

Appendix C

Mathematical Formulas

This appendix—by no means exhaustive—serves as a handy reference. It does contain all the formulas needed to solve circuit problems in this book.

C.1 Quadratic Formula

The roots of the quadratic equation $ax^2 + bx + c = 0$ are

$$x_1, x_2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

C.2 Trigonometric Identities

$$\sin(-x) = -\sin x$$

$$\cos(-x) = \cos x$$

$$\sec x = \frac{1}{\cos x}, \quad \csc x = \frac{1}{\sin x}$$

$$\tan x = \frac{\sin x}{\cos x}, \quad \cot x = \frac{1}{\tan x}$$

$$\sin(x \pm 90^\circ) = \pm \cos x$$

$$\cos(x \pm 90^\circ) = \mp \sin x$$

$$\sin(x \pm 180^\circ) = -\sin x$$

$$\cos(x \pm 180^\circ) = -\cos x$$

$$\cos^2 x + \sin^2 x = 1$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \quad (\text{law of sines})$$

$$a^2 = b^2 + c^2 - 2bc \cos A \quad (\text{law of cosines})$$

$$\frac{\tan \frac{1}{2}(A - B)}{\tan \frac{1}{2}(A + B)} = \frac{a - b}{a + b} \quad (\text{law of tangents})$$

$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$$

$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$

$$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$$

$$2 \sin x \sin y = \cos(x - y) - \cos(x + y)$$

$$2 \sin x \cos y = \sin(x + y) + \sin(x - y)$$

$$2 \cos x \cos y = \cos(x + y) + \cos(x - y)$$

$$\sin 2x = 2 \sin x \cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x = 2 \cos^2 x - 1 = 1 - 2 \sin^2 x$$

$$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$$

$$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$$

$$\cos^2 x = \frac{1}{2}(1 + \cos 2x)$$

$$K_1 \cos x + K_2 \sin x = \sqrt{K_1^2 + K_2^2} \cos \left(x + \tan^{-1} \frac{-K_2}{K_1} \right)$$

$$e^{jx} = \cos x + j \sin x \quad (\text{Euler's formula})$$

$$\cos x = \frac{e^{jx} + e^{-jx}}{2}$$

$$\sin x = \frac{e^{jx} - e^{-jx}}{2j}$$

$$1 \text{ rad} = 57.296^\circ$$

C.3 Hyperbolic Functions

$$\sinh x = \frac{1}{2}(e^x - e^{-x})$$

$$\cosh x = \frac{1}{2}(e^x + e^{-x})$$

$$\tanh x = \frac{\sinh x}{\cosh x}$$

$$\coth x = \frac{1}{\tanh x}$$

$$\operatorname{csch} x = \frac{1}{\sinh x}$$

$$\operatorname{sech} x = \frac{1}{\cosh x}$$

$$\sinh(x \pm y) = \sinh x \cosh y \pm \cosh x \sinh y$$

$$\cosh(x \pm y) = \cosh x \cosh y \pm \sinh x \sinh y$$

C.4 Derivatives

If $U = U(x)$, $V = V(x)$, and $a = \text{constant}$,

$$\frac{d}{dx}(aU) = a \frac{dU}{dx}$$

$$\frac{d}{dx}(UV) = U \frac{dV}{dx} + V \frac{dU}{dx}$$

$$\frac{d}{dx} \left(\frac{U}{V} \right) = \frac{V \frac{dU}{dx} - U \frac{dV}{dx}}{V^2}$$

$$\frac{d}{dx}(aU^n) = naU^{n-1}$$

$$\frac{d}{dx}(a^U) = a^U \ln a \frac{dU}{dx}$$

$$\frac{d}{dx}(e^U) = e^U \frac{dU}{dx}$$

$$\frac{d}{dx}(\sin U) = \cos U \frac{dU}{dx}$$

$$\frac{d}{dx}(\cos U) = -\sin U \frac{dU}{dx}$$

C.5 Indefinite Integrals

If $U = U(x)$, $V = V(x)$, and $a = \text{constant}$,

$$\int a \, dx = ax + C$$

$$\int U \, dV = UV - \int V \, dU \quad (\text{integration by parts})$$

$$\int U^n \, dU = \frac{U^{n+1}}{n+1} + C, \quad n \neq -1$$

$$\int \frac{dU}{U} = \ln U + C$$

$$\int a^U \, dU = \frac{a^U}{\ln a} + C, \quad a > 0, a \neq 1$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax} + C$$

$$\int x e^{ax} \, dx = \frac{e^{ax}}{a^2} (ax - 1) + C$$

$$\int x^2 e^{ax} \, dx = \frac{e^{ax}}{a^3} (a^2 x^2 - 2ax + 2) + C$$

$$\int \ln x \, dx = x \ln x - x + C$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax + C$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax + C$$

$$\int \sin^2 ax \, dx = \frac{x}{2} - \frac{\sin 2ax}{4a} + C$$

$$\int \cos^2 ax \, dx = \frac{x}{2} + \frac{\sin 2ax}{4a} + C$$

$$\int x \sin ax \, dx = \frac{1}{a^2} (\sin ax - ax \cos ax) + C$$

$$\int x \cos ax \, dx = \frac{1}{a^2} (\cos ax + ax \sin ax) + C$$

$$\int x^2 \sin ax \, dx = \frac{1}{a^3} (2ax \sin ax + 2 \cos ax - a^2 x^2 \cos ax) + C$$

$$\int x^2 \cos ax \, dx = \frac{1}{a^3} (2ax \cos ax - 2 \sin ax + a^2 x^2 \sin ax) + C$$

$$\int e^{ax} \sin bx \, dx = \frac{e^{ax}}{a^2 + b^2} (a \sin bx - b \cos bx) + C$$

$$\int e^{ax} \cos bx \, dx = \frac{e^{ax}}{a^2 + b^2} (a \cos bx + b \sin bx) + C$$

$$\int \sin ax \sin bx \, dx = \frac{\sin(a-b)x}{2(a-b)} - \frac{\sin(a+b)x}{2(a+b)} + C, \quad a^2 \neq b^2$$

$$\int \sin ax \cos bx \, dx = -\frac{\cos(a-b)x}{2(a-b)} - \frac{\cos(a+b)x}{2(a+b)} + C, \quad a^2 \neq b^2$$

$$\int \cos ax \cos bx \, dx = \frac{\sin(a-b)x}{2(a-b)} + \frac{\sin(a+b)x}{2(a+b)} + C, \quad a^2 \neq b^2$$

$$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + C$$

$$\int \frac{x^2 dx}{a^2 + x^2} = x - a \tan^{-1} \frac{x}{a} + C$$

$$\int \frac{dx}{(a^2 + x^2)^2} = \frac{1}{2a^2} \left(\frac{x}{x^2 + a^2} + \frac{1}{a} \tan^{-1} \frac{x}{a} \right) + C$$

C.6 Definite Integrals

If m and n are integers,

$$\int_0^{2\pi} \sin ax \, dx = 0$$

$$\int_0^{2\pi} \cos ax \, dx = 0$$

$$\int_0^{\pi} \sin^2 ax \, dx = \int_0^{\pi} \cos^2 ax \, dx = \frac{\pi}{2}$$

$$\int_0^{\pi} \sin mx \sin nx \, dx = \int_0^{\pi} \cos mx \cos nx \, dx = 0, \quad m \neq n$$

$$\int_0^{\pi} \sin mx \cos nx \, dx = \begin{cases} 0, & m+n = \text{even} \\ \frac{2m}{m^2 - n^2}, & m+n = \text{odd} \end{cases}$$

$$\int_0^{2\pi} \sin mx \sin nx \, dx = \int_{-\pi}^{\pi} \sin mx \sin nx \, dx = \begin{cases} 0, & m \neq n \\ \pi, & m = n \end{cases}$$

$$\int_0^{\infty} \frac{\sin ax}{x} \, dx = \begin{cases} \frac{\pi}{2}, & a > 0 \\ 0, & a = 0 \\ -\frac{\pi}{2}, & a < 0 \end{cases}$$

C.7 L'Hopital's Rule

If $f(0) = 0 = h(0)$, then

$$\lim_{x \rightarrow 0} \frac{f(x)}{h(x)} = \lim_{x \rightarrow 0} \frac{f'(x)}{h'(x)}$$

where the prime indicates differentiation.