CHAPTER 10

SINUSOIDAL STEADY-STATE ANALYSIS

An expert problem solver must be endowed with two incompatible quantities, a restless imagination and a patient pertinacity.

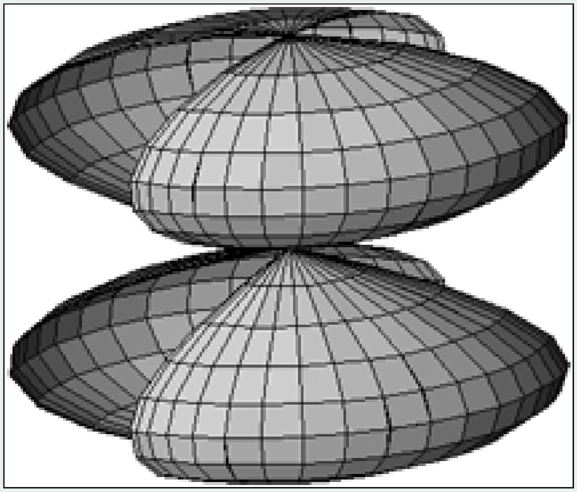
—Howard W. Eves

Enhancing Your Career

Career in Software Engineering Software engineering is that aspect of engineering that deals with the practical application of scientific knowledge in the design, construction, and validation of computer programs and the associated documentation required to develop, operate, and maintain them. It is a branch of electrical engineering that is becoming increasingly important as more and more disciplines require one form of software package or another to perform routine tasks and as programmable microelectronic systems are used in more and more applications.

The role of a software engineer should not be confused with that of a computer scientist; the software engineer is a practitioner, not a theoretician. A software engineer should have good computer-programming skill and be familiar with programming languages, in particular C^{++}, which is becoming increasingly popular. Because hardware and software are closely interlinked, it is essential that a software engineer have a thorough understanding of hardware design. Most important, the software engineer should have some specialized knowledge of the area in which the software development skill is to be applied.

All in all, the field of software engineering offers a great career to those who enjoy programming and developing software packages. The higher rewards will go to those having the best preparation, with the most interesting and challenging opportunities going to those with graduate education.



Output of a modeling software. (Courtesy of National Instruments.)

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10.1 INTRODUCTION

In Chapter 9, we learned that the forced or steady-state response of circuits to sinusoidal inputs can be obtained by using phasors. We also know that Ohm's and Kirchhoff's laws are applicable to ac circuits. In this chapter, we want to see how nodal analysis, mesh analysis, Thevenin's theorem, Norton's theorem, superposition, and source transformations are applied in analyzing ac circuits. Since these techniques were already introduced for dc circuits, our major effort here will be to illustrate with examples.

Analyzing ac circuits usually requires three steps.

Steps to Analyze ac Circuits:

1. Transform the circuit to the phasor or frequency domain.
2. Solve the problem using circuit techniques (nodal analysis, mesh analysis, superposition, etc.).
3. Transform the resulting phasor to the time domain.

Step 1 is not necessary if the problem is specified in the frequency domain. In step 2, the analysis is performed in the same manner as dc circuit analysis except that complex numbers are involved. Having read Chapter 9, we are adept at handling step 3.

Frequency-domain analysis of an ac circuit via phasors is much easier than analysis of the circuit in the time domain.

Toward the end of the chapter, we learn how to apply *PSpice* in solving ac circuit problems. We finally apply ac circuit analysis to two practical ac circuits: oscillators and ac transistor circuits.

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10.2 NODAL ANALYSIS

The basis of nodal analysis is Kirchhoff's current law. Since KCL is valid for phasors, as demonstrated in Section 9.6, we can analyze ac circuits by nodal analysis. The following examples illustrate this.

EXAMPLE 10.1

Find i_x in the circuit of Fig. 10.1 using nodal analysis.

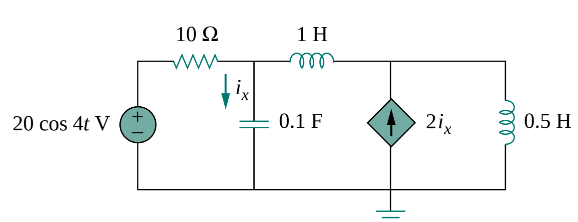

Figure 10.1 For Example 10.1.

Solution:

We first convert the circuit to the frequency domain:

$$\begin{aligned}
20\cos 4t &\implies 20\underline{/0^\circ}, \quad \omega = 4 \text{ rad/s} \\
1 \text{ H} &\implies j\omega L = j4 \\
0.5 \text{ H} &\implies j\omega L = j2 \\
0.1 \text{ F} &\implies \frac{1}{j\omega C} = -j2.5
\end{aligned}$$

Thus, the frequency-domain equivalent circuit is as shown in Fig. 10.2.

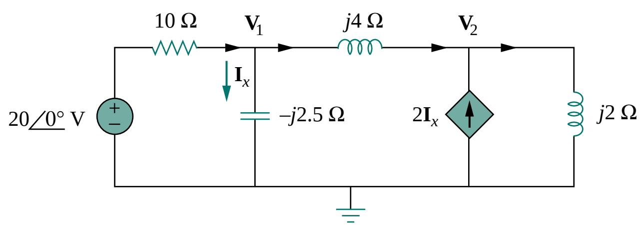

Figure 10.2 Frequency-domain equivalent of the circuit in Fig. 10.1.

Applying KCL at node 1,

$$\frac{20 - \mathbf{V}_1}{10} = \frac{\mathbf{V}_1}{-j2.5} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{j4}$$

or

$$(1 + j1.5)\mathbf{V}_1 + j2.5\mathbf{V}_2 = 20 \tag{10.1.1}$$

At node 2,

$$2\mathbf{I}_x + \frac{\mathbf{V}_1 - \mathbf{V}_2}{j4} = \frac{\mathbf{V}_2}{j2}$$

But $\mathbf{I}_x = \mathbf{V}_1/-j2.5$. Substituting this gives

$$\frac{2\mathbf{V}_1}{-j2.5} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{j4} = \frac{\mathbf{V}_2}{j2}$$

By simplifying, we get

$$11\mathbf{V}_1 + 15\mathbf{V}_2 = 0 \tag{10.1.2}$$

Equations (10.1.1) and (10.1.2) can be put in matrix form as

$$\begin{bmatrix} 1 + j1.5 & j2.5 \\ 11 & 15 \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} = \begin{bmatrix} 20 \\ 0 \end{bmatrix}$$

We obtain the determinants as

$$\Delta = \begin{vmatrix} 1 + j1.5 & j2.5 \\ 11 & 15 \end{vmatrix} = 15 - j5$$

$$\Delta_1 = \begin{vmatrix} 20 & j2.5 \\ 0 & 15 \end{vmatrix} = 300, \qquad \Delta_2 = \begin{vmatrix} 1 + j1.5 & 20 \\ 11 & 0 \end{vmatrix} = -220$$

$$\mathbf{V}_1 = \frac{\Delta_1}{\Delta} = \frac{300}{15 - j5} = 18.97\underline{/18.43^\circ} \text{ V}$$

$$\mathbf{V}_2 = \frac{\Delta_2}{\Delta} = \frac{-220}{15 - j5} = 13.91\underline{/198.3^\circ} \text{ V}$$

The current $\mathbf{I}_x$ is given by

$$\mathbf{I}_x = \frac{\mathbf{V}_1}{-j2.5} = \frac{18.97\underline{/18.43^\circ}}{2.5\underline{/-90^\circ}} = 7.59\underline{/108.4^\circ} \text{ A}$$

Transforming this to the time domain,

$$i_x = 7.59\cos(4t + 108.4^\circ) \text{ A}$$

PRACTICE PROBLEM 10.1

Using nodal analysis, find v_1 and v_2 in the circuit of Fig. 10.3.

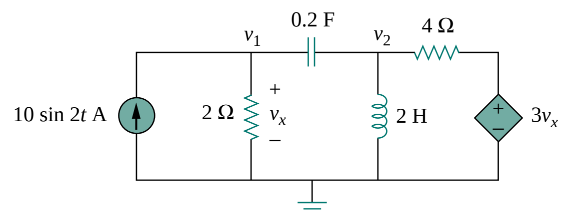

Figure 10.3 For Practice Prob. 10.1.

Answer: $v_1(t) = 20.96\sin(2t + 58^\circ)$ V, $v_2(t) = 44.11\sin(2t + 41^\circ)$ V.

EXAMPLE 10.2

Compute $\mathbf{V}_1$ and $\mathbf{V}_2$ in the circuit of Fig. 10.4.

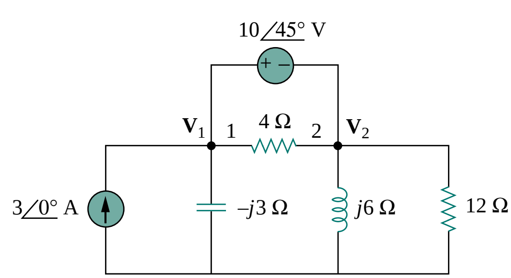

Figure 10.4 For Example 10.2.

Solution:

Nodes 1 and 2 form a supernode as shown in Fig. 10.5. Applying KCL at the supernode gives

$$3 = \frac{\mathbf{V}_1}{-j3} + \frac{\mathbf{V}_2}{j6} + \frac{\mathbf{V}_2}{12}$$

or

$$36 = j4\mathbf{V}_1 + (1 - j2)\mathbf{V}_2 \tag{10.2.1}$$

But a voltage source is connected between nodes 1 and 2, so that

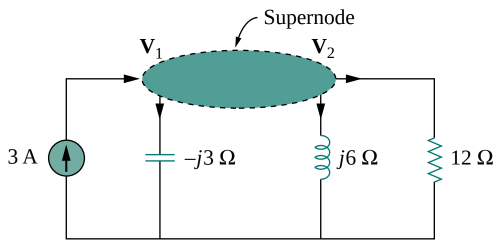

Figure 10.5 A supernode in the circuit of Fig. 10.4.

$$\mathbf{V}_1 = \mathbf{V}_2 + 10\angle 45^\circ \qquad (10.2.2)$$

Substituting Eq. (10.2.2) in Eq. (10.2.1) results in

$$36 - 40\angle 135^\circ = (1 + j2)\mathbf{V}_2 \qquad \Longrightarrow \qquad \mathbf{V}_2 = 31.41\angle -87.18^\circ \text{ V}$$

From Eq. (10.2.2),

$$\mathbf{V}_1 = \mathbf{V}_2 + 10\angle 45^\circ = 25.78\angle -70.48^\circ \text{ V}$$

PRACTICE PROBLEM 10.2

Calculate $\mathbf{V}_1$ and $\mathbf{V}_2$ in the circuit shown in Fig. 10.6.

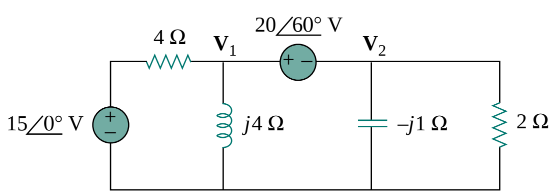

Figure 10.6 For Practice Prob. 10.2.

Answer: $\mathbf{V}_1 = 19.36\angle 69.67^\circ$ V, $\mathbf{V}_2 = 3.376\angle 165.7^\circ$ V.

10.3 MESH ANALYSIS

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Kirchhoff's voltage law (KVL) forms the basis of mesh analysis. The validity of KVL for ac circuits was shown in Section 9.6 and is illustrated in the following examples.

EXAMPLE 10.3

Determine current $\mathbf{I}_o$ in the circuit of Fig. 10.7 using mesh analysis.

Solution:

Applying KVL to mesh 1, we obtain

$$(8 + j10 - j2)\mathbf{I}_1 - (-j2)\mathbf{I}_2 - j10\mathbf{I}_3 = 0 \qquad (10.3.1)$$

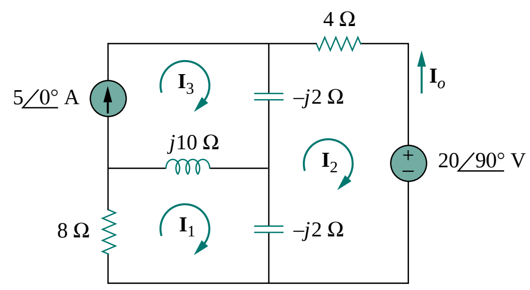

Figure 10.7 For Example 10.3.

For mesh 2,

$$(4 - j2 - j2)\mathbf{I}_2 - (-j2)\mathbf{I}_1 - (-j2)\mathbf{I}_3 + 20\underline{/90^\circ} = 0 \qquad (10.3.2)$$

For mesh 3, $\mathbf{I}_3 = 5$. Substituting this in Eqs. (10.3.1) and (10.3.2), we get

$$(8 + j8)\mathbf{I}_1 + j2\mathbf{I}_2 = j50 \qquad (10.3.3)$$

$$j2\mathbf{I}_1 + (4 - j4)\mathbf{I}_2 = -j20 - j10 \qquad (10.3.4)$$

Equations (10.3.3) and (10.3.4) can be put in matrix form as

$$\begin{bmatrix} 8 + j8 & j2 \\ j2 & 4 - j4 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} j50 \\ -j30 \end{bmatrix}$$

from which we obtain the determinants

$$\Delta = \begin{vmatrix} 8 + j8 & j2 \\ j2 & 4 - j4 \end{vmatrix} = 32(1 + j)(1 - j) + 4 = 68$$

$$\Delta_2 = \begin{vmatrix} 8 + j8 & j50 \\ j2 & -j30 \end{vmatrix} = 340 - j240 = 416.17\underline{/-35.22^\circ}$$

$$\mathbf{I}_2 = \frac{\Delta_2}{\Delta} = \frac{416.17\underline{/-35.22^\circ}}{68} = 6.12\underline{/-35.22^\circ} \text{ A}$$

The desired current is

$$\mathbf{I}_o = -\mathbf{I}_2 = 6.12\underline{/144.78^\circ} \text{ A}$$

PRACTICE PROBLEM 10.3

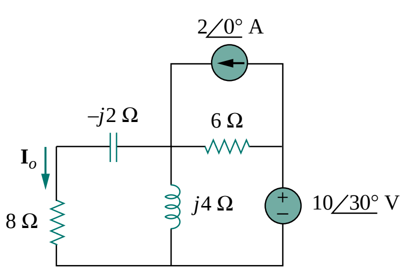

Figure 10.8 For Practice Prob. 10.3.

Find $\mathbf{I}_o$ in Fig. 10.8 using mesh analysis.

Answer: $1.194\underline{/65.45^\circ}$ A.

EXAMPLE 10.4

Solve for $\mathbf{V}_o$ in the circuit in Fig. 10.9 using mesh analysis.

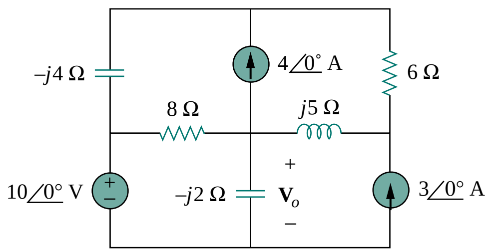

Figure 10.9 For Example 10.4.

Solution:

As shown in Fig. 10.10, meshes 3 and 4 form a supermesh due to the current source between the meshes. For mesh 1, KVL gives

$$-10 + (8 - j2)\mathbf{I}_1 - (-j2)\mathbf{I}_2 - 8\mathbf{I}_3 = 0$$

or

$$(8 - j2)\mathbf{I}_1 + j2\mathbf{I}_2 - 8\mathbf{I}_3 = 10 \tag{10.4.1}$$

For mesh 2,

$$\mathbf{I}_2 = -3 \tag{10.4.2}$$

For the supermesh,

$$(8 - j4)\mathbf{I}_3 - 8\mathbf{I}_1 + (6 + j5)\mathbf{I}_4 - j5\mathbf{I}_2 = 0 \tag{10.4.3}$$

Due to the current source between meshes 3 and 4, at node A,

$$\mathbf{I}_4 = \mathbf{I}_3 + 4 \tag{10.4.4}$$

Combining Eqs. (10.4.1) and (10.4.2),

$$(8 - j2)\mathbf{I}_1 - 8\mathbf{I}_3 = 10 + j6 \tag{10.4.5}$$

Combining Eqs. (10.4.2) to (10.4.4),

$$-8\mathbf{I}_1 + (14 + j)\mathbf{I}_3 = -24 - j35 \tag{10.4.6}$$

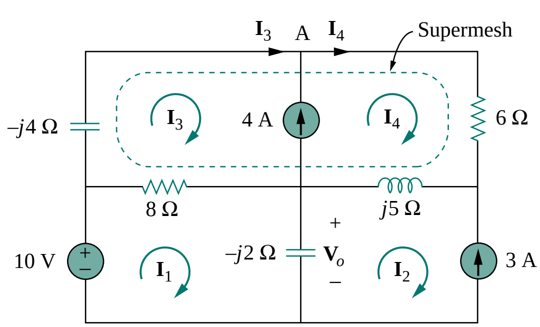

Figure 10.10 Analysis of the circuit in Fig. 10.9.

From Eqs. (10.4.5) and (10.4.6), we obtain the matrix equation

$$\begin{bmatrix} 8 - j2 & -8 \\ -8 & 14 + j \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_3 \end{bmatrix} = \begin{bmatrix} 10 + j6 \\ -24 - j35 \end{bmatrix}$$

We obtain the following determinants

$$\Delta = \begin{vmatrix} 8 - j2 & -8 \\ -8 & 14 + j \end{vmatrix} = 112 + j8 - j28 + 2 - 64 = 50 - j20$$

$$\Delta_1 = \begin{vmatrix} 10 + j6 & -8 \\ -24 - j35 & 14 + j \end{vmatrix} = 140 + j10 + j84 - 6 - 192 - j280$$
$$= -58 - j186$$

Current $\mathbf{I}_1$ is obtained as

$$\mathbf{I}_1 = \frac{\Delta_1}{\Delta} = \frac{-58 - j186}{50 - j20} = 3.618\underline{/274.5^\circ} \text{ A}$$

The required voltage $\mathbf{V}_o$ is

$$\mathbf{V}_o = -j2(\mathbf{I}_1 - \mathbf{I}_2) = -j2(3.618\underline{/274.5^\circ} + 3)$$
$$= -7.2134 - j6.568 = 9.756\underline{/222.32^\circ} \text{ V}$$

Network Analysis

PRACTICE PROBLEM 10.4

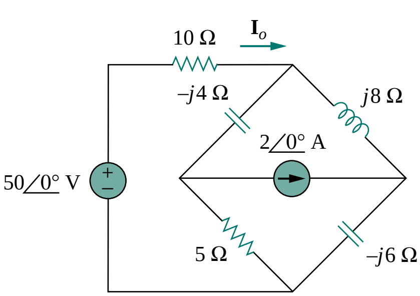

Figure 10.11 For Practice Prob. 10.4.

Calculate current $\mathbf{I}_o$ in the circuit of Fig. 10.11.

Answer: $5.075\underline{/5.943^\circ}$ A.

10.4 SUPERPOSITION THEOREM

Since ac circuits are linear, the superposition theorem applies to ac circuits the same way it applies to dc circuits. The theorem becomes important if the circuit has sources operating at *different* frequencies. In this case, since the impedances depend on frequency, we must have a different frequency-domain circuit for each frequency. The total response must be obtained by adding the individual responses in the *time* domain. It is incorrect to try to add the responses in the phasor or frequency domain. Why? Because the exponential factor $e^{j\omega t}$ is implicit in sinusoidal analysis, and that factor would change for every angular frequency ω. It would therefore not make sense to add responses at different frequencies in the phasor domain. Thus, when a circuit has sources operating at different frequencies, one must add the responses due to the individual frequencies in the time domain.

EXAMPLE 10.5

Use the superposition theorem to find $\mathbf{I}_o$ in the circuit in Fig. 10.7.

Solution:

Let

$$\mathbf{I}_o = \mathbf{I}'_o + \mathbf{I}''_o \tag{10.5.1}$$

where $\mathbf{I}'_o$ and $\mathbf{I}''_o$ are due to the voltage and current sources, respectively. To find $\mathbf{I}'_o$, consider the circuit in Fig. 10.12(a). If we let $\mathbf{Z}$ be the parallel combination of $-j2$ and $8 + j10$, then

$$\mathbf{Z} = \frac{-j2(8 + j10)}{-2j + 8 + j10} = 0.25 - j2.25$$

and current $\mathbf{I}'_o$ is

$$\mathbf{I}'_o = \frac{j20}{4 - j2 + \mathbf{Z}} = \frac{j20}{4.25 - j4.25}$$

or

$$\mathbf{I}'_o = -2.353 + j2.353 \tag{10.5.2}$$

To get $\mathbf{I}''_o$, consider the circuit in Fig. 10.12(b). For mesh 1,

$$(8 + j8)\mathbf{I}_1 - j10\mathbf{I}_3 + j2\mathbf{I}_2 = 0 \tag{10.5.3}$$

For mesh 2,

$$(4 - j4)\mathbf{I}_2 + j2\mathbf{I}_1 + j2\mathbf{I}_3 = 0 \tag{10.5.4}$$

For mesh 3,

$$\mathbf{I}_3 = 5 \tag{10.5.5}$$

From Eqs. (10.5.4) and (10.5.5),

$$(4 - j4)\mathbf{I}_2 + j2\mathbf{I}_1 + j10 = 0$$

Expressing $\mathbf{I}_1$ in terms of $\mathbf{I}_2$ gives

$$\mathbf{I}_1 = (2 + j2)\mathbf{I}_2 - 5 \tag{10.5.6}$$

Substituting Eqs. (10.5.5) and (10.5.6) into Eq. (10.5.3), we get

$$(8 + j8)[(2 + j2)\mathbf{I}_2 - 5] - j50 + j2\mathbf{I}_2 = 0$$

or

$$\mathbf{I}_2 = \frac{90 - j40}{34} = 2.647 - j1.176$$

Current $\mathbf{I}''_o$ is obtained as

$$\mathbf{I}''_o = -\mathbf{I}_2 = -2.647 + j1.176 \tag{10.5.7}$$

From Eqs. (10.5.2) and (10.5.7), we write

$$\mathbf{I}_o = \mathbf{I}'_o + \mathbf{I}''_o = -5 + j3.529 = 6.12\underline{/144.78^\circ} \text{ A}$$

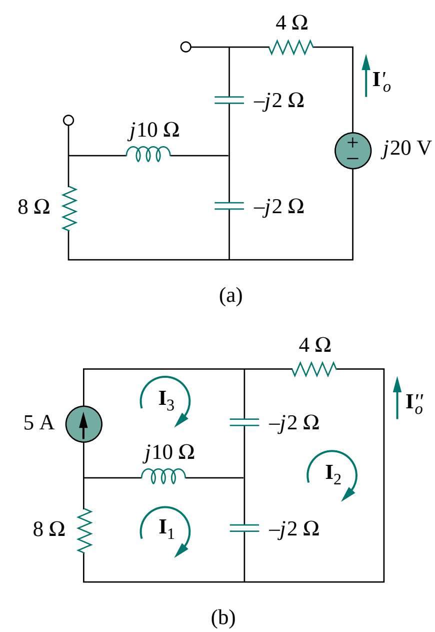

Figure 10.12 Solution of Example 10.5.

which agrees with what we got in Example 10.3. It should be noted that applying the superposition theorem is not the best way to solve this problem. It seems that we have made the problem twice as hard as the original one by using superposition. However, in Example 10.6, superposition is clearly the easiest approach.

PRACTICE PROBLEM 10.5

Find current $\mathbf{I}_o$ in the circuit of Fig. 10.8 using the superposition theorem.

Answer: $1.194\underline{/65.45^\circ}$ A.

EXAMPLE 10.6

Find v_o in the circuit in Fig. 10.13 using the superposition theorem.

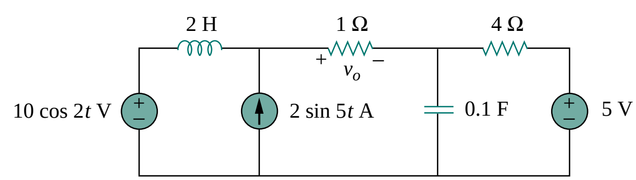

Figure 10.13 For Example 10.6.

Solution:

Since the circuit operates at three different frequencies ($\omega = 0$ for the dc voltage source), one way to obtain a solution is to use superposition, which breaks the problem into single-frequency problems. So we let

$$v_o = v_1 + v_2 + v_3 \tag{10.6.1}$$

where v_1 is due to the 5-V dc voltage source, v_2 is due to the $10 \cos 2t$ V voltage source, and v_3 is due to the $2 \sin 5t$ A current source.

To find v_1, we set to zero all sources except the 5-V dc source. We recall that at steady state, a capacitor is an open circuit to dc while an inductor is a short circuit to dc. There is an alternative way of looking at this. Since $\omega = 0$, $j\omega L = 0$, $1/j\omega C = \infty$. Either way, the equivalent circuit is as shown in Fig. 10.14(a). By voltage division,

$$-v_1 = \frac{1}{1+4}(5) = 1 \text{ V} \tag{10.6.2}$$

To find v_2, we set to zero both the 5-V source and the $2 \sin 5t$ current source and transform the circuit to the frequency domain.

$$\begin{aligned} 10 \cos 2t \quad &\Longrightarrow \quad 10\underline{/0^\circ}, \qquad \omega = 2 \text{ rad/s} \\ 2 \text{ H} \quad &\Longrightarrow \quad j\omega L = j4\ \Omega \\ 0.1 \text{ F} \quad &\Longrightarrow \quad \frac{1}{j\omega C} = -j5\ \Omega \end{aligned}$$

The equivalent circuit is now as shown in Fig. 10.14(b). Let

$$\mathbf{Z} = -j5 \parallel 4 = \frac{-j5 \times 4}{4 - j5} = 2.439 - j1.951$$

By voltage division,

$$\mathbf{V}_2 = \frac{1}{1 + j4 + \mathbf{Z}}(10\underline{/0^\circ}) = \frac{10}{3.439 + j2.049} = 2.498\underline{/-30.79^\circ}$$

In the time domain,

$$v_2 = 2.498\cos(2t - 30.79^\circ) \qquad (10.6.3)$$

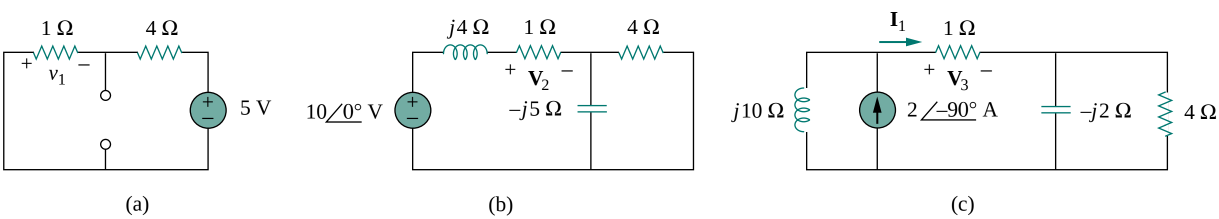

Figure 10.14 Solution of Example 10.6: (a) setting all sources to zero except the 5-V dc source, (b) setting all sources to zero except the ac voltage source, (c) setting all sources to zero except the ac current source.

To obtain v_3, we set the voltage sources to zero and transform what is left to the frequency domain.

$$2\sin 5t \quad \Longrightarrow \quad 2\underline{/-90^\circ}, \qquad \omega = 5 \text{ rad/s}$$

$$2 \text{ H} \quad \Longrightarrow \quad j\omega L = j10\ \Omega$$

$$0.1 \text{ F} \quad \Longrightarrow \quad \frac{1}{j\omega C} = -j2\ \Omega$$

The equivalent circuit is in Fig. 10.14(c). Let

$$\mathbf{Z}_1 = -j2 \parallel 4 = \frac{-j2 \times 4}{4 - j2} = 0.8 - j1.6\ \Omega$$

By current division,

$$\mathbf{I}_1 = \frac{j10}{j10 + 1 + \mathbf{Z}_1}(2\underline{/-90^\circ}) \text{ A}$$

$$\mathbf{V}_3 = \mathbf{I}_1 \times 1 = \frac{j10}{1.8 + j8.4}(-j2) = 2.328\underline{/-77.91^\circ} \text{ V}$$

In the time domain,

$$v_3 = 2.33\cos(5t - 80^\circ) = 2.33\sin(5t + 10^\circ) \text{ V} \qquad (10.6.4)$$

Substituting Eqs. (10.6.2) to (10.6.4) into Eq. (10.6.1), we have

$$v_o(t) = -1 + 2.498\cos(2t - 30.79^\circ) + 2.33\sin(5t + 10^\circ) \text{ V}$$

PRACTICE PROBLEM 10.6

Calculate v_o in the circuit of Fig. 10.15 using the superposition theorem.

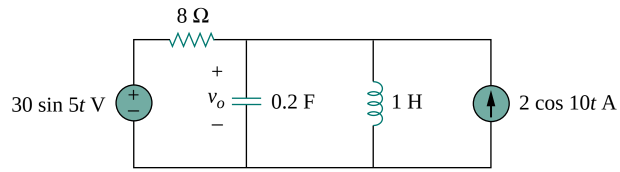

Figure 10.15 For Practice Prob. 10.6.

Answer: $4.631 \sin(5t - 81.12^\circ) + 1.051 \cos(10t - 86.24^\circ)$ V.

10.5 SOURCE TRANSFORMATION

As Fig. 10.16 shows, source transformation in the frequency domain involves transforming a voltage source in series with an impedance to a current source in parallel with an impedance, or vice versa. As we go from one source type to another, we must keep the following relationship in mind:

$$\boxed{\mathbf{V}_s = \mathbf{Z}_s \mathbf{I}_s \qquad \Longleftrightarrow \qquad \mathbf{I}_s = \frac{\mathbf{V}_s}{\mathbf{Z}_s}} \tag{10.1}$$

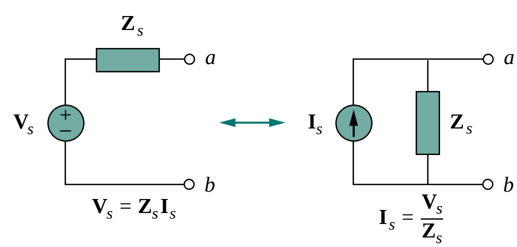

Figure 10.16 Source transformation.

EXAMPLE 10.7

Calculate $\mathbf{V}_x$ in the circuit of Fig. 10.17 using the method of source transformation.

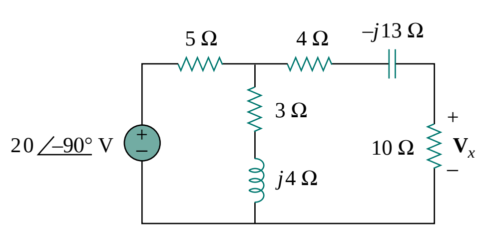

Figure 10.17 For Example 10.7.

Solution:

We transform the voltage source to a current source and obtain the circuit in Fig. 10.18(a), where

$$\mathbf{I}_s = \frac{20\angle -90^\circ}{5} = 4\angle -90^\circ = -j4 \text{ A}$$

The parallel combination of 5-Ω resistance and $(3 + j4)$ impedance gives

$$\mathbf{Z}_1 = \frac{5(3 + j4)}{8 + j4} = 2.5 + j1.25\ \Omega$$

Converting the current source to a voltage source yields the circuit in Fig. 10.18(b), where

$$\mathbf{V}_s = \mathbf{I}_s\mathbf{Z}_1 = -j4(2.5 + j1.25) = 5 - j10 \text{ V}$$

By voltage division,

$$\mathbf{V}_x = \frac{10}{10 + 2.5 + j1.25 + 4 - j13}(5 - j10) = 5.519\angle -28^\circ \text{ V}$$

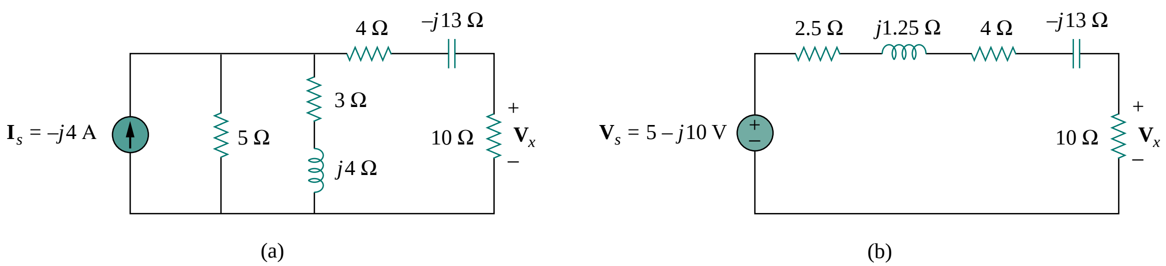

Figure 10.18 Solution of the circuit in Fig. 10.17.

PRACTICE PROBLEM 10.7

Find $\mathbf{I}_o$ in the circuit of Fig. 10.19 using the concept of source transformation.

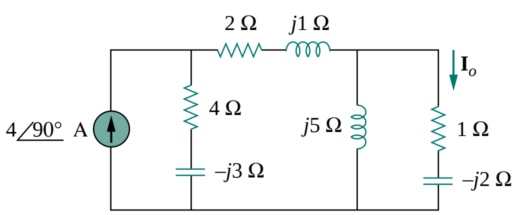

Figure 10.19 For Practice Prob. 10.7.

Answer: $3.288\angle 99.46^\circ$ A.

Electronic Testing Tutorials

10.6 THEVENIN AND NORTON EQUIVALENT CIRCUITS

Thevenin's and Norton's theorems are applied to ac circuits in the same way as they are to dc circuits. The only additional effort arises from the need to manipulate complex numbers. The frequency-domain version of a Thevenin equivalent circuit is depicted in Fig. 10.20, where a linear circuit is replaced by a voltage source in series with an impedance. The Norton equivalent circuit is illustrated in Fig. 10.21, where a linear circuit is replaced by a current source in parallel with an impedance. Keep in mind that the two equivalent circuits are related as

$$\boxed{\mathbf{V}_{\text{Th}} = \mathbf{Z}_N \mathbf{I}_N, \qquad \mathbf{Z}_{\text{Th}} = \mathbf{Z}_N} \tag{10.2}$$

just as in source transformation. $\mathbf{V}_{\text{Th}}$ is the open-circuit voltage while $\mathbf{I}_N$ is the short-circuit current.

If the circuit has sources operating at different frequencies (see Example 10.6, for example), the Thevenin or Norton equivalent circuit must be determined at each frequency. This leads to entirely different equivalent circuits, one for each frequency, not one equivalent circuit with equivalent sources and equivalent impedances.

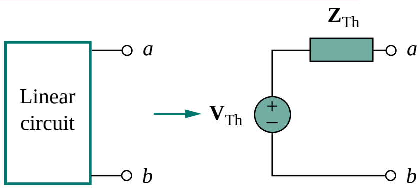

Figure 10.20 Thevenin equivalent.

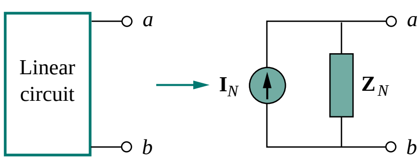

Figure 10.21 Norton equivalent.

EXAMPLE 10.8

Obtain the Thevenin equivalent at terminals *a*-*b* of the circuit in Fig. 10.22.

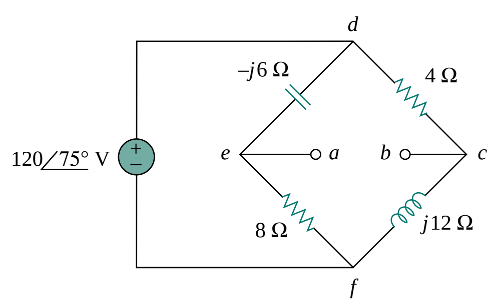

Figure 10.22 For Example 10.8.

Solution:

We find $\mathbf{Z}_{\text{Th}}$ by setting the voltage source to zero. As shown in Fig. 10.23(a), the 8-Ω resistance is now in parallel with the $-j6$ reactance, so that their combination gives

$$\mathbf{Z}_1 = -j6 \parallel 8 = \frac{-j6 \times 8}{8 - j6} = 2.88 - j3.84\ \Omega$$

Similarly, the 4-Ω resistance is in parallel with the $j12$ reactance, and their combination gives

$$\mathbf{Z}_2 = 4 \parallel j12 = \frac{j12 \times 4}{4 + j12} = 3.6 + j1.2\ \Omega$$

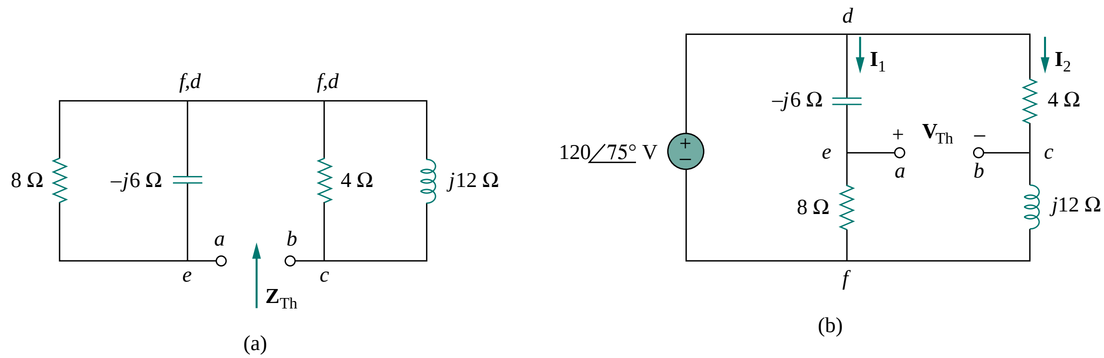

Figure 10.23 Solution of the circuit in Fig. 10.22: (a) finding $\mathbf{Z}_{\text{Th}}$, (b) finding $\mathbf{V}_{\text{Th}}$.

The Thevenin impedance is the series combination of $\mathbf{Z}_1$ and $\mathbf{Z}_2$; that is,

$$\mathbf{Z}_{\text{Th}} = \mathbf{Z}_1 + \mathbf{Z}_2 = 6.48 - j2.64\ \Omega$$

To find $\mathbf{V}_{\text{Th}}$, consider the circuit in Fig. 10.23(b). Currents $\mathbf{I}_1$ and $\mathbf{I}_2$ are obtained as

$$\mathbf{I}_1 = \frac{120\angle 75^\circ}{8 - j6}\ \text{A}, \qquad \mathbf{I}_2 = \frac{120\angle 75^\circ}{4 + j12}\ \text{A}$$

Applying KVL around loop $bcdeab$ in Fig. 10.23(b) gives

$$\mathbf{V}_{\text{Th}} - 4\mathbf{I}_2 + (-j6)\mathbf{I}_1 = 0$$

or

$$\begin{aligned}
\mathbf{V}_{\text{Th}} = 4\mathbf{I}_2 + j6\mathbf{I}_1 &= \frac{480\angle 75^\circ}{4 + j12} + \frac{720\angle 75^\circ + 90^\circ}{8 - j6} \\
&= 37.95\angle 3.43^\circ + 72\angle 201.87^\circ \\
&= -28.936 - j24.55 = 37.95\angle 220.31^\circ\ \text{V}
\end{aligned}$$

PRACTICE PROBLEM 10.8

Find the Thevenin equivalent at terminals a-b of the circuit in Fig. 10.24.

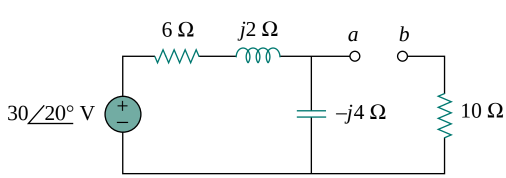

Figure 10.24 For Practice Prob. 10.8.

Answer: $\mathbf{Z}_{\text{Th}} = 12.4 - j3.2\ \Omega$, $\mathbf{V}_{\text{Th}} = 18.97\angle -51.57^\circ$ V.

EXAMPLE 10.9

Find the Thevenin equivalent of the circuit in Fig. 10.25 as seen from terminals *a*-*b*.

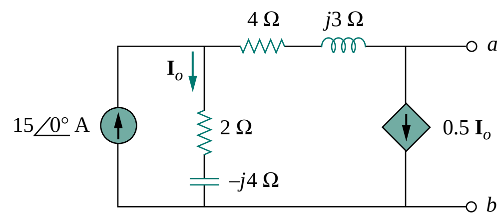

Figure 10.25 For Example 10.9.

Solution:

To find $\mathbf{V}_{\text{Th}}$, we apply KCL at node 1 in Fig. 10.26(a).

$$15 = \mathbf{I}_o + 0.5\mathbf{I}_o \qquad \Longrightarrow \qquad \mathbf{I}_o = 10 \text{ A}$$

Applying KVL to the loop on the right-hand side in Fig. 10.26(a), we obtain

$$-\mathbf{I}_o(2 - j4) + 0.5\mathbf{I}_o(4 + j3) + \mathbf{V}_{\text{Th}} = 0$$

or

$$\mathbf{V}_{\text{Th}} = 10(2 - j4) - 5(4 + j3) = -j55$$

Thus, the Thevenin voltage is

$$\mathbf{V}_{\text{Th}} = 55\angle{-90^\circ} \text{ V}$$

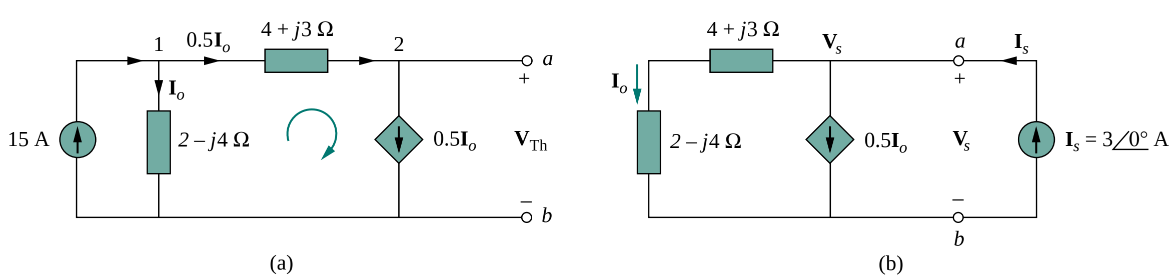

Figure 10.26 Solution of the problem in Fig. 10.25: (a) finding $\mathbf{V}_{\text{Th}}$, (b) finding $\mathbf{Z}_{\text{Th}}$.

To obtain $\mathbf{Z}_{\text{Th}}$, we remove the independent source. Due to the presence of the dependent current source, we connect a 3-A current source (3 is an arbitrary value chosen for convenience here, a number divisible by the sum of currents leaving the node) to terminals *a*-*b* as shown in Fig. 10.26(b). At the node, KCL gives

$$3 = \mathbf{I}_o + 0.5\mathbf{I}_o \qquad \Longrightarrow \qquad \mathbf{I}_o = 2 \text{ A}$$

Applying KVL to the outer loop in Fig. 10.26(b) gives

$$\mathbf{V}_s = \mathbf{I}_o(4 + j3 + 2 - j4) = 2(6 - j)$$

The Thevenin impedance is

$$\mathbf{Z}_{\text{Th}} = \frac{\mathbf{V}_s}{\mathbf{I}_s} = \frac{2(6-j)}{3} = 4 - j0.6667\ \Omega$$

PRACTICE PROBLEM 10.9

Determine the Thevenin equivalent of the circuit in Fig. 10.27 as seen from the terminals a-b.

Answer: $\mathbf{Z}_{\text{Th}} = 12.166\underline{/136.3^\circ}\ \Omega$, $\mathbf{V}_{\text{Th}} = 7.35\underline{/72.9^\circ}$ V.

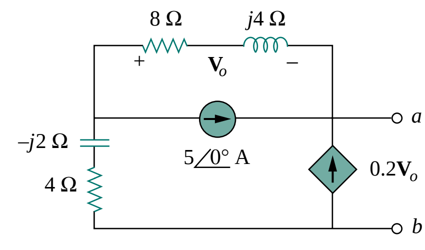

Figure 10.27 For Practice Prob. 10.9.

EXAMPLE 10.10

Obtain current $\mathbf{I}_o$ in Fig. 10.28 using Norton's theorem.

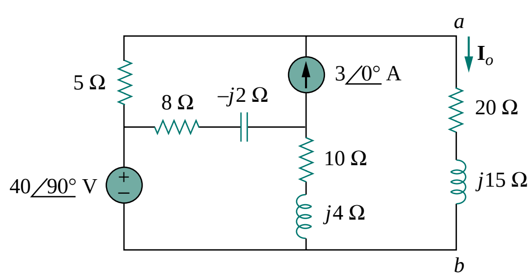

Figure 10.28 For Example 10.10.

Solution:

Our first objective is to find the Norton equivalent at terminals a-b. $\mathbf{Z}_N$ is found in the same way as $\mathbf{Z}_{\text{Th}}$. We set the sources to zero as shown in Fig. 10.29(a). As evident from the figure, the $(8 - j2)$ and $(10 + j4)$ impedances are short-circuited, so that

$$\mathbf{Z}_N = 5\ \Omega$$

To get $\mathbf{I}_N$, we short-circuit terminals a-b as in Fig. 10.29(b) and apply mesh analysis. Notice that meshes 2 and 3 form a supermesh because of the current source linking them. For mesh 1,

$$-j40 + (18 + j2)\mathbf{I}_1 - (8 - j2)\mathbf{I}_2 - (10 + j4)\mathbf{I}_3 = 0 \qquad (10.10.1)$$

For the supermesh,

$$(13 - j2)\mathbf{I}_2 + (10 + j4)\mathbf{I}_3 - (18 + j2)\mathbf{I}_1 = 0 \qquad (10.10.2)$$

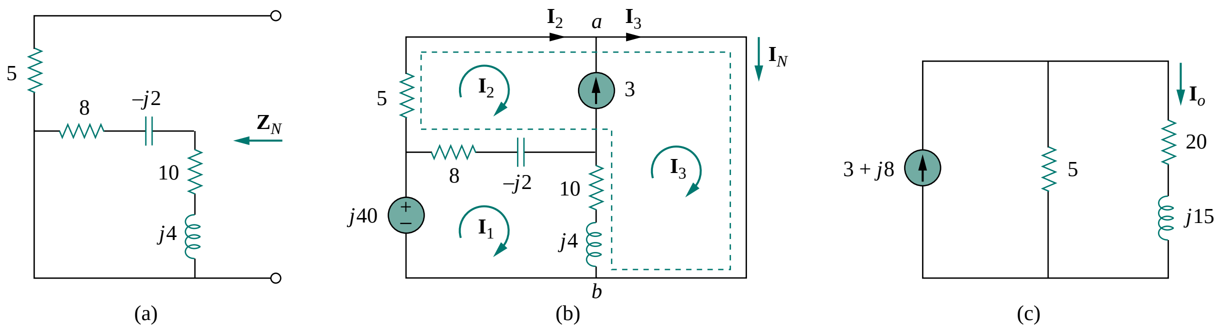

Figure 10.29 Solution of the circuit in Fig. 10.28: (a) finding $\mathbf{Z}_N$, (b) finding $\mathbf{V}_N$, (c) calculating $\mathbf{I}_o$.

At node a, due to the current source between meshes 2 and 3,

$$\mathbf{I}_3 = \mathbf{I}_2 + 3 \tag{10.10.3}$$

Adding Eqs. (10.10.1) and (10.10.2) gives

$$-j40 + 5\mathbf{I}_2 = 0 \qquad \Longrightarrow \qquad \mathbf{I}_2 = j8$$

From Eq. (10.10.3),

$$\mathbf{I}_3 = \mathbf{I}_2 + 3 = 3 + j8$$

The Norton current is

$$\mathbf{I}_N = \mathbf{I}_3 = (3 + j8)\text{ A}$$

Figure 10.29(c) shows the Norton equivalent circuit along with the impedance at terminals a-b. By current division,

$$\mathbf{I}_o = \frac{5}{5 + 20 + j15}\mathbf{I}_N = \frac{3 + j8}{5 + j3} = 1.465\underline{/38.48^\circ}\text{ A}$$

PRACTICE PROBLEM 10.10

Determine the Norton equivalent of the circuit in Fig. 10.30 as seen from terminals a-b. Use the equivalent to find $\mathbf{I}_o$.

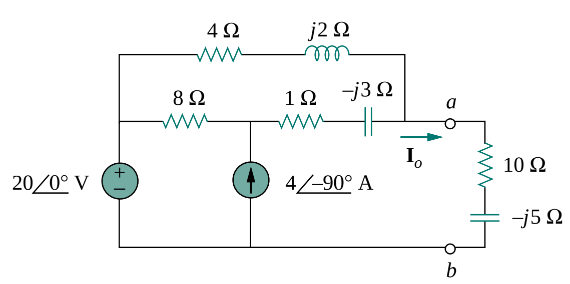

Figure 10.30 For Practice Prob. 10.10.

Answer: $\mathbf{Z}_N = 3.176 + j0.706\ \Omega$, $\mathbf{I}_N = 8.396\underline{/-32.68^\circ}$ A, $\mathbf{I}_o = 1.971\underline{/-2.101^\circ}$ A.

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10.7 OP AMP AC CIRCUITS

The three steps stated in Section 10.1 also apply to op amp circuits, as long as the op amp is operating in the linear region. As usual, we will assume ideal op amps. (See Section 5.2.) As discussed in Chapter 5, the key to analyzing op amp circuits is to keep two important properties of an ideal op amp in mind:

1. No current enters either of its input terminals.
2. The voltage across its input terminals is zero.

The following examples will illustrate these ideas.

EXAMPLE 10.11

Determine $v_o(t)$ for the op amp circuit in Fig. 10.31(a) if $v_s = 3\cos 1000t$ V.

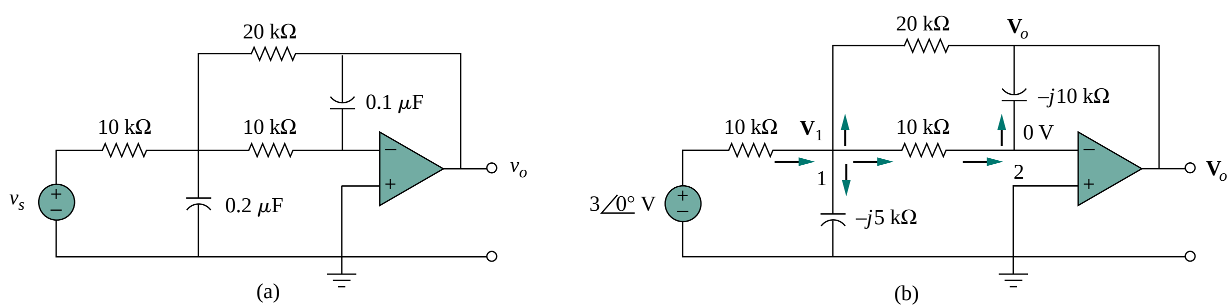

Figure 10.31 For Example 10.11: (a) the original circuit in the time domain, (b) its frequency-domain equivalent.

Solution:

We first transform the circuit to the frequency domain, as shown in Fig. 10.31(b), where $\mathbf{V}_s = 3\angle 0^\circ$, $\omega = 1000$ rad/s. Applying KCL at node 1, we obtain

$$\frac{3\angle 0^\circ - \mathbf{V}_1}{10} = \frac{\mathbf{V}_1}{-j5} + \frac{\mathbf{V}_1 - 0}{10} + \frac{\mathbf{V}_1 - \mathbf{V}_o}{20}$$

or

$$6 = (5 + j4)\mathbf{V}_1 - \mathbf{V}_o \tag{10.11.1}$$

At node 2, KCL gives

$$\frac{\mathbf{V}_1 - 0}{10} = \frac{0 - \mathbf{V}_o}{-j10}$$

which leads to

$$\mathbf{V}_1 = -j\mathbf{V}_o \tag{10.11.2}$$

Substituting Eq. (10.11.2) into Eq. (10.11.1) yields

$$6 = -j(5 + j4)\mathbf{V}_o - \mathbf{V}_o = (3 - j5)\mathbf{V}_o$$

$$\mathbf{V}_o = \frac{6}{3 - j5} = 1.029\angle 59.04^\circ$$

Hence,

$$v_o(t) = 1.029\cos(1000t + 59.04^\circ)\text{ V}$$

PRACTICE PROBLEM 10.11

Find v_o and i_o in the op amp circuit of Fig. 10.32. Let $v_s = 2\cos 5000t$ V.

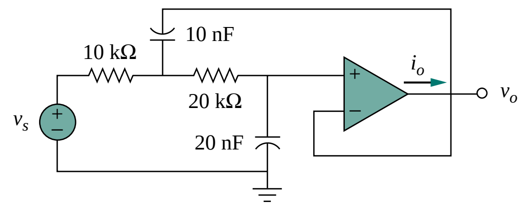

Figure 10.32 For Practice Prob. 10.11.

Answer: $0.667 \sin 5000t$ V, $66.67 \sin 5000t$ μA.

EXAMPLE 10.12

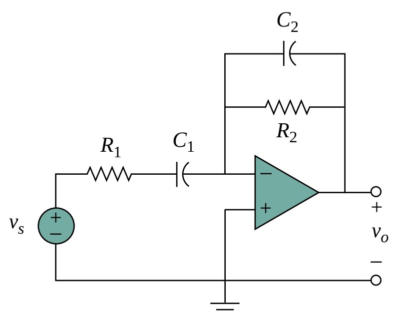

Figure 10.33 For Example 10.12.

Compute the closed-loop gain and phase shift for the circuit in Fig. 10.33. Assume that $R_1 = R_2 = 10$ kΩ, $C_1 = 2$ μF, $C_2 = 1$ μF, and $\omega = 200$ rad/s.

Solution:

The feedback and input impedances are calculated as

$$\mathbf{Z}_f = R_2 \left\| \frac{1}{j\omega C_2} = \frac{R_2}{1 + j\omega R_2 C_2} \right.$$

$$\mathbf{Z}_i = R_1 + \frac{1}{j\omega C_1} = \frac{1 + j\omega R_1 C_1}{j\omega C_1}$$

Since the circuit in Fig. 10.33 is an inverting amplifier, the closed-loop gain is given by

$$\mathbf{G} = \frac{\mathbf{V}_o}{\mathbf{V}_s} = -\frac{\mathbf{Z}_f}{\mathbf{Z}_i} = \frac{j\omega C_1 R_2}{(1 + j\omega R_1 C_1)(1 + j\omega R_2 C_2)}$$

Substituting the given values of R_1, R_2, C_1, C_2, and ω, we obtain

$$\mathbf{G} = \frac{j4}{(1 + j4)(1 + j2)} = 0.434 \underline{/-49.4^\circ}$$

Thus the closed-loop gain is 0.434 and the phase shift is -49.4°.

PRACTICE PROBLEM 10.12

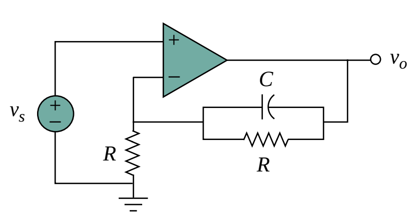

Figure 10.34 For Practice Prob. 10.12.

Obtain the closed-loop gain and phase shift for the circuit in Fig. 10.34. Let $R = 10$ kΩ, $C = 1$ μF, and $\omega = 1000$ rad/s.

Answer: 1.015, -5.599°.

10.8 AC ANALYSIS USING *PSPICE*

PSpice affords a big relief from the tedious task of manipulating complex numbers in ac circuit analysis. The procedure for using *PSpice* for ac analysis is quite similar to that required for dc analysis. The reader should read Section D.5 in Appendix D for a review of *PSpice* concepts for ac analysis. AC circuit analysis is done in the phasor or frequency domain, and all sources must have the same frequency. Although AC analysis with *PSpice* involves using AC Sweep, our analysis in this chapter requires a single frequency $f = \omega/2\pi$. The output file of *PSpice* contains voltage and current phasors. If necessary, the impedances can be calculated using the voltages and currents in the output file.

EXAMPLE 10.13

Obtain v_o and i_o in the circuit of Fig. 10.35 using *PSpice*.

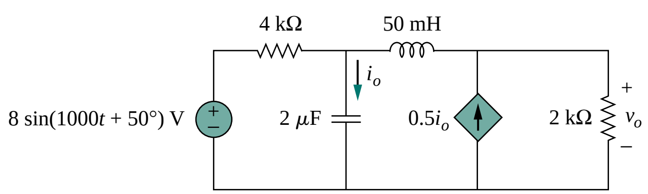

Figure 10.35 For Example 10.13.

Solution:

We first convert the sine function to cosine.

$$8\sin(1000t + 50^\circ) = 8\cos(1000t + 50^\circ - 90^\circ) = 8\cos(1000t - 40^\circ)$$

The frequency f is obtained from ω as

$$f = \frac{\omega}{2\pi} = \frac{1000}{2\pi} = 159.155 \text{ Hz}$$

The schematic for the circuit is shown in Fig. 10.36. Notice the current-controlled current source F1 is connected such that its current flows from

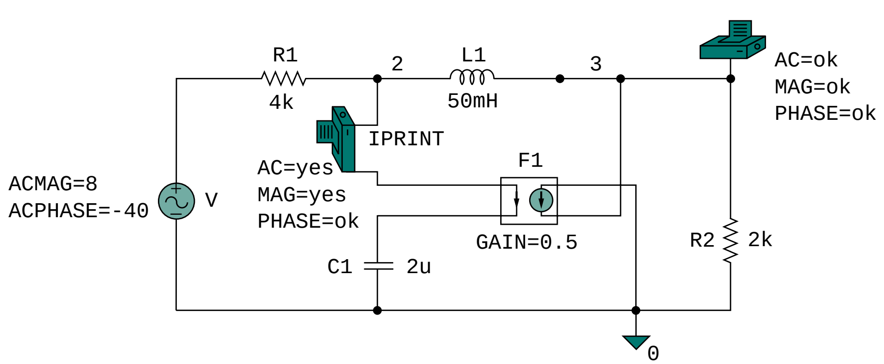

Figure 10.36 The schematic of the circuit in Fig. 10.35.

node 0 to node 3 in conformity with the original circuit in Fig. 10.35. Since we only want the magnitude and phase of v_o and i_o, we set the attributes of IPRINT AND VPRINT1 each to *AC* = *yes*, *MAG* = *yes*, *PHASE* = *yes*. As a single-frequency analysis, we select **Analysis/Setup/AC Sweep** and enter *Total Pts* = 1, *Start Freq* = 159.155, and *Final Freq* = 159.155. After saving the schematic, we simulate it by selecting **Analysis/Simulate**. The output file includes the source frequency in addition to the attributes checked for the pseudocomponents IPRINT and VPRINT1,

```
FREQ        IM(V_PRINT3)  IP(V_PRINT3)
1.592E+02   3.264E-03     -3.743E+01

FREQ        VM(3)         VP(3)
1.592E+02   1.550E+00     -9.518E+01
```

From this output file, we obtain

$$\mathbf{V}_o = 1.55\underline{/-95.18^\circ}\ \text{V}, \qquad \mathbf{I}_o = 3.264\underline{/-37.43^\circ}\ \text{mA}$$

which are the phasors for

$$v_o = 1.55\cos(1000t - 95.18^\circ) = 1.55\sin(1000t - 5.18^\circ)\ \text{V}$$

and

$$i_o = 3.264\cos(1000t - 37.43^\circ)\ \text{mA}$$

PRACTICE PROBLEM 10.13

Use *PSpice* to obtain v_o and i_o in the circuit of Fig. 10.37.

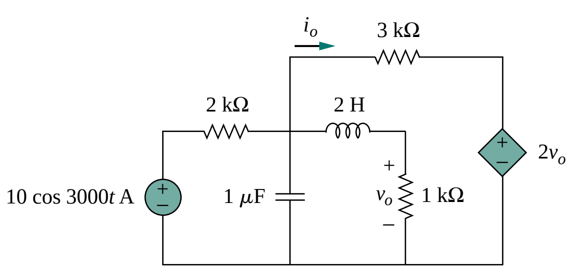

Figure 10.37 For Practice Prob. 10.13.

Answer: $0.2682\cos(3000t - 154.6^\circ)$ V, $0.544\cos(3000t - 55.12^\circ)$ mA.

EXAMPLE 10.14

Find $\mathbf{V}_1$ and $\mathbf{V}_2$ in the circuit of Fig. 10.38.

Solution:

The circuit in Fig. 10.35 is in the time domain, whereas the one in Fig. 10.38 is in the frequency domain. Since we are not given a particular

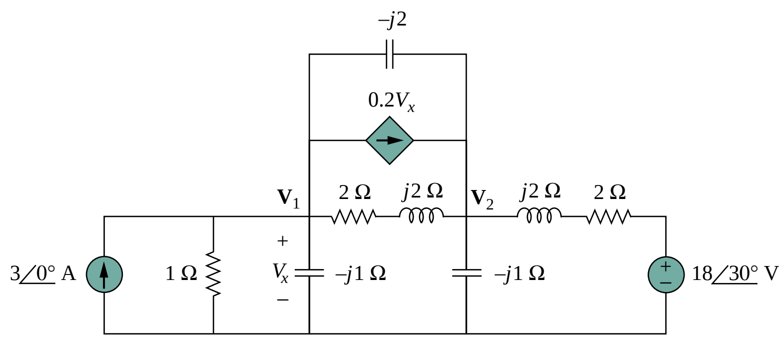

Figure 10.38 For Example 10.14.

frequency and *PSpice* requires one, we select any frequency consistent with the given impedances. For example, if we select $\omega = 1$ rad/s, the corresponding frequency is $f = \omega/2\pi = 0.159155$ Hz. We obtain the values of the capacitance ($C = 1/\omega X_C$) and inductances ($L = X_L/\omega$). Making these changes results in the schematic in Fig. 10.39. To ease wiring, we have exchanged the positions of the voltage-controlled current source G1 and the $2 + j2\ \Omega$ impedance. Notice that the current of G1 flows from node 1 to node 3, while the controlling voltage is across the capacitor c2, as required in Fig. 10.38. The attributes of pseudocomponents VPRINT1 are set as shown. As a single-frequency analysis, we select **Analysis/Setup/AC Sweep** and enter *Total Pts* = 1, *Start Freq* = 0.159155, and *Final Freq* = 0.159155. After saving the schematic, we select **Analysis/Simulate** to simulate the circuit. When this is done, the output file includes

```
FREQ        VM(1)       VP(1)
1.592E-01   2.708E+00   -5.673E+01

FREQ        VM(3)       VP(3)
1.592E-01   4.468E+00   -1.026E+02
```

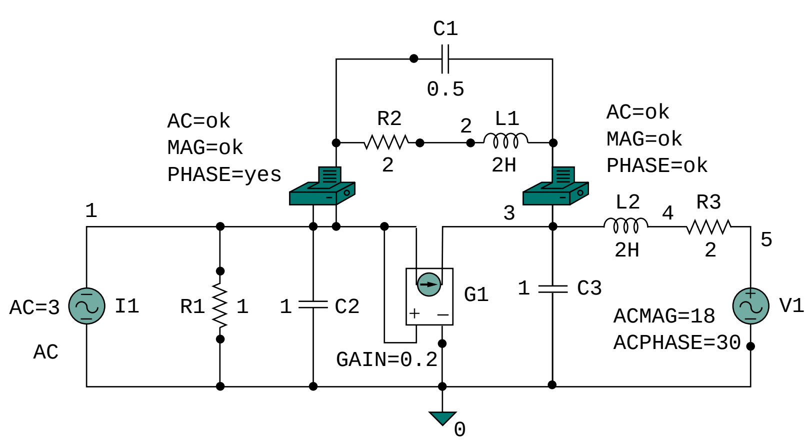

Figure 10.39 Schematic for the circuit in Fig. 10.38.

from which we obtain

$$\mathbf{V}_1 = 2.708\underline{/-56.73^\circ}\text{ V}, \qquad \mathbf{V}_2 = 4.468\underline{/-102.6^\circ}\text{ V}$$

PRACTICE PROBLEM 10.14

Obtain $\mathbf{V}_x$ and $\mathbf{I}_x$ in the circuit depicted in Fig. 10.40.

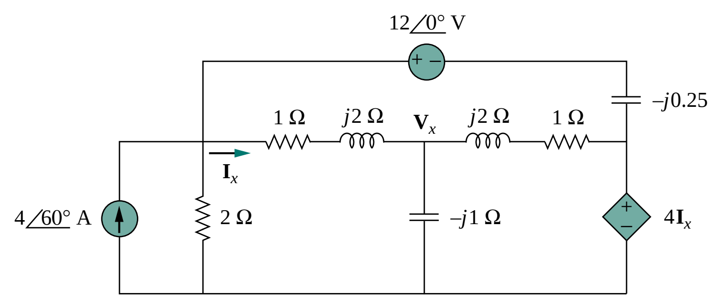

Figure 10.40 For Practice Prob. 10.14.

Answer: $13.02\underline{/-76.08^\circ}$ V, $8.234\underline{/-4.516^\circ}$ A.

†10.9 APPLICATIONS

The concepts learned in this chapter will be applied in later chapters to calculate electric power and determine frequency response. The concepts are also used in analyzing coupled circuits, three-phase circuits, ac transistor circuits, filters, oscillators, and other ac circuits. In this section, we apply the concepts to develop two practical ac circuits: the capacitance multiplier and the sine wave oscillators.

10.9.1 Capacitance Multiplier

The op amp circuit in Fig. 10.41 is known as a *capacitance multiplier*, for reasons that will become obvious. Such a circuit is used in integrated-circuit technology to produce a multiple of a small physical capacitance C when a large capacitance is needed. The circuit in Fig. 10.41 can be used to multiply capacitance values by a factor up to 1000. For example, a 10-pF capacitor can be made to behave like a 100-nF capacitor.

In Fig. 10.41, the first op amp operates as a voltage follower, while the second one is an inverting amplifier. The voltage follower isolates the capacitance formed by the circuit from the loading imposed by the inverting amplifier. Since no current enters the input terminals of the op amp, the input current $\mathbf{I}_i$ flows through the feedback capacitor. Hence, at node 1,

$$\mathbf{I}_i = \frac{\mathbf{V}_i - \mathbf{V}_o}{1/j\omega C} = j\omega C(\mathbf{V}_i - \mathbf{V}_o) \tag{10.3}$$

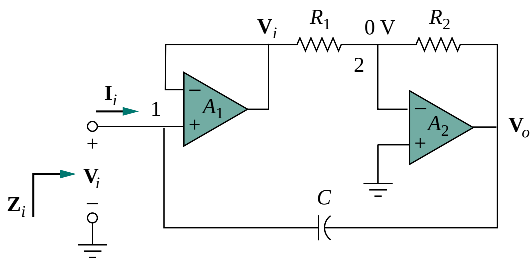

Figure 10.41 Capacitance multiplier.

Applying KCL at node 2 gives

$$\frac{\mathbf{V}_i - 0}{R_1} = \frac{0 - \mathbf{V}_o}{R_2}$$

or

$$\mathbf{V}_o = -\frac{R_2}{R_1}\mathbf{V}_i \tag{10.4}$$

Substituting Eq. (10.4) into (10.3) gives

$$\mathbf{I}_i = j\omega C\left(1 + \frac{R_2}{R_1}\right)\mathbf{V}_i$$

or

$$\frac{\mathbf{I}_i}{\mathbf{V}_i} = j\omega\left(1 + \frac{R_2}{R_1}\right)C \tag{10.5}$$

The input impedance is

$$\mathbf{Z}_i = \frac{\mathbf{V}_i}{\mathbf{I}_i} = \frac{1}{j\omega C_{\text{eq}}} \tag{10.6}$$

where

$$C_{\text{eq}} = \left(1 + \frac{R_2}{R_1}\right)C \tag{10.7}$$

Thus, by a proper selection of the values of R_1 and R_2, the op amp circuit in Fig. 10.41 can be made to produce an effective capacitance between the input terminal and ground, which is a multiple of the physical capacitance C. The size of the effective capacitance is practically limited by the inverted output voltage limitation. Thus, the larger the capacitance multiplication, the smaller is the allowable input voltage to prevent the op amps from reaching saturation.

A similar op amp circuit can be designed to simulate inductance. (See Prob. 10.69.) There is also an op amp circuit configuration to create a resistance multiplier.

EXAMPLE 10.15

Calculate C_{eq} in Fig. 10.41 when $R_1 = 10\,\text{k}\Omega$, $R_2 = 1\,\text{M}\Omega$, and $C = 1\,\text{nF}$.

Solution:

From Eq. (10.7)

$$C_{eq} = \left(1 + \frac{R_2}{R_1}\right) C = \left(1 + \frac{1 \times 10^6}{10 \times 10^3}\right) 1\text{ nF} = 101\text{ nF}$$

PRACTICE PROBLEM 10.15

Determine the equivalent capacitance of the op amp circuit in Fig. 10.41 if $R_1 = 10\ \text{k}\Omega$, $R_2 = 10\ \text{M}\Omega$, and $C = 10$ nF.

Answer: 10 μF.

10.9.2 Oscillators

We know that dc is produced by batteries. But how do we produce ac? One way is using *oscillators*, which are circuits that convert dc to ac.

> An oscillator is a circuit that produces an ac waveform as output when powered by a dc input.

The only external source an oscillator needs is the dc power supply. Ironically, the dc power supply is usually obtained by converting the ac supplied by the electric utility company to dc. Having gone through the trouble of conversion, one may wonder why we need to use the oscillator to convert the dc to ac again. The problem is that the ac supplied by the utility company operates at a preset frequency of 60 Hz in the United States (50 Hz in some other nations), whereas many applications such as electronic circuits, communication systems, and microwave devices require internally generated frequencies that range from 0 to 10 GHz or higher. Oscillators are used for generating these frequencies.

This corresponds to $\omega = 2\pi f = 377$ rad/s.

In order for sine wave oscillators to sustain oscillations, they must meet the *Barkhausen criteria:*

1. The overall gain of the oscillator must be unity or greater. Therefore, losses must be compensated for by an amplifying device.
2. The overall phase shift (from input to output and back to the input) must be zero.

Three common types of sine wave oscillators are phase-shift, twin T, and Wien-bridge oscillators. Here we consider only the Wien-bridge oscillator.

The *Wien-bridge oscillator* is widely used for generating sinusoids in the frequency range below 1 MHz. It is an RC op amp circuit with only a few components, easily tunable and easy to design. As shown in Fig. 10.42, the oscillator essentially consists of a noninverting amplifier with two feedback paths: the positive feedback path to the noninverting input creates oscillations, while the negative feedback path to the inverting

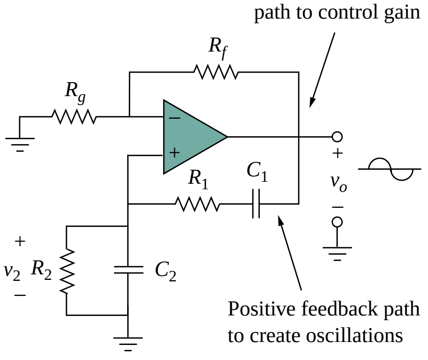

Figure 10.42 Wien-bridge oscillator.

input controls the gain. If we define the impedances of the RC series and parallel combinations as $\mathbf{Z}_s$ and $\mathbf{Z}_p$, then

$$\mathbf{Z}_s = R_1 + \frac{1}{j\omega C_1} = R_1 - \frac{j}{\omega C_1} \tag{10.8}$$

$$\mathbf{Z}_p = R_2 \parallel \frac{1}{j\omega C_2} = \frac{R_2}{1 + j\omega R_2 C_2} \tag{10.9}$$

The feedback ratio is

$$\frac{\mathbf{V}_2}{\mathbf{V}_o} = \frac{\mathbf{Z}_p}{\mathbf{Z}_s + \mathbf{Z}_p} \tag{10.10}$$

Substituting Eqs. (10.8) and (10.9) into Eq. (10.10) gives

$$\begin{aligned} \frac{\mathbf{V}_2}{\mathbf{V}_o} &= \frac{R_2}{R_2 + \left(R_1 - \dfrac{j}{\omega C_1}\right)(1 + j\omega R_2 C_2)} \\ &= \frac{\omega R_2 C_1}{\omega(R_2 C_1 + R_1 C_1 + R_2 C_2) + j(\omega^2 R_1 C_1 R_2 C_2 - 1)} \end{aligned} \tag{10.11}$$

To satisfy the second Barkhausen criterion, $\mathbf{V}_2$ must be in phase with $\mathbf{V}_o$, which implies that the ratio in Eq. (10.11) must be purely real. Hence, the imaginary part must be zero. Setting the imaginary part equal to zero gives the oscillation frequency ω_o as

$$\omega_o^2 R_1 C_1 R_2 C_2 - 1 = 0$$

or

$$\omega_o = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}} \tag{10.12}$$

In most practical applications, $R_1 = R_2 = R$ and $C_1 = C_2 = C$, so that

$$\omega_o = \frac{1}{RC} = 2\pi f_o \tag{10.13}$$

or

$$\boxed{f_o = \frac{1}{2\pi RC}} \tag{10.14}$$

Substituting Eq. (10.13) and $R_1 = R_2 = R$, $C_1 = C_2 = C$ into Eq. (10.11) yields

$$\frac{\mathbf{V}_2}{\mathbf{V}_o} = \frac{1}{3} \tag{10.15}$$

Thus in order to satisfy the first Barkhausen criterion, the op amp must compensate by providing a gain of 3 or greater so that the overall gain is at least 1 or unity. We recall that for a noninverting amplifier,

$$\frac{\mathbf{V}_o}{\mathbf{V}_2} = 1 + \frac{R_f}{R_g} = 3 \tag{10.16}$$

or

$$R_f = 2R_g \tag{10.17}$$

Due to the inherent delay caused by the op amp, Wien-bridge oscillators are limited to operating in the frequency range of 1 MHz or less.

EXAMPLE 10.16

Design a Wien-bridge circuit to oscillate at 100 kHz.

Solution:

Using Eq. (10.14), we obtain the time constant of the circuit as

$$RC = \frac{1}{2\pi f_o} = \frac{1}{2\pi \times 100 \times 10^3} = 1.59 \times 10^{-6} \qquad (10.16.1)$$

If we select $R = 10$ kΩ, then we can select $C = 159$ pF to satisfy Eq. (10.16.1). Since the gain must be 3, $R_f/R_g = 2$. We could select $R_f = 20$ kΩ while $R_g = 10$ kΩ.

PRACTICE PROBLEM 10.16

In the Wien-bridge oscillator circuit in Fig. 10.42, let $R_1 = R_2 = 2.5$ kΩ, $C_1 = C_2 = 1$ nF. Determine the frequency f_o of the oscillator.

Answer: 63.66 kHz.

10.10 SUMMARY

1. We apply nodal and mesh analysis to ac circuits by applying KCL and KVL to the phasor form of the circuits.
2. In solving for the steady-state response of a circuit that has independent sources with different frequencies, each independent source *must* be considered separately. The most natural approach to analyzing such circuits is to apply the superposition theorem. A separate phasor circuit for each frequency *must* be solved independently, and the corresponding response should be obtained in the time domain. The overall response is the sum of the time-domain responses of all the individual phasor circuits.
3. The concept of source transformation is also applicable in the frequency domain.
4. The Thevenin equivalent of an ac circuit consists of a voltage source $\mathbf{V}_{\text{Th}}$ in series with the Thevenin impedance $\mathbf{Z}_{\text{Th}}$.
5. The Norton equivalent of an ac circuit consists of a current source $\mathbf{I}_N$ in parallel with the Norton impedance $\mathbf{Z}_N$ $(= \mathbf{Z}_{\text{Th}})$.
6. *PSpice* is a simple and powerful tool for solving ac circuit problems. It relieves us of the tedious task of working with the complex numbers involved in steady-state analysis.
7. The capacitance multiplier and the ac oscillator provide two typical applications for the concepts presented in this chapter. A capacitance multiplier is an op amp circuit used in producing a multiple of a physical capacitance. An oscillator is a device that uses a dc input to generate an ac output.

REVIEW QUESTIONS

10.1 The voltage $\mathbf{V}_o$ across the capacitor in Fig. 10.43 is:

(a) $5\angle 0^\circ$ V (b) $7.071\angle 45^\circ$ V

(c) $7.071\angle -45^\circ$ V (d) $5\angle -45^\circ$ V

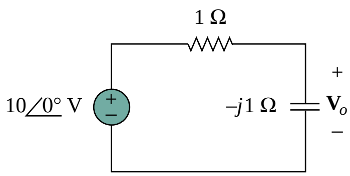

Figure 10.43 For Review Question 10.1.

10.2 The value of the current $\mathbf{I}_o$ in the circuit in Fig. 10.44 is:

(a) $4\angle 0^\circ$ A (b) $2.4\angle -90^\circ$ A

(c) $0.6\angle 0^\circ$ A (d) -1 A

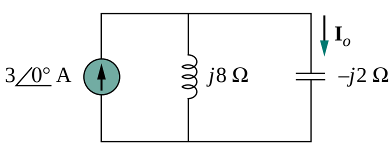

Figure 10.44 For Review Question 10.2.

10.3 Using nodal analysis, the value of $\mathbf{V}_o$ in the circuit of Fig. 10.45 is:

(a) -24 V (b) -8 V

(c) 8 V (d) 24 V

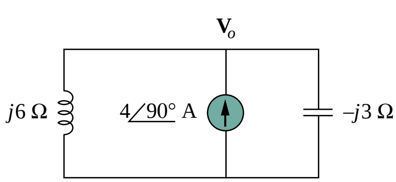

Figure 10.45 For Review Question 10.3.

10.4 In the circuit of Fig. 10.46, current $i(t)$ is:

(a) $10\cos t$ A (b) $10\sin t$ A (c) $5\cos t$ A

(d) $5\sin t$ A (e) $4.472\cos(t - 63.43^\circ)$ A

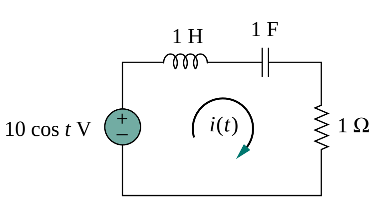

Figure 10.46 For Review Question 10.4.

10.5 Refer to the circuit in Fig. 10.47 and observe that the two sources do not have the same frequency. The current $i_x(t)$ can be obtained by:

(a) source transformation

(b) the superposition theorem

(c) *PSpice*

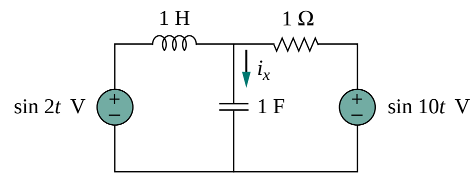

Figure 10.47 For Review Question 10.5.

10.6 For the circuit in Fig. 10.48, the Thevenin impedance at terminals a-b is:

(a) 1 Ω (b) $0.5 - j0.5$ Ω

(c) $0.5 + j0.5$ Ω (d) $1 + j2$ Ω

(e) $1 - j2$ Ω

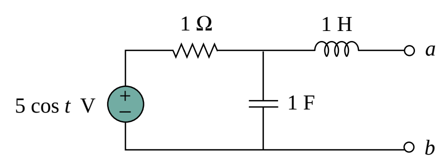

Figure 10.48 For Review Questions 10.6 and 10.7.

10.7 In the circuit of Fig. 10.48, the Thevenin voltage at terminals a-b is:

(a) $3.535\angle -45^\circ$ V (b) $3.535\angle 45^\circ$ V

(c) $7.071\angle -45^\circ$ V (d) $7.071\angle 45^\circ$ V

10.8 Refer to the circuit in Fig. 10.49. The Norton equivalent impedance at terminals a-b is:

(a) $-j4$ Ω (b) $-j2$ Ω

(c) $j2$ Ω (d) $j4$ Ω

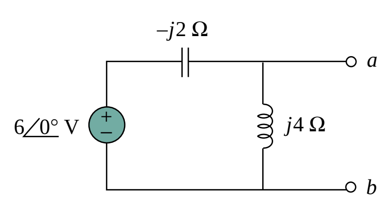

Figure 10.49 For Review Questions 10.8 and 10.9.

10.9 The Norton current at terminals a-b in the circuit of Fig. 10.49 is:

(a) $1\angle 0^\circ$ A (b) $1.5\angle -90^\circ$ A

(c) $1.5\angle 90^\circ$ A (d) $3\angle 90^\circ$ A

10.10 *PSpice* can handle a circuit with two independent sources of different frequencies.

(a) True (b) False

Answers: 10.1c, 10.2a, 10.3d, 10.4a, 10.5b, 10.6c, 10.7a, 10.8a, 10.9d, 10.10b.

PROBLEMS

Section 10.2 Nodal Analysis

10.1 Find v_o in the circuit in Fig. 10.50.

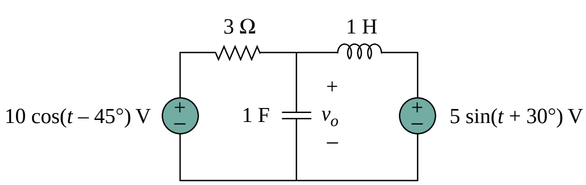

Figure 10.50 For Prob. 10.1.

10.2 For the circuit depicted in Fig. 10.51 below, determine i_o.

10.3 Determine v_o in the circuit of Fig. 10.52.

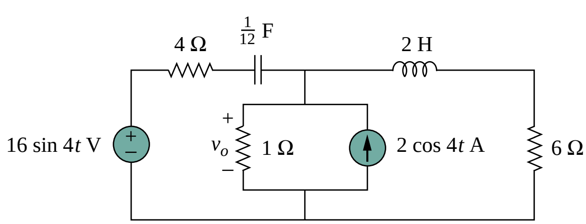

Figure 10.52 For Prob. 10.3.

10.4 Compute $v_o(t)$ in the circuit of Fig. 10.53.

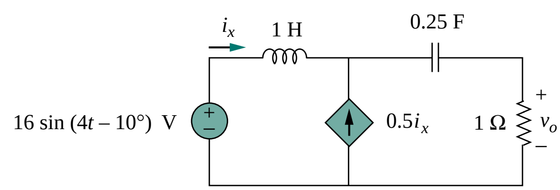

Figure 10.53 For Prob. 10.4.

10.5 Use nodal analysis to find v_o in the circuit of Fig. 10.54.

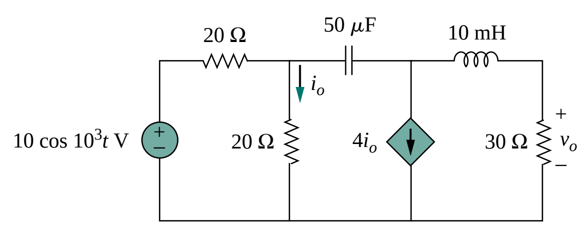

Figure 10.54 For Prob. 10.5.

10.6 Using nodal analysis, find $i_o(t)$ in the circuit in Fig. 10.55.

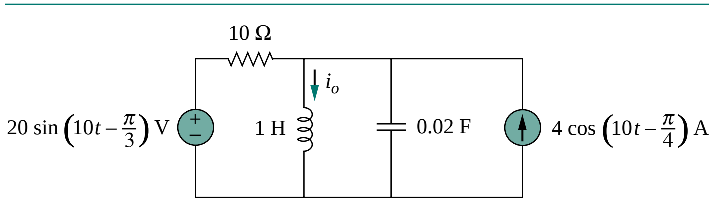

Figure 10.51 For Prob. 10.2.

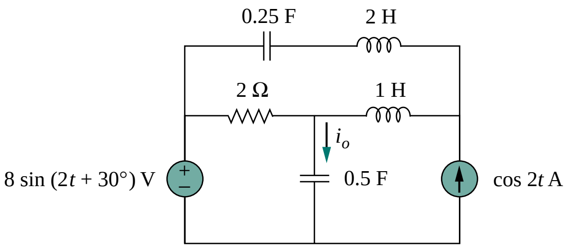

Figure 10.55 For Prob. 10.6.

10.7 By nodal analysis, find i_o in the circuit in Fig. 10.56.

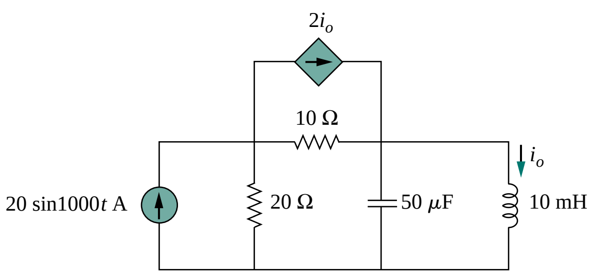

Figure 10.56 For Prob. 10.7.

10.8 Calculate the voltage at nodes 1 and 2 in the circuit of Fig. 10.57 using nodal analysis.

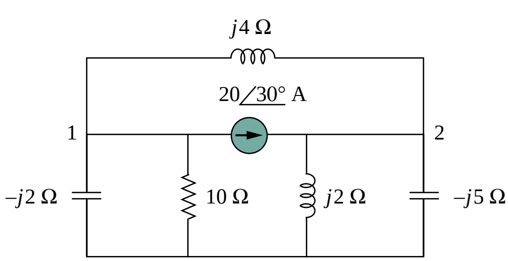

Figure 10.57 For Prob. 10.8.

10.9 Solve for the current **I** in the circuit of Fig. 10.58 using nodal analysis.

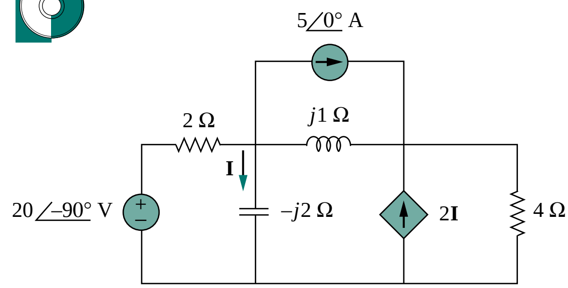

Figure 10.58 For Prob. 10.9.

10.10 Using nodal analysis, find $\mathbf{V}_1$ and $\mathbf{V}_2$ in the circuit of Fig. 10.59.

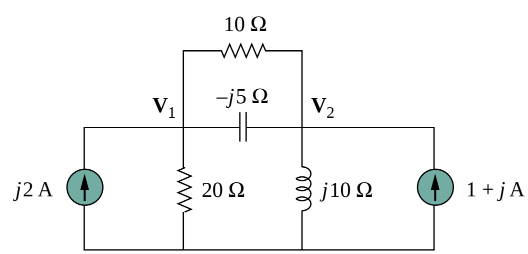

Figure 10.59 For Prob. 10.10.

10.11 By nodal analysis, obtain current $\mathbf{I}_o$ in the circuit in Fig. 10.60.

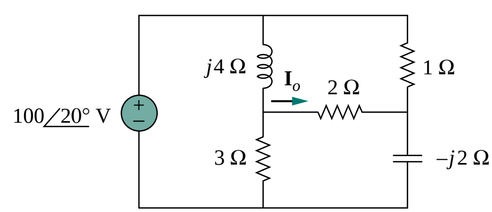

Figure 10.60 For Prob. 10.11.

10.12 Use nodal analysis to obtain $\mathbf{V}_o$ in the circuit of Fig. 10.61 below.

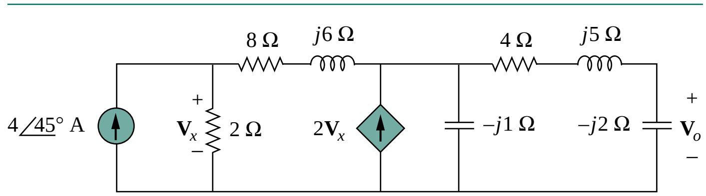

Figure 10.61 For Prob. 10.12.

10.13 Obtain $\mathbf{V}_o$ in Fig. 10.62 using nodal analysis.

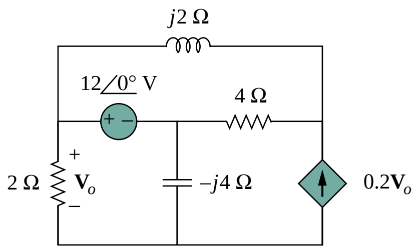

Figure 10.62 For Prob. 10.13.

10.14 Refer to Fig. 10.63. If $v_s(t) = V_m \sin \omega t$ and $v_o(t) = A \sin(\omega t + \phi)$, derive the expressions for A and ϕ.

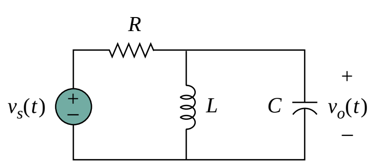

Figure 10.63 For Prob. 10.14.

10.15 For each of the circuits in Fig. 10.64, find $\mathbf{V}_o/\mathbf{V}_i$ for $\omega = 0$, $\omega \to \infty$, and $\omega^2 = 1/LC$.

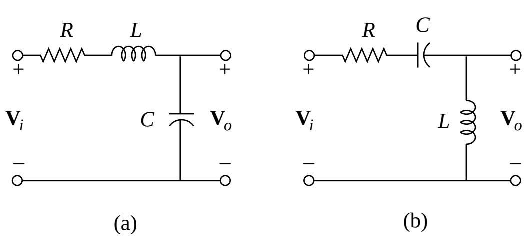

Figure 10.64 For Prob. 10.15.

10.16 For the circuit in Fig. 10.65, determine $\mathbf{V}_o/\mathbf{V}_s$.

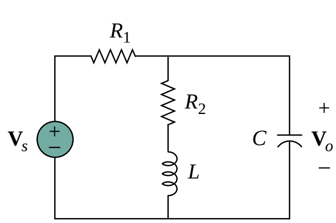

Figure 10.65 For Prob. 10.16.

Section 10.3 Mesh Analysis

10.17 Obtain the mesh currents $\mathbf{I}_1$ and $\mathbf{I}_2$ in the circuit of Fig. 10.66.

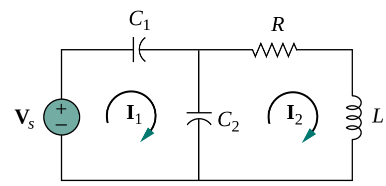

Figure 10.66 For Prob. 10.17.

10.18 Solve for i_o in Fig. 10.67 using mesh analysis.

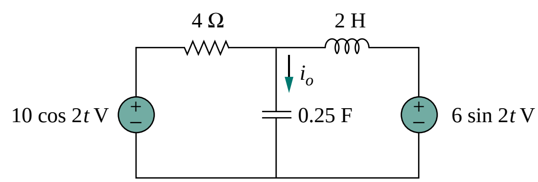

Figure 10.67 For Prob. 10.18.

10.19 Rework Prob. 10.5 using mesh analysis.

10.20 Using mesh analysis, find $\mathbf{I}_1$ and $\mathbf{I}_2$ in the circuit of Fig. 10.68.

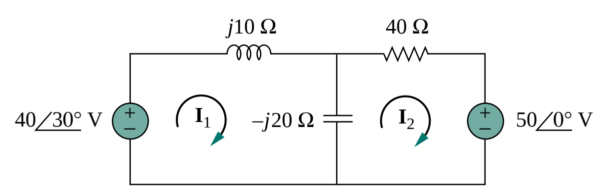

Figure 10.68 For Prob. 10.20.

10.21 By using mesh analysis, find $\mathbf{I}_1$ and $\mathbf{I}_2$ in the circuit depicted in Fig. 10.69.

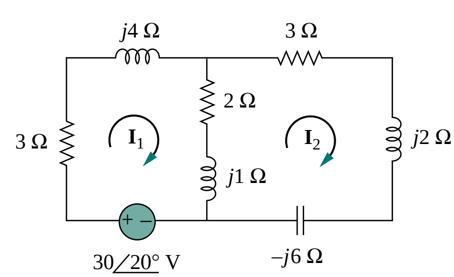

Figure 10.69 For Prob. 10.21.

10.22 Repeat Prob. 10.11 using mesh analysis.

10.23 Use mesh analysis to determine current $\mathbf{I}_o$ in the circuit of Fig. 10.70 below.

10.24 Determine $\mathbf{V}_o$ and $\mathbf{I}_o$ in the circuit of Fig. 10.71 using mesh analysis.

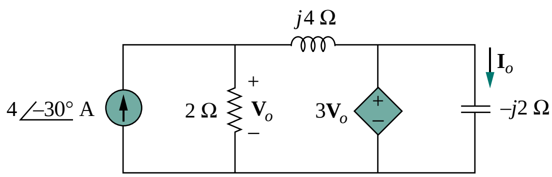

Figure 10.71 For Prob. 10.24.

10.25 Compute **I** in Prob. 10.9 using mesh analysis.

10.26 Use mesh analysis to find $\mathbf{I}_o$ in Fig. 10.28 (for Example 10.10).

10.27 Calculate $\mathbf{I}_o$ in Fig. 10.30 (for Practice Prob. 10.10) using mesh analysis.

10.28 Compute $\mathbf{V}_o$ in the circuit of Fig. 10.72 using mesh analysis.

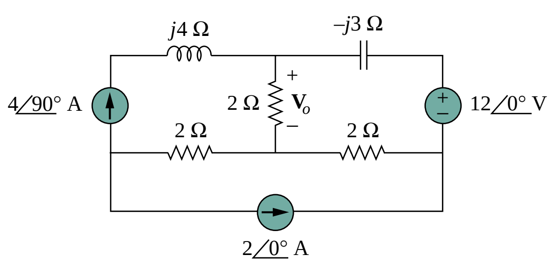

Figure 10.72 For Prob. 10.28.

10.29 Using mesh analysis, obtain $\mathbf{I}_o$ in the circuit shown in Fig. 10.73.

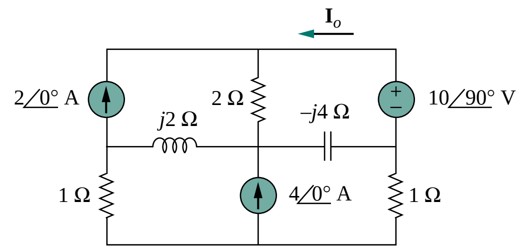

Figure 10.73 For Prob. 10.29.

Section 10.4 Superposition Theorem

10.30 Find i_o in the circuit shown in Fig. 10.74 using superposition.

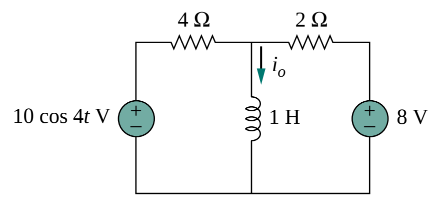

Figure 10.74 For Prob. 10.30.

10.31 Using the superposition principle, find i_x in the circuit of Fig. 10.75.

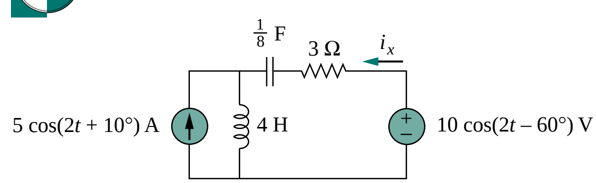

Figure 10.75 For Prob. 10.31.

10.32 Rework Prob. 10.2 using the superposition theorem.

10.33 Solve for $v_o(t)$ in the circuit of Fig. 10.76 using the superposition principle.

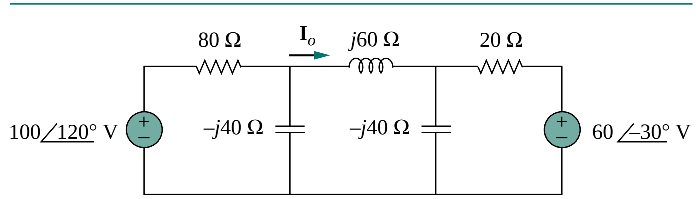

Figure 10.70 For Prob. 10.23.

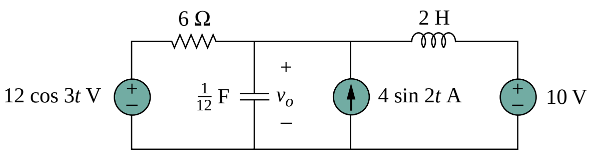

Figure 10.76 For Prob. 10.33.

10.34 Determine i_o in the circuit of Fig. 10.77.

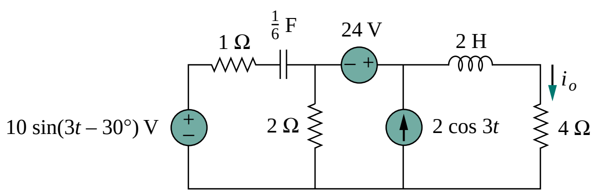

Figure 10.77 For Prob. 10.34.

10.35 Find i_o in the circuit in Fig. 10.78 using superposition.

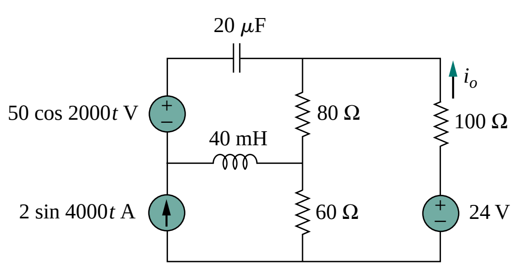

Figure 10.78 For Prob. 10.35.

Section 10.5 Source Transformation

10.36 Using source transformation, find i in the circuit of Fig. 10.79.

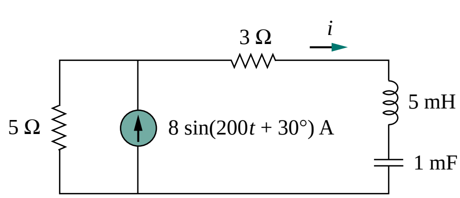

Figure 10.79 For Prob. 10.36.

10.37 Use source transformation to find v_o in the circuit in Fig. 10.80.

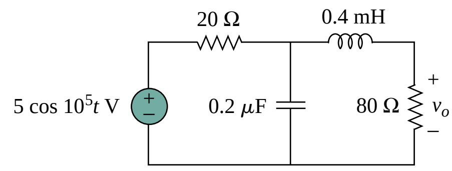

Figure 10.80 For Prob. 10.37.

10.38 Solve Prob. 10.20 using source transformation.

10.39 Use the method of source transformation to find $\mathbf{I}_x$ in the circuit of Fig. 10.81.

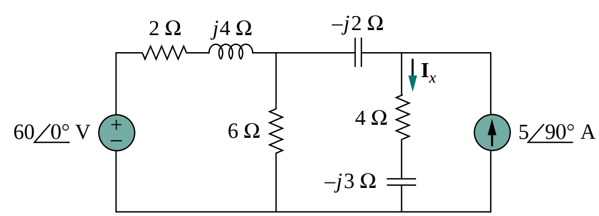

Figure 10.81 For Prob. 10.39.

10.40 Use the concept of source transformation to find $\mathbf{V}_o$ in the circuit of Fig. 10.82.

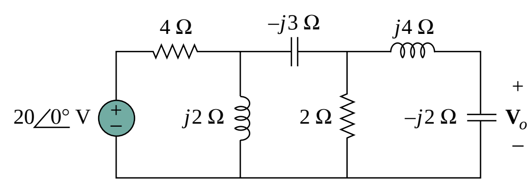

Figure 10.82 For Prob. 10.40.

Section 10.6 Thevenin and Norton Equivalent Circuits

10.41 Find the Thevenin and Norton equivalent circuits at terminals *a-b* for each of the circuits in Fig. 10.83.

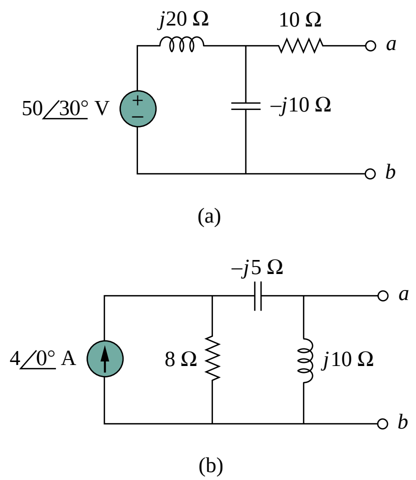

Figure 10.83 For Prob. 10.41.

10.42 For each of the circuits in Fig. 10.84, obtain Thevenin and Norton equivalent circuits at terminals *a*-*b*.

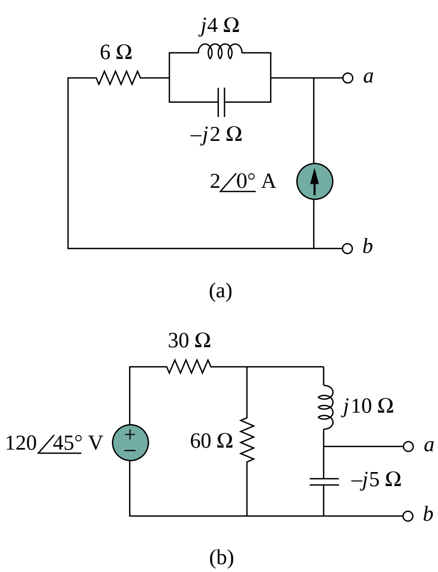

Figure 10.84 For Prob. 10.42.

10.43 Find the Thevenin and Norton equivalent circuits for the circuit shown in Fig. 10.85.

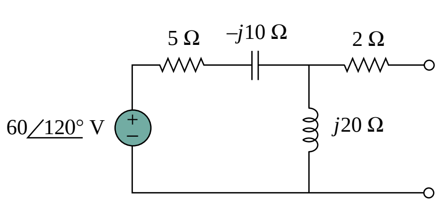

Figure 10.85 For Prob. 10.43.

10.44 For the circuit depicted in Fig. 10.86, find the Thevenin equivalent circuit at terminals *a*-*b*.

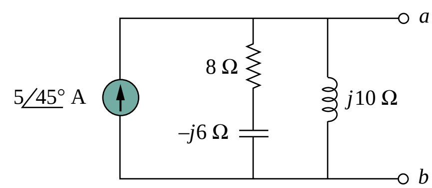

Figure 10.86 For Prob. 10.44.

10.45 Repeat Prob. 10.1 using Thevenin's theorem.

10.46 Find the Thevenin equivalent of the circuit in Fig. 10.87 as seen from:
(a) terminals *a*-*b* (b) terminals *c*-*d*

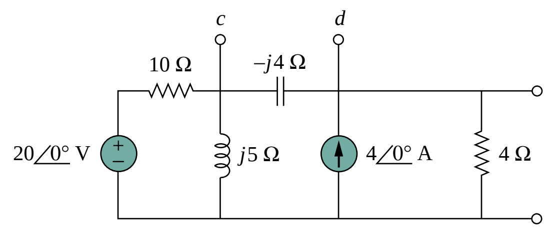

Figure 10.87 For Prob. 10.46.

10.47 Solve Prob. 10.3 using Thevenin's theorem.

10.48 Using Thevenin's theorem, find v_o in the circuit in Fig. 10.88.

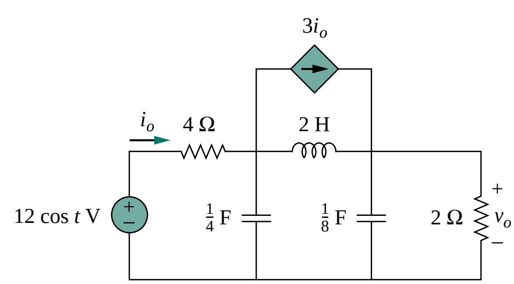

Figure 10.88 For Prob. 10.48.

10.49 Obtain the Norton equivalent of the circuit depicted in Fig. 10.89 at terminals *a*-*b*.

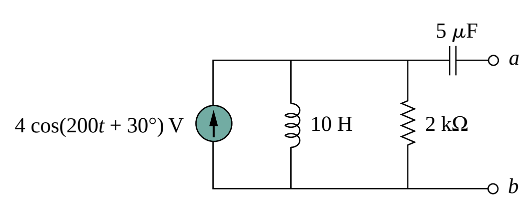

Figure 10.89 For Prob. 10.49.

10.50 For the circuit shown in Fig. 10.90, find the Norton equivalent circuit at terminals a-b.

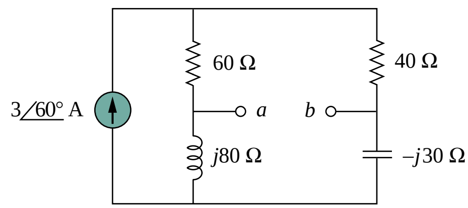

Figure 10.90 For Prob. 10.50.

10.51 Compute i_o in Fig. 10.91 using Norton's theorem.

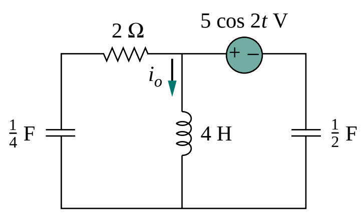

Figure 10.91 For Prob. 10.51.

10.52 At terminals a-b, obtain Thevenin and Norton equivalent circuits for the network depicted in Fig. 10.92. Take $\omega = 10$ rad/s.

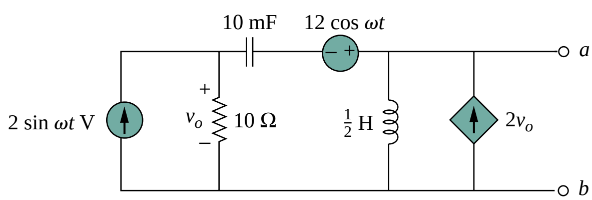

Figure 10.92 For Prob. 10.52.

Section 10.7 Op Amp AC Circuits

10.53 For the differentiator shown in Fig. 10.93, obtain $\mathbf{V}_o/\mathbf{V}_s$. Find $v_o(t)$ when $v_s(t) = V_m \sin \omega t$ and $\omega = 1/RC$.

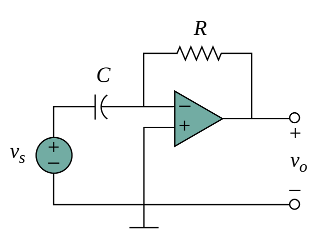

Figure 10.93 For Prob. 10.53.

10.54 The circuit in Fig. 10.94 is an integrator with a feedback resistor. Calculate $v_o(t)$ if $v_s = 2 \cos 4 \times 10^4 t$ V.

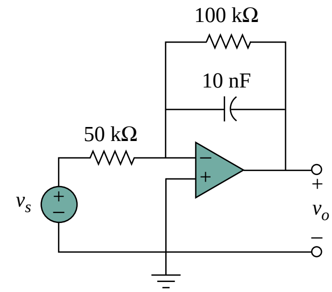

Figure 10.94 For Prob. 10.54.

10.55 Compute $i_o(t)$ in the op amp circuit in Fig. 10.95 if $v_s = 4 \cos 10^4 t$ V.

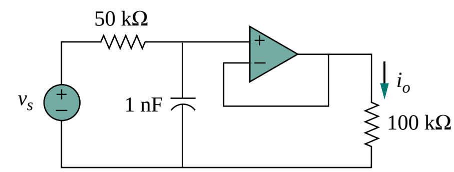

Figure 10.95 For Prob. 10.55.

10.56 If the input impedance is defined as $\mathbf{Z}_{\text{in}} = \mathbf{V}_s/\mathbf{I}_s$, find the input impedance of the op amp circuit in Fig. 10.96 when $R_1 = 10$ kΩ, $R_2 = 20$ kΩ, $C_1 = 10$ nF, $C_2 = 20$ nF, and $\omega = 5000$ rad/s.

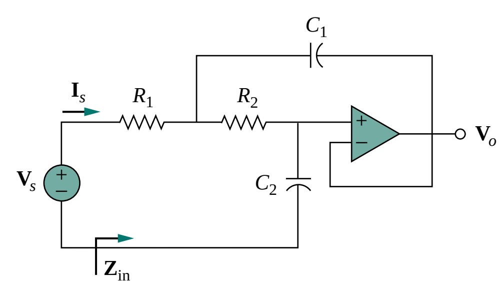

Figure 10.96 For Prob. 10.56.

10.57 Evaluate the voltage gain $\mathbf{A}_v = \mathbf{V}_o/\mathbf{V}_s$ in the op amp circuit of Fig. 10.97. Find $\mathbf{A}_v$ at $\omega = 0$, $\omega \to \infty$, $\omega = 1/R_1C_1$, and $\omega = 1/R_2C_2$.

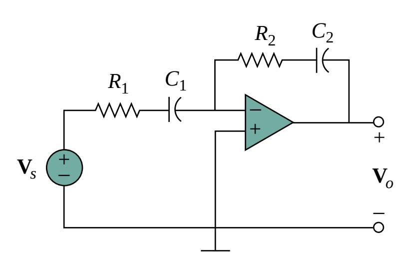

Figure 10.97 For Prob. 10.57.

10.58 In the op amp circuit of Fig. 10.98, find the closed-loop gain and phase shift if $C_1 = C_2 = 1$ nF, $R_1 = R_2 = 100$ kΩ, $R_3 = 20$ kΩ, $R_4 = 40$ kΩ, and $\omega = 2000$ rad/s.

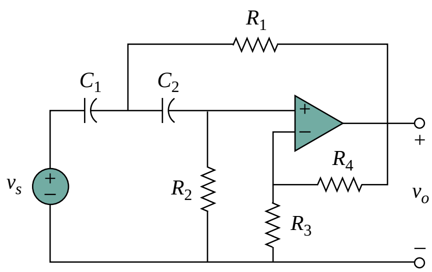

Figure 10.98 For Prob. 10.58.

10.59 Compute the closed-loop gain $\mathbf{V}_o/\mathbf{V}_s$ for the op amp circuit of Fig. 10.99.

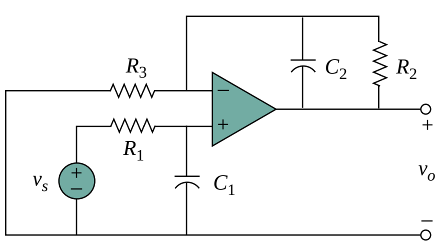

Figure 10.99 For Prob. 10.59.

10.60 Determine $v_o(t)$ in the op amp circuit in Fig. 10.100 below.

10.61 For the op amp circuit in Fig. 10.101, obtain $v_o(t)$.

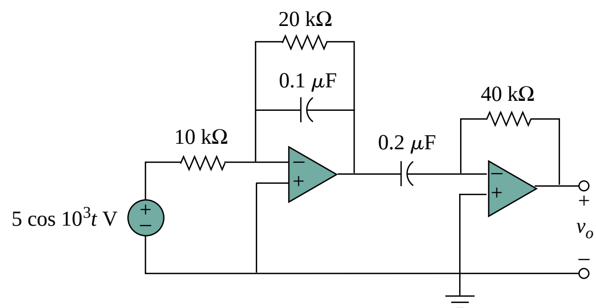

Figure 10.101 For Prob. 10.61.

10.62 Obtain $v_o(t)$ for the op amp circuit in Fig. 10.102 if $v_s = 4\cos(1000t - 60°)$ V.

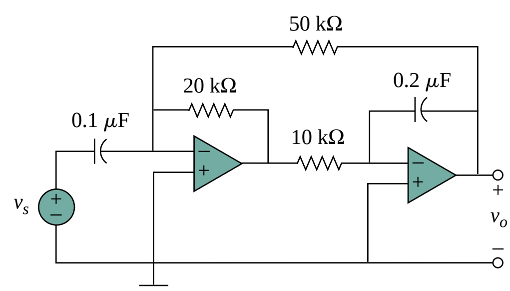

Figure 10.102 For Prob. 10.62.

Section 10.8 AC Analysis Using *PSpice*

10.63 Use *PSpice* to solve Example 10.10.

10.64 Solve Prob. 10.13 using *PSpice*.

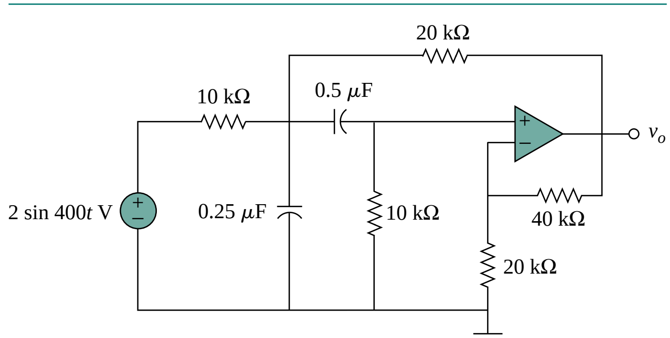

Figure 10.100 For Prob. 10.60.

10.65 Obtain $\mathbf{V}_o$ in the circuit of Fig. 10.103 using *PSpice*.

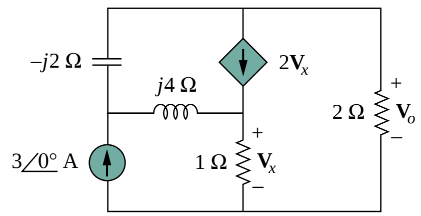

Figure 10.103 For Prob. 10.65.

10.66 Use *PSpice* to find $\mathbf{V}_1$, $\mathbf{V}_2$, and $\mathbf{V}_3$ in the network of Fig. 10.104.

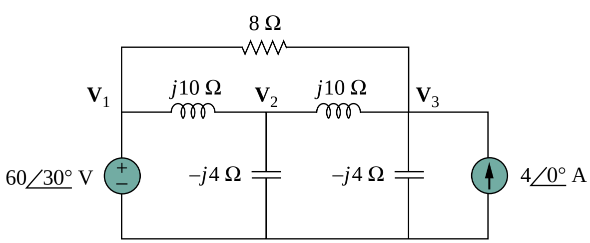

Figure 10.104 For Prob. 10.66.

10.67 Determine $\mathbf{V}_1$, $\mathbf{V}_2$, and $\mathbf{V}_3$ in the circuit of Fig. 10.105 using *PSpice*.

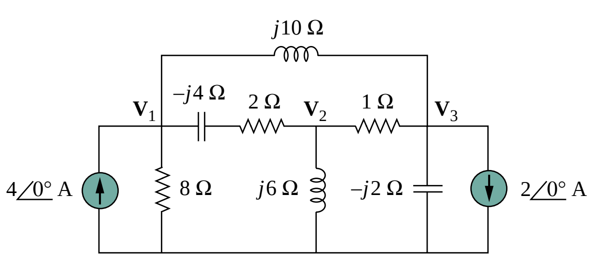

Figure 10.105 For Prob. 10.67.

10.68 Use *PSpice* to find v_o and i_o in the circuit of Fig. 10.106 below.

Section 10.9 Applications

10.69 The op amp circuit in Fig. 10.107 is called an *inductance simulator*. Show that the input impedance is given by

$$\mathbf{Z}_{\text{in}} = \frac{\mathbf{V}_{\text{in}}}{\mathbf{I}_{\text{in}}} = j\omega L_{\text{eq}}$$

where

$$L_{\text{eq}} = \frac{R_1 R_3 R_4}{R_2} C$$

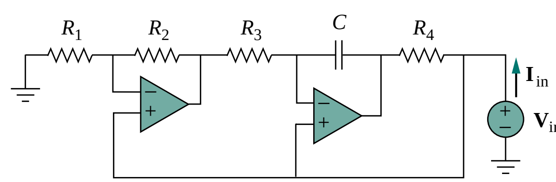

Figure 10.107 For Prob. 10.69.

10.70 Figure 10.108 shows a Wien-bridge network. Show that the frequency at which the phase shift between the input and output signals is zero is $f = \frac{1}{2}\pi RC$, and that the necessary gain is $\mathbf{A}_v = \mathbf{V}_o/\mathbf{V}_i = 3$ at that frequency.

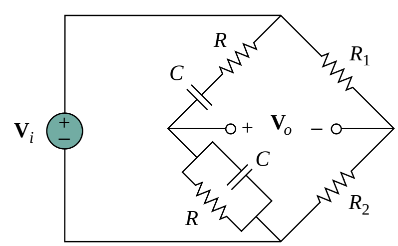

Figure 10.108 For Prob. 10.70.

10.71 Consider the oscillator in Fig. 10.109.

(a) Determine the oscillation frequency.

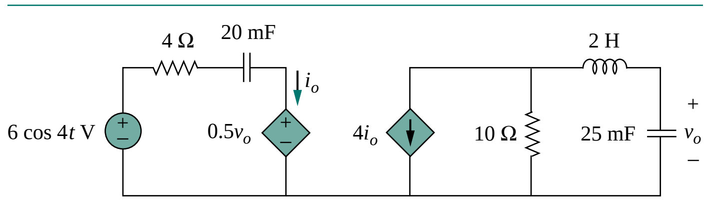

Figure 10.106 For Prob. 10.68.

(b) Obtain the minimum value of R for which oscillation takes place.

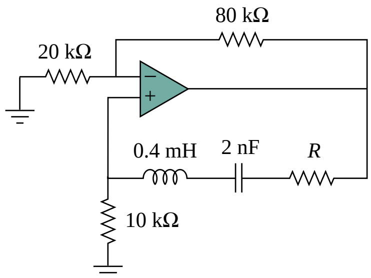

Figure 10.109 For Prob. 10.71.

10.72 The oscillator circuit in Fig. 10.110 uses an ideal op amp.

(a) Calculate the minimum value of R_o that will cause oscillation to occur.

(b) Find the frequency of oscillation.

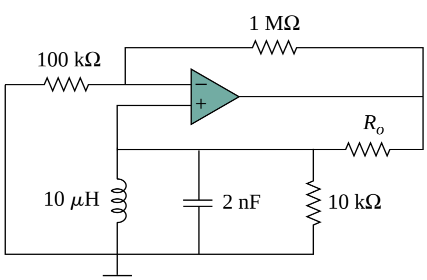

Figure 10.110 For Prob. 10.72.

10.73 Figure 10.111 shows a *Colpitts oscillator*. Show that the oscillation frequency is

$$f_o = \frac{1}{2\pi\sqrt{LC_T}}$$

where $C_T = C_1C_2/(C_1 + C_2)$. Assume $R_i \gg X_{C_2}$.

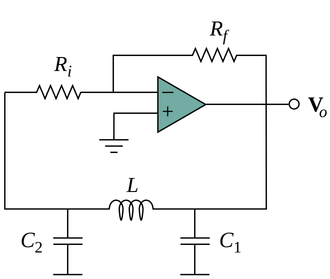

Figure 10.111 A Colpitts oscillator; for Prob. 10.73.

(*Hint:* Set the imaginary part of the impedance in the feedback circuit equal to zero.)

10.74 Design a Colpitts oscillator that will operate at 50 kHz.

10.75 Figure 10.112 shows a *Hartley oscillator*. Show that the frequency of oscillation is

$$f_o = \frac{1}{2\pi\sqrt{C(L_1 + L_2)}}$$

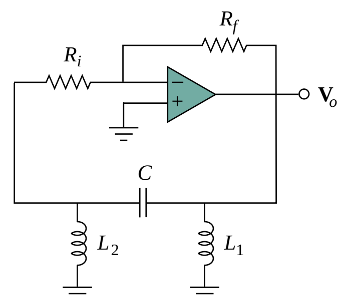

Figure 10.112 A Hartley oscillator; for Prob. 10.75.

10.76 Refer to the oscillator in Fig. 10.113.

(a) Show that

$$\frac{\mathbf{V}_2}{\mathbf{V}_o} = \frac{1}{3 + j(\omega L/R - R/\omega L)}$$

(b) Determine the oscillation frequency f_o.

(c) Obtain the relationship between R_1 and R_2 in order for oscillation to occur.

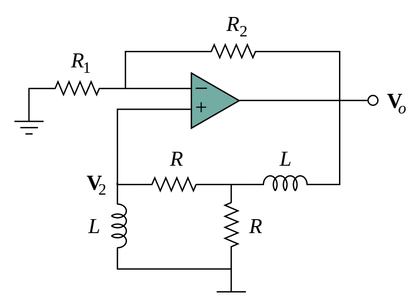

Figure 10.113 For Prob. 10.76.