CHAPTER 10 - SINUSOIDAL STEADY-STATE ANALYSIS

List of topics for this chapter :

- **Nodal Analysis**
- **Mesh Analysis**
- **Superposition Theorem**
- **Source Transformation**
- **Thevenin and Norton Equivalent Circuits**
- **AC Op Amp Circuits**

NODAL ANALYSIS

Problem 10.1 Given the circuit in Figure 10.1 and $i(t) = 5\,sin(1000t)$ amps, find $v_o(t)$ using nodal analysis.

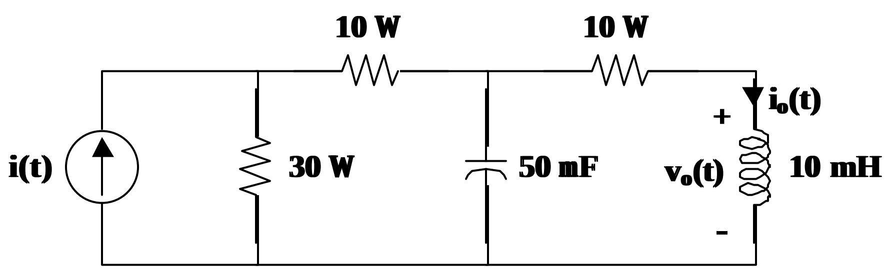


Figure 10.1

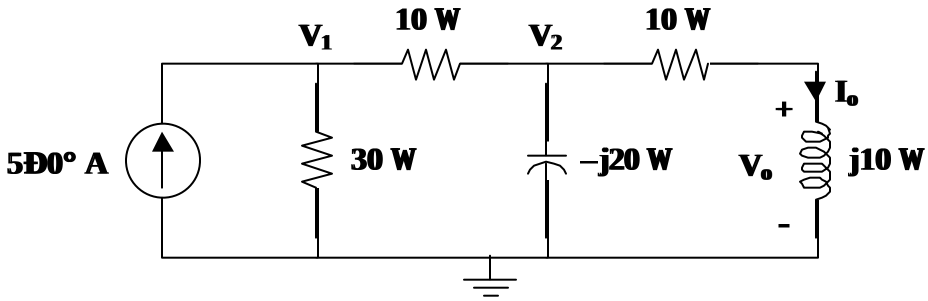
- **Carefully DEFINE the problem.**
  Each component is labeled, indicating the value and polarity. The problem is clear.

- **PRESENT everything you know about the problem.**
  The goal of the problem is to find $v_o(t)$, which is clearly labeled in Figure 10.1, using nodal analysis. Thus, we need to label the nodes and ground.

  To find $v_o(t)$ without using derivatives and integrals, we must transform the circuit to the frequency domain. This allows us to find the answer using algebra with complex numbers. We can transform the circuit to the frequency domain after setting a reference value. Let us use a reference of $A\,sin(1000t + \phi)$.

  In transforming to the frequency domain, remember that $X_L = j\omega L$ and $X_C = 1/(j\omega C)$. Hence, the inductor becomes $j\omega L = j(10^3)(10 \times 10^{-3}) = j10$ and the capacitor becomes $1/j\omega C = 1/[j(10^3)(50 \times 10^{-6})] = -j20$.

Let us draw the circuit after the transformation into the frequency domain and labeling the nodes and ground.



- **Establish a set of ALTERNATIVE solutions and determine the one that promises the greatest likelihood of success.**
  The problem clearly states that the problem be solved using nodal analysis. Thus, the technique to solve the problem is set. There is no reason to look at an alternative at this point.

- **ATTEMPT a problem solution.**

Let us begin by writing the node equations.

At node 1 :

$$-5 + \frac{V_1 - 0}{30} + \frac{V_1 - V_2}{10} = 0$$

At node 2 :

$$\frac{V_2 - V_1}{10} + \frac{V_2 - 0}{-j20} + \frac{V_2 - 0}{10 + j10} = 0$$

Simplifying,

$$V_1 + 3V_1 - 3V_2 = 150$$

$$4V_1 - 3V_2 = 150$$

$$\frac{V_2 - V_1}{10} + \frac{jV_2}{20} + \frac{V_2(1-j)}{20} = 0$$

$$(2)(V_2 - V_1) + jV_2 + V_2(1-j) = 0$$

$$-2V_1 + 3V_2 = 0$$

Thus, the system of simultaneous equations is

$$\begin{bmatrix} 4 & -3 \\ -2 & 3 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 150 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \frac{1}{(12-6)} \begin{bmatrix} 3 & 3 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} 150 \\ 0 \end{bmatrix} = \frac{1}{6} \begin{bmatrix} 450 \\ 300 \end{bmatrix} = \begin{bmatrix} 75 \\ 50 \end{bmatrix}$$

So,

$V_1 = 75\angle 0°$ **or** $v_1(t) = 75\sin(1000t)$ **volts**

$V_2 = 50\angle 0°$ **or** $v_2(t) = 50\sin(1000t)$ **volts**

Clearly,

$$V_o = j10\, I_o$$

and $I_o = \dfrac{V_2}{10+j10} = \dfrac{50}{(10)(1+j)} = \dfrac{(50)(1-j)}{20} = (2.5)(1-j) = (2.5)(\sqrt{2}\angle\text{-}45°)$ **amps**

Hence,

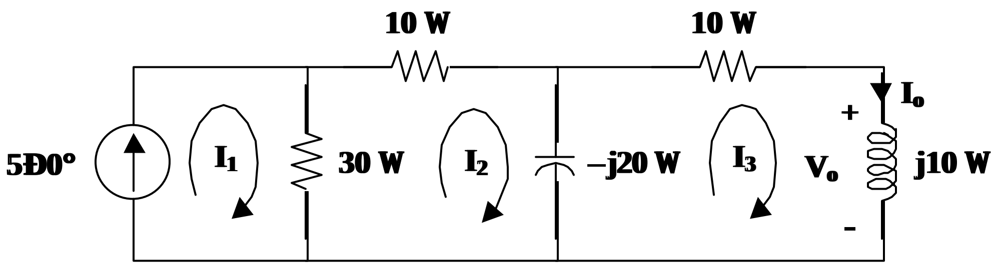
$$V_o = j10\,I_o = (10\angle 90°)(2.5)(\sqrt{2}\angle\text{-}45°) = 25\sqrt{2}\angle 45° = 35.36\angle 45° \text{ volts}$$

Therefore,

$$v_o(t) = 35.36\sin(1000t + 45°) \text{ volts}$$

- **EVALUATE the solution and check for accuracy.**
  Solving the problem with an alternate method, such as mesh analysis in this case, would show that the results of the problem solution are correct.

Let us draw the circuit defining the loop currents for mesh analysis.



Write the loop equations.

Loop 1 : $I_1 = 5\angle 0°$ **amps**

Loop 2 : $30(I_2 - I_1) + 10I_2 - j20(I_2 - I_3) = 0$

Loop 3 : $\text{-}j20(I_3 - I_2) + 10I_3 + j10I_3 = 0$

Simplifying the equations for loops 2 and 3,

Loop 2 : $\text{-}30I_1 + (40 - j20)I_2 + j20I_3 = 0$

Loop 3 : $j20I_2 + (10 - j10)I_3 = 0$

Solve the third loop equation for I_3 **in terms of** I_2**.**

$$I_3 = \frac{\text{-}j20}{10 - j10} I_2 = (1-j)I_2 = 1.4142\angle\text{-}45°\, I_2$$

Now, substitute I_1 **and** I_3 **into the second loop equation and simplify to find** I_2**.**

$$(\text{-}30)(5\angle 0°) + (40 - j20)I_2 + (j20)(1-j)I_2 = 0$$
$$(\text{-}30)(5\angle 0°) + (40 - j20)I_2 + (20 + j20)I_2 = 0$$
$$60I_2 = (30)(5\angle 0°)$$
$$I_2 = \frac{(30)(5\angle 0°)}{60} = 2.5\angle 0° \text{ amps}$$

Now, find I_3**.**

$$I_3 = (1.4142\angle -45°)\ I_2 = (1.4142\angle -45°)(2.5\angle 0°) = 3.536\angle -45° \text{ amps}$$

Clearly, $I_o = I_3 = 3.536\angle -45°$ **amps.**

Evidently,

$$V_o = j10 I_o = (10\angle 90°)(3.536\angle -45°) = 35.36\angle 45° \text{ volts}$$

and $v_o(t) = 35.36 \sin(1000t + 45°)$ **volts**

This answer is the same as the answer obtained using nodal analysis. Our check for accuracy was successful.

- **Has the problem been solved SATISFACTORILY? If so, present the solution; if not, then return "ALTERNATIVE solutions" and continue through the process again.**
  This problem has been solved satisfactorily.

$$v_o(t) = \underline{\mathbf{35.36\,sin(1000t + 45°)\ volts}}$$

Problem 10.2 *[10.5]* **Given the circuit in Figure 10.1 and** $v_i(t) = 10\cos(1000t)$ **volts, use nodal analysis to find** $v_o(t)$**.**

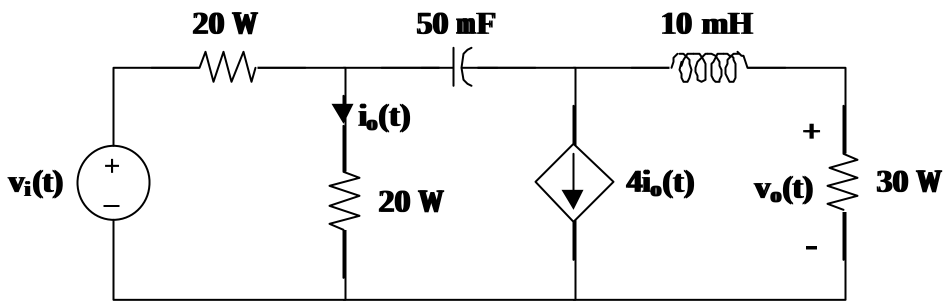
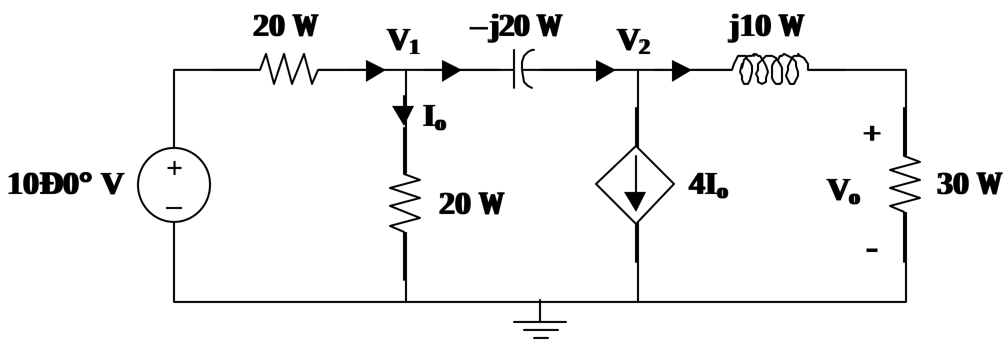


Figure 10.1

Let us start by building the ac circuit. The voltage source, $v_i(t) = 10\cos(1000t)$ **volts, becomes** $10\angle 0°$ **volts, with** $\omega = 1000$**. The inductive reactance becomes** $j\omega L = j10$**. The capacitive reactance becomes** $1/j\omega C = 1/[j(1000)(50\times 10^{-6})] = -j20$**. Now, draw the ac**



circuit.

At node 1 :

$$\frac{10 - V_1}{20} = \frac{V_1}{20} + \frac{V_1 - V_2}{-j20}$$

$$10 - V_1 = V_1 + j(V_1 - V_2)$$

$$10 = (2 + j)\, V_1 - j2 + j\, V_2$$

At node 2 :

$$\frac{V_1 - V_2}{-j20} = \frac{4V_1}{20} + \frac{V_2}{30 + j10}$$

$$(-4 + j)\, V_1 = (0.6 + j0.8)\, V_2$$

$$V_1 = \frac{0.6 + j0.8}{-4 + j}\, V_2$$

Note that $I_o = V_1/20$ was substituted when writing the equation for node 2.

Substituting the equation for node 2 into the equation for node 1,

$$10 = \frac{(2 + j)(0.6 + 0.8j)}{-4 + j}\, V_2 - j\, V_2$$

or $$V_2 = \frac{170}{0.6 - j26.2}$$

Clearly,

$$V_o = \frac{30}{30 + j10}\, V_2 = \left(\frac{3}{3 + j}\right)\left(\frac{170}{0.6 - j26.2}\right) = 6.154\angle 70.26^\circ \text{ volts}$$

With a reference of $A\, cos(1000t + \phi)$ **,**

$$v_o(t) = \underline{\mathbf{6.154\, cos(1000t + 70.26^\circ)\ volts}}$$

Problem 10.3 **Given the circuit in Figure 10.1 and** $v(t) = 20 cos(1000t)$ **volts, find** $i_o(t)$ **using nodal analysis.**

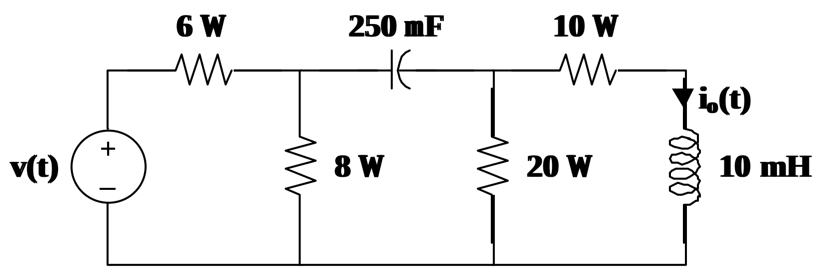


Figure 10.1

$$i_o(t) = \underline{\mathbf{632.5\, cos(1000t - 18.44^\circ)\ milliamps}}$$

MESH ANALYSIS

Problem 10.4 Given the circuit in Figure 10.1 and $i(t) = 2\cos(1000t)$ amps, find $i_o(t)$ using mesh analysis.

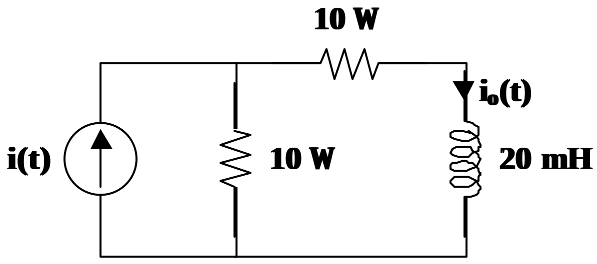
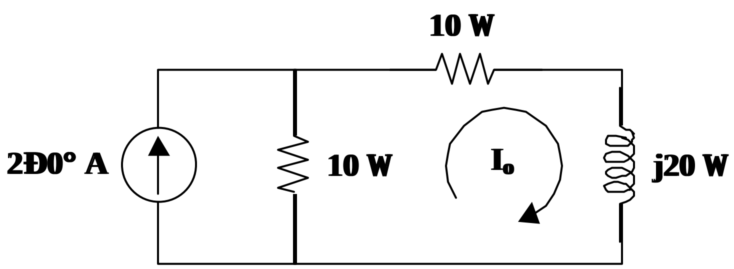


Figure 10.1

First, we transform the circuit to the frequency domain using a reference of $A\cos(1000t + \phi)$ and define the mesh currents as seen in the following circuit.

Remember that $X_L = j\omega L$ and $X_C = \dfrac{1}{j\omega C}$.



There is only one unknown loop current, I_o.
Writing the loop equation,

$$10(I_o - 2) + 10I_o + j20I_o = 0$$

$$(20 + j20)\,I_o = 20$$

$$I_o = \frac{20}{20 + j20} = \frac{1}{1 + j} = \frac{1\angle 0°}{\sqrt{2}\angle 45°} = 0.7071\angle -45° \text{ amps}$$

Therefore,

$$i_o(t) = \underline{707.1\cos(1000t - 45°) \text{ milliamps}}$$

Problem 10.5 Given the circuit in Figure 10.1 and $i(t) = 5\,sin(1000t)$ amps, find $i_o(t)$ using mesh analysis.

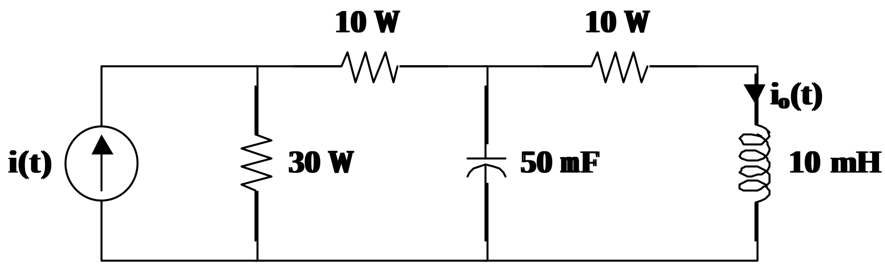
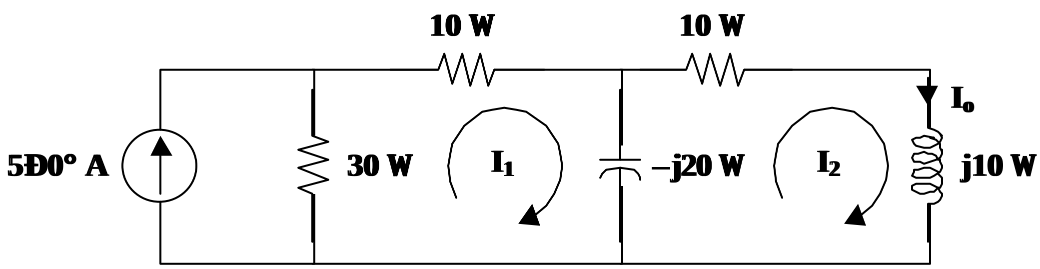


Figure 10.1

Transform the circuit to the frequency domain using a reference of $A\ sin(1000t + \phi)$ **and define the mesh currents.**



Clearly, $I_o = I_2$.

Use mesh analysis to find I_1 **and** I_2.

For loop 1 : $30(I_1 - 5) + 10I_1 - j20(I_1 - I_2) = 0$

For loop 2 : $-j20(I_2 - I_1) + 10I_2 + j10I_2 = 0$

Simplifying,

$$(40 - j20)\,I_1 + j20I_2 = 150$$
$$j20I_1 + (10 - j10)\,I_2 = 0$$

Simplifying further,

$$(2 - j)I_1 + jI_2 = 7.5$$
$$j2I_1 + (1 - j)\,I_2 = 0$$

Find I_1 **in terms of** I_2 **for the second loop equation.**

$$I_1 = \left(\frac{-1 + j}{j2}\right) I_2 = \left(\frac{1}{2} + j\frac{1}{2}\right) I_2 = \left(\frac{1}{2}\right)(1 + j)\,I_2$$

Substituting this equation into the first loop equation we get,

$$(2-j)\left(\frac{1+j}{2}\right)I_2 + jI_2 = 7.5$$

$$(2-j)(1+j)I_2 + j2I_2 = 15$$

$$(3+j3)\,I_2 = 15$$

$$I_2 = \frac{15}{3+j3} = \frac{5}{1+j} = \frac{5\angle 0°}{\sqrt{2}\angle 45°} = 3.536\angle\text{-}45°$$

Hence,

$I_o = 3.536\angle\text{-}45°$ **amps**

Therefore, $i_o(t) =$ **3.536 sin(1000t - 45°) amps**

SUPERPOSITION THEOREM

The superposition theorem applies to ac circuits the same as it does for dc circuits. This theorem is important if the circuit has sources operating at different frequencies. Since the impedances depend on frequency, we must have a different frequency-domain circuit for each source. The total response is obtained by adding the individual responses in the time domain.

Problem 10.6 **Given the circuit in Figure 10.1,** $i(t) = 2\cos(1000t)$ **amps and** $v(t) = 10\sin(4000/3t)$ **volts, find** $i_C(t)$**.**

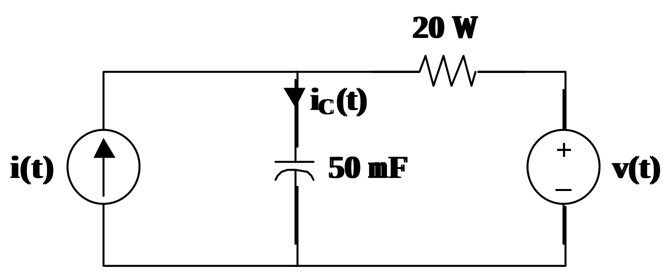
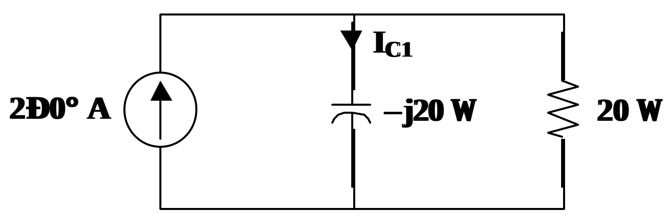


Figure 10.1

Because the two sources have different frequencies, we need to use superposition to solve this problem. Thus, $i_C(t) = i_{C1}(t) + i_{C2}(t)$**.**

Start with the current source and a reference of $A\cos(1000t + \phi)$**.**



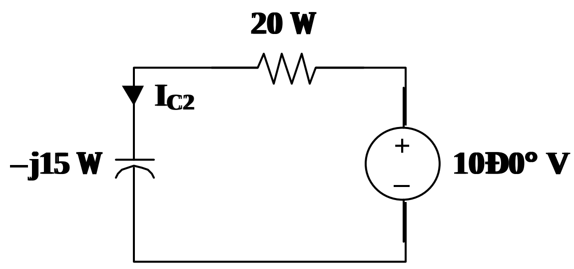
Using current division,

$$I_{C1} = \left(\frac{20}{20 - j20}\right)(2) = \frac{2}{1 - j} = \frac{(2)(1 + j)}{2} = 1 + j = \sqrt{2}\angle 45° = 1.4142\angle 45° \text{ amps}$$

Hence,

$$i_{C1}(t) = 1.4142\cos(1000t + 45°) \text{ amps}$$

Next, use the voltage source with a reference of $A\ sin(4000/3t + \phi)$.



Clearly,

$$I_{C2} = \frac{10}{20 - j15} = \frac{2}{4 - j3} = \frac{(2)(4 + j3)}{16 + 9} = \frac{(2)(5\angle 36.87°)}{25} = 0.4\angle 36.87° \text{ amps}$$

Hence,

$$i_{C2}(t) = 0.4\sin(4000/3t + 36.87°) \text{ amps}$$

Recall that $i_C(t) = i_{C1}(t) + i_{C2}(t)$.

Therefore,

$$i_C(t) = \underline{[1.4142\cos(1000t) + 0.4\sin(4000/3t + 36.87°)] \text{ amps}}$$

Problem 10.7 *[10.33]* **Given the circuit in Figure 10.1 and** $v_i(t) = 12\cos(3t)$ **volts, solve for** $v_o(t)$ **using the principle of superposition.**

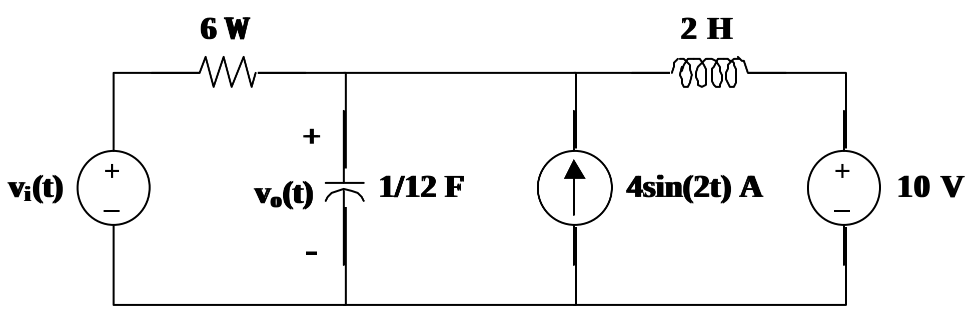
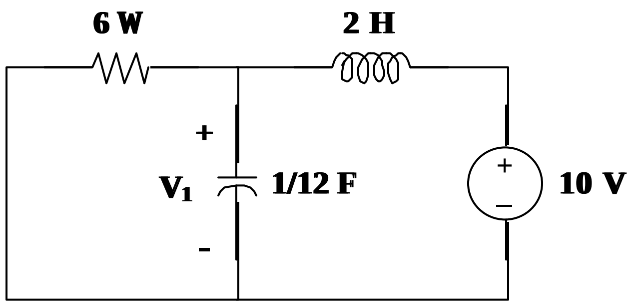


Figure 10.1

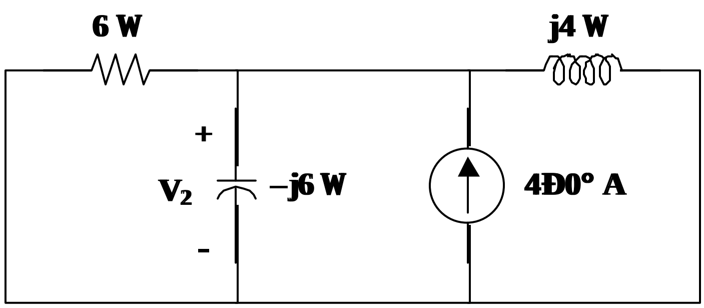
Let $V_o = V_1 + V_2 + V_3$ where V_1, V_2, and V_3 are respectively the voltages produced by the 10 volt dc source, the ac current source, and the ac voltage source acting independently. For V_1, consider the circuit shown in Figure (a).



(a)

The capacitor is an open circuit to dc while the inductor is a short circuit. Hence, $V_1 = 10$ **volts and** $v_1(t) = 10$ **volts.**

For V_2**, consider the circuit in Figure (b).** $\omega = 2$**, so the inductor becomes** $j\omega L = j4$**. Likewise, the capacitor becomes** $1/j\omega C = -j/(2/12) = -j6$**.**



(b)

Applying nodal analysis,

$$4 = \frac{V_2}{6} + \frac{V_2}{-j6} + \frac{V_2}{j4} = \left(\frac{1}{6} + j\frac{1}{6} - j\frac{1}{4}\right)V_2$$

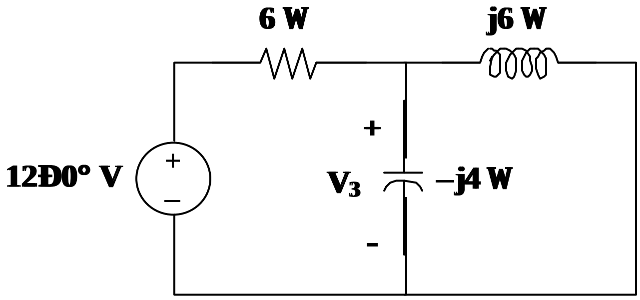
which leads to

$$V_2 = \frac{(4)(12)}{2-j} = 21.45\angle 26.56° \text{ volts}$$

With a reference of $A\,sin(2t + \phi)$**,**

$$v_2(t) = 21.45 sin(2t + 26.56°) \text{ volts}$$

For V_3, consider the circuit in Figure (c). $\omega = 3$ which leads to $j\omega L = j6$ for the inductor and $1/j\omega C = -j/(3/12) = -j4$ for the capacitor.



(c)

At the non-reference node,

$$\frac{12 - V_3}{6} = \frac{V_3}{-j4} + \frac{V_3}{j6}$$

which leads to

$$V_3 = \frac{(2)(12)}{2 + j} = 10.733\angle -26.56^\circ \text{ volts}$$

With a reference of $A\cos(3t + \phi)$,

$$v_3(t) = 10.733\cos(3t - 26.56^\circ) \text{ volts}$$

Recall that $v_o(t) = v_1(t) + v_2(t) + v_3(t)$.

Therefore,

$$v_o(t) = \underline{[10 + 21.45\sin(2t + 25.56^\circ) + 10.733\cos(3t - 25.56^\circ)] \text{ volts}}$$

Problem 10.8 Given the circuit in Figure 10.1 and $v_S(t) = \cos(1000t)$ volts, find $v(t)$.

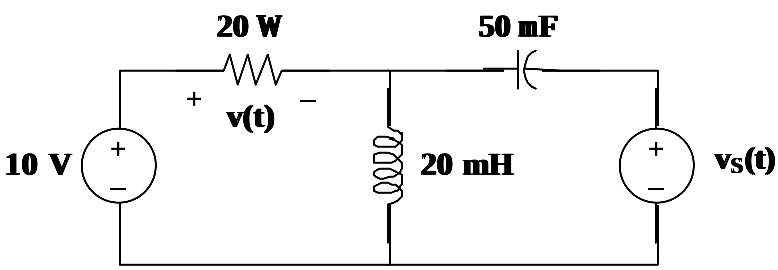


Figure 10.1

$$v(t) = \underline{10 + 20\cos(1000t - 90^\circ) \text{ volts}}$$

or $$v(t) = \underline{10 + 20\sin(1000t) \text{ volts}}$$

SOURCE TRANSFORMATION

Source transformation in the frequency domain involves transforming a voltage source in series with an impedance to a current source in parallel with an impedance or vice versa. We must keep the following relationship in mind when performing source transformations.

$$V_S = Z_S I_S \quad \Longleftrightarrow \quad I_S = \frac{V_S}{Z_S}$$

Problem 10.9 Given the circuit in Figure 10.1 and $v(t) = 20\cos(1000t)$ volts, find $v_o(t)$ using source transformations.

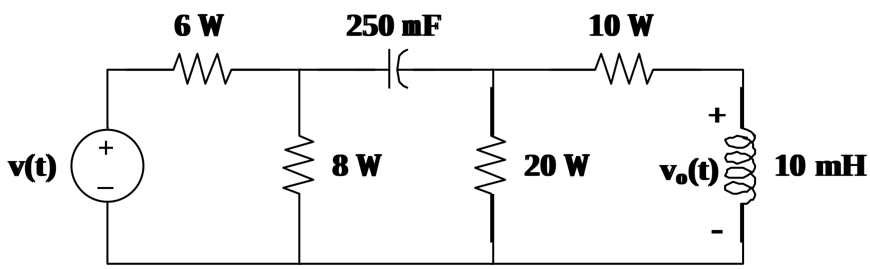
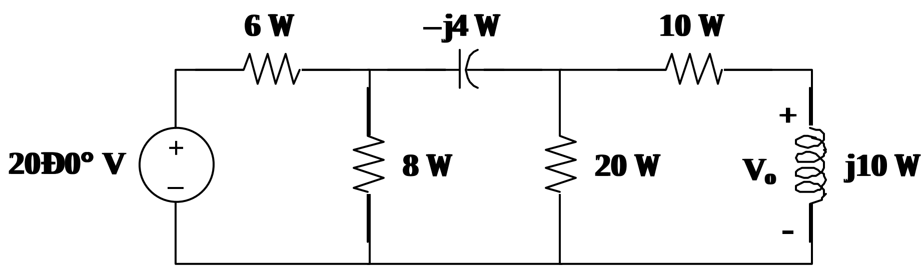
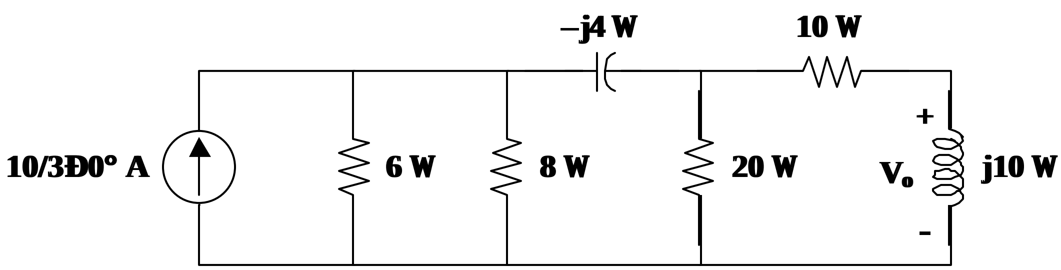


Figure 10.1

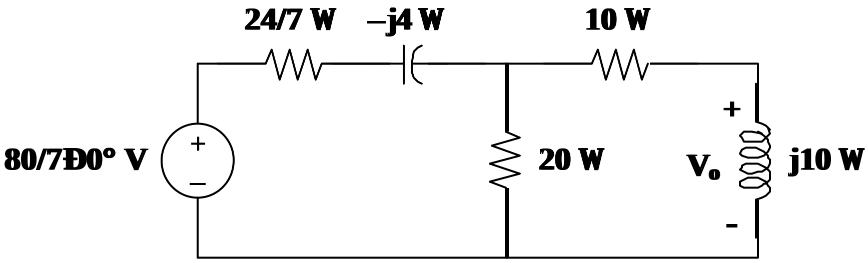
Transform the circuit to the frequency domain using a reference of $A\cos(1000t + \phi)$.



Reduce the circuit using source transformations. Begin with the $20\angle 0°$V source in series with 6Ω which becomes a $10/3\angle 0°$A source in parallel with 6Ω.



Now combine 6Ω || 8Ω = 24/7Ω. The 10/3 ∠0°A source in parallel with 24/7Ω becomes an 80/7 ∠0°V source in series with 24/7 Ω.



To perform the next source transformation, we need to find the parallel equivalent of the resistor and capacitor in series. We know that two series impedances and two parallel impedances are

$$Z_{Seq} = Z_{S1} + Z_{S2} \qquad \textbf{and} \qquad \frac{1}{Z_{Peq}} = \frac{1}{Z_{P1}} + \frac{1}{Z_{P2}}$$

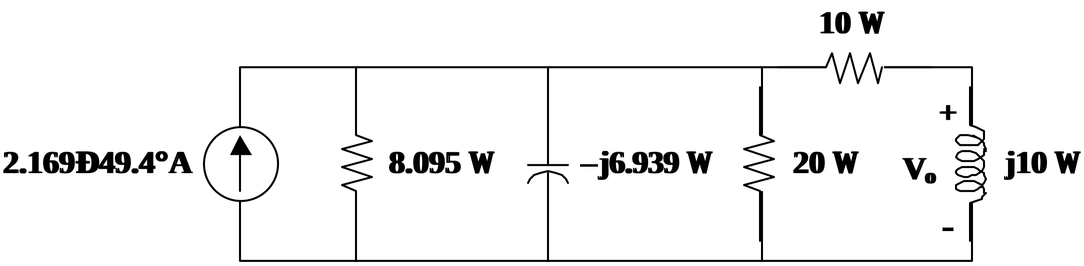
To find the parallel equivalence of two series impedances, let $Z_{Seq} = Z_{Peq}$**. Then,**

$$\frac{1}{Z_{S1} + Z_{S2}} = \frac{1}{Z_{P1}} + \frac{1}{Z_{P2}}$$

It can be shown that

$$\frac{1}{24/7 - j4} = \frac{1}{8.095} + \frac{1}{-j6.939}$$

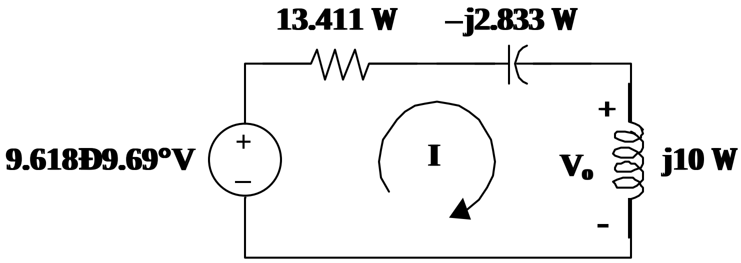
Thus, the 80/7 ∠0°V source in series with 24/7 – j4Ω becomes a 2.169∠49.4°A source in parallel with 8.095 – j6.939Ω.



Combining 8.095Ω || 20Ω = 5.763Ω. We now need to find the series equivalent of the resistor in parallel with the capacitor. It can be shown that

$$\frac{1}{5.763} + \frac{1}{-j6.939} = \frac{1}{3.411 - j2.833}$$

Thus, the 2.169∠49.4°A source in parallel with 5.763 – j6.939Ω becomes a 9.618∠9.69°V source in series with 3.411 – j2.833Ω. Before redrawing the circuit, let's combine the series resistances of 3.411Ω + 10Ω = 13.411Ω.



Now that we have found a less complex circuit via source transformations, it is simple to solve for $v_o(t)$.

$$I = \frac{9.618\angle 9.69°}{13.411 - j2.833 + j10} = \frac{9.618\angle 9.69°}{13.411 + j7.167} = \frac{9.618\angle 9.69°}{15.206\angle 28.12°} = 0.6325\angle -18.43°$$

and

$$V_o = j10\,I = (10\angle 90°)(0.6325\angle -18.43°) = 6.325\angle 71.57°$$

Recall that we used a reference of $A\,cos(1000t + \phi)$. Therefore,

$$v_o(t) = \underline{\mathbf{6.325\cos(1000t + 71.57°)\ volts}}$$

THEVENIN AND NORTON EQUIVALENT CIRCUITS

Problem 10.10 Given the circuit in Figure 10.1 and $v(t) = 100\,cos(1000t)$ volts, find V_{Th}, I_N, and Z_{eq} looking into terminals a and b.

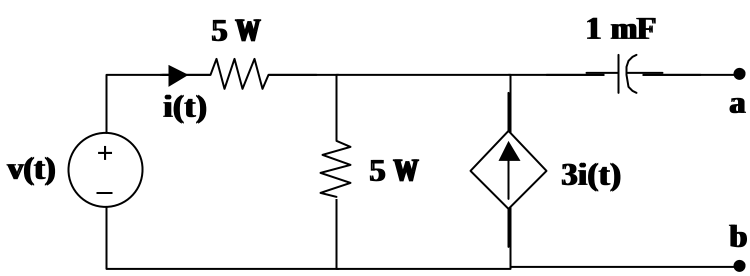
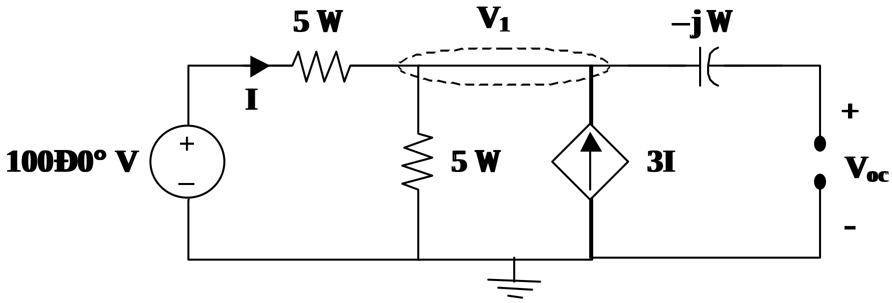


Figure 10.1

Because the circuit in Figure 10.1 has a capacitor, we can only determine V_{Th}, I_N, and Z_{eq} for an ac circuit in the frequency domain. Hence, the circuit becomes,



Begin by finding the open-circuit voltage, V_{oc}. Note that there is no current flowing through the capacitor. Thus, $V_{oc} = V_1$.

Using nodal analysis,

$$\frac{V_1 - 100}{5} + \frac{V_1 - 0}{5} - 3I = 0$$

where $I = \dfrac{100 - V_1}{5}$.

So,

$$\frac{V_1 - 100}{5} + \frac{V_1 - 0}{5} - (3)\left(\frac{100 - V_1}{5}\right) = 0$$

$$(V_1 - 100) + V_1 - (3)(100 - V_1) = 0$$

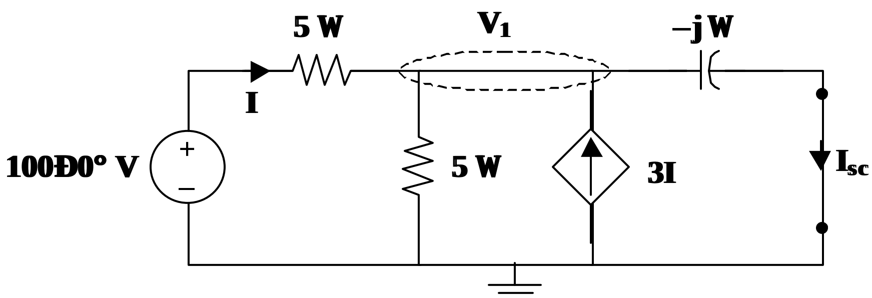
$$5\,V_1 = 400$$

$V_1 = 80$ volts

Hence,

$V_{oc} = 80\angle 0°$ volts

Now, find the short-circuit current, I_{sc}.



Use nodal analysis to find V_1 and then

$$I_{sc} = \frac{V_1}{-j}$$

Writing the nodal equation,

$$\frac{V_1 - 100}{5} + \frac{V_1 - 0}{5} - 3I + \frac{V_1 - 0}{-j} = 0$$

where $I = \dfrac{100 - V_1}{5}$.

So,

$$\frac{V_1 - 100}{5} + \frac{V_1 - 0}{5} - (3)\left(\frac{100 - V_1}{5}\right) + jV_1 = 0$$

$$(V_1 - 100) + V_1 - (3)(100 - V_1) + j5V_1 = 0$$

$$(5 + j5)V_1 = 400$$

$$V_1 = \frac{400}{5 + j5} = \frac{80}{1 + j} = \frac{(80)(1 - j)}{2} = (40)(1 - j)$$

Thus,

$$I_{sc} = \frac{V_1}{-j} = \frac{(40)(1 - j)}{-j} = (40)(1 + j) = 40\sqrt{2}\angle 45° \text{ amps}$$

Finally,

$$Z_{eq} = \frac{V_{oc}}{I_{sc}} = \frac{80\angle 0°}{40\sqrt{2}\angle 45°} = \sqrt{2}\angle -45° \text{ ohms}$$

Therefore,

$V_{Th} = V_{oc} = \underline{\mathbf{80\angle 0°}}$	**or**	$v_{Th}(t) = \underline{\mathbf{80\cos(1000t)\ volts}}$
$I_N = I_{sc} = \underline{\mathbf{40\sqrt{2}\angle 45°}}$.	**or**	$i_N(t) = \underline{\mathbf{56.57\cos(1000t + 45°)\ amps}}$
$Z_{eq} = \underline{\mathbf{\sqrt{2}\angle -45°}}$	**or**	$Z_{eq} = \underline{\mathbf{(1 - j)\ ohms}}$

Problem 10.11*[10.43]* **Find the Thevenin and Norton equivalent circuits for the circuit shown in Figure 10.1.**

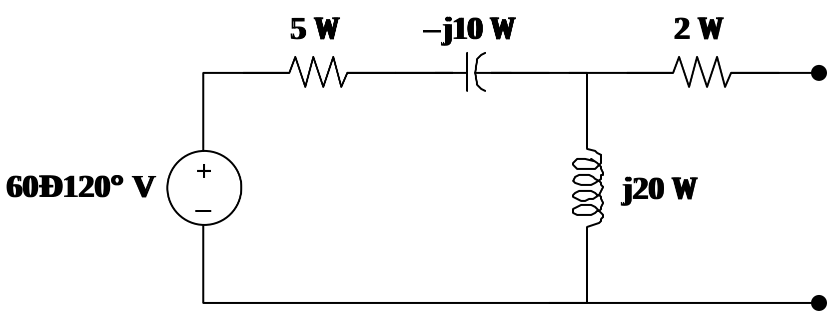
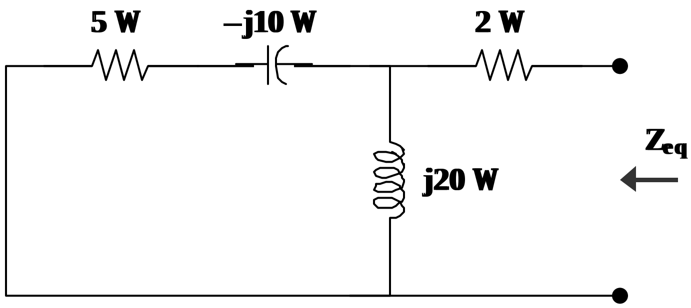


Figure 10.1

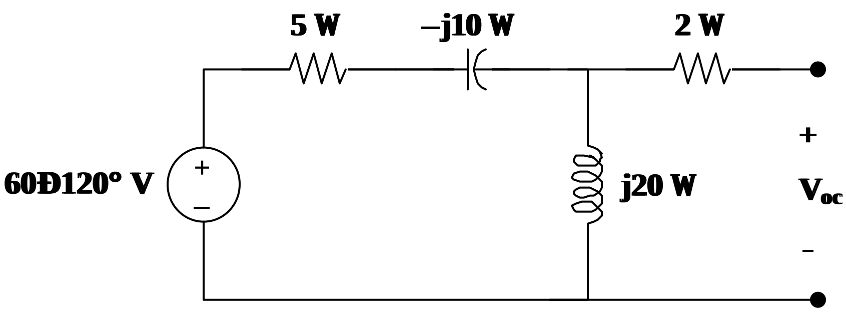
To find Z_{Th} **, consider the circuit shown in Figure (a).**



(a)

$$Z_{eq} = j20 \| (5 - j10) + 2 = (16 - j12) + 2 = 18 - j12 = 21.63\angle -33.69° \text{ ohms}$$

To obtain V_{Th} **, consider the circuit in Figure (b).**



(b)

$$V_{oc} = \frac{j20}{5 - j10 + j20} 60\angle 120° = \frac{j4}{1 + j2} 60\angle 120°$$
$$V_{oc} = (1.7889\angle 26.57°)(60\angle 120°) = 107.33\angle 146.57° \text{ volts}$$

$$I_{sc} = \frac{V_{oc}}{Z_{eq}} = \frac{107.33\angle 146.57°}{21.63\angle -33.69°} = 4.961\angle 180.26° \text{ amps}$$

where I_{sc} **is the current flowing downward through a short across the terminals.**

Recall that $Z_{Th} = Z_N = Z_{eq}$, $V_{Th} = V_{oc}$, **and** $I_N = I_{sc}$.

Therefore,

$Z_{Th} = Z_N =$ **(18 – j12) ohms**

$V_{Th} =$ **107.33Ð146.57° volts**

$I_N = \mathbf{4.961\angle 180.26°}$ **amps**

Problem 10.12 Given the circuit in Figure 10.1 and $v(t) = 100\cos(1000t)$ volts, find $i(t)$, the current through R, for $R = 0\Omega, 1\Omega, 10\Omega, 100\Omega, \text{ and } 1000\Omega$.

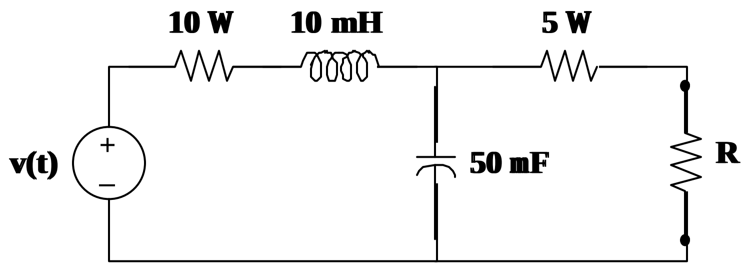


Figure 10.1

R	I	$i(t)$
$0\,\Omega$	$5.657\angle -45°$ A	**5.657 cos(1000t - 45°) A**
$1\,\Omega$	$5.439\angle -45°$ A	**5.439 cos(1000t - 45°) A**
$10\,\Omega$	$4.041\angle -45°$ A	**4.041 cos(1000t - 45°) A**
$100\,\Omega$	$1.1314\angle -45°$ A	**1.1314 cos(1000t - 45°) A**
$1000\,\Omega$	$0.138\angle -45°$ A	**138 cos(1000t - 45°) mA**

AC OP AMP CIRCUITS

Problem 10.13 Given the ac circuit in Figure 10.1, find V_{out} as a function of V_{in}.

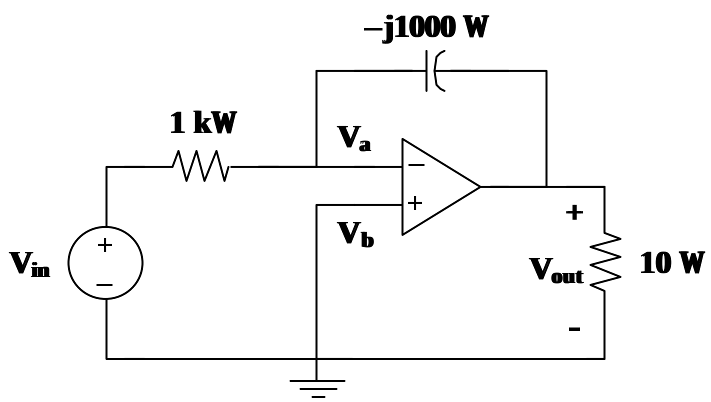


Figure 10.1

Using nodal analysis at node a,

$$\frac{V_a - V_{in}}{10^3} + \frac{V_a - V_{out}}{-j10^3} = 0$$

where $V_a = V_b = 0$.

So,

$$\frac{-V_{in}}{10^3} + \frac{-V_{out}}{-j10^3} = 0$$

$$-jV_{out} = V_{in}$$

$$V_{out} = \frac{V_{in}}{-j} = jV_{in}$$

Therefore,

$$V_{out} = \underline{\mathbf{V_{in}Ð90°}}$$

The output is equal to the input except for a phase shift of 90°.

Problem 10.14 **Given the circuit in Figure 10.1 and** $v_{in}(t) = 10sin(\omega t)$ **volts, find** $v_{out}(t)$ **for** $\omega = 1, 10, 100, 1000$ **rad/sec.**

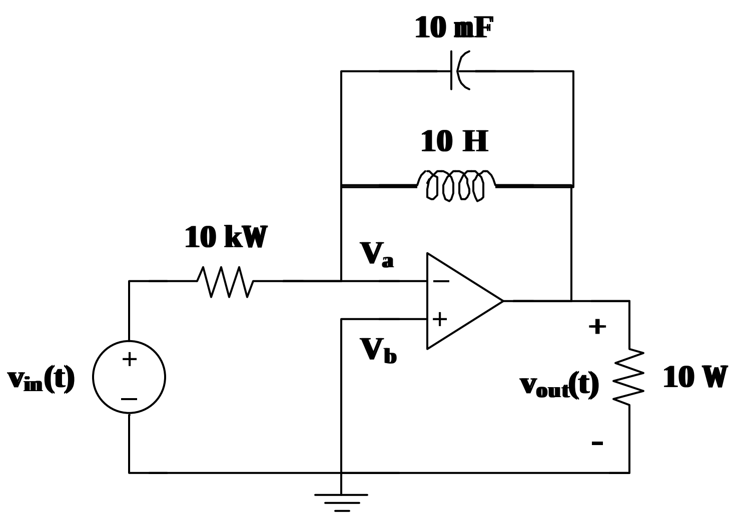
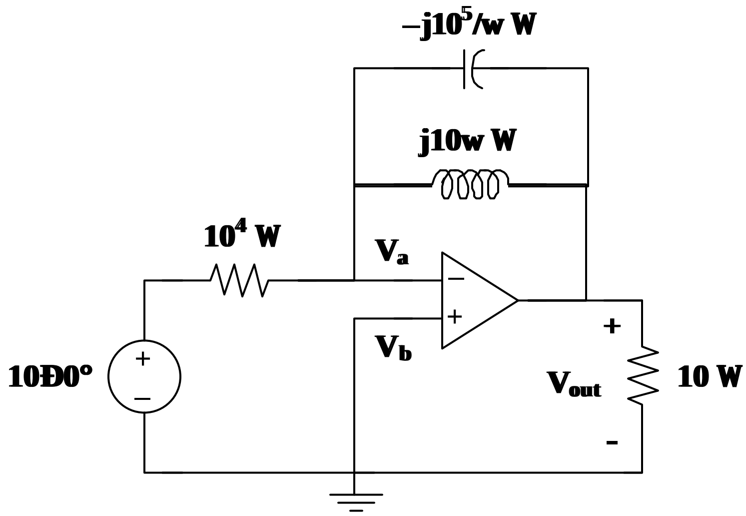


Figure 10.1

Use the following ac circuit, with a reference of $A\, sin(\omega t + \phi)$, to find V_{out} in terms of ω.



Using nodal analysis at node a,

$$\frac{V_a - 10}{10^4} + \frac{V_a - V_{out}}{-j10^5/\omega} + \frac{V_a - V_{out}}{j10\,\omega} = 0$$

where $V_a = V_b = 0$.

So,

$$\frac{-10}{10^4} + \frac{-V_{out}}{-j10^5/\omega} + \frac{-V_{out}}{j10\omega} = 0$$

$$-100 - j\omega V_{out} + \frac{j10^4}{\omega} V_{out} = 0$$

$$\left(-j\omega + \frac{j10^4}{\omega}\right) V_{out} = 100$$

$$V_{out} = \frac{100}{-j\omega + j10^4/\omega} = \frac{j\omega 100}{\omega^2 - 10^4} = \frac{\omega 100}{\omega^2 - 10^4} \angle 90°$$

Now, substitute the values for ω into the equation for V_{out}.

At $\omega = 1$ rad/sec, $$V_{out} \cong \frac{100}{-10^4} \angle 90° = 0.01 \angle -90°$$

$$v_{out}(t) = \underline{\mathbf{10\, sin(t - 90°)\ mV}}$$

At $\omega = 10$ rad/sec, $$V_{out} \cong \frac{10^3}{10^2 - 10^4} \angle 90° = \frac{10^3}{-9900} \angle 90° = 0.10101 \angle -90°$$

$$v_{out}(t) = \underline{\mathbf{101.01\sin(10t - 90°)\ mV}}$$

At $\omega = 100$ **rad/sec,** $\quad V_{out} \cong \dfrac{10^4}{10^4 - 10^4}\angle 90° = \dfrac{10^4}{0}\angle 90°$

$$v_{out}(t) = \underline{\mathbf{\infty}}$$

This corresponds to the case where the LC combination forms a parallel resonant circuit and the output goes to infinity.

At $\omega = 1000$ **rad/sec,** $\quad V_{out} \cong \dfrac{10^5}{10^6 - 10^4}\angle 90° = \dfrac{10^5}{9.9\times 10^5}\angle 90° = 0.10101\angle 90°$

$$v_{out}(t) = \underline{\mathbf{101.01\sin(1000t + 90°)\ mV}}$$

In conclusion, the output, $v_{out}(t)$**, has a –90° phase shift for all values of** ω **less than 100 and has a 90° phase shift for values of** ω **greater than 100.**