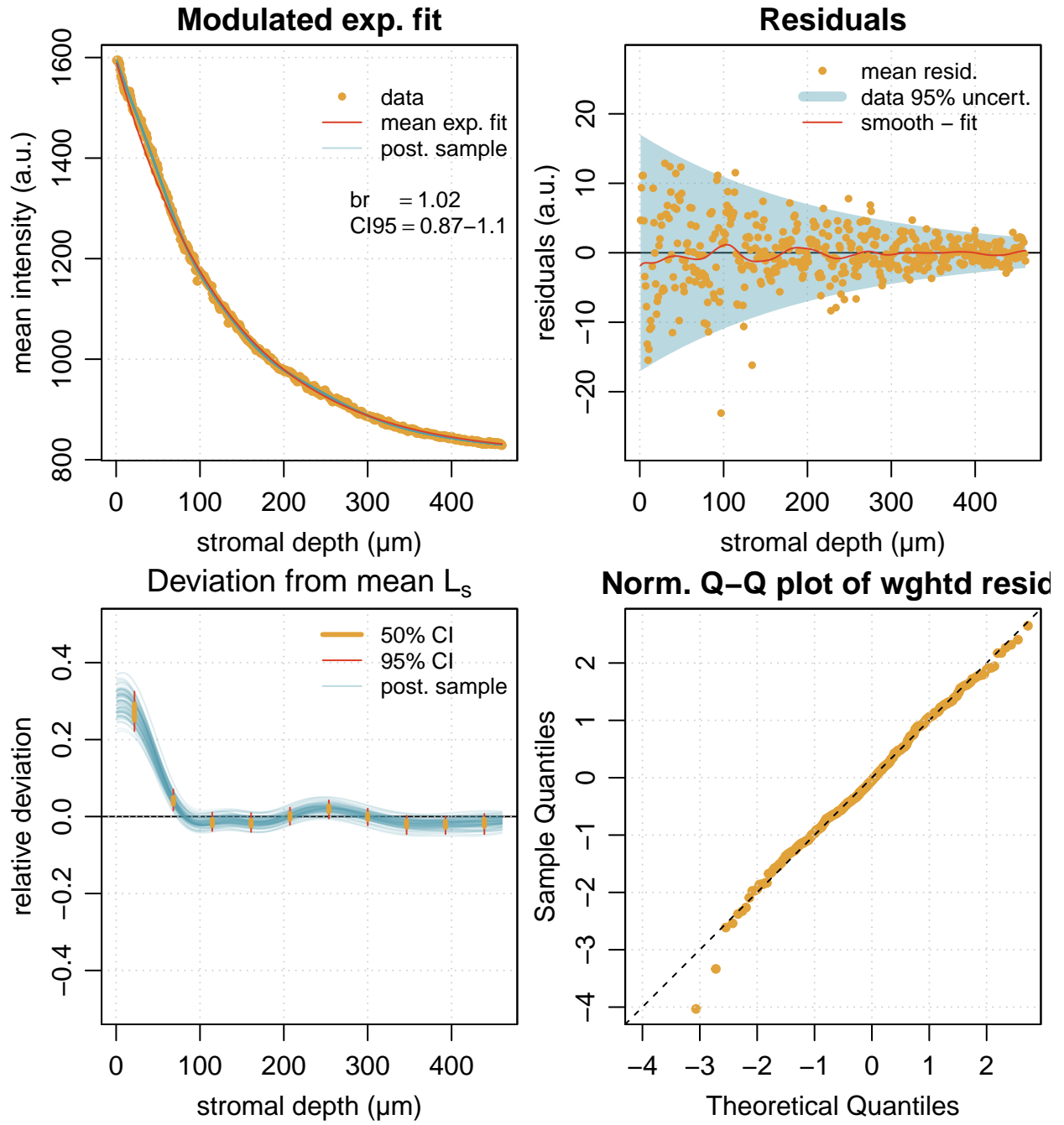
Figure 4: Modulated-exponential fit, residuals, Modulation function and Normal QQ-plot of residuals.
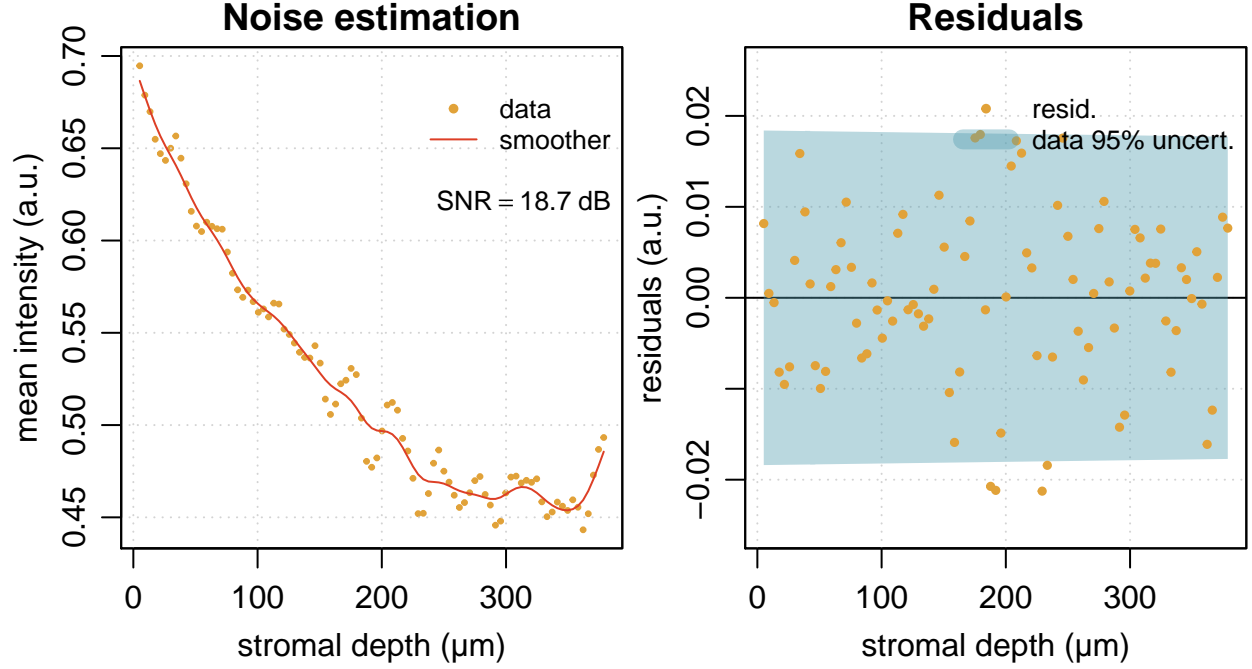
Figure 5: Splines smoothing and noise estimation

```
## b_1 :   0.434
## b_2 :   0.265
## b_3 :   279
##
##
## ndf        : 88
## br         : 2.1
## CI95(br)   : 0.73-1.3
## !!! WARNING: br out of interval !!!
```

It can be seen in Fig. 6 that the residuals present notable serial correlation.

1.2.3 Modulated-exponential fit

Considering that the mono-exponential is deemed inadequate, the model with mean-depts modulation is fit to the data. The default parameters are used, notably with a number of control points $M = 10$.

1.2.3.1 Prior sampling

The moments-matching prior is used for the exponential parameters, and one can check the dispersion of the exponential curves generated from this prior (prior predictive sampling) in
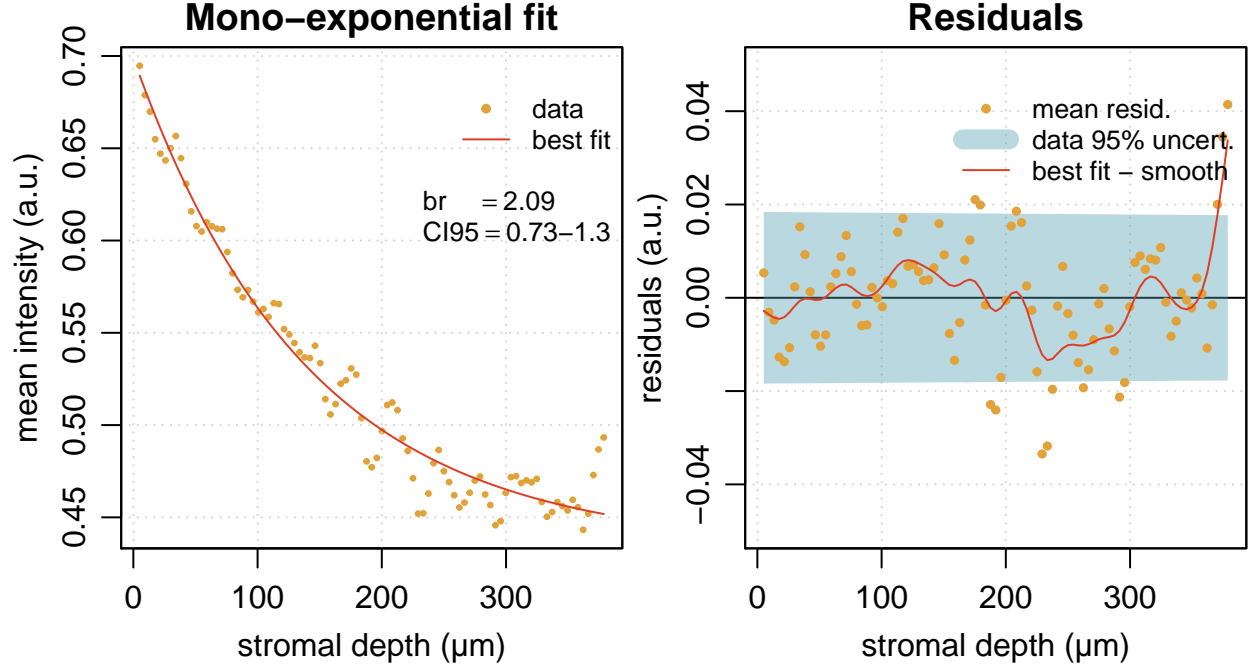
Figure 6: Mono-exponential fit and residuals.

Fig. 7. Prior probability intervals for the control values are also shown in this figure (right panel).

1.2.3.2 Posterior sampling

The priors pdf seems correct, and one can now draw samples from the posterior pdf.

The Birge ratio analysis invalidates this model.

```
## Active pts.: 1 / 10
## ndf        : 85
## br         : 1.3
## CI95(br)   : 0.72-1.3
##
```

As can be seen in Fig. 8, the residuals have not been notably improved from the mono-exponential model, and the present design does not enable to fit a sharp rise in the OCT signal at a depth of about $400\,\mu$m. An option is to increase the number of control points.

1.2.4 Add control points

The number of control points has been raised to $M=15$ on a grid covering the extremities of the depts range, and the posterior distribution has been resampled.
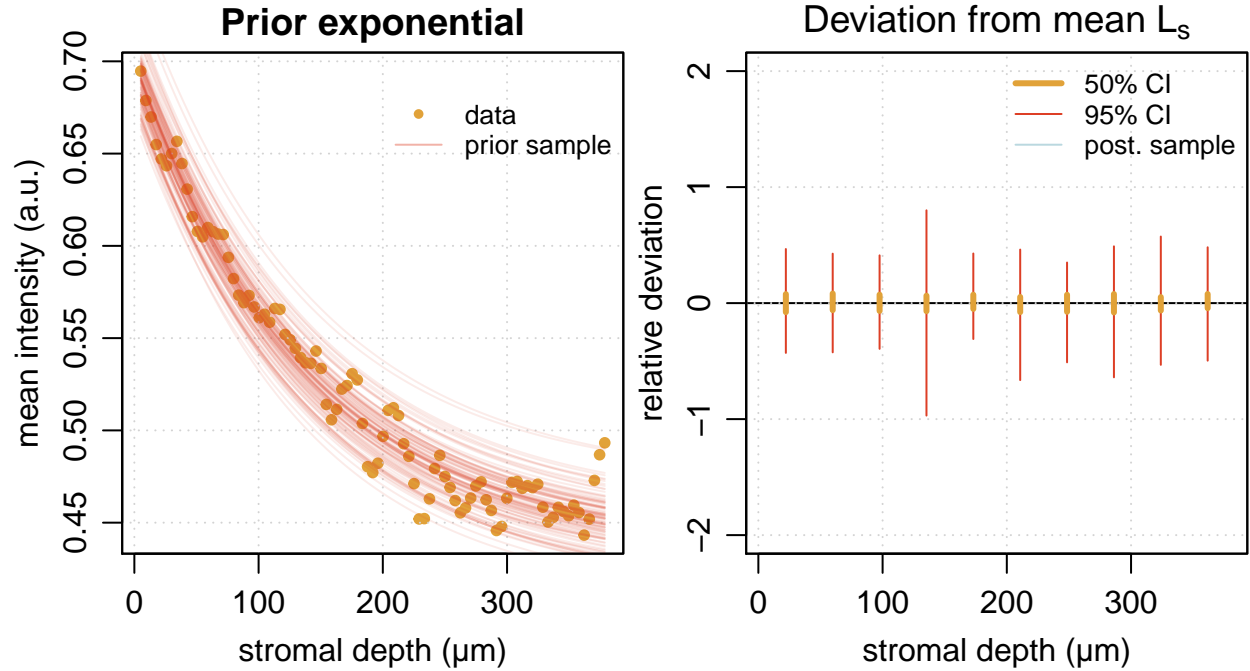
Figure 7: Samples from prior pdf.

```
##
##   ExpGP parameters:
## Inference for Stan model: modFitExpGP.
## 4 chains, each with iter=200; warmup=100; thin=1;
## post-warmup draws per chain=100, total post-warmup draws=400.
##
##            mean se_mean   sd    2.5%     25%     50%     75%   97.5% n_eff Rhat
## theta[1]   0.43    0.00 0.00    0.43    0.43    0.43    0.44    0.44   506 1.00
## theta[2]   0.26    0.00 0.00    0.26    0.26    0.26    0.27    0.27   431 1.01
## theta[3] 279.89    0.32 6.28  267.59  275.80  279.68  283.81  292.33   393 0.99
## yGP[1]    -0.03    0.01 0.12   -0.28   -0.10   -0.02    0.03    0.23   445 0.99
## yGP[2]    -0.02    0.00 0.06   -0.16   -0.06   -0.01    0.02    0.11   474 0.99
## yGP[3]     0.04    0.00 0.06   -0.06    0.00    0.03    0.07    0.16   420 1.00
## yGP[4]     0.00    0.00 0.04   -0.07   -0.02    0.00    0.02    0.09   349 1.00
## yGP[5]     0.07    0.00 0.05   -0.02    0.03    0.07    0.10    0.19   366 1.00
## yGP[6]     0.05    0.00 0.04   -0.03    0.02    0.05    0.08    0.14   365 0.99
## yGP[7]     0.03    0.00 0.04   -0.04    0.00    0.02    0.05    0.11   357 1.00
## yGP[8]     0.00    0.00 0.04   -0.08   -0.02    0.00    0.02    0.08   513 1.00
## yGP[9]    -0.03    0.00 0.04   -0.11   -0.05   -0.02    0.00    0.05   379 1.00
## yGP[10]   -0.14    0.00 0.06   -0.25   -0.18   -0.14   -0.09   -0.03   401 1.00
## yGP[11]   -0.06    0.00 0.06   -0.18   -0.09   -0.05   -0.02    0.04   430 1.00
## yGP[12]   -0.05    0.00 0.06   -0.17   -0.09   -0.04    0.00    0.04   425 1.00
```
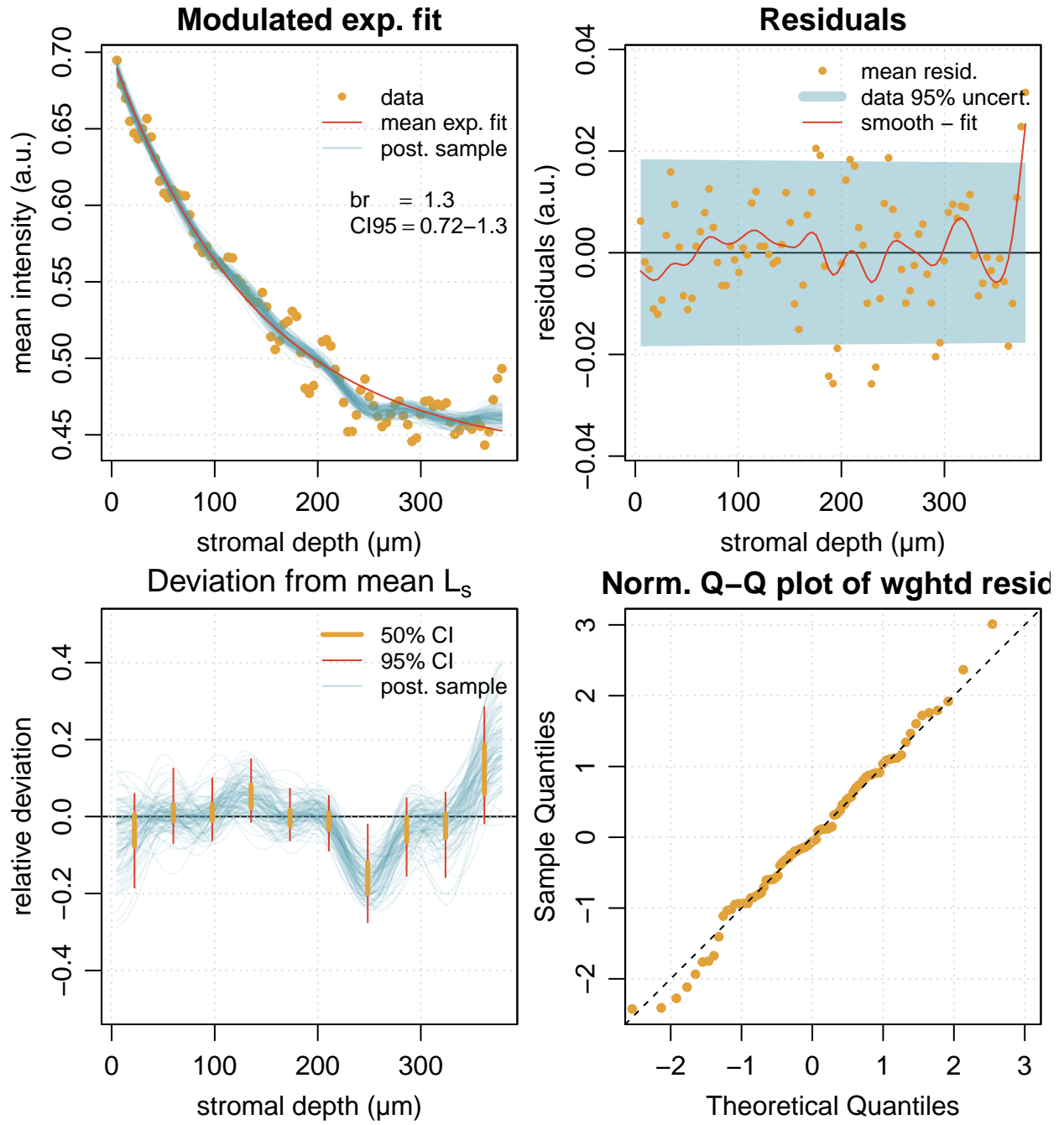
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Figure 8: Modulated-exponential fit, residuals, Modulation function and Normal QQ-plot of residuals.

```
## yGP[13]     0.09     0.00 0.07  -0.04   0.03   0.09   0.14   0.23   257 1.00
## yGP[14]    -0.06     0.00 0.08  -0.25  -0.11  -0.05   0.00   0.05   386 0.99
## yGP[15]     0.56     0.01 0.13   0.31   0.46   0.55   0.65   0.81   288 1.00
## lambda    19.19     0.47 6.45  10.20  14.69  18.33  22.32  34.23   185 1.01
## sigma      1.11     0.00 0.06   0.99   1.07   1.12   1.16   1.25   477 1.01
## br         1.45     0.01 0.16   1.18   1.35   1.45   1.56   1.78   435 1.00
##
## Samples were drawn using NUTS(diag_e) at Fri Dec  7 16:17:59 2018.
## For each parameter, n_eff is a crude measure of effective sample size,
## and Rhat is the potential scale reduction factor on split chains (at
## convergence, Rhat=1).
##
##
## Active pts.: 2 / 15
## ndf       : 84
## br        : 1.2
## CI95(br)   : 0.72-1.3
##
```

The Birge ratio is now acceptable, although there are only 3 active control points. The others do not contribute significantly to the fit. The residuals are also improved (Fig. 9). The model is statistically valid, and the modulation curve can be used for physical interpretation.
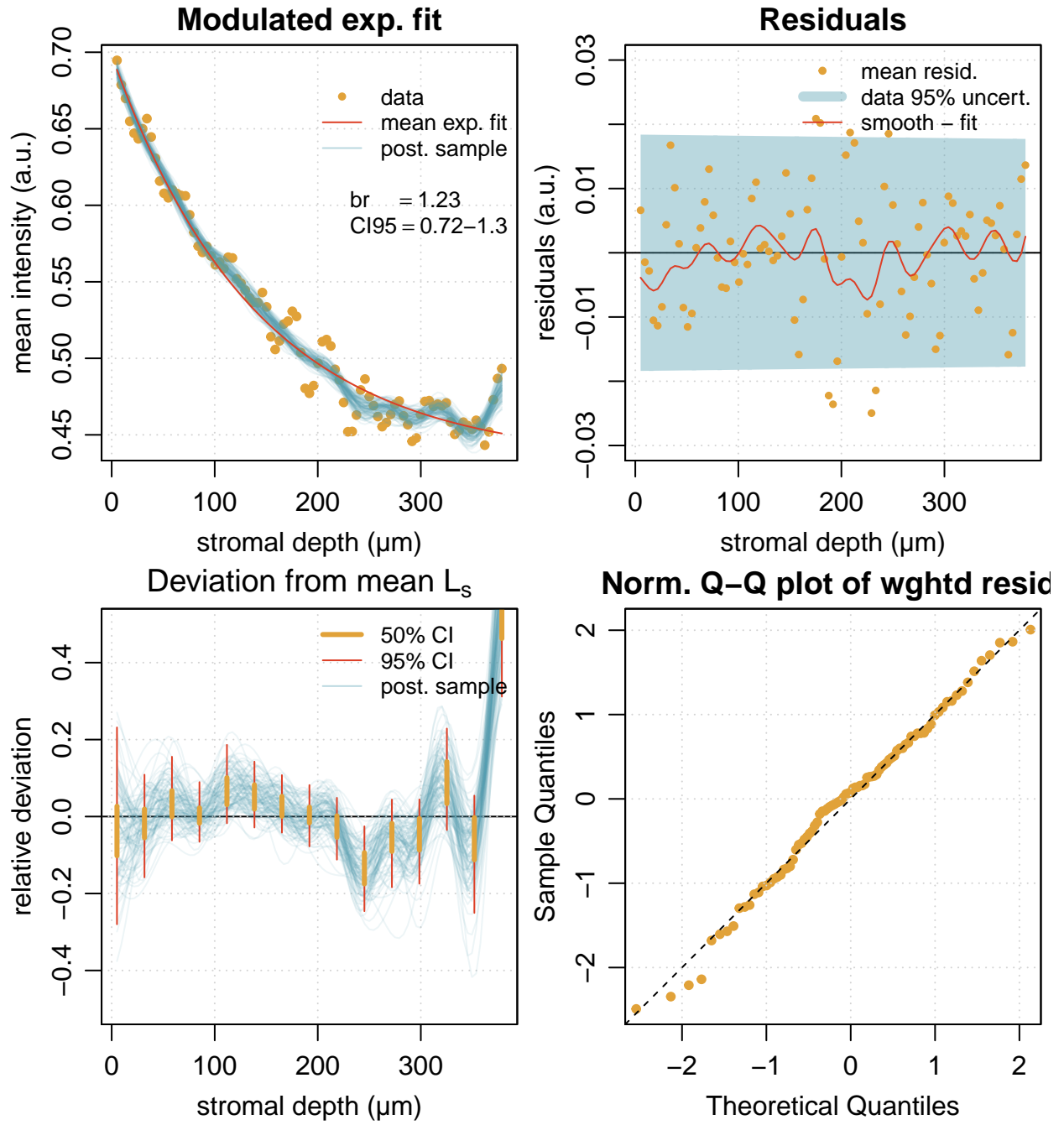
2   References

Figure 9: Modulated-exponential fit, residuals, Modulation function and Normal QQ-plot of residuals.