CHAPTER 13 - MAGNETICALLY COUPLED CIRCUITS

List of topics for this chapter :
Mutual Inductance
Energy in a Coupled Circuit
Linear Transformers
Ideal Transformers
Three-Phase Transformers
PSpice Analysis of Magnetically Coupled Circuits
Applications

MUTUAL INDUCTANCE

Problem 13.1   Given the circuit in Figure 13.1 and $k = 1$, find I_1 and I_2.
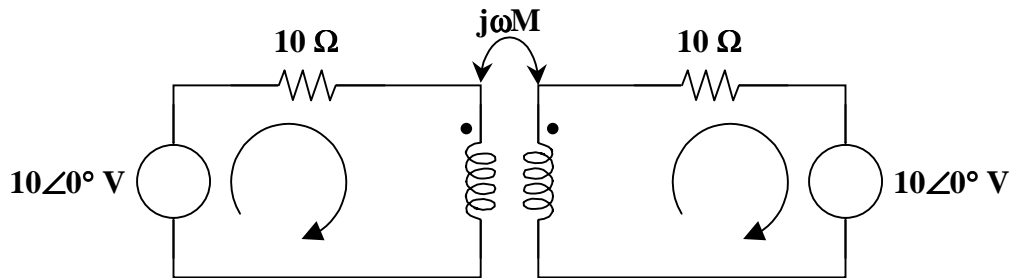

Figure 13.1

➢ **Carefully DEFINE the problem.**
Each component is labeled completely. The problem is clear.

➢ **PRESENT everything you know about the problem.**
We know all values of the independent sources. We also know the values of all the elements.

In order to find the equivalent circuit containing the induced voltages, we need to know the mutual inductance, M.

We know that the coupling coefficient is

$$k = \frac{M}{\sqrt{L_1 L_2}} = 1$$

Then,

$$M = k\sqrt{L_1 L_2} = \sqrt{L_1 L_2}$$

➢ **Establish a set of ALTERNATIVE solutions and determine the one that promises the greatest likelihood of success.**
The three solution techniques that can be used are nodal analysis, mesh analysis, and basic circuit analysis. The goal of the problem is to find two currents. Hence, mesh analysis will be used.

➢ **ATTEMPT a problem solution.**
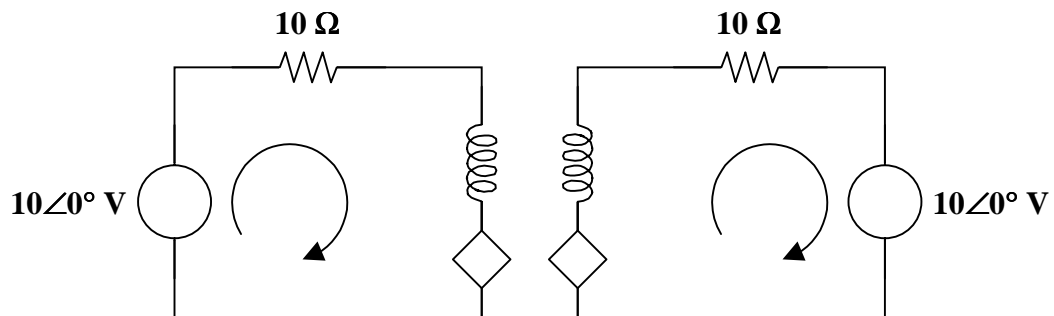From the circuit in Figure 13.1, we can see that $j\omega L_1 = j\omega L_2 = j10$.

Thus,
$$\omega L_1 = \omega L_2 = 10 \text{ and } L_1 = L_2 = L.$$
Hence,
$$M = \sqrt{L_1 L_2} = L \text{ and } j\omega M = j\omega L = j10.$$

So, the equivalent circuit is

Now, using mesh analysis,
Loop 1 : $-10 + 10 I_1 + j10 I_1 - j10 I_2 = 0$
$\quad\quad\quad (10 + j10) I_1 - j10 I_2 = 10$
$\quad\quad\quad (1 + j) I_1 - j I_2 = 1$

Loop 2 : $- j10 I_1 + j10 I_2 + 10 I_2 + 10 = 0$
$\quad\quad\quad - j10 I_1 + (10 + j10) I_2 = -10$
$\quad\quad\quad - j I_1 + (1 + j) I_2 = -1$

In matrix form,
$$\begin{bmatrix} 1+j & -j \\ -j & 1+j \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$
or
$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \frac{\begin{bmatrix} 1+j & j \\ j & 1+j \end{bmatrix}}{\Delta} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

where $\Delta = (1 + j)^2 - (-j)^2 = (1 + j2 + j^2) - j^2 = 1 + j2$.

Now,

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} \dfrac{1+j}{1+j2} & \dfrac{j}{1+j2} \\ \dfrac{j}{1+j2} & \dfrac{1+j}{1+j2} \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

Therefore,

$$I_1 = \frac{1+j-j}{1+j2} = \frac{1}{1+j2} = \frac{1\angle 0°}{\sqrt{5}\angle 63.43°} = 0.4472\angle -63.43° \text{ A}$$

$$I_2 = \frac{j-(1+j)}{1+j2} = \frac{-1}{1+j2} = \frac{1\angle 180°}{\sqrt{5}\angle 63.43°} = 0.4472\angle 116.57° \text{ A}$$

➢ **EVALUATE the solution and check for accuracy.**
Use KVL to check the solution.

The equation produced by KVL of the left loop is
$$-10 + 10I_1 + j10I_1 - j10I_2 = 0$$

The equation produced by KVL of the right loop is
$$10 - j10I_1 + j10I_2 + 10I_2 = 0$$

Inserting the values for I_1 and I_2 results in valid equations. Thus, our check for accuracy was successful.

➢ **Has the problem been solved SATISFACTORILY? If so, present the solution; if not, then return to "ALTERNATIVE solutions" and continue through the process again.**
This problem has been solved satisfactorily.

$$I_1 = \textbf{0.4472}\angle \textbf{-63.43° A} \qquad\qquad I_2 = \textbf{0.4472}\angle \textbf{116.57° A}$$

Problem 13.2   *[13.1]*   For the three coupled coils in Figure 13.1, calculate the total inductance.
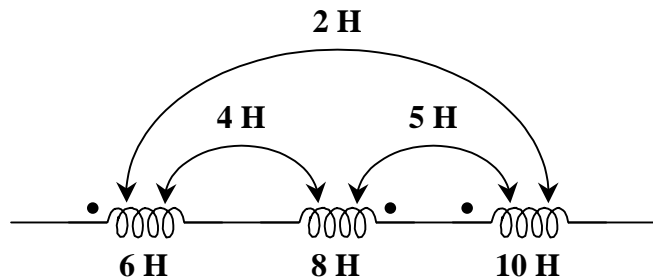
Figure 13.1

For coil 1, $L_1 - M_{12} + M_{13} = 6 - 4 + 2 = 4$

For coil 2, $L_2 - M_{21} - M_{23} = 8 - 4 - 5 = -1$

For coil 3, $L_3 + M_{31} - M_{32} = 10 + 2 - 5 = 7$

$$L_T = 4 - 1 + 7 = \underline{\mathbf{10 \ H}}$$

or

$$L_T = L_1 + L_2 + L_3 - 2M_{12} - 2M_{23} + 2M_{12}$$
$$L_T = 6 + 8 + 10 - (2)(4) - (2)(5) + (2)(2)$$
$$L_T = 6 + 8 + 10 - 8 - 10 + 4 = \underline{\mathbf{10 \ H}}$$

Problem 13.3   For the frequency domain circuit shown in Figure 13.1, determine the value of $v_{out}(t)$ for $v_{in}(t) = 10\cos(377t)$ and a coupling coefficient $k = 0.8$.
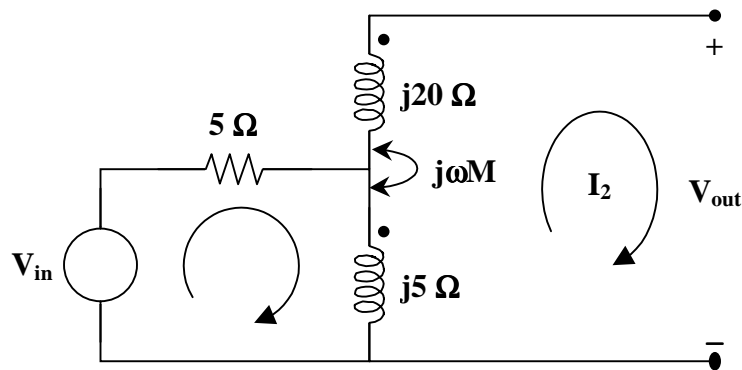

Figure 13.1

Before an equivalent circuit can be drawn, we must determine the value of ωM. Using $k = 0.8$,

$$k = \frac{M}{\sqrt{L_1 L_2}}.$$

Because the circuit is in the frequency domain rather than the time domain, we know the value of ωL rather than the value of L. So, transform the equation for k to include ω. Then,

$$k = \frac{\omega M}{\sqrt{(\omega L_1)(\omega L_2)}}.$$

Hence,

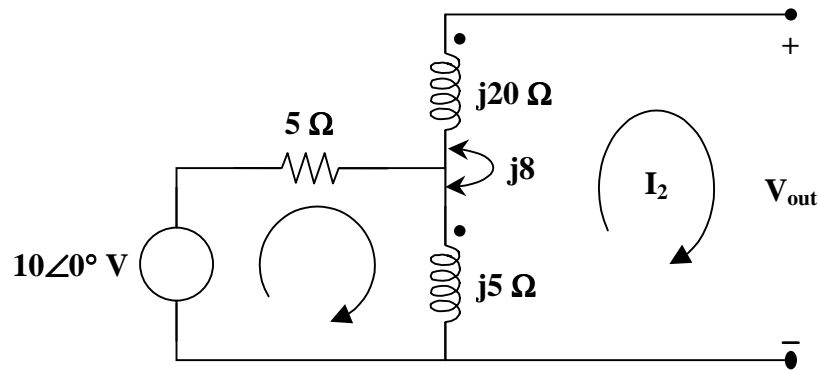$$\omega M = k\sqrt{(\omega L_1)(\omega L_2)} = (0.8)\sqrt{(5)(20)} = 8$$

We also need to transform the voltage source from the time domain to the frequency domain. Let's assume a reference of
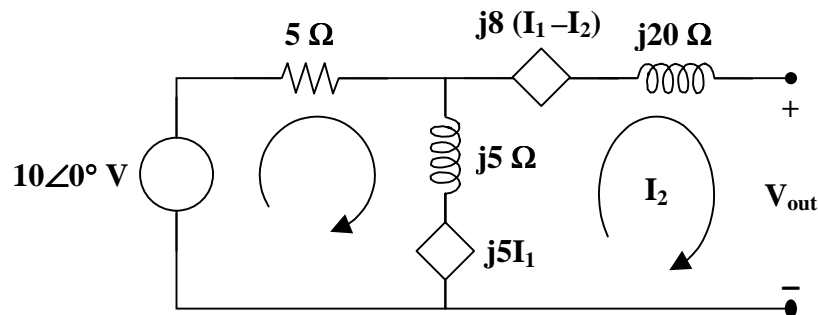
$$A\cos(377t + \phi).$$

Then,

$$V_{in} = 10\angle 0°.$$

The circuit can be redrawn as

Using the dot convention, we can draw an equivalent circuit to incorporate the induced voltages from the coupling effects.

With this circuit, we can use mesh analysis to find the answer in the frequency domain.

Loop #1 : $-10 + 5I_1 + j5(I_1 - I_2) - j8I_2 = 0$

Loop #2 : $j8I_2 + j5(I_2 - I_1) - j8(I_1 - I_2) + j20I_2 + V_{out} = 0$

This is a system of two equations and three unknowns. We need a constraint equation. Due to the open circuit, it is obvious that $I_2 = 0$.

Combining like terms and introducing the constraint, the equations become

$$(5 + j5)I_1 = 10$$

and

$$V_{out} = j13I_1$$

Clearly,

$$I_1 = \frac{10}{5 + j5} = \frac{10\angle 0°}{5\sqrt{2}\angle 45°} = \sqrt{2}\angle -45°$$

and

$$V_{out} = j13I_1 = (13\angle 90°)(\sqrt{2}\angle -45°) = 13\sqrt{2}\angle 45°$$

Using the reference assumed above, the voltage converts to the time domain as

$$v_{out}(t) = \mathbf{13\sqrt{2}\cos(377t + 45°)\ V}$$

Problem 13.4   Given the circuit in Figure 13.1, find the coupling coefficient, k, and the voltage across the 1-Ω resistor.

Figure 13.1
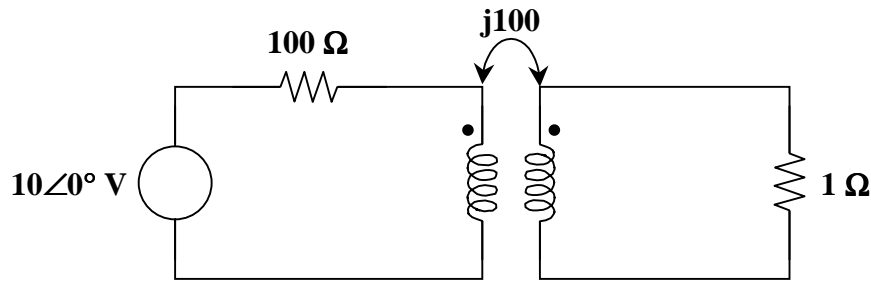
$$k = \underline{\mathbf{1}} \qquad\qquad V_{1\Omega} = \underline{\mathbf{0.1\angle0° \, V}}$$

ENERGY IN A COUPLED CIRCUIT

Problem 13.5   Given the circuit in Figure 13.1, $V_1 = V_2 = 10 \, \text{volts}$, $R_1 = R_2 = 10 \, \text{ohms}$, $\omega L_1 = \omega L_2 = 10$, and $\omega M = 5$, find the coupling coefficient, k, the currents in the primary and secondary circuits, I_1 and I_2, and the power absorbed.
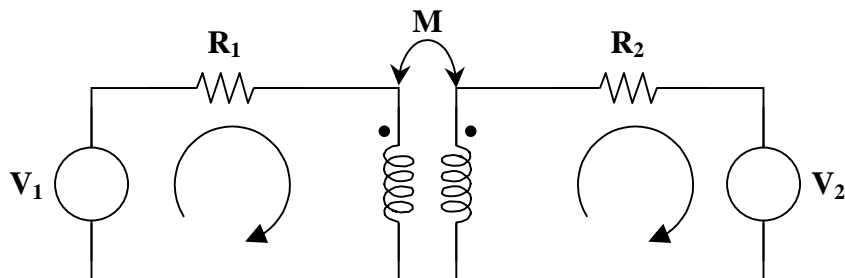

Figure 13.1

The coupling coefficient is $k = \dfrac{M}{\sqrt{L_1 L_2}}$.

Given values for ωM, ωL_1, and ωL_2, we need to modify the equation for k to be

$$k = \frac{\omega M}{\sqrt{(\omega L_1)(\omega L_2)}} \, .$$

$$k = \frac{5}{\sqrt{(10)(10)}} = \frac{5}{10} = \underline{\mathbf{0.5}}$$

To find the currents, begin by finding an equivalent circuit that takes into account the coupling effects, i.e., the induced voltages.

Use mesh analysis to find I_1 and I_2.

Loop 1 : $\qquad 10 = (10 + j10) I_1 - j5 I_2$

Loop 2 : $\qquad -10 = -j5 I_1 + (10 + j10) I_2$

In matrix form,

$$\begin{bmatrix} 10 + j10 & -j5 \\ -j5 & 10 + j10 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 10 \\ -10 \end{bmatrix}$$

where $\Delta = (10 + j10)(10 + j10) - (-j5)(-j5) = j200 + 25 = (25)(1 + j8)$.

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \frac{\begin{bmatrix} 10 + j10 & j5 \\ j5 & 10 + j10 \end{bmatrix}}{(25)(1 + j8)} \begin{bmatrix} 10 \\ -10 \end{bmatrix}$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} \dfrac{10 + j10}{(25)(1 + j8)} & \dfrac{j5}{(25)(1 + j8)} \\ \dfrac{j5}{(25)(1 + j8)} & \dfrac{10 + j10}{(25)(1 + j8)} \end{bmatrix} \begin{bmatrix} 10 \\ -10 \end{bmatrix} = \begin{bmatrix} \dfrac{100 + j100 - j50}{(25)(1 + j8)} \\ \dfrac{j50 - 100 - j100}{(25)(1 + j8)} \end{bmatrix}$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} \dfrac{(50)(2 + j)}{(25)(1 + j8)} \\ \dfrac{(50)(-2 - j)}{(25)(1 + j8)} \end{bmatrix} = \begin{bmatrix} \dfrac{4 + j2}{1 + j8} \\ \dfrac{-4 - j2}{1 + j8} \end{bmatrix}$$

Thus,

$$I_1 = \frac{4 + j2}{1 + j8} = \frac{4.4721\angle 26.57°}{8.0623\angle 82.88°} = \underline{\mathbf{0.5547\angle - 56.31° \ A}}$$

$$I_2 = \frac{-4 - j2}{1 + j8} = \frac{4.4721\angle -153.43°}{8.0623\angle 82.88°} = \underline{\mathbf{0.5547\angle 123.69° \ A}}$$

Now, find the power absorbed in the circuit. Look at the power absorbed by each element.

Starting with the primary circuit,

$$p_{V1} = -V I \cos\theta = -(10)(0.5547)\cos(0° - (-56.31°)) = -3.0769 \text{ W}$$

$$p_{R1} = |I_1|^2 R_1 = (0.5547)^2(10) = 3.0769 \text{ W}$$

$$p_1 = -(2.7735)(0.5547)\cos(213.69° - (-56.31)°) = 1.5385\cos(270°) = 0 \text{ W}$$

where p_1 is the power absorbed by the induced voltage of L_1.

Ending with the secondary circuit,

$$p_{V2} = V I \cos\theta = (10)(0.5547)\cos(0° - 123.69°) = -3.0769 \text{ W}$$

$$p_{R2} = |I_2|^2 R_2 = (0.5547)^2(10) = 3.0769 \text{ W}$$

$$p_2 = -(2.7735)(0.5547)\cos(33.69° - 123.69°) = 1.5385\cos(90°) = 0 \text{ W}$$

where p_2 is the power absorbed by the induced voltage of L_2.

The voltage sources absorb –3.0769 watts, or deliver +3.0769 watts, the resistances absorb 3.0769 watts, and the induced voltages absorb 0 watts. The inductors do not absorb power.

Problem 13.6   *[13.13]*   Determine the currents I_1, I_2, and I_3 in the circuit of Figure 13.1. Find the energy stored in the coupled coils at $t = 2$ ms. Take $\omega = 1000$ rad/s.
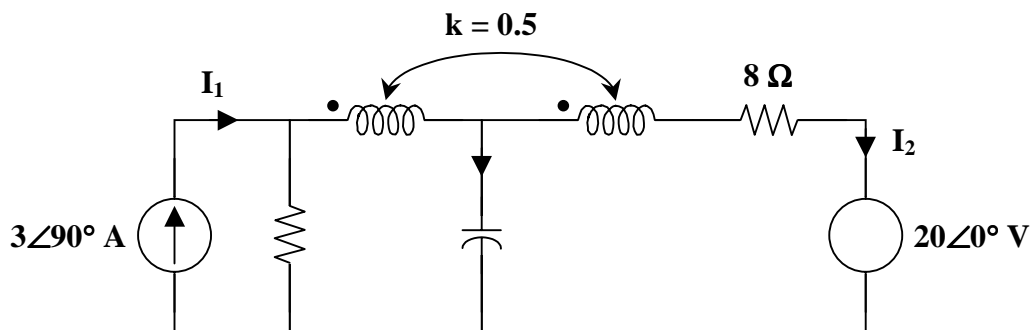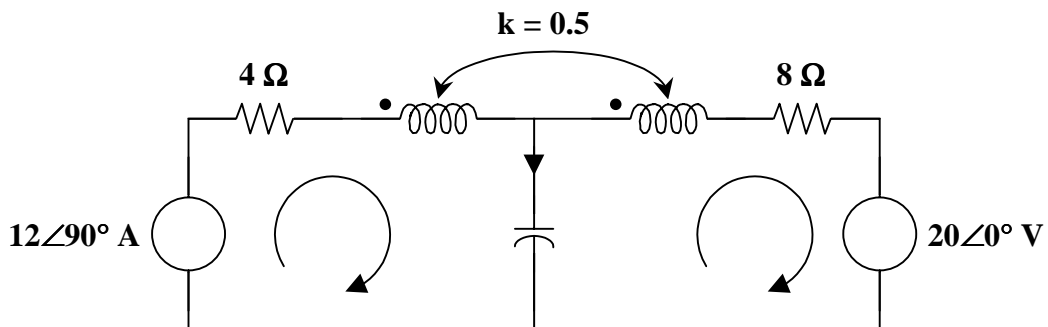

Figure 13.1

Transform the current source to a voltage source as shown below.

$$k = \frac{M}{\sqrt{L_1 L_2}} \qquad \text{or} \qquad M = \frac{k}{\sqrt{L_1 L_2}}$$

$$\omega M = k\sqrt{(\omega L_1)(\omega L_2)} = (0.5)(10) = 5$$

Using mesh analysis,

Mesh 1, $\qquad j12 = (4 + j10 - j5)I_1 + j5I_2 + j5I_2 = (4 + j5)I_1 + j10I_2$ $\qquad$ (1)

Mesh 2, $\qquad 0 = 20 + (8 + j10 - j5)I_2 + j5I_1 + j5I_1$

$\qquad\qquad -20 = j10I_1 + (8 + j5)I_2$ $\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ (2)

From (1) and (2),

$$\begin{bmatrix} j12 \\ -20 \end{bmatrix} = \begin{bmatrix} 4 + j5 & j10 \\ j10 & 8 + j5 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\Delta = 107 + j60, \qquad \Delta_1 = -60 - j296, \qquad \Delta_2 = 40 - j100$$

$$I_1 = \frac{\Delta_1}{\Delta} = \mathbf{2.462\angle 72.18° \ A}$$

$$I_2 = \frac{\Delta_2}{\Delta} = \mathbf{0.878\angle -97.48° \ A}$$

$$I_3 = I_1 - I_2 = \mathbf{3.329\angle 74.89° \ A}$$

$$i_1(t) = 2.462\cos(1000t + 72.18°) \ A$$
$$i_2(t) = 0.878\cos(1000t - 97.48°) \ A$$

At $t = 2$ ms, $\qquad\qquad 1000t = 2$ rad $= 114.6°$

$$i_1(2 \text{ ms}) = 2.462\cos(114.6° + 72.18°) = -2.445$$
$$i_2(2 \text{ ms}) = 0.878\cos(114.6° - 97.48°) = 0.8391$$

The total energy stored in the coupled coils is
$$w = 0.5L_1 i_1^2 + 0.5L_2 i_2^2 + M i_1 i_2$$

Since $\omega L_1 = 10$ and $\omega = 1000$,

$$L_1 = L_2 = 10 \text{ mH}, \qquad M = 0.5L_1 = 5 \text{ mH}$$

$$w = (0.5)(10)(-2.445)^2 + (0.5)(10)(0.8391)^2 + (5)(-2.445)(0.8391)$$
$$w = \mathbf{23.15 \ mJ}$$

Problem 13.7 Given the circuit in Figure 13.1, $V_1 = V_2 = 10$ volts, $R_1 = R_2 = 10$ ohms, $\omega L_1 = \omega L_2 = 10$, and $\omega M = 5$, find the coupling coefficient, k, the currents in the primary and secondary circuits, I_1 and I_2, and the power absorbed.
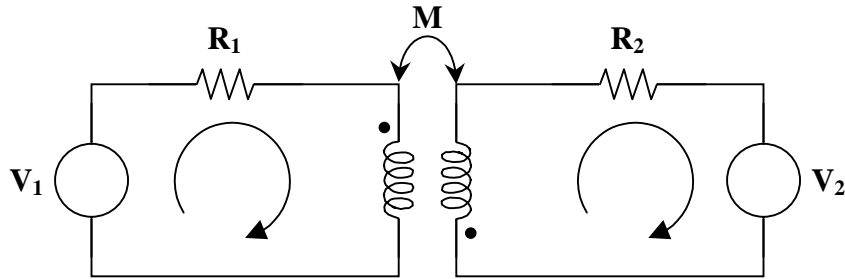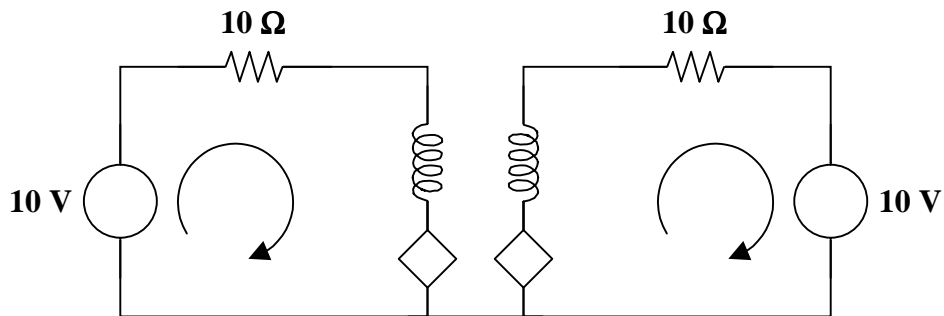

Figure 13.1

As seen in Problem 13.5,

$$k = \frac{\omega M}{\sqrt{(\omega L_1)(\omega L_2)}} = \frac{5}{\sqrt{(10)(10)}} = \frac{5}{10} = \underline{\mathbf{0.5}}$$

To find the currents, begin by finding an equivalent circuit which takes into account the coupling effects, i.e., the induced voltages.

Use mesh analysis to find I_1 and I_2.

Loop 1 : $\qquad 10 = (10 + j10) I_1 + j5 I_2$

Loop 2 : $\qquad -10 = j5 I_1 + (10 + j10) I_2$

In matrix form,

$$\begin{bmatrix} 10 + j10 & j5 \\ j5 & 10 + j10 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 10 \\ -10 \end{bmatrix}$$

where $\Delta = (10 + j10)(10 + j10) - (j5)(j5) = j200 + 25 = (25)(1 + j8)$.

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \dfrac{\begin{bmatrix} 10 + j10 & -j5 \\ -j5 & 10 + j10 \end{bmatrix}}{(25)(1 + j8)} \begin{bmatrix} 10 \\ -10 \end{bmatrix}$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} \dfrac{10 + j10}{(25)(1 + j8)} & \dfrac{-j5}{(25)(1 + j8)} \\ \dfrac{-j5}{(25)(1 + j8)} & \dfrac{10 + j10}{(25)(1 + j8)} \end{bmatrix} \begin{bmatrix} 10 \\ -10 \end{bmatrix} = \begin{bmatrix} \dfrac{100 + j100 + j50}{(25)(1 + j8)} \\ \dfrac{-j50 - 100 - j100}{(25)(1 + j8)} \end{bmatrix}$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} \dfrac{(50)(2 + j3)}{(25)(1 + j8)} \\ \dfrac{(50)(-2 - j3)}{(25)(1 + j8)} \end{bmatrix} = \begin{bmatrix} \dfrac{4 + j6}{1 + j8} \\ \dfrac{-4 - j6}{1 + j8} \end{bmatrix}$$

Thus,

$$I_1 = \frac{4 + j6}{1 + j8} = \frac{7.2111\angle 56.31°}{8.0623\angle 82.88°} = \textbf{0.8944}\angle\textbf{- 26.57° A}$$

$$I_2 = \frac{-4 - j6}{1 + j8} = \frac{7.2111\angle -123.69°}{8.0623\angle 82.88°} = \textbf{0.8944}\angle\textbf{153.43° A}$$

Now, find the power absorbed in the circuit. Look at the power absorbed by each element.

Starting with the primary circuit,

$$p_{V1} = -V I \cos\theta = -(10)(0.8944)\cos(0° - (-26.57°)) = -7.9994 \text{ W}$$

$$p_{R1} = |I_1|^2 R_1 = (0.8944)^2(10) = 7.9995 \text{ W}$$

$$p_1 = (4.4720)(0.8944)\cos(243.43° - (-26.57°)) = 3.9998\cos(270°) = 0 \text{ W}$$

where p_1 is the power absorbed by the induced voltage of L_1.

Ending with the secondary circuit,

$$p_{V2} = V I \cos\theta = (10)(0.8944)\cos(0 - 153.43°) = -7.9994 \text{ W}$$

$$p_{R2} = |I_2|^2 R_2 = (0.8944)^2(10) = 7.9995 \text{ W}$$

$$p_2 = (4.4720)(0.8944)\cos(63.43° - 153.43°) = 3.9998\cos(-90°) = 0 \text{ W}$$

where p_2 is the power absorbed by the induced voltage of L_2.

The voltage sources absorb –7.9994 watts, or deliver +7.9994 watts, the resistances absorb 7.9995 watts, and the induced voltages absorb 0 watts. The inductors do not absorb power.

Problem 13.8   Given the circuit in Figure 13.1, $V_1 = V_2 = 10$ volts, $R_1 = R_2 = 10$ ohms, $\omega L_1 = \omega L_2 = 10$, and $\omega M = 5$, find the coupling coefficient, k, the currents in the primary and secondary circuits, I_1 and I_2, and the power absorbed.

Figure 13.1

$$k = \underline{\textbf{0.5}}$$

$$I_1 = \underline{\textbf{0.5547}\angle\textbf{- 56.31° A}}$$

$$I_2 = \underline{\textbf{0.5547}\angle\textbf{123.69° A}}$$

The voltage sources absorb –3.0769 watts, or deliver +3.0769 watts, the resistances absorb 3.0769 watts, and the induced voltages absorb 0 watts. The inductors do not absorb power.

Problem 13.9   Given the circuit in Figure 13.1, $V_1 = V_2 = 10$ volts, $R_1 = R_2 = 10$ ohms, $\omega L_1 = \omega L_2 = 10$, and $\omega M = 5$, find the coupling coefficient, k, the currents in the primary and secondary circuits, I_1 and I_2, and the power absorbed.
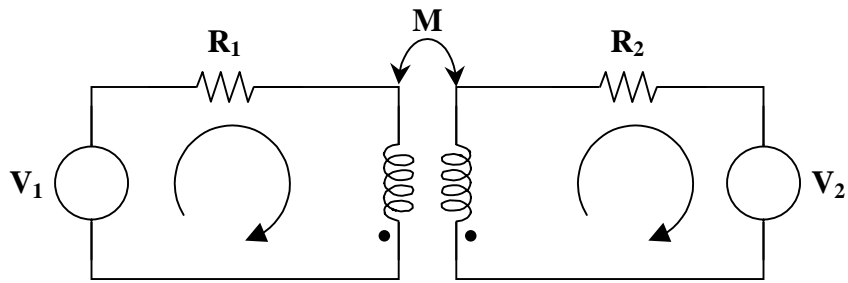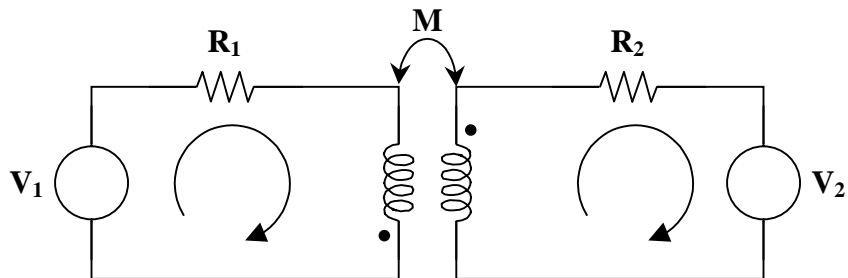

Figure 13.1

$$k = \underline{\textbf{0.5}}$$

$$I_1 = \underline{\textbf{0.8944}\angle\textbf{- 26.57° A}}$$

$$I_2 = \underline{\textbf{0.8944}\angle\textbf{153.43° A}}$$

The voltage sources absorb –7.9994 watts, or deliver +7.9994 watts, the resistances absorb 7.9995 watts, and the induced voltages absorb 0 watts. The inductors do not absorb power.

LINEAR TRANSFORMERS

Problem 13.10 *[13.23]* For the circuit in Figure 13.1, find :
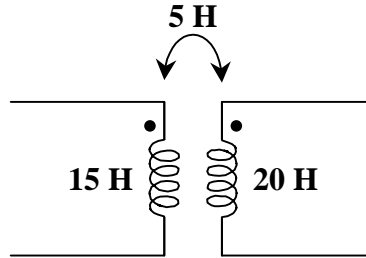(a) the T-equivalent circuit,
(b) the Π-equivalent circuit.

Figure 13.1

(a) $L_a = L_1 - M = \underline{\textbf{10 H}}$

$L_b = L_2 - M = \underline{\textbf{15 H}}$

$L_c = M = \underline{\textbf{5 H}}$

(b) $L_1 L_2 - M^2 = 300 - 25 = 275$

$L_A = \dfrac{L_1 L_2 - M^2}{L_2 - M} = \dfrac{275}{15} = \underline{\textbf{18.33 H}}$

$L_B = \dfrac{L_1 L_2 - M^2}{L_1 - M} = \underline{\textbf{27.5 H}}$

$L_C = \dfrac{L_1 L_2 - M^2}{M} = \dfrac{275}{5} = \underline{\textbf{55 H}}$

IDEAL TRANSFORMERS

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Problem 13.11   Given the ideal transformer circuit in Figure 13.1, find $V_{10\Omega}$.
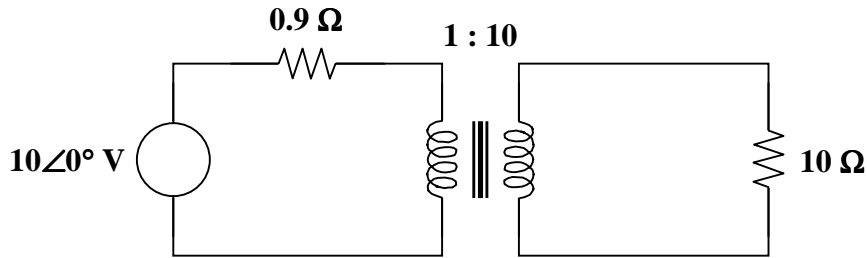

Figure 13.1

➢ **Carefully DEFINE the problem.**
   Each component is labeled completely. The problem is clear.

➢ **PRESENT everything you know about the problem.**
   We know the values of the independent source. We also know the values of all the elements.

➢ **Establish a set of ALTERNATIVE solutions and determine the one that promises the greatest likelihood of success.**
   The three solution techniques that can be used are nodal analysis, mesh analysis, and basic circuit analysis. Using either nodal or mesh analysis will produce two equations with four unknowns. In either case, we need to find two constraint equations. So, let's use mesh analysis for the initial attempt to find a solution. Then, nodal analysis will be used to check the solution.

➢ **ATTEMPT a problem solution.**
   We need to make some assumptions. First, assume that a positive voltage for the inductor in the primary circuit yields a positive voltage for the inductor in the secondary circuit. Second, assume that a positive (or clockwise) current in the primary circuit yields a positive (or clockwise) current in the secondary circuit. The assumptions are shown in the following circuit.
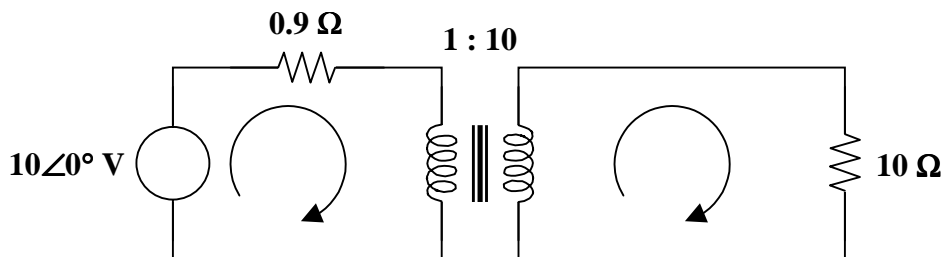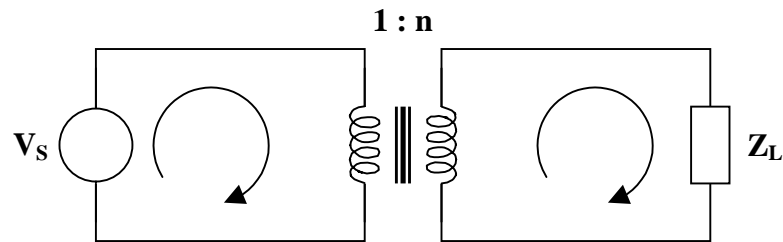
Mesh analysis yields,

Loop 1 : $\quad 10 = 0.9I_1 + V_1$

Loop 2 : $\quad V_2 = 10I_2$

This is a set of two equations and four unknowns. Two constraint equations are needed.

From the ideal transformer, as shown, we know that $V_2 = nV_1$ and $I_1 = nI_2$.

1 : n

This implies that
$$V_2 = 10V_1 \qquad \text{and} \qquad I_1 = 10I_2$$
which are the two constraint equations.

There are many ways to find the values of V_1, V_2, I_1, and I_2. Let's find I_1. To do this, find V_1 in terms of I_1 and substitute into the equation for loop 1.

$$V_1 = 0.1V_2 = (0.1)(10I_2) = I_2 = 0.1I_1$$

and the equation for loop 1 becomes
$$10 = 0.9I_1 + 0.1I_1$$
$$10 = I_1$$

Hence,
$$I_1 = 10 \text{ A} \qquad\qquad V_1 = 1 \text{ V}$$
$$I_2 = 1 \text{ A} \qquad\qquad V_2 = 10 \text{ V}$$

Therefore,
$$V_{10\Omega} = V_2 = 10 \text{ V}$$

➢ **EVALUATE the solution and check for accuracy.**
Using nodal analysis,

At node 1, $\quad \dfrac{V_1 - 10}{0.9} + I_1 = 0 \qquad$ or $\qquad V_1 - 10 + 0.9I_1 = 0 \qquad\qquad$ (1)

At node 2, $\quad \dfrac{V_2 - 0}{10} + (-I_2) = 0 \qquad$ or $\qquad V_2 - 10I_2 = 0 \qquad\qquad$ (2)

Again, this is a set of two equations and four unknowns. Two constraint equations are needed. From the ideal transformer, we have

$$V_2 = 10V_1 \qquad\qquad (3)$$
$$I_1 = 10I_2 \qquad\qquad (4)$$

From (2) and (4),
$$V_2 = 10I_2 = (10)(1/10)I_1 = I_1 \qquad\qquad (5)$$

From (1), (3), and (5),
$$V_1 - 10 + 0.9\,V_2 = V_1 - 10 + (0.9)(10)\,V_1 = (1+9)\,V_1 - 10 = 0$$
$$10\,V_1 = 10 \quad\longrightarrow\quad V_1 = 1\text{ V}$$

Then, $\quad V_2 = 10\,V_1 = 10\text{ V} \qquad I_1 = V_2 = 10\text{ A} \qquad$ and $\qquad I_2 = (1/10)\,I_1 = 1\text{ A}$

Our check for accuracy was successful.

➢ **Has the problem been solved SATISFACTORILY? If so, present the solution; if not, then return to "ALTERNATIVE solutions" and continue through the process again.**
This problem has been solved satisfactorily.

$$V_{10\Omega} = \underline{\textbf{10 V}}$$

Problem 13.12   *[13.33]*   For the circuit in Figure 13.1, find V_o. Switch the dot on the secondary side and find V_o again.
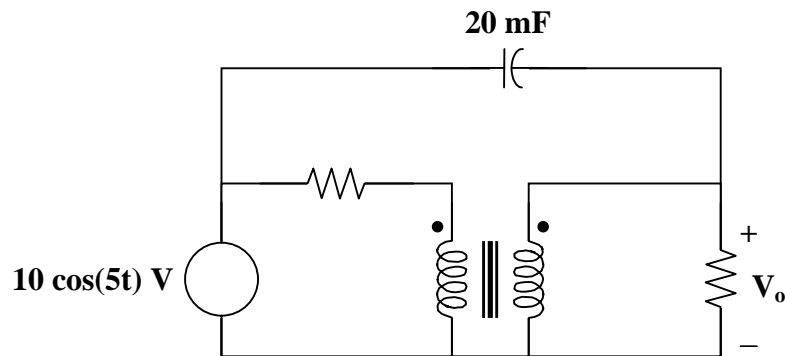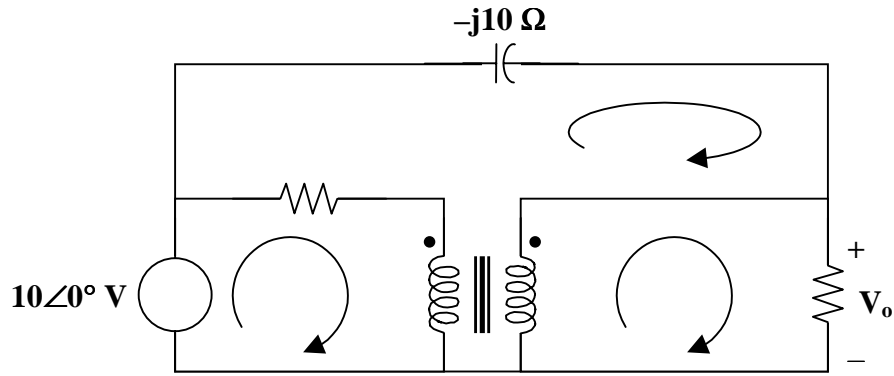

Figure 13.1

$$C = 0.02\text{ F becomes } \frac{1}{j\omega C} = \frac{1}{j(5)(0.02)} = -j10$$

We apply mesh analysis to the circuit shown below.

Using mesh analysis,

For mesh 1, $\qquad$ $10 = 10I_1 - 10I_3 + V_1$ $\qquad$ (1)

For mesh 2, $\qquad$ $V_2 = 2I_2 = V_o$ $\qquad$ (2)

For mesh 3, $\qquad$ $0 = (10 - j10)I_3 - 10I_1 + V_2 - V_1$ $\qquad$ (3)

At the terminals, $\qquad$ $V_2 = nV_1 = V_1/3$ $\qquad$ (4)

$\qquad\qquad\qquad\quad$ $I_1 = nI_2 = I_2/3$ $\qquad$ (5)

From (2) and (4),

$\qquad\qquad\qquad\quad$ $V_1 = 6I_2$ $\qquad$ (6)

Substituting this into (1),

$\qquad\qquad\qquad\quad$ $10 = 10I_1 - 10I_3$ $\qquad$ (7)

Substituting (4) and (6) into (3) yields

$\qquad\qquad\qquad\quad$ $0 = -10I_1 - 4I_2 + (10)(1 - j)I_3$ $\qquad$ (8)

From (5), (7), and (8),

$$\begin{bmatrix} 1 & -0.333 & 0 \\ 10 & 6 & -10 \\ -10 & -4 & 10 - j10 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 10 \\ 0 \end{bmatrix}$$

$$I_2 = \frac{\Delta_2}{\Delta} = \frac{100 - j100}{-20 - j93.33} = 1.482\angle 32.9° \text{ A}$$

$$V_o = 2I_2 = \underline{\mathbf{2.963\angle 32.9° \text{ V}}}$$

Switching the dot on the secondary side affects only equations (4) and (5).

$$V_2 = -V_1/3 \qquad (9)$$

$$I_1 = -I_2/3 \tag{10}$$

From (2) and (9),
$$V_1 = -6I_2$$

Substituting this into (1),
$$10 = 10I_1 - 10I_3 - 6I_2 = (23 - j5)I_1 \tag{11}$$

Substituting (9) and (10) into (3),
$$0 = -10I_1 + 4I_2 + (10)(1-j)I_3 \tag{12}$$

From (10) to (12), we get

$$\begin{bmatrix} 1 & 0.333 & 0 \\ 10 & -6 & -10 \\ -10 & 4 & 10-j10 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 10 \\ 0 \end{bmatrix}$$

$$I_2 = \frac{\Delta_2}{\Delta} = \frac{100 - j100}{-20 + j93.33} = 1.482\angle -147.1° \text{ A}$$

$$V_o = 2I_2 = \mathbf{2.963\angle \text{-}147.1° \text{ V}}$$

THREE-PHASE TRANSFORMERS

Problem 13.13 *[13.53]* In order to meet an emergency, three single-phase transformers with $12,470/7200$ V rms are connected in $\Delta - Y$ to form a three-phase transformer which is fed by a 12,470-V transmission line. If the transformer supplies 60 MVA to a load, find :

      (a)   the turns ratio for each transformer,
      (b)   the currents in the primary and secondary windings of the transformer,
      (c)   the incoming and outgoing transmission line currents.

      (a)   Consider just one phase at a time.
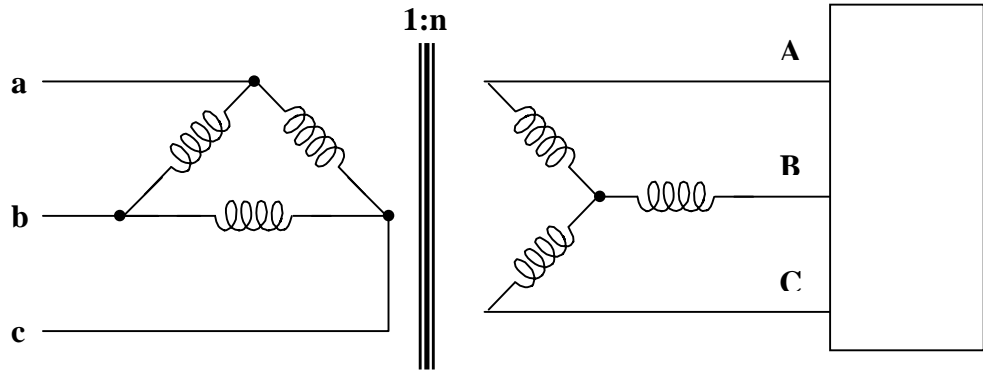
$$n = \frac{V_L}{\sqrt{3}V_{Lp}} = \frac{7200}{12470\sqrt{3}} = \underline{\frac{1}{3}}$$

(b)   The load carried by each transformer is $60/3 = 20$ MVA

Hence, $I_{Lp} = \dfrac{20 \text{ MVA}}{12.47 \text{ kV}} = \underline{\textbf{1,604 A}}$

$I_{Ls} = \dfrac{20 \text{ MVA}}{7.2 \text{kV}} = \underline{\textbf{2,778 A}}$

(c)   The current in each incoming line a, b, c is

$$\sqrt{3}\,I_{Lp} = \sqrt{3} \times 1603.85 = \underline{\textbf{2,778 A}}$$

The current in each outgoing line A, B, C is
$$\frac{2778}{n\sqrt{3}} = \underline{\textbf{4,812 A}}$$

Problem 13.14 *[13.63]* Use PSpice to find V_1, V_2, and I_o in the circuit in Figure 13.1.
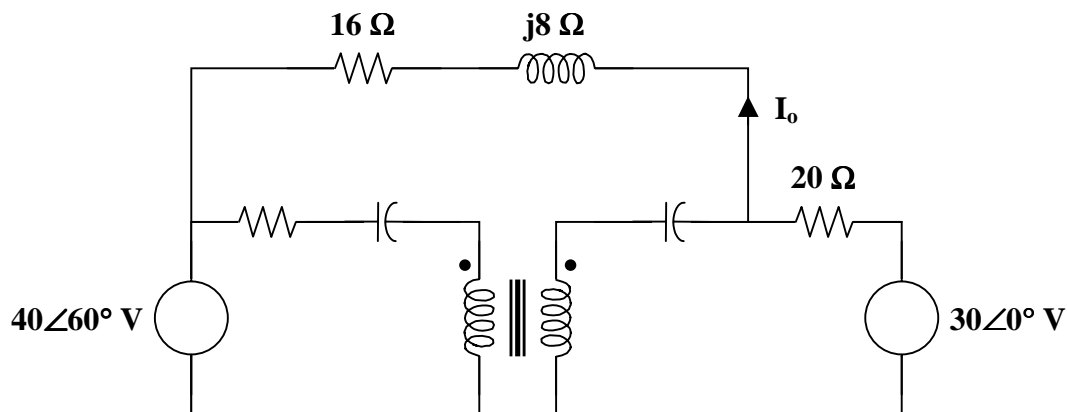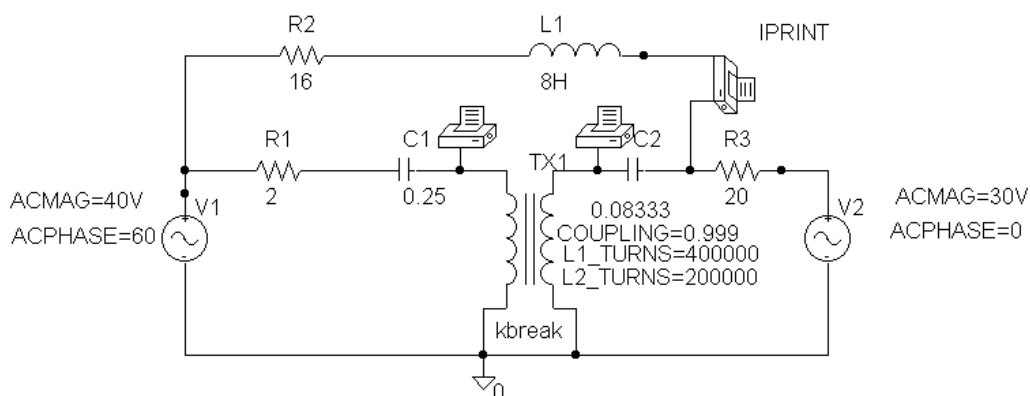

Figure 13.1

The schematic is shown below.

In the AC Sweep box, we type Total Pts $=1$, Start Freq $=0.1592$, and End Freq $=0.1592$. After simulation, we obtain the output file which includes

FREQ	IM(V_PRINT1)	IP(V_PRINT1)
1.592 E–01	1.955 E+01	8.332 E+01

FREQ	IM(V_PRINT2)	IP(V_PRINT2)
1.592 E–01	6.847 E+01	4.640 E+01

FREQ	IM(V_PRINT3)	IP(V_PRINT3)
1.592 E–01	4.434 E–01	–9.260 E+01

Thus,

$$V_1 = \mathbf{19.55 \angle 83.32° \ V} \qquad\qquad V_2 = \mathbf{68.47 \angle 46.4° \ V}$$

$$I_o = \mathbf{443.4 \angle - 92.6° \ mA}$$

APPLICATIONS

Problem 13.15   *[13.73]*   A 4800-V rms transmission line feeds a distribution transformer with 1200 turns on the primary and 28 turns on the secondary. When a 10-Ω load is connected across the secondary, find :

    (a)   the secondary voltage,
    (b)   the primary and secondary currents,
    (c)   the power supplied to the load.

(a) $\qquad \dfrac{V_2}{V_1} = \dfrac{N_2}{N_1} = n$

$\qquad\qquad V_2 = \dfrac{N_2}{N_1} V_1 = \left(\dfrac{28}{1200}\right)(4800) = \underline{\mathbf{112 \ V}}$

(b) $\qquad I_2 = \dfrac{V_2}{R} = \dfrac{112}{10} = \underline{\mathbf{11.2 \ A}}$

$\qquad\qquad I_1 = n\, I_2, \qquad \text{where} \quad n = 28/1200$

$\qquad\qquad I_1 = \left(\dfrac{28}{1200}\right)(11.2) = \underline{\mathbf{261.3 \ mA}}$

(c) $\qquad p = \left| I_2 \right|^2 R = (11.2)^2 (10) = \underline{\mathbf{1254 \ W}}$