

# CHAPTER 14

## FREQUENCY RESPONSE

*One machine can do the work of fifty ordinary men. No machine can do the work of one extraordinary man.*

— Elbert G. Hubbard

### Enhancing Your Career

**Career in Control Systems** Control systems are another area of electrical engineering where circuit analysis is used. A control system is designed to regulate the behavior of one or more variables in some desired manner. Control systems play major roles in our everyday life. Household appliances such as heating and air-conditioning systems, switch-controlled thermostats, washers and dryers, cruise controllers in automobiles, elevators, traffic lights, manufacturing plants, navigation systems—all utilize control systems. In the aerospace field, precision guidance of space probes, the wide range of operational modes of the space shuttle, and the ability to maneuver space vehicles remotely from earth all require knowledge of control systems. In the manufacturing sector, repetitive production line operations are increasingly performed by robots, which are programmable control systems designed to operate for many hours without fatigue.

Control engineering integrates circuit theory and communication theory. It is not limited to any specific engineering discipline but may involve environmental, chemical, aeronautical, mechanical, civil, and electrical engineering. For example, a typical task for a control system engineer might be to design a speed regulator for a disk drive head.

A thorough understanding of control systems techniques is essential to the electrical engineer and is of great value for designing control systems to perform the desired task.



*A welding robot.  
(Courtesy of Shela Terry/Science Photo Library.)*

## 14.1 INTRODUCTION

In our sinusoidal circuit analysis, we have learned how to find voltages and currents in a circuit with a constant frequency source. If we let the amplitude of the sinusoidal source remain constant and vary the frequency, we obtain the circuit's *frequency response*. The frequency response may be regarded as a complete description of the sinusoidal steady-state behavior of a circuit as a function of frequency.

The *frequency response* of a circuit is the variation in its behavior with change in signal frequency.

The frequency response of a circuit may also be considered as the variation of the gain and phase with frequency.

The sinusoidal steady-state frequency responses of circuits are of significance in many applications, especially in communications and control systems. A specific application is in electric filters that block out or eliminate signals with unwanted frequencies and pass signals of the desired frequencies. Filters are used in radio, TV, and telephone systems to separate one broadcast frequency from another.

We begin this chapter by considering the frequency response of simple circuits using their transfer functions. We then consider Bode plots, which are the industry-standard way of presenting frequency response. We also consider series and parallel resonant circuits and encounter important concepts such as resonance, quality factor, cutoff frequency, and bandwidth. We discuss different kinds of filters and network scaling. In the last section, we consider one practical application of resonant circuits and two applications of filters.

## 14.2 TRANSFER FUNCTION

The transfer function  $\mathbf{H}(\omega)$  (also called the *network function*) is a useful analytical tool for finding the frequency response of a circuit. In fact, the frequency response of a circuit is the plot of the circuit's transfer function  $\mathbf{H}(\omega)$  versus  $\omega$ , with  $\omega$  varying from  $\omega = 0$  to  $\omega = \infty$ .

A transfer function is the frequency-dependent ratio of a forced function to a forcing function (or of an output to an input). The idea of a transfer function was implicit when we used the concepts of impedance and admittance to relate voltage and current. In general, a linear network can be represented by the block diagram shown in Fig. 14.1.

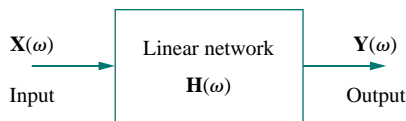


Figure 14.1 A block diagram representation of a linear network.

In this context,  $\mathbf{X}(\omega)$  and  $\mathbf{Y}(\omega)$  denote the input and output phasors of a network; they should not be confused with the same symbolism used for reactance and admittance. The multiple usage of symbols is conventionally permissible due to lack of enough letters in the English language to express all circuit variables distinctly.

The *transfer function*  $\mathbf{H}(\omega)$  of a circuit is the frequency-dependent ratio of a phasor output  $\mathbf{Y}(\omega)$  (an element voltage or current) to a phasor input  $\mathbf{X}(\omega)$  (source voltage or current).

Thus,

$$\mathbf{H}(\omega) = \frac{\mathbf{Y}(\omega)}{\mathbf{X}(\omega)} \quad (14.1)$$

assuming zero initial conditions. Since the input and output can be either voltage or current at any place in the circuit, there are four possible transfer functions:

$$\mathbf{H}(\omega) = \text{Voltage gain} = \frac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)} \quad (14.2a)$$

$$\mathbf{H}(\omega) = \text{Current gain} = \frac{\mathbf{I}_o(\omega)}{\mathbf{I}_i(\omega)} \quad (14.2b)$$

$$\mathbf{H}(\omega) = \text{Transfer Impedance} = \frac{\mathbf{V}_o(\omega)}{\mathbf{I}_i(\omega)} \quad (14.2c)$$

$$\mathbf{H}(\omega) = \text{Transfer Admittance} = \frac{\mathbf{I}_o(\omega)}{\mathbf{V}_i(\omega)} \quad (14.2d)$$

where subscripts  $i$  and  $o$  denote input and output values. Being a complex quantity,  $\mathbf{H}(\omega)$  has a magnitude  $H(\omega)$  and a phase  $\phi$ ; that is,  $\mathbf{H}(\omega) = H(\omega) \angle \phi$ .

To obtain the transfer function using Eq. (14.2), we first obtain the frequency-domain equivalent of the circuit by replacing resistors, inductors, and capacitors with their impedances  $R$ ,  $j\omega L$ , and  $1/j\omega C$ . We then use any circuit technique(s) to obtain the appropriate quantity in Eq. (14.2). We can obtain the frequency response of the circuit by plotting the magnitude and phase of the transfer function as the frequency varies. A computer is a real time-saver for plotting the transfer function.

The transfer function  $\mathbf{H}(\omega)$  can be expressed in terms of its numerator polynomial  $\mathbf{N}(\omega)$  and denominator polynomial  $\mathbf{D}(\omega)$  as

$$\mathbf{H}(\omega) = \frac{\mathbf{N}(\omega)}{\mathbf{D}(\omega)} \quad (14.3)$$

where  $\mathbf{N}(\omega)$  and  $\mathbf{D}(\omega)$  are not necessarily the same expressions for the input and output functions, respectively. The representation of  $\mathbf{H}(\omega)$  in Eq. (14.3) assumes that common numerator and denominator factors in  $\mathbf{H}(\omega)$  have canceled, reducing the ratio to lowest terms. The roots of  $\mathbf{N}(\omega) = 0$  are called the *zeros* of  $\mathbf{H}(\omega)$  and are usually represented as  $j\omega = z_1, z_2, \dots$ . Similarly, the roots of  $\mathbf{D}(\omega) = 0$  are the *poles* of  $\mathbf{H}(\omega)$  and are represented as  $j\omega = p_1, p_2, \dots$ .

A **zero**, as a root of the numerator polynomial, is a value that results in a zero value of the function. A **pole**, as a root of the denominator polynomial, is a value for which the function is infinite.

Some authors use  $\mathbf{H}(j\omega)$  for transfer instead of  $\mathbf{H}(\omega)$ , since  $\omega$  and  $j$  are an inseparable pair.

A zero may also be regarded as the value of  $s = j\omega$  that makes  $\mathbf{H}(s)$  zero, and a pole as the value of  $s = j\omega$  that makes  $\mathbf{H}(s)$  infinite.

To avoid complex algebra, it is expedient to replace  $j\omega$  temporarily with  $s$  when working with  $\mathbf{H}(\omega)$  and replace  $s$  with  $j\omega$  at the end.

## EXAMPLE 14.1

For the  $RC$  circuit in Fig. 14.2(a), obtain the transfer function  $\mathbf{V}_o/\mathbf{V}_s$  and its frequency response. Let  $v_s = V_m \cos \omega t$ .

**Solution:**

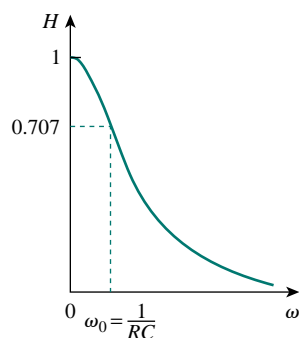
The frequency-domain equivalent of the circuit is in Fig. 14.2(b). By voltage division, the transfer function is given by

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_s} = \frac{1/j\omega C}{R + 1/j\omega C} = \frac{1}{1 + j\omega RC}$$

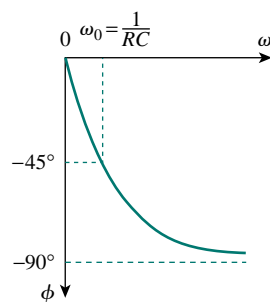
Comparing this with Eq. (9.18e), we obtain the magnitude and phase of  $\mathbf{H}(\omega)$  as

$$H = \frac{1}{\sqrt{1 + (\omega/\omega_0)^2}}, \quad \phi = -\tan^{-1} \frac{\omega}{\omega_0}$$

where  $\omega_0 = 1/RC$ . To plot  $H$  and  $\phi$  for  $0 < \omega < \infty$ , we obtain their values at some critical points and then sketch.

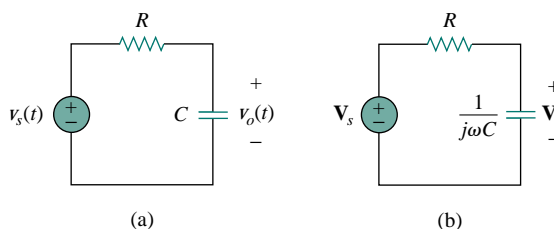


(a)



(b)

**Figure 14.3** Frequency response of the  $RC$  circuit: (a) amplitude response, (b) phase response.



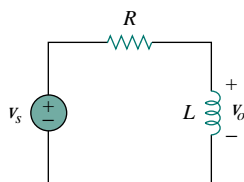
**Figure 14.2** For Example 14.1: (a) time-domain  $RC$  circuit, (b) frequency-domain  $RC$  circuit.

At  $\omega = 0$ ,  $H = 1$  and  $\phi = 0$ . At  $\omega = \infty$ ,  $H = 0$  and  $\phi = -90^\circ$ . Also, at  $\omega = \omega_0$ ,  $H = 1/\sqrt{2}$  and  $\phi = -45^\circ$ . With these and a few more points as shown in Table 14.1, we find that the frequency response is as shown in Fig. 14.3. Additional features of the frequency response in Fig. 14.3 will be explained in Section 14.6.1 on lowpass filters.

**TABLE 14.1** For Example 14.1.

$\omega/\omega_0$	$H$	$\phi$	$\omega/\omega_0$	$H$	$\phi$
0	1	0	10	0.1	$-84^\circ$
1	0.71	$-45^\circ$	20	0.05	$-87^\circ$
2	0.45	$-63^\circ$	100	0.01	$-89^\circ$
3	0.32	$-72^\circ$	$\infty$	0	$-90^\circ$

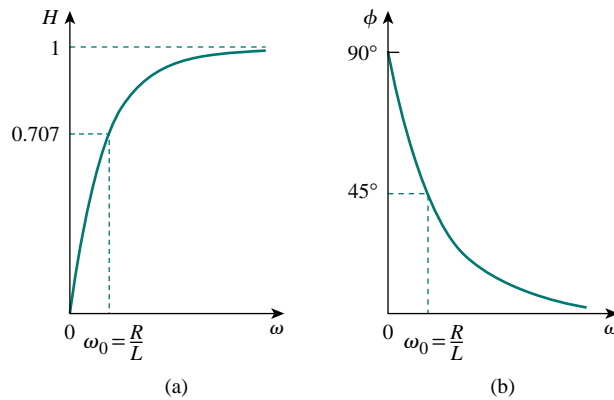
## PRACTICE PROBLEM 14.1



**Figure 14.4**  $RL$  circuit for Practice Prob. 14.1.

Obtain the transfer function  $\mathbf{V}_o/\mathbf{V}_s$  of the  $RL$  circuit in Fig. 14.4, assuming  $v_s = V_m \cos \omega t$ . Sketch its frequency response.

**Answer:**  $j\omega L/(R + j\omega L)$ ; see Fig. 14.5 for the response.

Figure 14.5 Frequency response of the  $RL$  circuit in Fig. 14.4.**EXAMPLE 14.2**

For the circuit in Fig. 14.6, calculate the gain  $\mathbf{I}_o(\omega)/\mathbf{I}_i(\omega)$  and its poles and zeros.

**Solution:**

By current division,

$$\mathbf{I}_o(\omega) = \frac{4 + j2\omega}{4 + j2\omega + 1/j0.5\omega} \mathbf{I}_i(\omega)$$

or

$$\frac{\mathbf{I}_o(\omega)}{\mathbf{I}_i(\omega)} = \frac{j0.5\omega(4 + j2\omega)}{1 + j2\omega + (j\omega)^2} = \frac{s(s+2)}{s^2 + 2s + 1}, \quad s = j\omega$$

The zeros are at

$$s(s+2) = 0 \quad \Rightarrow \quad z_1 = 0, \quad z_2 = -2$$

The poles are at

$$s^2 + 2s + 1 = (s+1)^2 = 0$$

Thus, there is a repeated pole (or double pole) at  $p = -1$ .

**PRACTICE PROBLEM 14.2**

Find the transfer function  $\mathbf{V}_o(\omega)/\mathbf{I}_i(\omega)$  for the circuit of Fig. 14.7. Obtain its poles and zeros.

**Answer:**  $\frac{5(s+2)(s+1.5)}{s^2 + 4s + 5}$ ,  $s = j\omega$ ; poles:  $-2, -1.5$ ; zeros:  $-2 \pm j$ .

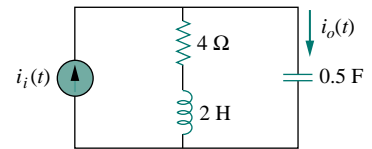


Figure 14.6 For Example 14.2.

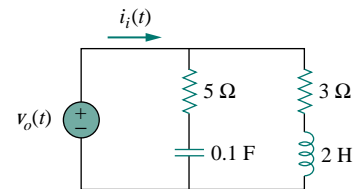


Figure 14.7 For Practice Prob. 14.2.

### †14.3 THE DECIBEL SCALE

It is not always easy to get a quick plot of the magnitude and phase of the transfer function as we did above. A more systematic way of obtaining the frequency response is to use Bode plots. Before we begin to construct Bode plots, we should take care of two important issues: the use of logarithms and decibels in expressing gain.

Since Bode plots are based on logarithms, it is important that we keep the following properties of logarithms in mind:

1.  $\log P_1 P_2 = \log P_1 + \log P_2$
2.  $\log P_1 / P_2 = \log P_1 - \log P_2$
3.  $\log P^n = n \log P$
4.  $\log 1 = 0$

In communications systems, gain is measured in *bels*. Historically, the bel is used to measure the ratio of two levels of power or power gain  $G$ ; that is,

$$G = \text{Number of bels} = \log_{10} \frac{P_2}{P_1} \quad (14.4)$$

The *decibel* (dB) provides us with a unit of less magnitude. It is 1/10th of a bel and is given by

$$G_{\text{dB}} = 10 \log_{10} \frac{P_2}{P_1} \quad (14.5)$$

When  $P_1 = P_2$ , there is no change in power and the gain is 0 dB. If  $P_2 = 2P_1$ , the gain is

$$G_{\text{dB}} = 10 \log_{10} 2 = 3 \text{ dB} \quad (14.6)$$

and when  $P_2 = 0.5P_1$ , the gain is

$$G_{\text{dB}} = 10 \log_{10} 0.5 = -3 \text{ dB} \quad (14.7)$$

Equations (14.6) and (14.7) show another reason why logarithms are greatly used: The logarithm of the reciprocal of a quantity is simply negative the logarithm of that quantity.

Alternatively, the gain  $G$  can be expressed in terms of voltage or current ratio. To do so, consider the network shown in Fig. 14.8. If  $P_1$  is the input power,  $P_2$  is the output (load) power,  $R_1$  is the input resistance, and  $R_2$  is the load resistance, then  $P_1 = 0.5V_1^2/R_1$  and  $P_2 = 0.5V_2^2/R_2$ , and Eq. (14.5) becomes

$$\begin{aligned} G_{\text{dB}} &= 10 \log_{10} \frac{P_2}{P_1} = 10 \log_{10} \frac{V_2^2/R_2}{V_1^2/R_1} \\ &= 10 \log_{10} \left( \frac{V_2}{V_1} \right)^2 + 10 \log_{10} \frac{R_1}{R_2} \end{aligned} \quad (14.8)$$

$$G_{\text{dB}} = 20 \log_{10} \frac{V_2}{V_1} - 10 \log_{10} \frac{R_2}{R_1} \quad (14.9)$$

*Historical note:* The *bel* is named after Alexander Graham Bell, the inventor of the telephone.

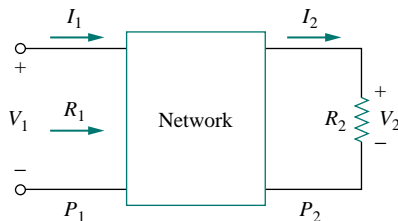


Figure 14.8 Voltage-current relationships for a four-terminal network.

For the case when  $R_2 = R_1$ , a condition that is often assumed when comparing voltage levels, Eq. (14.9) becomes

$$G_{\text{dB}} = 20 \log_{10} \frac{V_2}{V_1} \quad (14.10)$$

Instead, if  $P_1 = I_1^2 R_1$  and  $P_2 = I_2^2 R_2$ , for  $R_1 = R_2$ , we obtain

$$G_{\text{dB}} = 20 \log_{10} \frac{I_2}{I_1} \quad (14.11)$$

Two things are important to note from Eqs. (14.5), (14.10), and (14.11):

1. That  $10 \log$  is used for power, while  $20 \log$  is used for voltage or current, because of the square relationship between them ( $P = V^2/R = I^2 R$ ).
2. That the dB value is a logarithmic measurement of the *ratio* of one variable to another *of the same type*. Therefore, it applies in expressing the transfer function  $H$  in Eqs. (14.2a) and (14.2b), which are dimensionless quantities, but not in expressing  $H$  in Eqs. (14.2c) and (14.2d).

With this in mind, we now apply the concepts of logarithms and decibels to construct Bode plots.

## 14.4 BODE PLOTS

Obtaining the frequency response from the transfer function as we did in Section 14.2 is an uphill task. The frequency range required in frequency response is often so wide that it is inconvenient to use a linear scale for the frequency axis. Also, there is a more systematic way of locating the important features of the magnitude and phase plots of the transfer function. For these reasons, it has become standard practice to use a logarithmic scale for the frequency axis and a linear scale in each of the separate plots of magnitude and phase. Such semilogarithmic plots of the transfer function—known as *Bode plots*—have become the industry standard.

**Bode plots** are semilog plots of the magnitude (in decibels) and phase (in degrees) of a transfer function versus frequency.

*Historical note:* Named after Hendrik W. Bode (1905–1982), an engineer with the Bell Telephone Laboratories, for his pioneering work in the 1930s and 1940s.

Bode plots contain the same information as the nonlogarithmic plots discussed in the previous section, but they are much easier to construct, as we shall see shortly.

The transfer function can be written as

$$\mathbf{H} = H \angle \phi = H e^{j\phi} \quad (14.12)$$

Taking the natural logarithm of both sides,

$$\ln \mathbf{H} = \ln H + \ln e^{j\phi} = \ln H + j\phi \quad (14.13)$$

**TABLE 14.2** Specific gains and their decibel values.

Magnitude $H$	$20 \log_{10} H$ (dB)
0.001	-60
0.01	-40
0.1	-20
0.5	-6
$1/\sqrt{2}$	-3
1	0
$\sqrt{2}$	3
2	6
10	20
20	26
100	40
1000	60

The origin is where  $\omega = 1$  or  $\log \omega = 0$  and the gain is zero.

Thus, the real part of  $\ln \mathbf{H}$  is a function of the magnitude while the imaginary part is the phase. In a Bode magnitude plot, the gain

$$H_{\text{dB}} = 20 \log_{10} H \quad (14.14)$$

is plotted in decibels (dB) versus frequency. Table 14.2 provides a few values of  $H$  with the corresponding values in decibels. In a Bode phase plot,  $\phi$  is plotted in degrees versus frequency. Both magnitude and phase plots are made on semilog graph paper.

A transfer function in the form of Eq. (14.3) may be written in terms of factors that have real and imaginary parts. One such representation might be

$$\mathbf{H}(\omega) = \frac{K(j\omega)^{\pm 1}(1 + j\omega/z_1)[1 + j2\zeta_1\omega/\omega_k + (j\omega/\omega_k)^2] \cdots}{(1 + j\omega/p_1)[1 + j2\zeta_2\omega/\omega_n + (j\omega/\omega_n)^2] \cdots} \quad (14.15)$$

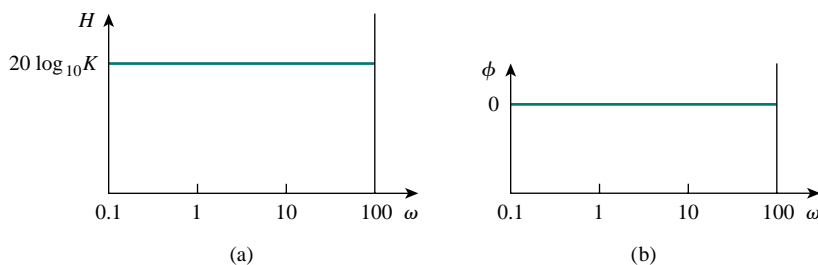
which is obtained by dividing out the poles and zeros in  $\mathbf{H}(\omega)$ . The representation of  $\mathbf{H}(\omega)$  as in Eq. (14.15) is called the *standard form*. In this particular case,  $\mathbf{H}(\omega)$  has seven different factors that can appear in various combinations in a transfer function. These are:

1. A gain  $K$
2. A pole  $(j\omega)^{-1}$  or zero  $(j\omega)$  at the origin
3. A simple pole  $1/(1 + j\omega/p_1)$  or zero  $(1 + j\omega/z_1)$
4. A quadratic pole  $1/[1 + j2\zeta_2\omega/\omega_n + (j\omega/\omega_n)^2]$  or zero  $[1 + j2\zeta_1\omega/\omega_k + (j\omega/\omega_k)^2]$

In constructing a Bode plot, we plot each factor separately and then combine them graphically. The factors can be considered one at a time and then combined additively because of the logarithms involved. It is this mathematical convenience of the logarithm that makes Bode plots a powerful engineering tool.

We will now make straight-line plots of the factors listed above. We shall find that these straight-line plots known as Bode plots approximate the actual plots to a surprising degree of accuracy.

**Constant term:** For the gain  $K$ , the magnitude is  $20 \log_{10} K$  and the phase is  $0^\circ$ ; both are constant with frequency. Thus the magnitude and phase plots of the gain are shown in Fig. 14.9. If  $K$  is negative, the magnitude remains  $20 \log_{10} |K|$  but the phase is  $\pm 180^\circ$ .



**Figure 14.9** Bode plots for gain  $K$ : (a) magnitude plot, (b) phase plot.



**Pole/zero at the origin:** For the zero ( $j\omega$ ) at the origin, the magnitude is  $20 \log_{10} \omega$  and the phase is  $90^\circ$ . These are plotted in Fig. 14.10, where we notice that the slope of the magnitude plot is 20 dB/decade, while the phase is constant with frequency.

The Bode plots for the pole  $(j\omega)^{-1}$  are similar except that the slope of the magnitude plot is  $-20$  dB/decade while the phase is  $-90^\circ$ . In general, for  $(j\omega)^N$ , where  $N$  is an integer, the magnitude plot will have a slope of  $20N$  dB/decade, while the phase is  $90N$  degrees.

**Simple pole/zero:** For the simple zero  $(1 + j\omega/z_1)$ , the magnitude is  $20 \log_{10} |1 + j\omega/z_1|$  and the phase is  $\tan^{-1} \omega/z_1$ . We notice that

$$H_{dB} = 20 \log_{10} \left| 1 + \frac{j\omega}{z_1} \right| \Rightarrow 20 \log_{10} 1 = 0 \quad (14.16)$$

as  $\omega \rightarrow 0$

$$H_{dB} = 20 \log_{10} \left| 1 + \frac{j\omega}{z_1} \right| \Rightarrow 20 \log_{10} \frac{\omega}{z_1} \quad (14.17)$$

as  $\omega \rightarrow \infty$

showing that we can approximate the magnitude as zero (a straight line with zero slope) for small values of  $\omega$  and by a straight line with slope 20 dB/decade for large values of  $\omega$ . The frequency  $\omega = z_1$  where the two asymptotic lines meet is called the *corner frequency* or *break frequency*. Thus the approximate magnitude plot is shown in Fig. 14.11(a), where the actual plot is also shown. Notice that the approximate plot is close to the actual plot except at the break frequency, where  $\omega = z_1$  and the deviation is  $20 \log_{10} |(1 + j1)| = 20 \log_{10} \sqrt{2} = 3$  dB.

The phase  $\tan^{-1}(\omega/z_1)$  can be expressed as

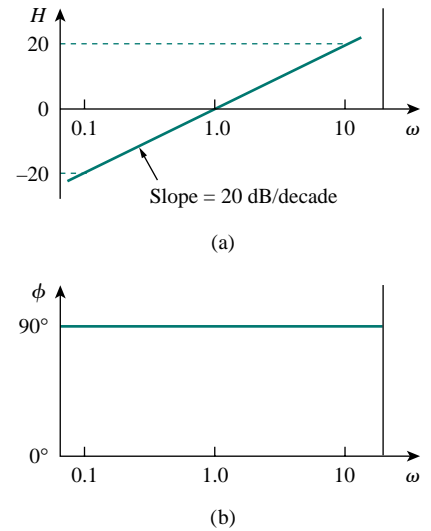
$$\phi = \tan^{-1} \left( \frac{\omega}{z_1} \right) = \begin{cases} 0, & \omega = 0 \\ 45^\circ, & \omega = z_1 \\ 90^\circ, & \omega \rightarrow \infty \end{cases} \quad (14.18)$$

As a straight-line approximation, we let  $\phi \simeq 0$  for  $\omega \leq z_1/10$ ,  $\phi \simeq 45^\circ$  for  $\omega = z_1$ , and  $\phi \simeq 90^\circ$  for  $\omega \geq 10z_1$ . As shown in Fig. 14.11(b) along with the actual plot, the straight-line plot has a slope of  $45^\circ$  per decade.

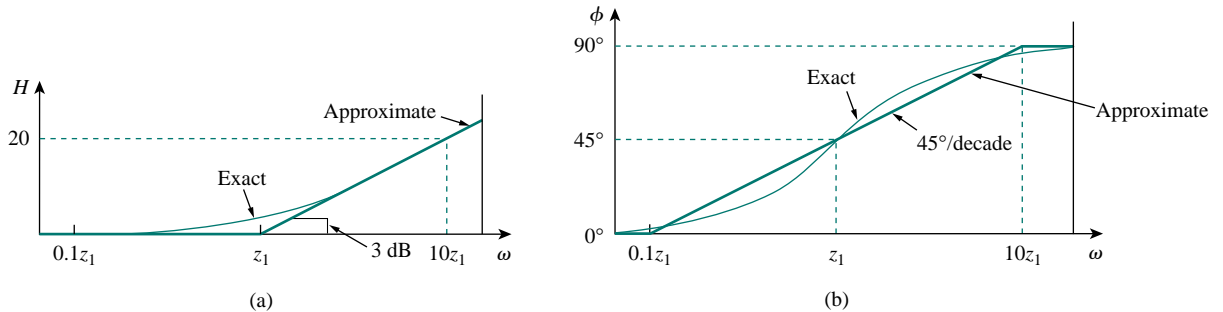
The Bode plots for the pole  $1/(1 + j\omega/p_1)$  are similar to those in Fig. 14.11 except that the corner frequency is at  $\omega = p_1$ , the magnitude

A decade is an interval between two frequencies with a ratio of 10; e.g., between  $\omega_0$  and  $10\omega_0$ , or between 10 and 100 Hz. Thus, 20 dB/decade means that the magnitude changes 20 dB whenever the frequency changes tenfold or one decade.

The special case of dc ( $\omega = 0$ ) does not appear on Bode plots because  $\log 0 = -\infty$ , implying that zero frequency is infinitely far to the left of the origin of Bode plots.



**Figure 14.10** Bode plot for a zero ( $j\omega$ ) at the origin: (a) magnitude plot, (b) phase plot.



**Figure 14.11** Bode plots of zero  $(1 + j\omega/z_1)$ : (a) magnitude plot, (b) phase plot.

has a slope of  $-20$  dB/decade, and the phase has a slope of  $-45^\circ$  per decade.

**Quadratic pole/zero:** The magnitude of the quadratic pole  $1/[1 + j2\zeta_2\omega/\omega_n + (j\omega/\omega_n)^2]$  is  $-20 \log_{10} |1 + j2\zeta_2\omega/\omega_n + (j\omega/\omega_n)^2|$  and the phase is  $-\tan^{-1}(2\zeta_2\omega/\omega_n)/(1 - \omega/\omega_n^2)$ . But

$$H_{\text{dB}} = -20 \log_{10} \left| 1 + \frac{j2\zeta_2\omega}{\omega_n} + \left( \frac{j\omega}{\omega_n} \right)^2 \right| \Rightarrow 0 \quad (14.19)$$

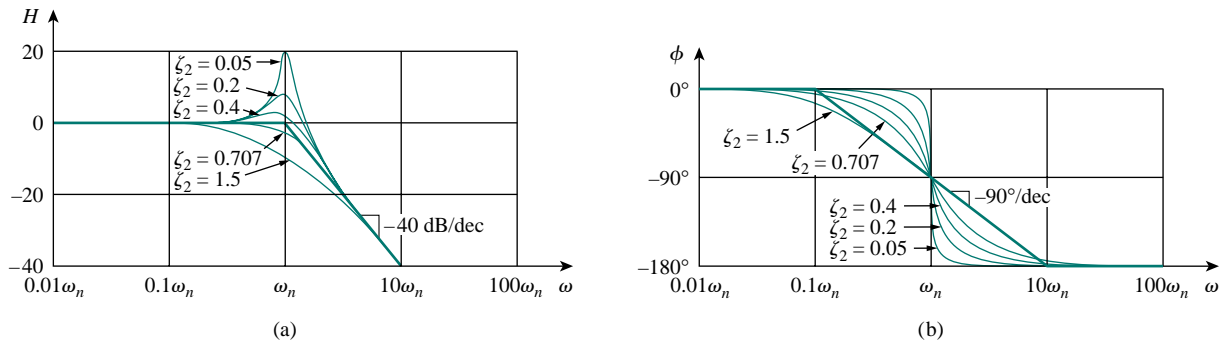
as  $\omega \rightarrow 0$

and

$$H_{\text{dB}} = -20 \log_{10} \left| 1 + \frac{j2\zeta_2\omega}{\omega_n} + \left( \frac{j\omega}{\omega_n} \right)^2 \right| \Rightarrow -40 \log_{10} \frac{\omega}{\omega_n} \quad (14.20)$$

as  $\omega \rightarrow \infty$

Thus, the amplitude plot consists of two straight asymptotic lines: one with zero slope for  $\omega < \omega_n$  and the other with slope  $-40$  dB/decade for  $\omega > \omega_n$ , with  $\omega_n$  as the corner frequency. Figure 14.12(a) shows the approximate and actual amplitude plots. Note that the actual plot depends on the damping factor  $\zeta_2$  as well as the corner frequency  $\omega_n$ . The significant peaking in the neighborhood of the corner frequency should be added to the straight-line approximation if a high level of accuracy is desired. However, we will use the straight-line approximation for the sake of simplicity.



**Figure 14.12** Bode plots of quadratic pole  $[1 + j2\zeta\omega/\omega_n - \omega^2/\omega_n^2]^{-1}$ : (a) magnitude plot, (b) phase plot.

The phase can be expressed as

$$\phi = -\tan^{-1} \frac{2\zeta_2\omega/\omega_n}{1 - \omega^2/\omega_n^2} = \begin{cases} 0, & \omega = 0 \\ -90^\circ, & \omega = \omega_n \\ -180^\circ, & \omega \rightarrow \infty \end{cases} \quad (14.21)$$


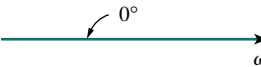
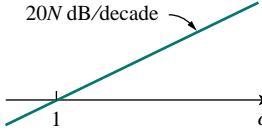
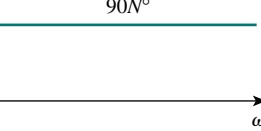
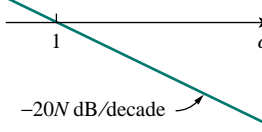
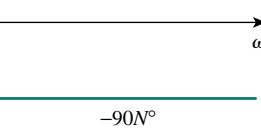
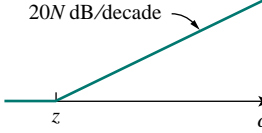
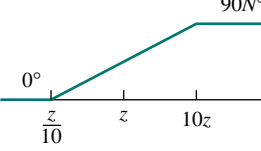

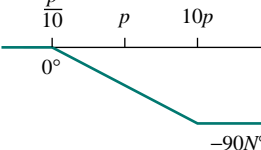
The phase plot is a straight line with a slope of  $90^\circ$  per decade starting at  $\omega_n/10$  and ending at  $10\omega_n$ , as shown in Fig. 14.12(b). We see again that the difference between the actual plot and the straight-line plot is due to the damping factor. Notice that the straight-line approximations for both magnitude and phase plots for the quadratic pole are the same

as those for a double pole, i.e.  $(1 + j\omega/\omega_n)^{-2}$ . We should expect this because the double pole  $(1 + j\omega/\omega_n)^{-2}$  equals the quadratic pole  $1/[1 + j2\zeta_2\omega/\omega_n + (j\omega/\omega_n)^2]$  when  $\zeta_2 = 1$ . Thus, the quadratic pole can be treated as a double pole as far as straight-line approximation is concerned.

For the quadratic zero  $[1 + j2\zeta_1\omega/\omega_k + (j\omega/\omega_k)^2]$ , the plots in Fig. 14.12 are inverted because the magnitude plot has a slope of 40 dB/decade while the phase plot has a slope of  $90^\circ$  per decade.

Table 14.3 presents a summary of Bode plots for the seven factors. To sketch the Bode plots for a function  $\mathbf{H}(\omega)$  in the form of Eq. (14.15), for example, we first record the corner frequencies on the semilog graph paper, sketch the factors one at a time as discussed above, and then combine additively the graphs of the factors. The combined graph is often drawn from left to right, changing slopes appropriately each time a corner frequency is encountered. The following examples illustrate this procedure.

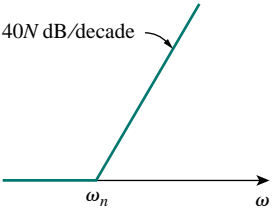
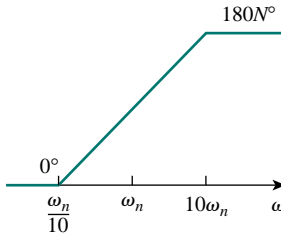
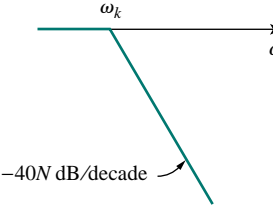
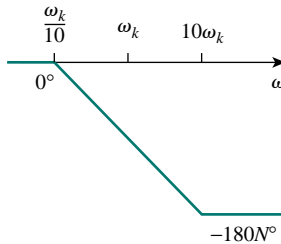
**TABLE 14.3** Summary of Bode straight-line magnitude and phase plots.

Factor	Magnitude	Phase
$K$		
$(j\omega)^N$		
$\frac{1}{(j\omega)^N}$		
$\left(1 + \frac{j\omega}{z}\right)^N$		
$\frac{1}{(1 + j\omega/p)^N}$		

There is another procedure for obtaining Bode plots that is faster and perhaps more efficient than the one we have just discussed. It consists in realizing that zeros cause an increase in slope, while poles cause a decrease. By starting with the low-frequency asymptote of the Bode plot, moving along the frequency axis, and increasing or decreasing the slope at each corner frequency, one can sketch the Bode plot immediately from the transfer function without the effort of making individual plots and adding them. This procedure can be used once you become proficient in the one discussed here.

Digital computers have rendered the procedure discussed here almost obsolete. Several software packages such as PSpice, Matlab, Mathcad, and Micro-Cap can be used to generate frequency response plots. We will discuss PSpice later in the chapter.

TABLE 14.3 (continued)

Factor	Magnitude	Phase
$\left[1 + \frac{2j\omega\zeta}{\omega_n} + \left(\frac{j\omega}{\omega_n}\right)^2\right]^N$		
$\frac{1}{[1 + 2j\omega\zeta/\omega_k + (j\omega/\omega_k)^2]^N}$		

## EXAMPLE 14.3

Construct the Bode plots for the transfer function

$$\mathbf{H}(\omega) = \frac{200j\omega}{(j\omega + 2)(j\omega + 10)}$$

**Solution:**

We first put  $\mathbf{H}(\omega)$  in the standard form by dividing out the poles and zeros. Thus,

$$\begin{aligned}\mathbf{H}(\omega) &= \frac{10j\omega}{(1 + j\omega/2)(1 + j\omega/10)} \\ &= \frac{10|j\omega|}{|1 + j\omega/2| |1 + j\omega/10|} \angle 90^\circ - \tan^{-1} \omega/2 - \tan^{-1} \omega/10\end{aligned}$$

Hence the magnitude and phase are

$$\begin{aligned}H_{\text{dB}} &= 20 \log_{10} 10 + 20 \log_{10} |j\omega| - 20 \log_{10} \left|1 + \frac{j\omega}{2}\right| \\ &\quad - 20 \log_{10} \left|1 + \frac{j\omega}{10}\right| \\ \phi &= 90^\circ - \tan^{-1} \frac{\omega}{2} - \tan^{-1} \frac{\omega}{10}\end{aligned}$$

We notice that there are two corner frequencies at  $\omega = 2, 10$ . For both the magnitude and phase plots, we sketch each term as shown by the dotted lines in Fig. 14.13. We add them up graphically to obtain the overall plots shown by the solid curves.

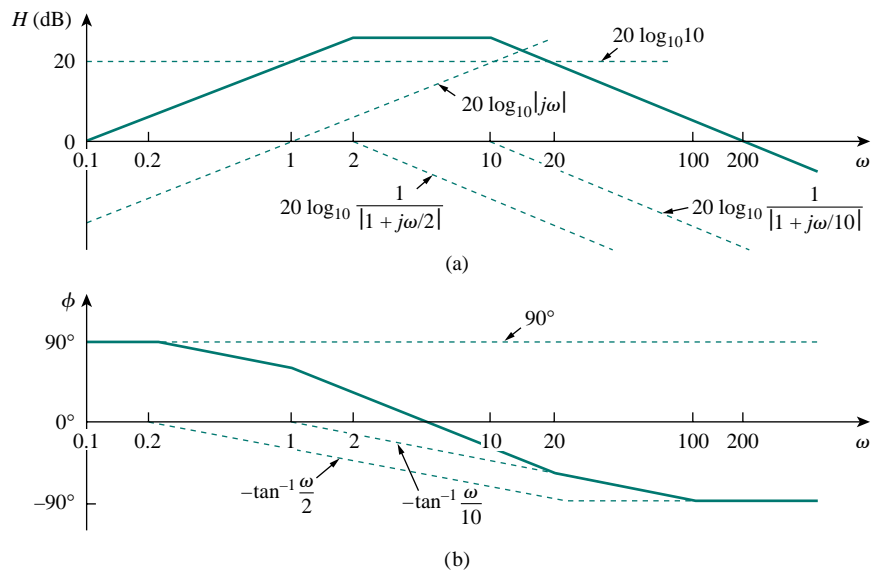


Figure 14.13 For Example 14.3: (a) magnitude plot, (b) phase plot.

### PRACTICE PROBLEM 14.3

Draw the Bode plots for the transfer function

$$\mathbf{H}(\omega) = \frac{5(j\omega + 2)}{j\omega(j\omega + 10)}$$

**Answer:** See Fig. 14.14.

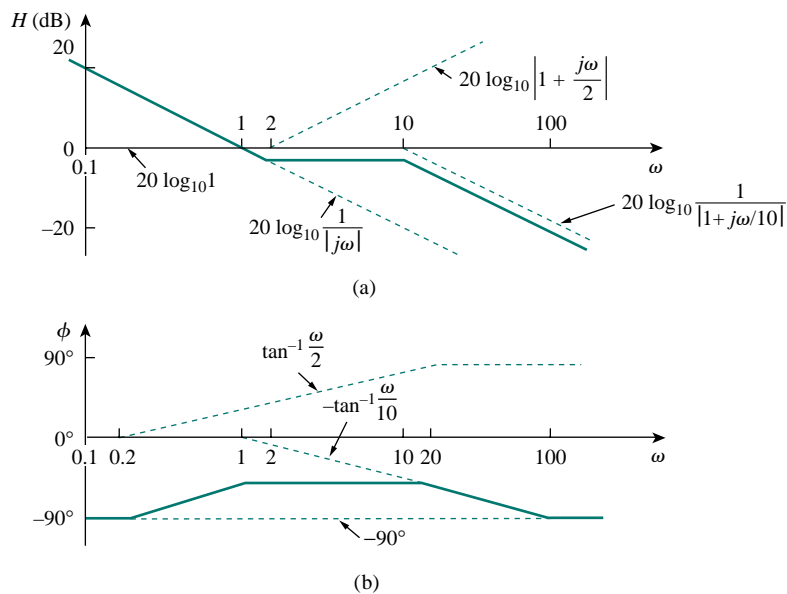


Figure 14.14 For Practice Prob. 14.3: (a) magnitude plot, (b) phase plot.

## EXAMPLE 14.4

Obtain the Bode plots for

$$\mathbf{H}(\omega) = \frac{j\omega + 10}{j\omega(j\omega + 5)^2}$$

**Solution:**

Putting  $\mathbf{H}(\omega)$  in the standard form, we get

$$\mathbf{H}(\omega) = \frac{0.4(1 + j\omega/10)}{j\omega(1 + j\omega/5)^2}$$

From this, we obtain the magnitude and phase as

$$H_{\text{dB}} = 20 \log_{10} 0.4 + 20 \log_{10} \left| 1 + \frac{j\omega}{10} \right| - 20 \log_{10} |j\omega|$$

$$- 40 \log_{10} \left| 1 + \frac{j\omega}{5} \right|$$

$$\phi = 0^\circ + \tan^{-1} \frac{\omega}{10} - 90^\circ - 2 \tan^{-1} \frac{\omega}{5}$$

There are two corner frequencies at  $\omega = 5, 10$  rad/s. For the pole with corner frequency at  $\omega = 5$ , the slope of the magnitude plot is  $-40$  dB/decade and that of the phase plot is  $-90^\circ$  per decade due to the power of 2. The magnitude and the phase plots for the individual terms (in dotted lines) and the entire  $\mathbf{H}(j\omega)$  (in solid lines) are in Fig. 14.15.

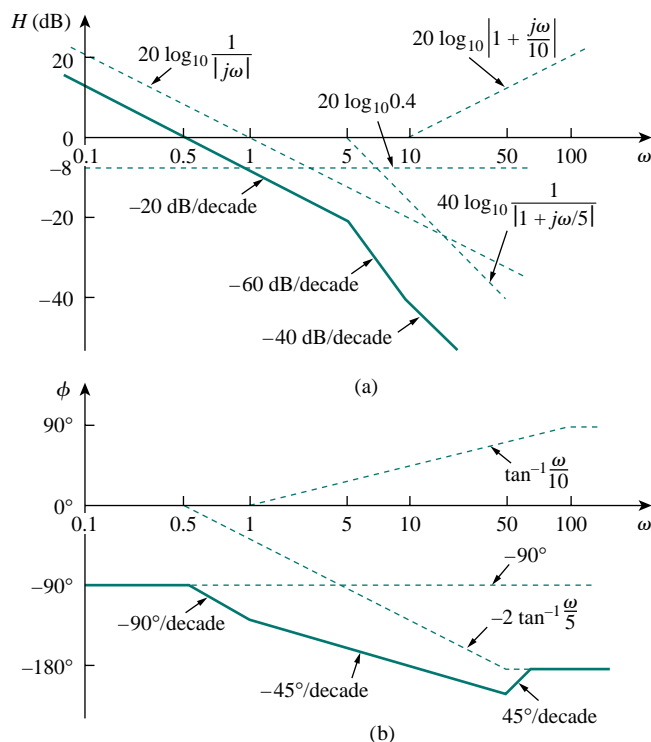


Figure 14.15 Bode plots for Example 14.4: (a) magnitude plot, (b) phase plot.

## PRACTICE PROBLEM 14.4

Sketch the Bode plots for

$$\mathbf{H}(\omega) = \frac{50j\omega}{(j\omega + 4)(j\omega + 10)^2}$$

**Answer:** See Fig. 14.16.

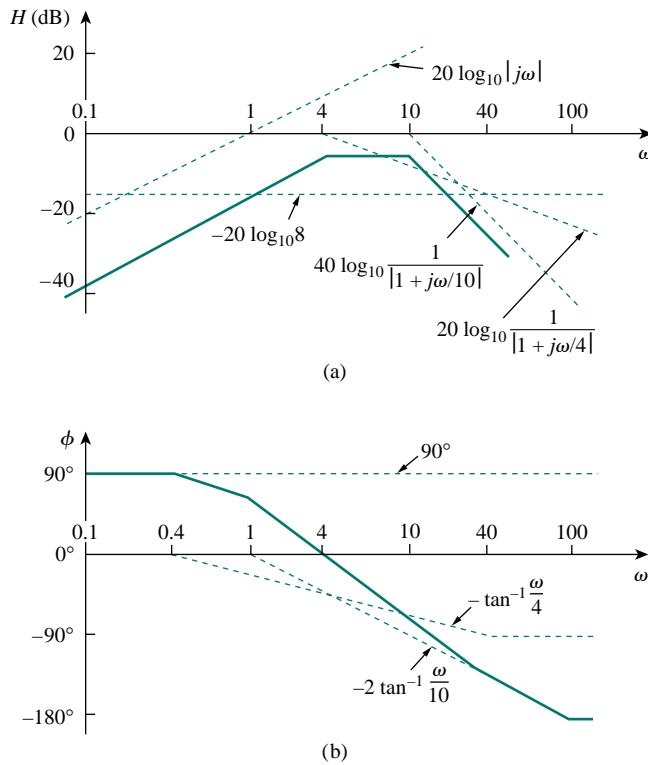


Figure 14.16 For Practice Prob. 14.4: (a) magnitude plot, (b) phase plot.

## EXAMPLE 14.5

Draw the Bode plots for

$$\mathbf{H}(s) = \frac{s + 1}{s^2 + 60s + 100}$$

**Solution:**

We express  $\mathbf{H}(s)$  as

$$\mathbf{H}(\omega) = \frac{1/100(1 + j\omega)}{1 + j\omega 6/10 + (j\omega/10)^2}$$

For the quadratic pole,  $\omega_n = 10$  rad/s, which serves as the corner fre-

quency. The magnitude and phase are

$$H_{\text{dB}} = -20 \log_{10} 100 + 20 \log_{10} |1 + j\omega|$$

$$- 20 \log_{10} \left| 1 + \frac{j\omega 6}{10} - \frac{\omega^2}{100} \right|$$

$$\phi = 0^\circ + \tan^{-1} \omega - \tan^{-1} \left[ \frac{\omega 6/10}{1 - \omega^2/100} \right]$$

Figure 14.17 shows the Bode plots. Notice that the quadratic pole is treated as a repeated pole at  $\omega_k$ , that is,  $(1 + j\omega/\omega_k)^2$ , which is an approximation.

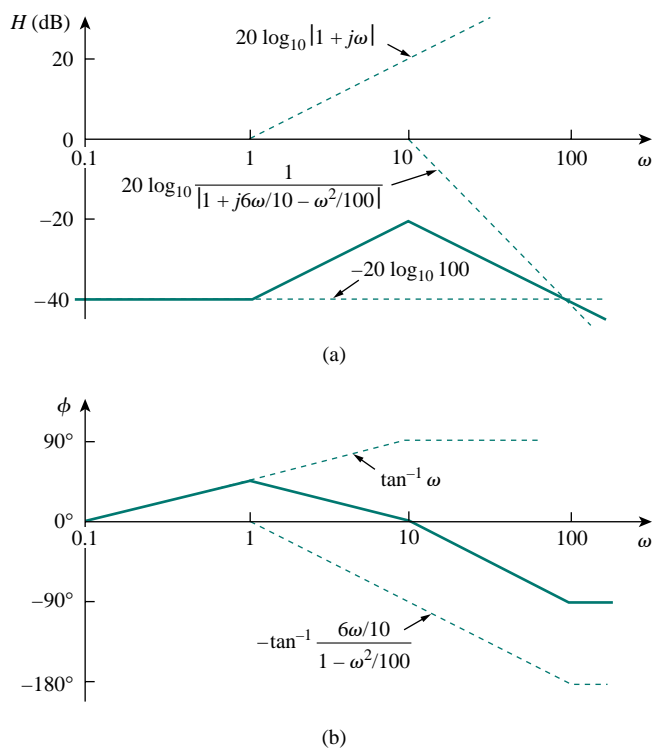


Figure 14.17 Bode plots for Example 14.5: (a) magnitude plot, (b) phase plot.

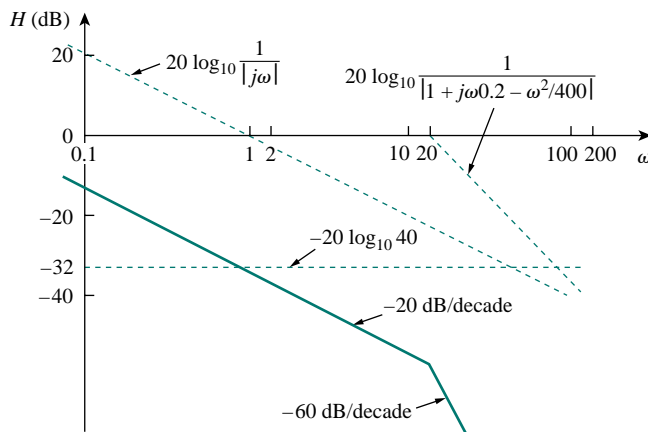
### PRACTICE PROBLEM 14.5

Construct the Bode plots for

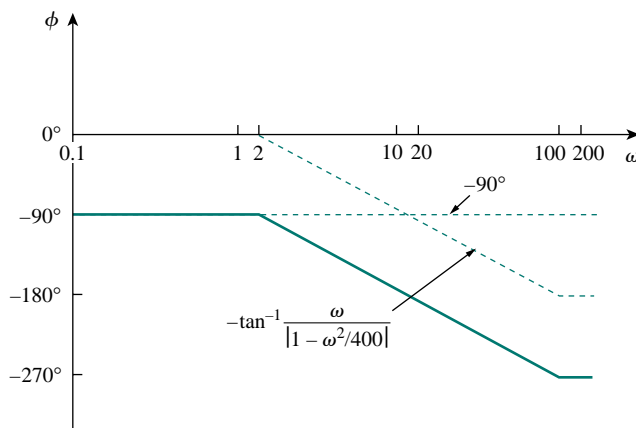
$$H(s) = \frac{10}{s(s^2 + 80s + 400)}$$

**Answer:** See Fig. 14.18.





(a)



(b)

Figure 14.18 For Practice Prob. 14.5: (a) magnitude plot, (b) phase plot.

**EXAMPLE 14.6**

Given the Bode plot in Fig. 14.19, obtain the transfer function  $\mathbf{H}(\omega)$ .

**Solution:**

To obtain  $\mathbf{H}(\omega)$  from the Bode plot, we keep in mind that a zero always causes an upward turn at a corner frequency, while a pole causes a downward turn. We notice from Fig. 14.19 that there is a zero  $j\omega$  at the origin which should have intersected the frequency axis at  $\omega = 1$ . This is indicated by the straight line with slope  $+20$  dB/decade. The fact that this straight line is shifted by 40 dB indicates that there is a 40-dB gain; that is,

$$40 = 20 \log_{10} K \quad \Rightarrow \quad \log_{10} K = 2$$

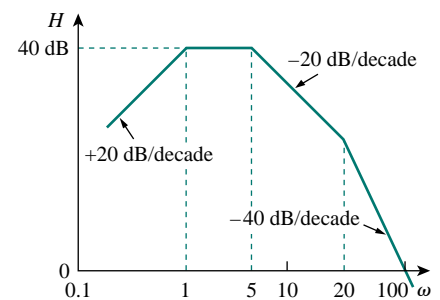


Figure 14.19 For Example 14.6.

or

$$K = 10^2 = 100$$

In addition to the zero  $j\omega$  at the origin, we notice that there are three factors with corner frequencies at  $\omega = 1, 5$ , and  $20$  rad/s. Thus, we have:

1. A pole at  $p = 1$  with slope  $-20$  dB/decade to cause a downward turn and counteract the pole at the origin. The pole at  $z = 1$  is determined as  $1/(1 + j\omega/1)$ .
2. Another pole at  $p = 5$  with slope  $-20$  dB/decade causing a downward turn. The pole is  $1/(1 + j\omega/5)$ .
3. A third pole at  $p = 20$  with slope  $-20$  dB/decade causing a further downward turn. The pole is  $1/(1 + j\omega/20)$ .

Putting all these together gives the corresponding transfer function as

$$\begin{aligned} \mathbf{H}(\omega) &= \frac{100j\omega}{(1 + j\omega/1)(1 + j\omega/5)(1 + j\omega/20)} \\ &= \frac{j\omega 10^4}{(j\omega + 1)(j\omega + 5)(j\omega + 20)} \end{aligned}$$

or

$$\mathbf{H}(s) = \frac{10^4 s}{(s + 1)(s + 5)(s + 20)}, \quad s = j\omega$$

## PRACTICE PROBLEM 14.6

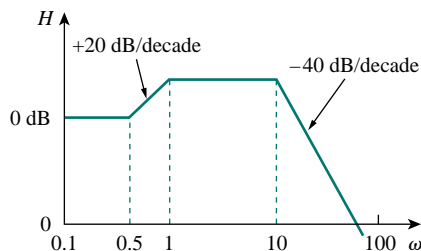


Figure 14.20 For Practice Prob. 14.6.

Obtain the transfer function  $\mathbf{H}(\omega)$  corresponding to the Bode plot in Fig. 14.20.

**Answer:**  $\mathbf{H}(\omega) = \frac{200(s + 0.5)}{(s + 1)(s + 10)^2}.$

## 14.5 SERIES RESONANCE

The most prominent feature of the frequency response of a circuit may be the sharp peak (or *resonant peak*) exhibited in its amplitude characteristic. The concept of resonance applies in several areas of science and engineering. Resonance occurs in any system that has a complex conjugate pair of poles; it is the cause of oscillations of stored energy from one form to another. It is the phenomenon that allows frequency discrimination in communications networks. Resonance occurs in any circuit that has at least one inductor and one capacitor.

**Resonance** is a condition in an  $RLC$  circuit in which the capacitive and inductive reactances are equal in magnitude, thereby resulting in a purely resistive impedance.

Resonant circuits (series or parallel) are useful for constructing filters, as their transfer functions can be highly frequency selective. They are used in many applications such as selecting the desired stations in radio and TV receivers.

Consider the series  $RLC$  circuit shown in Fig. 14.21 in the frequency domain. The input impedance is

$$\mathbf{Z} = \mathbf{H}(\omega) = \frac{\mathbf{V}_s}{\mathbf{I}} = R + j\omega L + \frac{1}{j\omega C} \quad (14.22)$$

or

$$\mathbf{Z} = R + j \left( \omega L - \frac{1}{\omega C} \right) \quad (14.23)$$

Resonance results when the imaginary part of the transfer function is zero, or

$$\text{Im}(\mathbf{Z}) = \omega L - \frac{1}{\omega C} = 0 \quad (14.24)$$

The value of  $\omega$  that satisfies this condition is called the *resonant frequency*  $\omega_0$ . Thus, the resonance condition is

$$\omega_0 L = \frac{1}{\omega_0 C} \quad (14.25)$$

or

$$\omega_0 = \frac{1}{\sqrt{LC}} \text{ rad/s} \quad (14.26)$$

Since  $\omega_0 = 2\pi f_0$ ,

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \text{ Hz} \quad (14.27)$$

Note that at resonance:

1. The impedance is purely resistive, thus,  $\mathbf{Z} = R$ . In other words, the  $LC$  series combination acts like a short circuit, and the entire voltage is across  $R$ .
2. The voltage  $\mathbf{V}_s$  and the current  $\mathbf{I}$  are in phase, so that the power factor is unity.
3. The magnitude of the transfer function  $\mathbf{H}(\omega) = \mathbf{Z}(\omega)$  is minimum.
4. The inductor voltage and capacitor voltage can be much more than the source voltage.

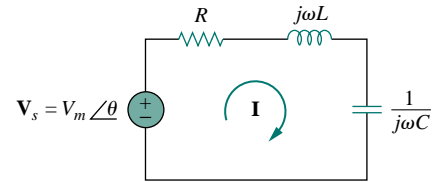


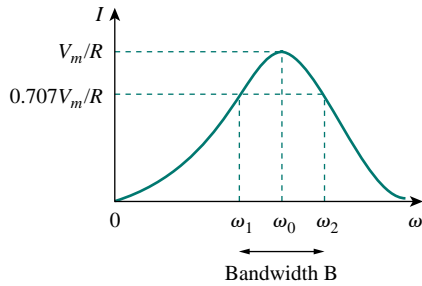
Figure 14.21 The series resonant circuit.

Note No. 4 becomes evident from the fact that

$$|V_L| = \frac{V_m}{R} \omega_0 L = QV_m$$

$$|V_C| = \frac{V_m}{R} \frac{1}{\omega_0 C} = QV_m$$

where  $Q$  is the quality factor, defined in Eq. (14.38).



**Figure 14.22** The current amplitude versus frequency for the series resonant circuit of Fig. 14.21.

The frequency response of the circuit's current magnitude

$$I = |\mathbf{I}| = \frac{V_m}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}} \quad (14.28)$$

is shown in Fig. 14.22; the plot only shows the symmetry illustrated in this graph when the frequency axis is a logarithm. The average power dissipated by the  $RLC$  circuit is

$$P(\omega) = \frac{1}{2} I^2 R \quad (14.29)$$

The highest power dissipated occurs at resonance, when  $I = V_m/R$ , so that

$$P(\omega_0) = \frac{1}{2} \frac{V_m^2}{R} \quad (14.30)$$

At certain frequencies  $\omega = \omega_1, \omega_2$ , the dissipated power is half the maximum value; that is,

$$P(\omega_1) = P(\omega_2) = \frac{(V_m/\sqrt{2})^2}{2R} = \frac{V_m^2}{4R} \quad (14.31)$$

Hence,  $\omega_1$  and  $\omega_2$  are called the *half-power frequencies*.

The half-power frequencies are obtained by setting  $Z$  equal to  $\sqrt{2}R$ , and writing

$$\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} = \sqrt{2}R \quad (14.32)$$

Solving for  $\omega$ , we obtain

$$\begin{aligned} \omega_1 &= -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} \\ \omega_2 &= \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} \end{aligned} \quad (14.33)$$

We can relate the half-power frequencies with the resonant frequency. From Eqs. (14.26) and (14.33),

$$\omega_0 = \sqrt{\omega_1 \omega_2} \quad (14.34)$$

showing that the resonant frequency is the geometric mean of the half-power frequencies. Notice that  $\omega_1$  and  $\omega_2$  are in general not symmetrical around the resonant frequency  $\omega_0$ , because the frequency response is not generally symmetrical. However, as will be explained shortly, symmetry of the half-power frequencies around the resonant frequency is often a reasonable approximation.

Although the height of the curve in Fig. 14.22 is determined by  $R$ , the width of the curve depends on other factors. The width of the response curve depends on the *bandwidth*  $B$ , which is defined as the difference between the two half-power frequencies,

$$B = \omega_2 - \omega_1 \quad (14.35)$$

This definition of bandwidth is just one of several that are commonly used. Strictly speaking,  $B$  in Eq. (14.35) is a half-power bandwidth, because it is the width of the frequency band between the half-power frequencies.

The “sharpness” of the resonance in a resonant circuit is measured quantitatively by the *quality factor*  $Q$ . At resonance, the reactive energy in the circuit oscillates between the inductor and the capacitor. The quality factor relates the maximum or peak energy stored to the energy dissipated in the circuit per cycle of oscillation:

$$Q = 2\pi \frac{\text{Peak energy stored in the circuit}}{\text{Energy dissipated by the circuit in one period at resonance}} \quad (14.36)$$

It is also regarded as a measure of the energy storage property of a circuit in relation to its energy dissipation property. In the series  $RLC$  circuit, the peak energy stored is  $\frac{1}{2}LI^2$ , while the energy dissipated in one period is  $\frac{1}{2}(I^2R)(1/f)$ . Hence,

$$Q = 2\pi \frac{\frac{1}{2}LI^2}{\frac{1}{2}I^2R(1/f)} = \frac{2\pi fL}{R} \quad (14.37)$$

or

$$Q = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 C R} \quad (14.38)$$

Notice that the quality factor is dimensionless. The relationship between the bandwidth  $B$  and the quality factor  $Q$  is obtained by substituting Eq. (14.33) into Eq. (14.35) and utilizing Eq. (14.38).

$$B = \frac{R}{L} = \frac{\omega_0}{Q} \quad (14.39)$$

or  $B = \omega_0^2 C R$ . Thus

The **quality factor** of a resonant circuit is the ratio of its resonant frequency to its bandwidth.

Keep in mind that Eqs. (14.26), (14.33), (14.38), and (14.39) only apply to a series  $RLC$  circuit.

As illustrated in Fig. 14.23, the higher the value of  $Q$ , the more selective the circuit is but the smaller the bandwidth. The *selectivity* of an  $RLC$  circuit is the ability of the circuit to respond to a certain frequency and discriminate against all other frequencies. If the band of frequencies to be selected or rejected is narrow, the quality factor of the resonant circuit must be high. If the band of frequencies is wide, the quality factor must be low.

A resonant circuit is designed to operate at or near its resonant frequency. It is said to be a *high- $Q$  circuit* when its quality factor is

Although the same symbol  $Q$  is used for the reactive power, the two are not equal and should not be confused.  $Q$  here is dimensionless, whereas reactive power  $Q$  is in VAR. This may help distinguish between the two.

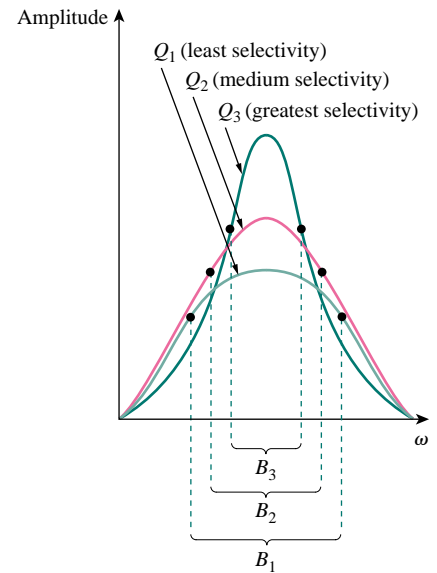


Figure 14.23

The higher the circuit  $Q$ , the smaller the bandwidth.

The quality factor is a measure of the selectivity (or “sharpness” of resonance) of the circuit.

equal to or greater than 10. For high- $Q$  circuits ( $Q \geq 10$ ), the half-power frequencies are, for all practical purposes, symmetrical around the resonant frequency and can be approximated as

$$\omega_1 \simeq \omega_0 - \frac{B}{2}, \quad \omega_2 \simeq \omega_0 + \frac{B}{2} \quad (14.40)$$

High- $Q$  circuits are used often in communications networks.

We see that a resonant circuit is characterized by five related parameters: the two half-power frequencies  $\omega_1$  and  $\omega_2$ , the resonant frequency  $\omega_0$ , the bandwidth  $B$ , and the quality factor  $Q$ .

### EXAMPLE 14.7

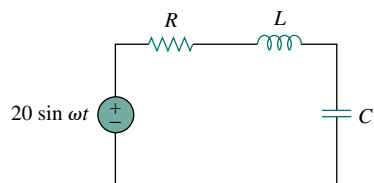


Figure 14.24 For Example 14.7.

In the circuit in Fig. 14.24,  $R = 2 \, \Omega$ ,  $L = 1 \, \text{mH}$ , and  $C = 0.4 \, \mu\text{F}$ . (a) Find the resonant frequency and the half-power frequencies. (b) Calculate the quality factor and bandwidth. (c) Determine the amplitude of the current at  $\omega_0$ ,  $\omega_1$ , and  $\omega_2$ .

**Solution:**

(a) The resonant frequency is

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10^{-3} \times 0.4 \times 10^{-6}}} = 50 \, \text{krad/s}$$

**METHOD 1** The lower half-power frequency is

$$\begin{aligned} \omega_1 &= -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} \\ &= -\frac{2}{2 \times 10^{-3}} + \sqrt{(10^3)^2 + (50 \times 10^3)^2} \\ &= -1 + \sqrt{1 + 2500} \, \text{krad/s} = 49 \, \text{krad/s} \end{aligned}$$

Similarly, the upper half-power frequency is

$$\omega_2 = 1 + \sqrt{1 + 2500} \, \text{krad/s} = 51 \, \text{krad/s}$$

(b) The bandwidth is

$$B = \omega_2 - \omega_1 = 2 \, \text{krad/s}$$

or

$$B = \frac{R}{L} = \frac{2}{10^{-3}} = 2 \, \text{krad/s}$$

The quality factor is

$$Q = \frac{\omega_0}{B} = \frac{50}{2} = 25$$

**METHOD 2** Alternatively, we could find

$$Q = \frac{\omega_0 L}{R} = \frac{50 \times 10^3 \times 10^{-3}}{2} = 25$$

From  $Q$ , we find

$$B = \frac{\omega_0}{Q} = \frac{50 \times 10^3}{25} = 2 \, \text{krad/s}$$

Since  $Q > 10$ , this is a high- $Q$  circuit and we can obtain the half-power frequencies as

$$\omega_1 = \omega_0 - \frac{B}{2} = 50 - 1 = 49 \text{ krad/s}$$

$$\omega_2 = \omega_0 + \frac{B}{2} = 50 + 1 = 51 \text{ krad/s}$$

as obtained earlier.

(c) At  $\omega = \omega_0$ ,

$$I = \frac{V_m}{R} = \frac{20}{2} = 10 \text{ A}$$

At  $\omega = \omega_1, \omega_2$ ,

$$I = \frac{V_m}{\sqrt{2}R} = \frac{10}{\sqrt{2}} = 7.071 \text{ A}$$

### PRACTICE PROBLEM 14.7

A series-connected circuit has  $R = 4 \Omega$  and  $L = 25 \text{ mH}$ . (a) Calculate the value of  $C$  that will produce a quality factor of 50. (b) Find  $\omega_1$ ,  $\omega_2$ , and  $B$ . (c) Determine the average power dissipated at  $\omega = \omega_0$ ,  $\omega_1$ ,  $\omega_2$ . Take  $V_m = 100 \text{ V}$ .

**Answer:** (a)  $0.625 \mu\text{F}$ , (b)  $7920 \text{ rad/s}$ ,  $8080 \text{ rad/s}$ ,  $160 \text{ rad/s}$ , (c)  $1.25 \text{ kW}$ ,  $0.625 \text{ kW}$ ,  $0.625 \text{ kW}$ .

## 14.6 PARALLEL RESONANCE

The parallel  $RLC$  circuit in Fig. 14.25 is the dual of the series  $RLC$  circuit. So we will avoid needless repetition. The admittance is

$$\mathbf{Y} = H(\omega) = \frac{\mathbf{I}}{\mathbf{V}} = \frac{1}{R} + j\omega C + \frac{1}{j\omega L} \quad (14.41)$$

or

$$\mathbf{Y} = \frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right) \quad (14.42)$$

Resonance occurs when the imaginary part of  $\mathbf{Y}$  is zero,

$$\omega C - \frac{1}{\omega L} = 0 \quad (14.43)$$

or

$$\omega_0 = \frac{1}{\sqrt{LC}} \text{ rad/s} \quad (14.44)$$

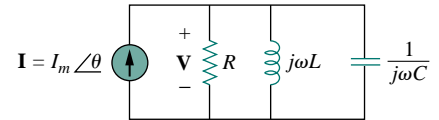


Figure 14.25 The parallel resonant circuit.

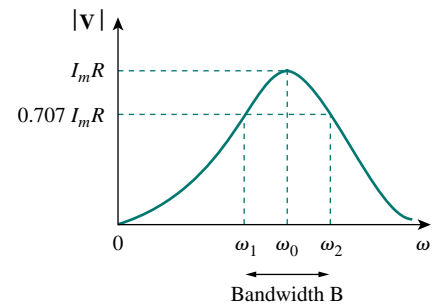


Figure 14.26 The current amplitude versus frequency for the series resonant circuit of Fig. 14.25.

which is the same as Eq. (14.26) for the series resonant circuit. The voltage  $|\mathbf{V}|$  is sketched in Fig. 14.26 as a function of frequency. Notice that at resonance, the parallel  $LC$  combination acts like an open circuit, so

We can see this from the fact that

$$|I_L| = \frac{I_m R}{\omega_0 L} = Q I_m$$

$$|I_C| = \omega_0 C I_m R = Q I_m$$

where  $Q$  is the quality factor, defined in Eq. (14.47).

that the entire currents flows through  $R$ . Also, the inductor and capacitor current can be much more than the source current at resonance.

We exploit the duality between Figs. 14.21 and 14.25 by comparing Eq. (14.42) with Eq. (14.23). By replacing  $R$ ,  $L$ , and  $C$  in the expressions for the series circuit with  $1/R$ ,  $1/C$ , and  $1/L$  respectively, we obtain for the parallel circuit

$$\begin{aligned}\omega_1 &= -\frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}} \\ \omega_2 &= \frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}\end{aligned}\quad (14.45)$$

$$B = \omega_2 - \omega_1 = \frac{1}{RC} \quad (14.46)$$

$$Q = \frac{\omega_0}{B} = \omega_0 RC = \frac{R}{\omega_0 L} \quad (14.47)$$

Using Eqs. (14.45) and (14.47), we can express the half-power frequencies in terms of the quality factor. The result is

$$\omega_1 = \omega_0 \sqrt{1 + \left(\frac{1}{2Q}\right)^2} - \frac{\omega_0}{2Q}, \quad \omega_2 = \omega_0 \sqrt{1 + \left(\frac{1}{2Q}\right)^2} + \frac{\omega_0}{2Q} \quad (14.48)$$

Again, for high- $Q$  circuits ( $Q \geq 10$ )

$$\omega_1 \simeq \omega_0 - \frac{B}{2}, \quad \omega_2 \simeq \omega_0 + \frac{B}{2} \quad (14.49)$$

Table 14.4 presents a summary of the characteristics of the series and parallel resonant circuits. Besides the series and parallel  $RLC$  considered here, other resonant circuits exist. Example 14.9 treats a typical example.

**TABLE 14.4** Summary of the characteristics of resonant  $RLC$  circuits.

Characteristic	Series circuit	Parallel circuit
Resonant frequency, $\omega_0$	$\frac{1}{\sqrt{LC}}$	$\frac{1}{\sqrt{LC}}$
Quality factor, $Q$	$\frac{\omega_0 L}{R}$ or $\frac{1}{\omega_0 RC}$	$\frac{R}{\omega_0 L}$ or $\omega_0 RC$
Bandwidth, $B$	$\frac{\omega_0}{Q}$	$\frac{\omega_0}{Q}$
Half-power frequencies, $\omega_1, \omega_2$	$\omega_0 \sqrt{1 + \left(\frac{1}{2Q}\right)^2} \pm \frac{\omega_0}{2Q}$	$\omega_0 \sqrt{1 + \left(\frac{1}{2Q}\right)^2} \pm \frac{\omega_0}{2Q}$
For $Q \geq 10$ , $\omega_1, \omega_2$	$\omega_0 \pm \frac{B}{2}$	$\omega_0 \pm \frac{B}{2}$



**EXAMPLE 14.8**

In the parallel  $RLC$  circuit in Fig. 14.27, let  $R = 8\text{ k}\Omega$ ,  $L = 0.2\text{ mH}$ , and  $C = 8\text{ }\mu\text{F}$ . (a) Calculate  $\omega_0$ ,  $Q$ , and  $B$ . (b) Find  $\omega_1$  and  $\omega_2$ . (c) Determine the power dissipated at  $\omega_0$ ,  $\omega_1$ , and  $\omega_2$ .

**Solution:**

(a)

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.2 \times 10^{-3} \times 8 \times 10^{-6}}} = \frac{10^5}{4} = 25\text{ krad/s}$$

$$Q = \frac{R}{\omega_0 L} = \frac{8 \times 10^3}{25 \times 10^3 \times 0.2 \times 10^{-3}} = 1600$$

$$B = \frac{\omega_0}{Q} = 15.625\text{ rad/s}$$

(b) Due to the high value of  $Q$ , we can regard this as a high- $Q$  circuit. Hence,

$$\omega_1 = \omega_0 - \frac{B}{2} = 25,000 - 7.812 = 24,992\text{ rad/s}$$

$$\omega_2 = \omega_0 + \frac{B}{2} = 25,000 + 7.8125 = 25,008\text{ rad/s}$$

(c) At  $\omega = \omega_0$ ,  $\mathbf{Y} = 1/R$  or  $\mathbf{Z} = R = 8\text{ k}\Omega$ . Then

$$\mathbf{I}_o = \frac{\mathbf{V}}{\mathbf{Z}} = \frac{10 \angle -90^\circ}{8000} = 1.25 \angle -90^\circ\text{ mA}$$

Since the entire current flows through  $R$  at resonance, the average power dissipated at  $\omega = \omega_0$  is

$$P = \frac{1}{2} |\mathbf{I}_o|^2 R = \frac{1}{2} (1.25 \times 10^{-3})^2 (8 \times 10^3) = 6.25\text{ mW}$$

or

$$P = \frac{V_m^2}{2R} = \frac{100}{2 \times 8 \times 10^3} = 6.25\text{ mW}$$

At  $\omega = \omega_1, \omega_2$ ,

$$P = \frac{V_m^2}{4R} = 3.125\text{ mW}$$

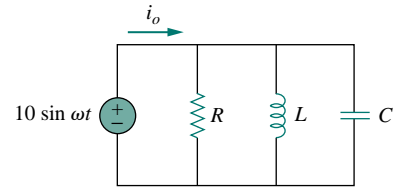


Figure 14.27 For Example 14.8.

**PRACTICE PROBLEM 14.8**

A parallel resonant circuit has  $R = 100\text{ k}\Omega$ ,  $L = 20\text{ mH}$ , and  $C = 5\text{ nF}$ . Calculate  $\omega_0$ ,  $\omega_1$ ,  $\omega_2$ ,  $Q$ , and  $B$ .

**Answer:** 100 krad/s, 99 krad/s, 101 krad/s, 50, 2 krad/s.

**EXAMPLE 14.9**

Determine the resonant frequency of the circuit in Fig. 14.28.

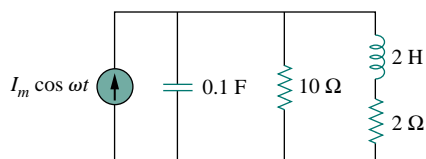


Figure 14.28 For Example 14.9.

**Solution:**

The input admittance is

$$\mathbf{Y} = j\omega 0.1 + \frac{1}{10} + \frac{1}{2 + j\omega 2} = 0.1 + j\omega 0.1 + \frac{2 - j\omega 2}{4 + 4\omega^2}$$

At resonance,  $\text{Im}(\mathbf{Y}) = 0$  and

$$\omega_0 0.1 - \frac{2\omega_0}{4 + 4\omega_0^2} = 0 \quad \Rightarrow \quad \omega_0 = 2 \text{ rad/s}$$

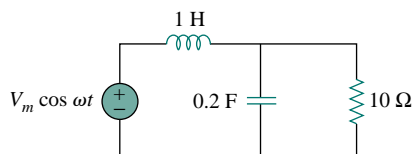
**PRACTICE PROBLEM 14.9**

Figure 14.29 For Practice Prob. 14.9.

Calculate the resonant frequency of the circuit in Fig. 14.29.

**Answer:** 2.179 rad/s.

**14.7 PASSIVE FILTERS**

The concept of filters has been an integral part of the evolution of electrical engineering from the beginning. Several technological achievements would not have been possible without electrical filters. Because of this prominent role of filters, much effort has been expended on the theory, design, and construction of filters and many articles and books have been written on them. Our discussion in this chapter should be considered introductory.

A **filter** is a circuit that is designed to pass signals with desired frequencies and reject or attenuate others.

As a frequency-selective device, a filter can be used to limit the frequency spectrum of a signal to some specified band of frequencies. Filters are the circuits used in radio and TV receivers to allow us to select one desired signal out of a multitude of broadcast signals in the environment.

A filter is a *passive filter* if it consists of only passive elements  $R$ ,  $L$ , and  $C$ . It is said to be an *active filter* if it consists of active elements (such as transistors and op amps) in addition to passive elements  $R$ ,  $L$ , and  $C$ . We consider passive filters in this section and active filters in the next section. Besides the filters we study in these sections, there are other kinds of filters—such as digital filters, electromechanical filters, and microwave filters—which are beyond the level of the text.

As shown in Fig. 14.30, there are four types of filters whether passive or active:

1. A *lowpass filter* passes low frequencies and stops high frequencies, as shown ideally in Fig. 14.30(a).
2. A *highpass filter* passes high frequencies and rejects low frequencies, as shown ideally in Fig. 14.30(b).
3. A *bandpass filter* passes frequencies within a frequency band and blocks or attenuates frequencies outside the band, as shown ideally in Fig. 14.30(c).
4. A *bandstop filter* passes frequencies outside a frequency band and blocks or attenuates frequencies within the band, as shown ideally in Fig. 14.30(d).

Table 14.5 presents a summary of the characteristics of these filters. Be aware that the characteristics in Table 14.5 are only valid for first- or second-order filters—but one should not have the impression that only these kinds of filter exist. We now consider typical circuits for realizing the filters shown in Table 14.5.

**TABLE 14.5** Summary of the characteristics of filters.

Type of Filter	$H(0)$	$H(\infty)$	$H(\omega_c)$ or $H(\omega_0)$
Lowpass	1	0	$1/\sqrt{2}$
Highpass	0	1	$1/\sqrt{2}$
Bandpass	0	0	1
Bandstop	1	1	0

$\omega_c$  is the cutoff frequency for lowpass and highpass filters;  $\omega_0$  is the center frequency for bandpass and bandstop filters.

### 14.7.1 Lowpass Filter

A typical lowpass filter is formed when the output of an  $RC$  circuit is taken off the capacitor as shown in Fig. 14.31. The transfer function (see also Example 14.1) is

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{1/j\omega C}{R + 1/j\omega C}$$

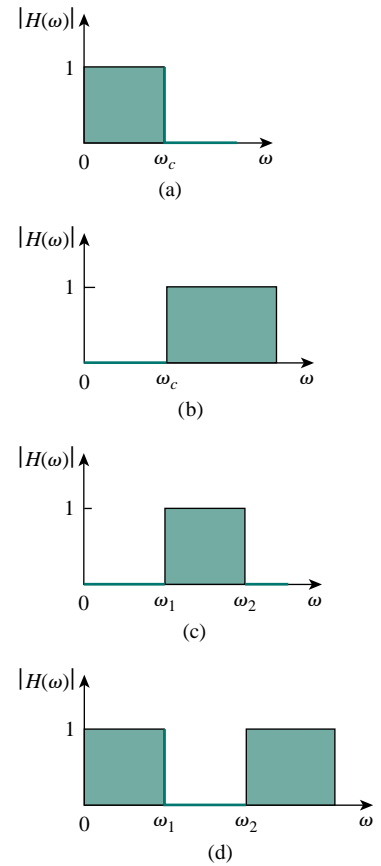
$$\mathbf{H}(\omega) = \frac{1}{1 + j\omega RC} \quad (14.50)$$

Note that  $\mathbf{H}(0) = 1$ ,  $\mathbf{H}(\infty) = 0$ . Figure 14.32 shows the plot of  $|H(\omega)|$ , along with the ideal characteristic. The half-power frequency, which is equivalent to the corner frequency on the Bode plots but in the context of filters is usually known as the *cutoff frequency*  $\omega_c$ , is obtained by setting the magnitude of  $\mathbf{H}(\omega)$  equal to  $1/\sqrt{2}$ , thus

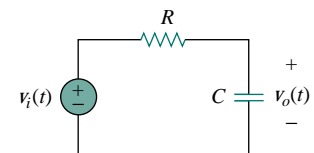
$$H(\omega_c) = \frac{1}{\sqrt{1 + \omega_c^2 R^2 C^2}} = \frac{1}{\sqrt{2}}$$

or

$$\omega_c = \frac{1}{RC} \quad (14.51)$$



**Figure 14.30** Ideal frequency response of four types of filter: (a) lowpass filter, (b) highpass filter, (c) bandpass filter, (d) bandstop filter.



**Figure 14.31** A lowpass filter.

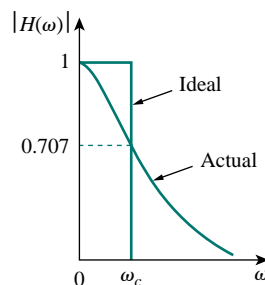


Figure 14.32 Ideal and actual frequency response of a lowpass filter.

The cutoff frequency is the frequency at which the transfer function  $\mathbf{H}$  drops in magnitude to 70.71% of its maximum value. It is also regarded as the frequency at which the power dissipated in a circuit is half of its maximum value.

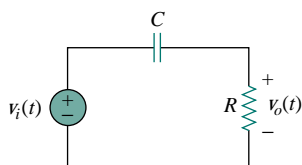


Figure 14.33 A highpass filter.

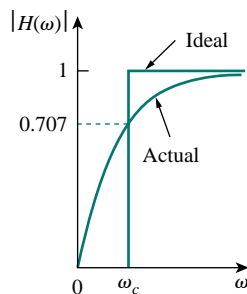


Figure 14.34 Ideal and actual frequency response of a highpass filter.

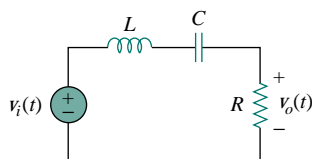


Figure 14.35 A bandpass filter.

The cutoff frequency is also called the *rolloff frequency*.

A **lowpass filter** is designed to pass only frequencies from dc up to the cutoff frequency  $\omega_c$ .

A lowpass filter can also be formed when the output of an  $RL$  circuit is taken off the resistor. Of course, there are many other circuits for lowpass filters.

### 14.7.2 Highpass Filter

A highpass filter is formed when the output of an  $RC$  circuit is taken off the resistor as shown in Fig. 14.33. The transfer function is

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{R}{R + 1/j\omega C}$$

$$\mathbf{H}(\omega) = \frac{j\omega RC}{1 + j\omega RC} \quad (14.52)$$

Note that  $\mathbf{H}(0) = 0$ ,  $\mathbf{H}(\infty) = 1$ . Figure 14.34 shows the plot of  $|H(\omega)|$ . Again, the corner or cutoff frequency is

$$\omega_c = \frac{1}{RC} \quad (14.53)$$

A **highpass filter** is designed to pass all frequencies above its cutoff frequency  $\omega_c$ .

A highpass filter can also be formed when the output of an  $RL$  circuit is taken off the inductor.

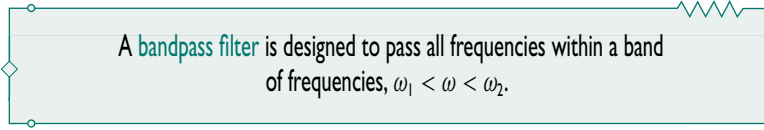
### 14.7.3 Bandpass Filter

The  $RLC$  series resonant circuit provides a bandpass filter when the output is taken off the resistor as shown in Fig. 14.35. The transfer function is

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{R}{R + j(\omega L - 1/\omega C)} \quad (14.54)$$

We observe that  $\mathbf{H}(0) = 0$ ,  $\mathbf{H}(\infty) = 0$ . Figure 14.36 shows the plot of  $|H(\omega)|$ . The bandpass filter passes a band of frequencies ( $\omega_1 < \omega < \omega_2$ ) centered on  $\omega_0$ , the center frequency, which is given by

$$\omega_0 = \frac{1}{\sqrt{LC}} \quad (14.55)$$



Since the bandpass filter in Fig. 14.35 is a series resonant circuit, the half-power frequencies, the bandwidth, and the quality factor are determined as in Section 14.5. A bandpass filter can also be formed by cascading the lowpass filter (where  $\omega_2 = \omega_c$ ) in Fig. 14.31 with the highpass filter (where  $\omega_1 = \omega_c$ ) in Fig. 14.33.

#### 14.7.4 Bandstop Filter

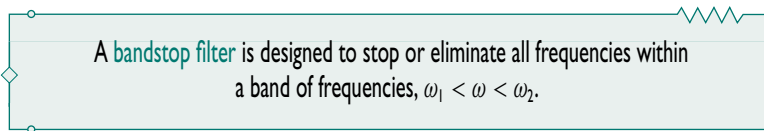
A filter that prevents a band of frequencies between two designated values ( $\omega_1$  and  $\omega_2$ ) from passing is variably known as a *bandstop*, *bandreject*, or *notch* filter. A bandstop filter is formed when the output *RLC* series resonant circuit is taken off the *LC* series combination as shown in Fig. 14.37. The transfer function is

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{j(\omega L - 1/\omega C)}{R + j(\omega L - 1/\omega C)} \quad (14.56)$$

Notice that  $\mathbf{H}(0) = 1$ ,  $\mathbf{H}(\infty) = 1$ . Figure 14.38 shows the plot of  $|H(\omega)|$ . Again, the center frequency is given by

$$\omega_0 = \frac{1}{\sqrt{LC}} \quad (14.57)$$

while the half-power frequencies, the bandwidth, and the quality factor are calculated using the formulas in Section 14.5 for a series resonant circuit. Here,  $\omega_0$  is called the *frequency of rejection*, while the corresponding bandwidth ( $B = \omega_2 - \omega_1$ ) is known as the *bandwidth of rejection*. Thus,



Notice that adding the transfer functions of the bandpass and the bandstop gives unity at any frequency for the same values of  $R$ ,  $L$ , and  $C$ . Of course, this is not true in general but true for the circuits treated here. This is due to the fact that the characteristic of one is the inverse of the other.

In concluding this section, we should note that:

1. From Eqs. (14.50), (14.52), (14.54), and (14.56), the maximum gain of a passive filter is unity. To generate a gain greater than unity, one should use an active filter as the next section shows.

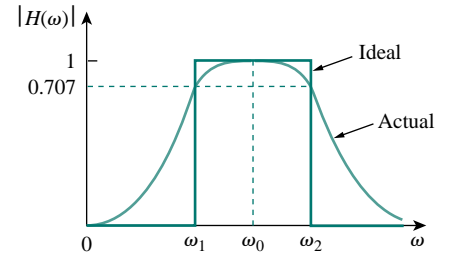


Figure 14.36 Ideal and actual frequency response of a bandpass filter.

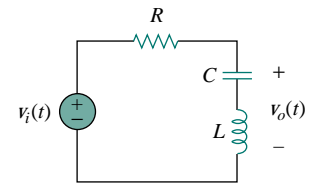


Figure 14.37 A bandstop filter.

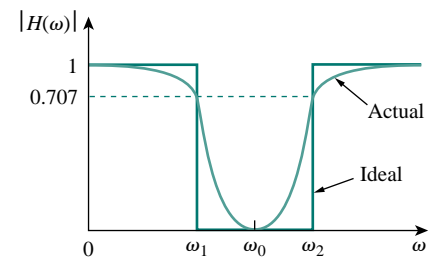


Figure 14.38 Ideal and actual frequency response of a bandstop filter.

2. There are other ways to get the types of filters treated in this section.
3. The filters treated here are the simple types. Many other filters have sharper and complex frequency responses.

### EXAMPLE 14.10

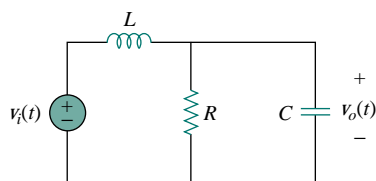


Figure 14.39 For Example 14.10.

Determine what type of filter is shown in Fig. 14.39. Calculate the corner or cutoff frequency. Take  $R = 2 \text{ k}\Omega$ ,  $L = 2 \text{ H}$ , and  $C = 2 \text{ }\mu\text{F}$ .

**Solution:**

The transfer function is

$$\mathbf{H}(s) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{R \parallel 1/sC}{sL + R \parallel 1/sC}, \quad s = j\omega \quad (14.10.1)$$

But

$$R \parallel \frac{1}{sC} = \frac{R/sC}{R + 1/sC} = \frac{R}{1 + sRC}$$

Substituting this into Eq. (14.10.1) gives

$$\mathbf{H}(s) = \frac{R/(1 + sRC)}{sL + R/(1 + sRC)} = \frac{R}{s^2RLC + sL + R}, \quad s = j\omega$$

or

$$\mathbf{H}(\omega) = \frac{R}{-\omega^2RLC + j\omega L + R} \quad (14.10.2)$$

Since  $\mathbf{H}(0) = 1$  and  $\mathbf{H}(\infty) = 0$ , we conclude from Table 14.5 that the circuit in Fig. 14.39 is a second-order lowpass filter. The magnitude of  $\mathbf{H}$  is

$$H = \frac{R}{\sqrt{(R - \omega^2RLC)^2 + \omega^2L^2}} \quad (14.10.3)$$

The corner frequency is the same as the half-power frequency, i.e., where  $\mathbf{H}$  is reduced by a factor of  $1/\sqrt{2}$ . Since the dc value of  $H(\omega)$  is 1, at the corner frequency, Eq. (14.10.3) becomes after squaring

$$H^2 = \frac{1}{2} = \frac{R^2}{(R - \omega_c^2RLC)^2 + \omega_c^2L^2}$$

or

$$2 = (1 - \omega_c^2LC)^2 + \left(\frac{\omega_cL}{R}\right)^2$$

Substituting the values of  $R$ ,  $L$ , and  $C$ , we obtain

$$2 = (1 - \omega_c^2 4 \times 10^{-6})^2 + (\omega_c 10^{-3})^2$$

Assuming that  $\omega_c$  is in krad/s,

$$2 = (1 - 4\omega_c^2)^2 + \omega_c^2 \quad \text{or} \quad 16\omega_c^4 - 7\omega_c^2 - 1 = 0$$

Solving the quadratic equation in  $\omega_c^2$ , we get  $\omega_c^2 = 0.5509$ , or

$$\omega_c = 0.742 \text{ krad/s} = 742 \text{ rad/s}$$

**PRACTICE PROBLEM 14.10**

For the circuit in Fig. 14.40, obtain the transfer function  $\mathbf{V}_o(\omega)/\mathbf{V}_i(\omega)$ . Identify the type of filter the circuit represents and determine the corner frequency. Take  $R_1 = 100\ \Omega = R_2$ ,  $L = 2\text{ mH}$ .

**Answer:** Highpass filter,  $\frac{R_2}{R_1 + R_2} \left( \frac{j\omega}{j\omega + \omega_c} \right)$ ,

$$\omega_c = \frac{R_1 R_2}{(R_1 + R_2)L} = 25\text{ krad/s}.$$

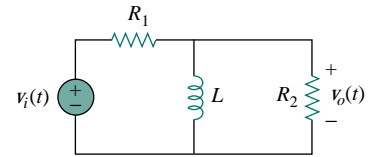


Figure 14.40 For Practice Prob. 14.10.

**EXAMPLE 14.11**

If the bandstop filter in Fig. 14.37 is to reject a 200-Hz sinusoid while passing other frequencies, calculate the values of  $L$  and  $C$ . Take  $R = 150\ \Omega$  and the bandwidth as 100 Hz.

**Solution:**

We use the formulas for a series resonant circuit in Section 14.5.

$$B = 2\pi(100) = 200\pi\text{ rad/s}$$

But

$$B = \frac{R}{L} \quad \Rightarrow \quad L = \frac{R}{B} = \frac{150}{200\pi} = 0.2387\text{ H}$$

Rejection of the 200-Hz sinusoid means that  $f_0$  is 200 Hz, so that  $\omega_0$  in Fig. 14.38 is

$$\omega_0 = 2\pi f_0 = 2\pi(200) = 400\pi$$

Since  $\omega_0 = 1/\sqrt{LC}$ ,

$$C = \frac{1}{\omega_0^2 L} = \frac{1}{(400\pi)^2(0.2387)} = 2.66\ \mu\text{F}$$

**PRACTICE PROBLEM 14.11**

Design a bandpass filter of the form in Fig. 14.35 with a lower cutoff frequency of 20.1 kHz and an upper cutoff frequency of 20.3 kHz. Take  $R = 20\text{ k}\Omega$ . Calculate  $L$ ,  $C$ , and  $Q$ .

**Answer:** 7.96 H, 3.9 pF, 101.

**14.8 ACTIVE FILTERS**

There are three major limits to the passive filters considered in the previous section. First, they cannot generate gain greater than 1; passive elements cannot add energy to the network. Second, they may require bulky and expensive inductors. Third, they perform poorly at frequencies below the audio frequency range ( $300\text{ Hz} < f < 3000\text{ Hz}$ ). Nevertheless, passive filters are useful at high frequencies.

Active filters consist of combinations of resistors, capacitors, and op amps. They offer some advantages over passive  $RLC$  filters. First, they are often smaller and less expensive, because they do not require inductors. This makes feasible the integrated circuit realizations of filters. Second, they can provide amplifier gain in addition to providing the same frequency response as  $RLC$  filters. Third, active filters can be combined with buffer amplifiers (voltage followers) to isolate each stage of the filter from source and load impedance effects. This isolation allows designing the stages independently and then cascading them to realize the desired transfer function. (Bode plots, being logarithmic, may be added when transfer functions are cascaded.) However, active filters are less reliable and less stable. The practical limit of most active filters is about 100 kHz—most active filters operate well below that frequency.

Filters are often classified according to their order (or number of poles) or their specific design type.

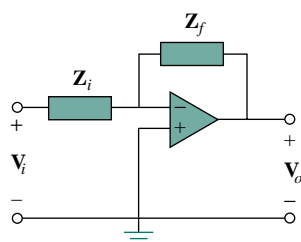


Figure 14.41 A general first-order active filter.

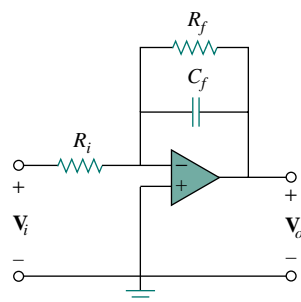


Figure 14.42 Active first-order lowpass filter.

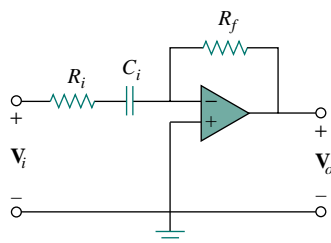


Figure 14.43 Active first-order highpass filter.

### 14.8.1 First-Order Lowpass Filter

One type of first-order filter is shown in Fig. 14.41. The components selected for  $Z_i$  and  $Z_f$  determine whether the filter is lowpass or highpass, but one of the components must be reactive.

Figure 14.42 shows a typical active low-pass filter. For this filter, the transfer function is

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = -\frac{\mathbf{Z}_f}{\mathbf{Z}_i} \quad (14.58)$$

where  $\mathbf{Z}_i = R_i$  and

$$\mathbf{Z}_f = R_f \parallel \frac{1}{j\omega C_f} = \frac{R_f / j\omega C_f}{R_f + 1/j\omega C_f} = \frac{R_f}{1 + j\omega C_f R_f} \quad (14.59)$$

Therefore,

$$\mathbf{H}(\omega) = -\frac{R_f}{R_i} \frac{1}{1 + j\omega C_f R_f} \quad (14.60)$$

We notice that Eq. (14.60) is similar to Eq. (14.50), except that there is a low frequency ( $\omega \rightarrow 0$ ) gain or dc gain of  $-R_f/R_i$ . Also, the corner frequency is

$$\omega_c = \frac{1}{R_f C_f} \quad (14.61)$$

which does not depend on  $R_i$ . This means that several inputs with different  $R_i$  could be summed if required, and the corner frequency would remain the same for each input.

### 14.8.2 First-Order Highpass Filter

Figure 14.43 shows a typical highpass filter. As before,

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = -\frac{\mathbf{Z}_f}{\mathbf{Z}_i} \quad (14.62)$$

where  $\mathbf{Z}_i = R_i + 1/j\omega C_i$  and  $\mathbf{Z}_f = R_f$  so that

$$\mathbf{H}(\omega) = -\frac{R_f}{R_i + 1/j\omega C_i} = -\frac{j\omega C_i R_f}{1 + j\omega C_i R_i} \quad (14.63)$$



This is similar to Eq. (14.52), except that at very high frequencies ( $\omega \rightarrow \infty$ ), the gain tends to  $-R_f/R_i$ . The corner frequency is

$$\omega_c = \frac{1}{R_i C_i} \quad (14.64)$$

### 14.8.3 Bandpass Filter

The circuit in Fig. 14.42 may be combined with that in Fig. 14.43 to form a bandpass filter that will have a gain  $K$  over the required range of frequencies. By cascading a unity-gain lowpass filter, a unity-gain highpass filter, and an inverter with gain  $-R_f/R_i$ , as shown in the block diagram of Fig. 14.44(a), we can construct a bandpass filter whose frequency response is that in Fig. 14.44(b). The actual construction of the bandpass filter is shown in Fig. 14.45.

This way of creating a bandpass filter, not necessarily the best, is perhaps the easiest to understand.

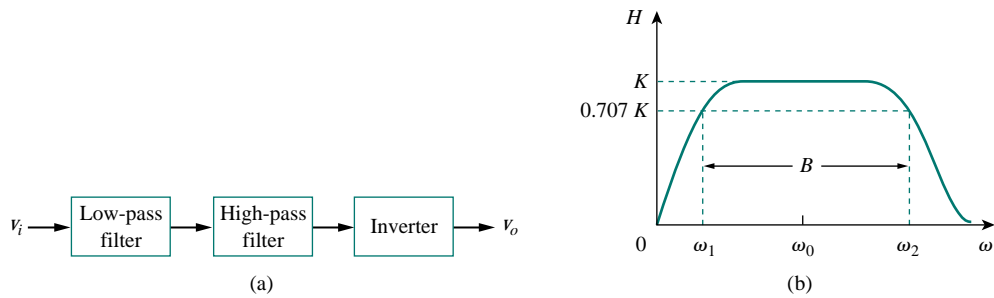


Figure 14.44 Active bandpass filter: (a) block diagram, (b) frequency response.

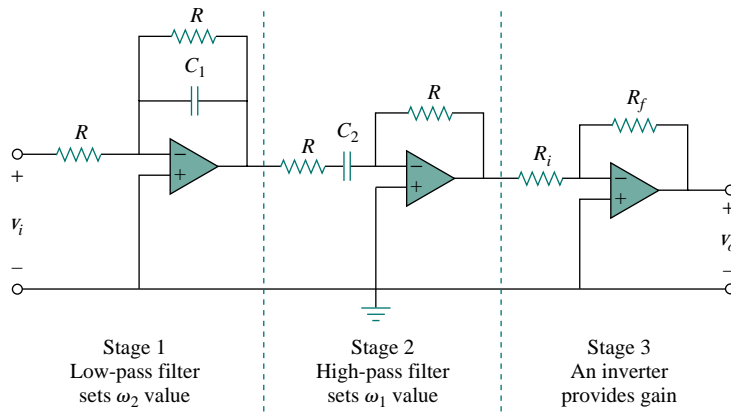


Figure 14.45 Active bandpass filter.

The analysis of the bandpass filter is relatively simple. Its transfer function is obtained by multiplying Eqs. (14.60) and (14.63) with the gain of the inverter; that is

$$\begin{aligned}
 \mathbf{H}(\omega) &= \frac{\mathbf{V}_o}{\mathbf{V}_i} = \left( -\frac{1}{1 + j\omega C_1 R} \right) \left( -\frac{j\omega C_2 R}{1 + j\omega C_2 R} \right) \left( -\frac{R_f}{R_i} \right) \\
 &= -\frac{R_f}{R_i} \frac{1}{1 + j\omega C_1 R} \frac{j\omega C_2 R}{1 + j\omega C_2 R}
 \end{aligned} \quad (14.65)$$

The lowpass section sets the upper corner frequency as

$$\omega_2 = \frac{1}{RC_1} \quad (14.66)$$

while the highpass section sets the lower corner frequency as

$$\omega_1 = \frac{1}{RC_2} \quad (14.67)$$

With these values of  $\omega_1$  and  $\omega_2$ , the center frequency, bandwidth, and quality factor are found as follows:

$$\omega_0 = \sqrt{\omega_1 \omega_2} \quad (14.68)$$

$$B = \omega_2 - \omega_1 \quad (14.69)$$

$$Q = \frac{\omega_0}{B} \quad (14.70)$$

To find the passband gain  $K$ , we write Eq. (14.65) in the standard form of Eq. (14.15),

$$\mathbf{H}(\omega) = \frac{-Kj\omega/\omega_1}{(1 + j\omega/\omega_1)(1 + j\omega/\omega_2)} = \frac{-Kj\omega\omega_2}{(\omega_1 + j\omega)(\omega_2 + j\omega)} \quad (14.71)$$

At the center frequency  $\omega_0 = \sqrt{\omega_1 \omega_2}$ , the magnitude of the transfer function is

$$H(\omega_0) = \left| \frac{-Kj\omega_0\omega_2}{(\omega_1 + j\omega_0)(\omega_2 + j\omega_0)} \right| = \frac{K\omega_2}{\omega_1 + \omega_2} \quad (14.72)$$

We set this equal to the gain of the inverting amplifier, as

$$\frac{K\omega_2}{\omega_1 + \omega_2} = \frac{R_f}{R_i} \quad (14.73)$$

from which the gain  $K$  can be determined.

#### 14.8.4 Bandreject (or Notch) Filter

A bandreject filter may be constructed by parallel combination of a low-pass filter and a highpass filter and a summing amplifier, as shown in the block diagram of Fig. 14.46(a). The circuit is designed such that the

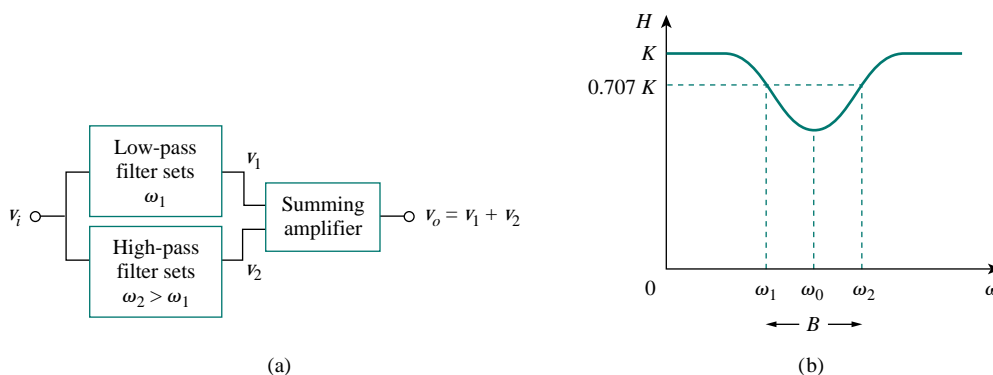


Figure 14.46 Active bandreject filter: (a) block diagram, (b) frequency response.

lower cutoff frequency  $\omega_1$  is set by the lowpass filter while the upper cutoff frequency  $\omega_2$  is set by the highpass filter. The gap between  $\omega_1$  and  $\omega_2$  is the bandwidth of the filter. As shown in Fig. 14.46(b), the filter passes frequencies below  $\omega_1$  and above  $\omega_2$ . The block diagram in Fig. 14.46(a) is actually constructed as shown in Fig. 14.47. The transfer function is

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = -\frac{R_f}{R_i} \left( -\frac{1}{1 + j\omega C_1 R} - \frac{j\omega C_2 R}{1 + j\omega C_2 R} \right) \quad (14.74)$$

The formulas for calculating the values of  $\omega_1$ ,  $\omega_2$ , the center frequency, bandwidth, and quality factor are the same as in Eqs. (14.66) to (14.70).

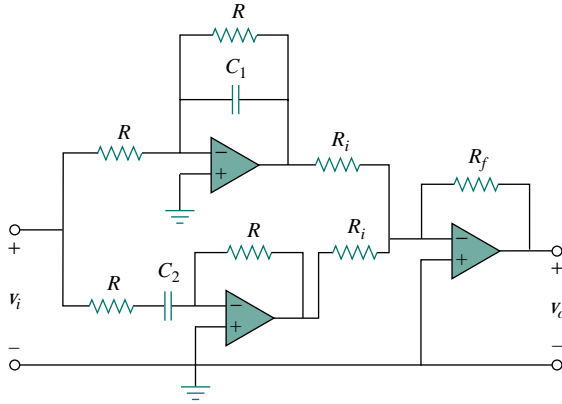


Figure 14.47 Active bandreject filter.

To determine the passband gain  $K$  of the filter, we can write Eq. (14.74) in terms of the upper and lower corner frequencies as

$$\begin{aligned} \mathbf{H}(\omega) &= \frac{R_f}{R_i} \left( \frac{1}{1 + j\omega/\omega_2} + \frac{j\omega/\omega_1}{1 + j\omega/\omega_1} \right) \\ &= \frac{R_f}{R_i} \frac{(1 + j2\omega/\omega_1 + (j\omega)^2/\omega_1\omega_1)}{(1 + j\omega/\omega_2)(1 + j\omega/\omega_1)} \end{aligned} \quad (14.75)$$

Comparing this with the standard form in Eq. (14.15) indicates that in the two passbands ( $\omega \rightarrow 0$  and  $\omega \rightarrow \infty$ ) the gain is

$$K = \frac{R_f}{R_i} \quad (14.76)$$

We can also find the gain at the center frequency by finding the magnitude of the transfer function at  $\omega_0 = \sqrt{\omega_1\omega_2}$ , writing

$$\begin{aligned} H(\omega_0) &= \left| \frac{R_f}{R_i} \frac{(1 + j2\omega_0/\omega_1 + (j\omega_0)^2/\omega_1\omega_1)}{(1 + j\omega_0/\omega_2)(1 + j\omega_0/\omega_1)} \right| \\ &= \frac{R_f}{R_i} \frac{2\omega_1}{\omega_1 + \omega_2} \end{aligned} \quad (14.77)$$

Again, the filters treated in this section are only typical. There are many other active filters that are more complex.

**EXAMPLE 14.12**

Design a low-pass active filter with a dc gain of 4 and a corner frequency of 500 Hz.

**Solution:**

From Eq. (14.61), we find

$$\omega_c = 2\pi f_c = 2\pi(500) = \frac{1}{R_f C_f} \quad (14.12.1)$$

The dc gain is

$$H(0) = -\frac{R_f}{R_i} = -4 \quad (14.12.2)$$

We have two equations and three unknowns. If we select  $C_f = 0.2 \mu\text{F}$ , then

$$R_f = \frac{1}{2\pi(500)0.2 \times 10^{-6}} = 1.59 \text{ k}\Omega$$

and

$$R_i = \frac{R_f}{4} = 397.5 \Omega$$

We use a 1.6-k $\Omega$  resistor for  $R_f$  and a 400- $\Omega$  resistor for  $R_i$ . Figure 14.42 shows the filter.

**PRACTICE PROBLEM 14.12**

Design a highpass filter with a high-frequency gain of 5 and a corner frequency of 2 kHz. Use a 0.1- $\mu\text{F}$  capacitor in your design.

**Answer:**  $R_i = 800 \Omega$  and  $R_f = 4 \text{ k}\Omega$ .

**EXAMPLE 14.13**

Design a bandpass filter in the form of Fig. 14.45 to pass frequencies between 250 Hz and 3000 Hz and with  $K = 10$ . Select  $R = 20 \text{ k}\Omega$ .

**Solution:**

Since  $\omega_1 = 1/RC_2$ , we obtain

$$C_2 = \frac{1}{R\omega_1} = \frac{1}{2\pi f_1 R} = \frac{1}{2\pi \times 250 \times 20 \times 10^3} = 31.83 \text{ nF}$$

Similarly, since  $\omega_2 = 1/RC_1$ ,

$$C_1 = \frac{1}{R\omega_2} = \frac{1}{2\pi f_2 R} = \frac{1}{2\pi \times 3000 \times 20 \times 10^3} = 2.65 \text{ nF}$$

From Eq. (14.73),

$$\frac{R_f}{R_i} = \frac{K\omega_2}{\omega_1 + \omega_2} = \frac{Kf_2}{f_1 + f_2} = 10 \frac{3000}{3250} = 9.223$$

If we select  $R_i = 10 \text{ k}\Omega$ , then  $R_f = 9.223 R_i \simeq 92 \text{ k}\Omega$ .

**PRACTICE PROBLEM 14.13**

Design a notch filter based on Fig. 14.47 for  $\omega_0 = 20$  krad/s,  $K = 5$ , and  $Q = 10$ . Use  $R = R_i = 10$  k $\Omega$ .

**Answer:**  $C_1 = 47.62$  nF,  $C_2 = 52.63$  nF, and  $R_f = 50$  k $\Omega$ .

**†14.9 SCALING**

In designing and analyzing filters and resonant circuits or in circuit analysis in general, it is sometimes convenient to work with element values of 1  $\Omega$ , 1 H, or 1 F, and then transform the values to realistic values by *scaling*. We have taken advantage of this idea by not using realistic element values in most of our examples and problems; mastering circuit analysis is made easy by using convenient component values. We have thus eased calculations, knowing that we could use scaling to then make the values realistic.

There are two ways of scaling a circuit: *magnitude* or *impedance scaling*, and *frequency scaling*. Both are useful in scaling responses and circuit elements to values within the practical ranges. While magnitude scaling leaves the frequency response of a circuit unaltered, frequency scaling shifts the frequency response up or down the frequency spectrum.

**14.9.1 Magnitude Scaling**

**Magnitude scaling** is the process of increasing all impedance in a network by a factor, the frequency response remaining unchanged.

Recall that impedances of individual elements  $R$ ,  $L$ , and  $C$  are given by

$$\mathbf{Z}_R = R, \quad \mathbf{Z}_L = j\omega L, \quad \mathbf{Z}_C = \frac{1}{j\omega C} \quad (14.78)$$

In magnitude scaling, we multiply the impedance of each circuit element by a factor  $K_m$  and let the frequency remain constant. This gives the new impedances as

$$\begin{aligned} \mathbf{Z}'_R &= K_m \mathbf{Z}_R = K_m R, & \mathbf{Z}'_L &= K_m \mathbf{Z}_L = j\omega K_m L \\ \mathbf{Z}'_C &= K_m \mathbf{Z}_C = \frac{1}{j\omega C/K_m} \end{aligned} \quad (14.79)$$

Comparing Eq. (14.79) with Eq. (14.78), we notice the following changes in the element values:  $R \rightarrow K_m R$ ,  $L \rightarrow K_m L$ , and  $C \rightarrow C/K_m$ . Thus, in magnitude scaling, the new values of the elements and frequency are

$$\begin{aligned} R' &= K_m R, & L' &= K_m L \\ C' &= \frac{C}{K_m}, & \omega' &= \omega \end{aligned} \quad (14.80)$$

The primed variables are the new values and the unprimed variables are the old values. Consider the series or parallel  $RLC$  circuit. We now have

$$\omega'_0 = \frac{1}{\sqrt{L'C'}} = \frac{1}{\sqrt{K_m LC/K_m}} = \frac{1}{\sqrt{LC}} = \omega_0 \quad (14.81)$$

showing that the resonant frequency, as expected, has not changed. Similarly, the quality factor and the bandwidth are not affected by magnitude scaling. Also, magnitude scaling does not affect transfer functions in the forms of Eqs. (14.2a) and (14.2b), which are dimensionless quantities.

### 14.9.2 Frequency Scaling

Frequency scaling is equivalent to relabeling the frequency axis of a frequency response plot. It is needed when translating such frequencies such as a resonant frequency, a corner frequency, a bandwidth, etc., to a realistic level. It can be used to bring capacitance and inductance values into a range that is convenient to work with.

Frequency scaling is the process of shifting the frequency response of a network up or down the frequency axis while leaving the impedance the same.

We achieve frequency scaling by multiplying the frequency by a factor  $K_f$  while keeping the impedance the same.

From Eq. (14.78), we see that the impedances of  $L$  and  $C$  are frequency-dependent. If we apply frequency scaling to  $\mathbf{Z}_L(\omega)$  and  $\mathbf{Z}_C(\omega)$  in Eq. (14.78), we obtain

$$\mathbf{Z}_L = j(\omega K_f)L' = j\omega L \quad \Rightarrow \quad L' = \frac{L}{K_f} \quad (14.82a)$$

$$\mathbf{Z}_C = \frac{1}{j(\omega K_f)C'} = \frac{1}{j\omega C} \quad \Rightarrow \quad C' = \frac{C}{K_f} \quad (14.82b)$$

since the impedance of the inductor and capacitor must remain the same after frequency scaling. We notice the following changes in the element values:  $L \rightarrow L/K_f$  and  $C \rightarrow C/K_f$ . The value of  $R$  is not affected, since its impedance does not depend on frequency. Thus, in frequency scaling, the new values of the elements and frequency are

$$\begin{aligned} R' &= R, & L' &= \frac{L}{K_f} \\ C' &= \frac{C}{K_f}, & \omega' &= K_f \omega \end{aligned} \quad (14.83)$$

Again, if we consider the series or parallel  $RLC$  circuit, for the resonant frequency

$$\omega'_0 = \frac{1}{\sqrt{L'C'}} = \frac{1}{\sqrt{(L/K_f)(C/K_f)}} = \frac{K_f}{\sqrt{LC}} = K_f \omega_0 \quad (14.84)$$

and for the bandwidth

$$B' = K_f B \quad (14.85)$$

but the quality factor remains the same ( $Q' = Q$ ).

### 14.9.3 Magnitude and Frequency Scaling

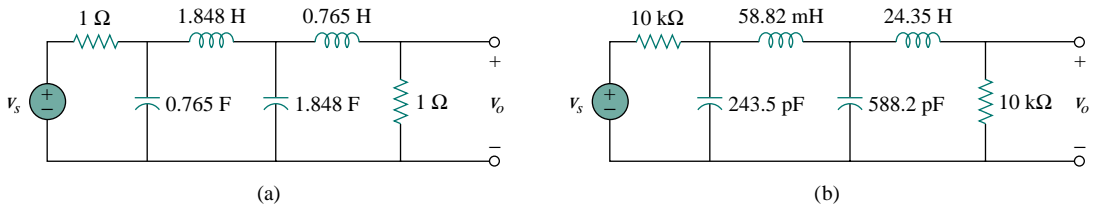
If a circuit is scaled in magnitude and frequency at the same time, then

$$\begin{aligned} R' &= K_m R, & L' &= \frac{K_m}{K_f} L \\ C' &= \frac{1}{K_m K_f} C, & \omega' &= K_f \omega \end{aligned} \quad (14.86)$$

These are more general formulas than those in Eqs. (14.80) and (14.83). We set  $K_m = 1$  in Eq. (14.86) when there is no magnitude scaling or  $K_f = 1$  when there is no frequency scaling.

#### EXAMPLE 14.14

A fourth-order Butterworth lowpass filter is shown in Fig. 14.48(a). The filter is designed such that the cutoff frequency  $\omega_c = 1$  rad/s. Scale the circuit for a cutoff frequency of 50 kHz using 10-k $\Omega$  resistors.



**Figure 14.48** For Example 14.14: (a) Normalized Butterworth lowpass filter, (b) scaled version of the same lowpass filter.

#### Solution:

If the cutoff frequency is to shift from  $\omega_c = 1$  rad/s to  $\omega'_c = 2\pi(50)$  krad/s, then the frequency scale factor is

$$K_f = \frac{\omega'_c}{\omega_c} = \frac{100\pi \times 10^3}{1} = \pi \times 10^5$$

Also, if each 1- $\Omega$  resistor is to be replaced by a 10-k $\Omega$  resistor, then the magnitude scale factor must be

$$K_m = \frac{R'}{R} = \frac{10 \times 10^3}{1} = 10^4$$

Using Eq. (14.86),

$$L'_1 = \frac{K_m}{K_f} L_1 = \frac{10^4}{\pi \times 10^5} (1.848) = 58.82 \text{ mH}$$

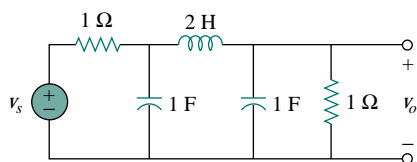
$$L'_2 = \frac{K_m}{K_f} L_2 = \frac{10^4}{\pi \times 10^5} (0.765) = 24.35 \text{ mH}$$

$$C'_1 = \frac{C_1}{K_m K_f} = \frac{0.765}{\pi \times 10^9} = 243.5 \text{ pF}$$

$$C'_2 = \frac{C_2}{K_m K_f} = \frac{1.848}{\pi \times 10^9} = 588.2 \text{ pF}$$

The scaled circuit is as shown in Fig. 14.48(b). This circuit uses practical values and will provide the same transfer function as the prototype in Fig. 14.48(a), but shifted in frequency.

### PRACTICE PROBLEM 14.14



A third-order Butterworth filter normalized to  $\omega_c = 1$  rad/s is shown in Fig. 14.49. Scale the circuit to a cutoff frequency of 10 kHz. Use 15-nF capacitors.

**Answer:**  $R'_1 = R'_2 = 1.061$  k $\Omega$ ,  $C'_1 = C'_2 = 15$  nF,  $L' = 33.77$  mH.

Figure 14.49 For Practice Prob. 14.14.

## 14.10 FREQUENCY RESPONSE USING PSpICE

*PSpice* is a useful tool in the hands of the modern circuit designer for obtaining the frequency response of circuits. The frequency response is obtained using the AC Sweep as discussed in Section D.5 (Appendix D). This requires that we specify in the AC Sweep dialog box *Total Pts*, *Start Freq*, *End Freq*, and the sweep type. *Total Pts* is the number of points in the frequency sweep, and *Start Freq* and *End Freq* are, respectively, the starting and final frequencies, in hertz. In order to know what frequencies to select for *Start Freq* and *End Freq*, one must have an idea of the frequency range of interest by making a rough sketch of the frequency response. In a complex circuit where this may not be possible, one may use a trial-and-error approach.

There are three types of sweeps:

**Linear:** The frequency is varied linearly from *Start Freq* to *End Freq* with *Total* equally spaced points (or responses).

**Octave:** The frequency is swept logarithmically by octaves from *Start Freq* to *End Freq* with *Total* points per octave. An octave is a factor of 2 (e.g., 2 to 4, 4 to 8, 8 to 16).

**Decade:** The frequency is varied logarithmically by decades from *Start Freq* to *End Freq* with *Total* points per decade. A decade is a factor of 10 (e.g., from 2 Hz to 20 Hz, 20 Hz to 200 Hz, 200 Hz to 2 kHz).

It is best to use a linear sweep when displaying a narrow frequency range of interest, as a linear sweep displays the frequency range well in a narrow range. Conversely, it is best to use a logarithmic (octave or decade) sweep for displaying a wide frequency range of interest—if a linear sweep is used for a wide range, all the data will be crowded at the high- or low-frequency end and insufficient data at the other end.

With the above specifications, *PSpice* performs a steady-state sinusoidal analysis of the circuit as the frequency of all the independent sources is varied (or swept) from *Start Freq* to *End Freq*.



The Probe program produces a graphical output. The output data type may be specified in the *Trace Command Box* by adding one of the following suffixes to V or I:

M	Amplitude of the sinusoid.
P	Phase of the sinusoid.
dB	Amplitude of the sinusoid in decibels, i.e., $20 \log_{10}$ (amplitude).

### EXAMPLE 14.15

Determine the frequency response of the circuit shown in Fig. 14.50.

#### Solution:

We let the input voltage  $v_s$  be a sinusoid of amplitude 1 V and phase  $0^\circ$ . Figure 14.51 is the schematic for the circuit. The capacitor is rotated  $270^\circ$  counterclockwise to ensure that pin 1 (the positive terminal) is on top. The voltage marker is inserted to the output voltage across the capacitor. To perform a linear sweep for  $1 < f < 1000$  Hz with 50 points, we select **Analysis/Setup/AC Sweep, DCLICK Linear**, type 50 in the *Total Pts* box, type 1 in the *Start Freq* box, and type 1000 in the *End Freq* box. After saving the file, we select **Analysis/Simulate** to simulate the circuit. If there are no errors, the Probe window will display the plot of V(C1:1), which is the same as  $V_o$  or  $H(\omega) = V_o/1$ , as shown in Fig. 14.52(a). This is the magnitude plot, since V(C1:1) is the same as VM(C1:1). To obtain the phase plot, select **Trace/Add** in the Probe menu and type VP(C1:1) in the **Trace Command** box. Figure 14.52(b) shows the result. Notice that the plots in Fig. 14.52 are similar to those in Fig. 14.3. By hand, the transfer function is

$$H(\omega) = \frac{V_o}{V_s} = \frac{1000}{9000 + j\omega 8}$$

or

$$H(\omega) = \frac{1}{9 + j16\pi \times 10^{-3}}$$

showing that the circuit is a lowpass filter as demonstrated in Fig. 14.52.

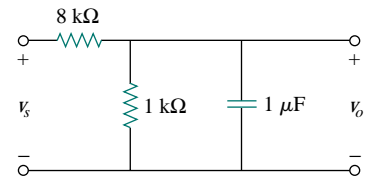


Figure 14.50 For Example 14.15.

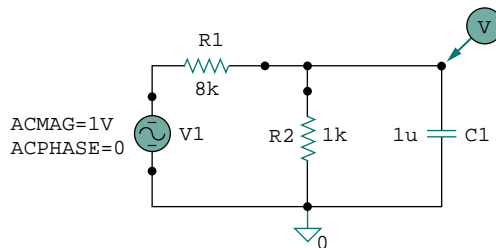


Figure 14.51 The schematic for the circuit in Fig. 14.50.

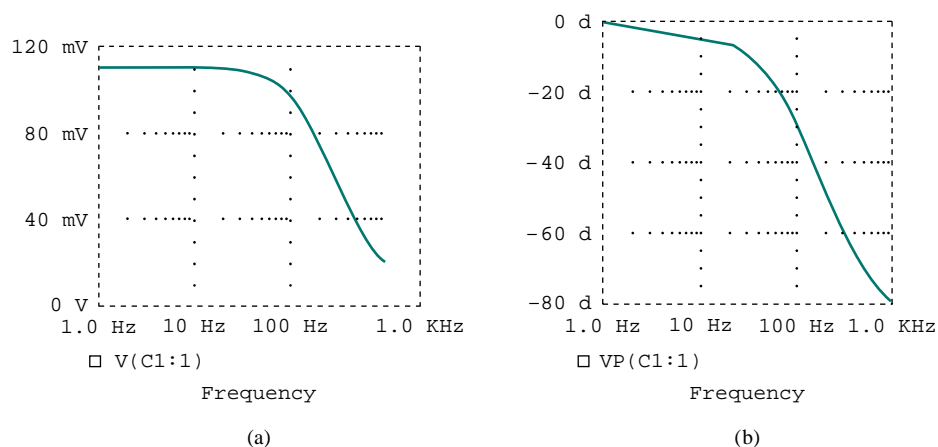
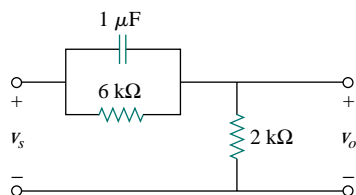


Figure 14.52 For Example 14.15: (a) magnitude plot, (b) phase plot of the frequency response.

### PRACTICE PROBLEM 14.15



Obtain the frequency response of the circuit in Fig. 14.53 using *PSpice*. Use a linear frequency sweep and consider  $1 < f < 1000$  Hz with 100 points.

**Answer:** See Fig. 14.54.

Figure 14.53 For Practice Prob. 14.15.

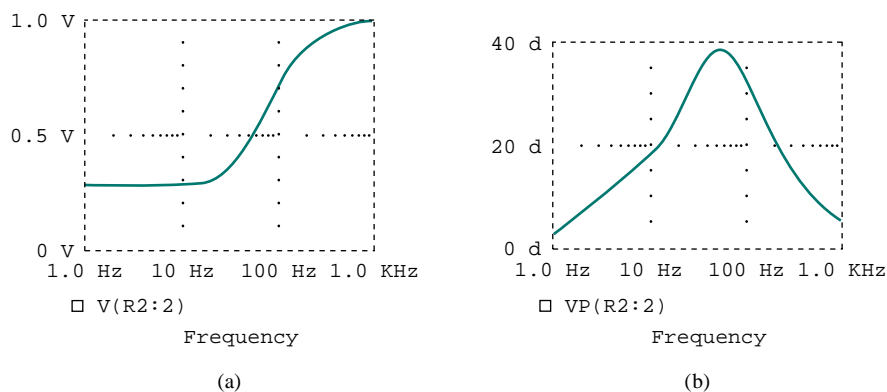


Figure 14.54 For Practice Problem 14.15: (a) magnitude plot, (b) phase plot of the frequency response.

### EXAMPLE 14.16

Use *PSpice* to generate the gain and phase Bode plots of  $V_o$  in the circuit of Fig. 14.55.

**Solution:**

The circuit treated in Example 14.15 is first-order while the one in this example is second-order. Since we are interested in Bode plots, we use decade frequency sweep for  $300 < f < 3000$  Hz with 50 points per decade. We select this range because we know that the resonant frequency of the circuit is within the range. Recall that

$$\omega_0 = \frac{1}{\sqrt{LC}} = 5 \text{ krad/s} \quad \text{or} \quad f_0 = \frac{\omega}{2\pi} = 795.8 \text{ Hz}$$

After drawing the circuit as in Fig. 14.55, we select **Analysis/Setup/AC Sweep, DCLICK Linear**, enter 50 as the *Total Pts* box, 300 as the *Start Freq*, and 3000 as the *End Freq* box. Upon saving the file, we simulate it by selecting **Analysis/Simulate**. This will automatically bring up the Probe window and display V(C1:1) if there are no errors. Since we are interested in the Bode plot, we select **Trace/Add** in the Probe menu and type dB(V(C1:1)) in the **Trace Command** box. The result is the Bode magnitude plot in Fig. 14.56(a). For the phase plot, we select **Trace/Add** in the Probe menu and type VP(C1:1) in the **Trace Command** box. The result is the Bode phase plot of Fig. 14.56(b). Notice that the plots confirm the resonant frequency of 795.8 Hz.

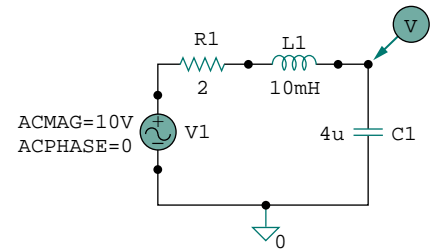


Figure 14.55 For Example 14.16.

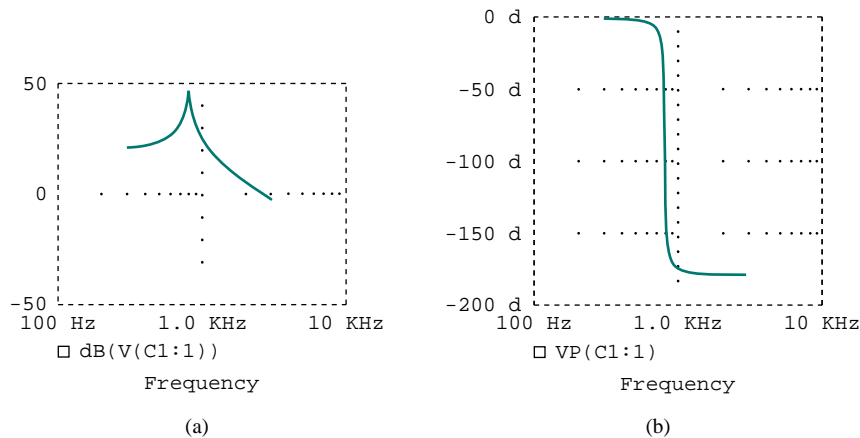


Figure 14.56 For Example 14.16: (a) Bode plot, (b) phase plot of the response.

### PRACTICE PROBLEM 14.16

Consider the network in Fig. 14.57. Use *PSpice* to obtain the Bode plots for  $V_o$  over a frequency from 1 kHz to 100 kHz using 20 points per decade.

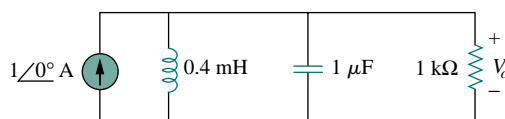


Figure 14.57 For Practice Prob. 14.16.

**Answer:** See Fig. 14.58.

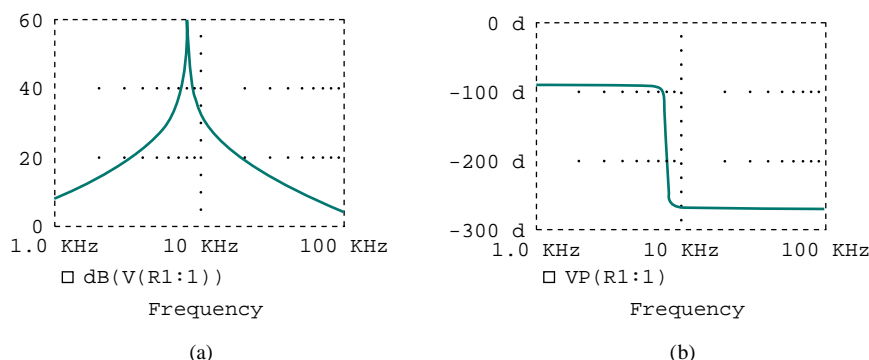


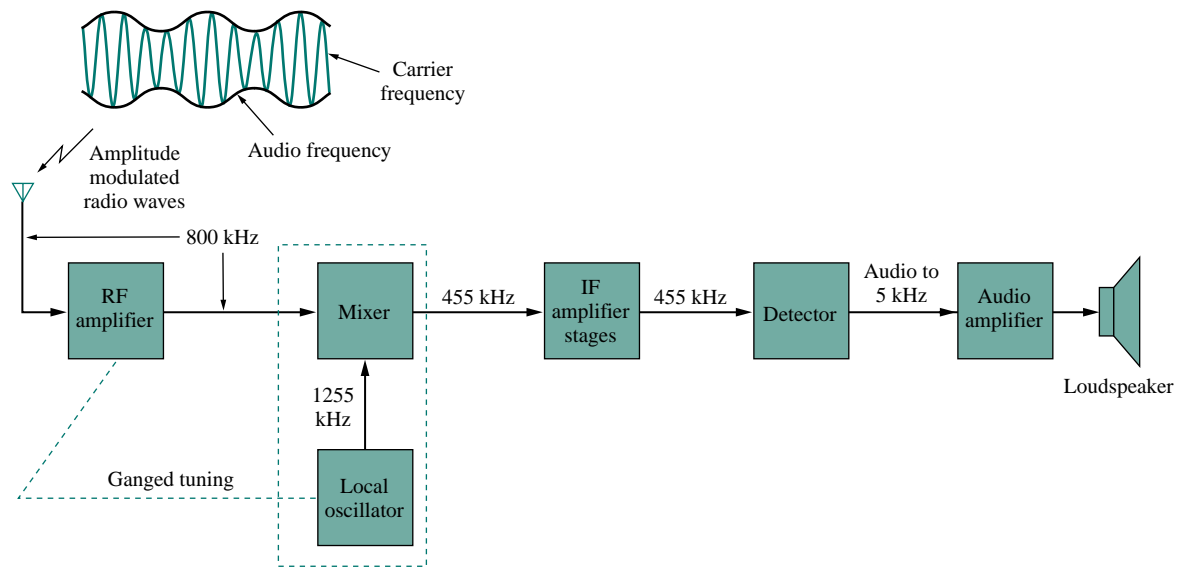
Figure 14.58 For Practice Prob. 14.16: Bode (a) magnitude plot, (b) phase plot.

## †14.11 APPLICATIONS

Resonant circuits and filters are widely used, particularly in electronics, power systems, and communications systems. For example, a Notch filter with cutoff frequency above 60 Hz may be used to eliminate the 60-Hz power line noise in various communications electronics. Filtering of signals in communications systems is necessary in order to select the desired signal from a host of others in the same range (as in the case of radio receivers discussed next) and also to minimize the effects of noise and interference on the desired signal. In this section, we consider one practical application of resonant circuits and two applications of filters. The focus of each application is not to understand the details of how each device works but to see how the circuits considered in this chapter are applied in the practical devices.

### 14.11.1 Radio Receiver

Series and parallel resonant circuits are commonly used in radio and TV receivers to tune in stations and to separate the audio signal from the radio-frequency carrier wave. As an example, consider the block diagram of an AM radio receiver shown in Fig. 14.59. Incoming amplitude-modulated radio waves (thousands of them at different frequencies from different broadcasting stations) are received by the antenna. A resonant circuit (or a bandpass filter) is needed to select just one of the incoming waves. The selected signal is very weak and is amplified in stages in order to generate an audible audio-frequency wave. Thus we have the radio frequency (RF) amplifier to amplify the selected broadcast signal, the intermediate frequency (IF) amplifier to amplify an internally generated signal based on the RF signal, and the audio amplifier to amplify the audio signal just before it reaches the loudspeaker. It is much easier to amplify the signal at three stages than to build an amplifier to provide the same amplification for the entire band.

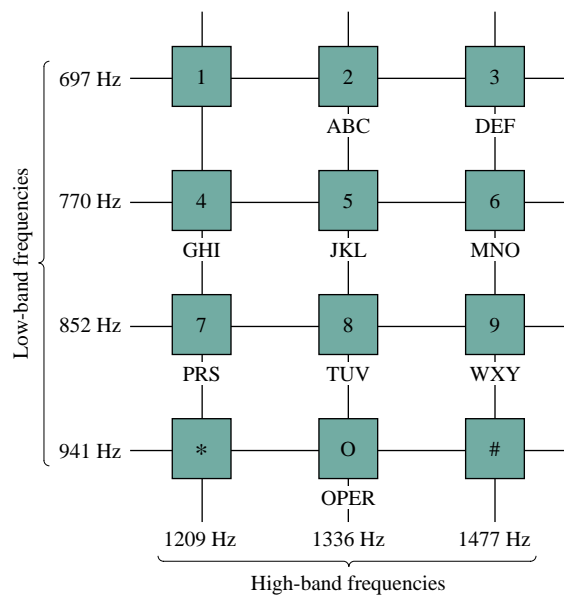


**Figure 14.59** A simplified block diagram of a superheterodyne AM radio receiver.

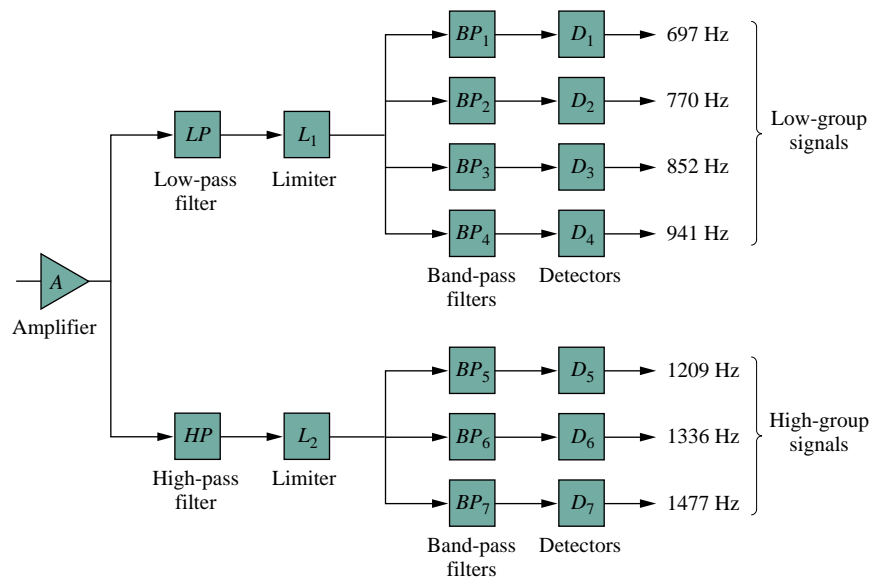
The type of AM receiver shown in Fig. 14.59 is known as the *superheterodyne receiver*. In the early development of radio, each amplification stage had to be tuned to the frequency of the incoming signal. This way, each stage must have several tuned circuits to cover the entire AM band (540 to 1600 kHz). To avoid the problem of having several resonant circuits, modern receivers use a *frequency mixer* or *heterodyne* circuit, which always produces the same IF signal (455 kHz) but retains the audio frequencies carried on the incoming signal. To produce the constant IF frequency, the rotors of two separate variable capacitors are mechanically coupled with one another so that they can be rotated simultaneously with a single control; this is called *ganged tuning*. A *local oscillator* ganged with the RF amplifier produces an RF signal that is combined with the incoming wave by the frequency mixer to produce an output signal that contains the sum and the difference frequencies of the two signals. For example, if the resonant circuit is tuned to receive an 800-kHz incoming signal, the local oscillator must produce a 1255-kHz signal, so that the sum ( $1255 + 800 = 2055$  kHz) and the difference ( $1255 - 800 = 455$  kHz) of frequencies are available at the output of the mixer. However, only the difference, 455 kHz, is used in practice. This is the only frequency to which all the IF amplifier stages are tuned, regardless of the station dialed. The original audio signal (containing the “intelligence”) is extracted in the detector stage. The detector basically removes the IF signal, leaving the audio signal. The audio signal is amplified to drive the loudspeaker, which acts as a transducer converting the electrical signal to sound.

Our major concern here is the tuning circuit for the AM radio receiver. The operation of the FM radio receiver is different from that of the AM receiver discussed here, and in a much different range of frequencies, but the tuning is similar.





**Figure 14.61** Frequency assignments for Touch-Tone dialing.  
(Adapted from G. Daryanani, *Principles of Active Network Synthesis and Design* [New York: John Wiley & Sons], 1976, p. 79.)



**Figure 14.62** Block diagram of detection scheme.  
(Source: G. Daryanani, *Principles of Active Network Synthesis and Design* [New York: John Wiley & Sons], 1976, p. 79.)

diagram for the detection scheme. The signals are first amplified and separated into their respective groups by the lowpass (LP) and highpass (HP) filters. The limiters (L) are used to convert the separated tones into square waves. The individual tones are identified using seven highpass (HP) filters, each filter passing one tone and rejecting other tones. Each filter is followed by a detector (D), which is energized when its input

voltage exceeds a certain level. The outputs of the detectors provide the required dc signals needed by the switching system to connect the caller to the party being called.

### EXAMPLE 14.18

Using the standard  $600\text{-}\Omega$  resistor used in telephone circuits and a series  $RLC$  circuit, design the bandpass filter  $BP_2$  in Fig. 14.62.

#### Solution:

The bandpass filter is the series  $RLC$  circuit in Fig. 14.35. Since  $BP_2$  passes frequencies 697 Hz to 852 Hz and is centered at  $f_0 = 770$  Hz, its bandwidth is

$$B = 2\pi(f_2 - f_1) = 2\pi(852 - 697) = 973.89 \text{ rad/s}$$

From Eq. (14.39),

$$L = \frac{R}{B} = \frac{600}{973.89} = 0.616 \text{ H}$$

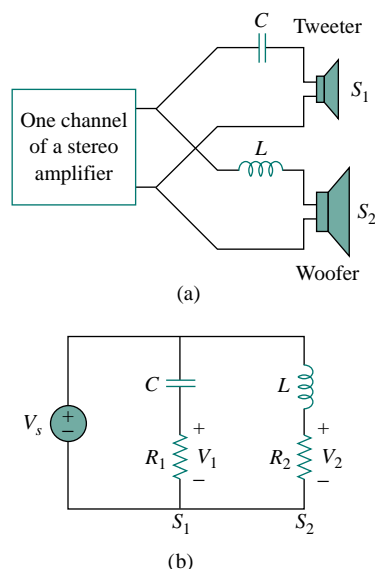
From Eq. (14.27) or (14.57),

$$C = \frac{1}{\omega_0^2 L} = \frac{1}{4\pi^2 f_0^2 L} = \frac{1}{4\pi^2 \times 770^2 \times 0.616} = 69.36 \text{ nF}$$

### PRACTICE PROBLEM 14.18

Repeat Example 14.18 for bandpass filter  $BP_6$ .

**Answer:** 0.356 H, 39.83 nF.



**Figure 14.63** (a) A crossover network for two loudspeakers, (b) equivalent model.

#### 14.11.3 Crossover Network

Another typical application of filters is the *crossover network* that couples an audio amplifier to woofer and tweeter speakers, as shown in Fig. 14.63(a). The network basically consists of one highpass  $RC$  filter and one lowpass  $RL$  filter. It routes frequencies higher than a prescribed crossover frequency  $f_c$  to the tweeter (high-frequency loudspeaker) and frequencies below  $f_c$  into the woofer (low-frequency loudspeaker). These loudspeakers have been designed to accommodate certain frequency responses. A woofer is a low-frequency loudspeaker designed to reproduce the lower part of the frequency range, up to about 3 kHz. A tweeter can reproduce audio frequencies from about 3 kHz to about 20 kHz. The two speaker types can be combined to reproduce the entire audio range of interest and provide the optimum in frequency response.

By replacing the amplifier with a voltage source, the approximate equivalent circuit of the crossover network is shown in Fig. 14.63(b), where the loudspeakers are modeled by resistors. As a highpass filter, the transfer function  $V_1/V_s$  is given by

$$H_1(\omega) = \frac{V_1}{V_s} = \frac{j\omega R_1 C}{1 + j\omega R_1 C} \quad (14.87)$$

Similarly, the transfer function of the lowpass filter is given by

$$H_2(\omega) = \frac{V_2}{V_s} = \frac{R_2}{R_2 + j\omega L} \quad (14.88)$$



The values of  $R_1$ ,  $R_2$ ,  $L$ , and  $C$  may be selected such that the two filters have the same cutoff frequency, known as the *crossover frequency*, as shown in Fig. 14.64.

The principle behind the crossover network is also used in the resonant circuit for a TV receiver, where it is necessary to separate the video and audio bands of RF carrier frequencies. The lower-frequency band (picture information in the range from about 30 Hz to about 4 MHz) is channeled into the receiver's video amplifier, while the high-frequency band (sound information around 4.5 MHz) is channeled to the receiver's sound amplifier.

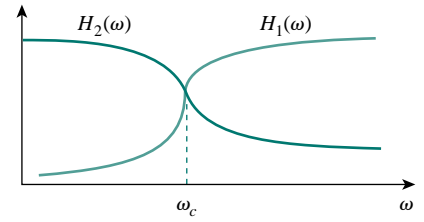


Figure 14.64 Frequency responses of the crossover network in Fig. 14.63.

### EXAMPLE 14.19

In the crossover network of Fig. 14.63, suppose each speaker acts as a  $6\text{-}\Omega$  resistance. Find  $C$  and  $L$  if the crossover frequency is  $2.5\text{ kHz}$ .

**Solution:**

For the highpass filter,

$$\omega_c = 2\pi f_c = \frac{1}{R_1 C}$$

or

$$C = \frac{1}{2\pi f_c R_1} = \frac{1}{2\pi \times 2.5 \times 10^3 \times 6} = 10.61\text{ }\mu\text{F}$$

For the lowpass filter,

$$\omega_c = 2\pi f_c = \frac{R_2}{L}$$

or

$$L = \frac{R_2}{2\pi f_c} = \frac{6}{2\pi \times 2.5 \times 10^3} = 382\text{ }\mu\text{H}$$

### PRACTICE PROBLEM 14.19

If each speaker in Fig. 14.63 has an  $8\text{-}\Omega$  resistance and  $C = 10\text{ }\mu\text{F}$ , find  $L$  and the crossover frequency.

**Answer:**  $0.64\text{ mH}$ ,  $1.989\text{ kHz}$ .

## 14.12 SUMMARY

1. The transfer function  $\mathbf{H}(\omega)$  is the ratio of the output response  $\mathbf{Y}(\omega)$  to the input excitation  $\mathbf{X}(\omega)$ ; that is,  $\mathbf{H}(\omega) = \mathbf{Y}(\omega)/\mathbf{X}(\omega)$ .
2. The frequency response is the variation of the transfer function with frequency.
3. Zeros of a transfer function  $\mathbf{H}(s)$  are the values of  $s = j\omega$  that make  $H(s) = 0$ , while poles are the values of  $s$  that make  $H(s) \rightarrow \infty$ .
4. The decibel is the unit of logarithmic gain. For a gain  $G$ , its decibel equivalent is  $G_{\text{dB}} = 20 \log_{10} G$ .

5. Bode plots are semilog plots of the magnitude and phase of the transfer function as it varies with frequency. The straight-line approximations of  $H$  (in dB) and  $\phi$  (in degrees) are constructed using the corner frequencies defined by the poles and zeros of  $\mathbf{H}(\omega)$ .
6. The resonant frequency is that frequency at which the imaginary part of a transfer function vanishes. For series and parallel  $RLC$  circuits,

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

7. The half-power frequencies ( $\omega_1, \omega_2$ ) are those frequencies at which the power dissipated is one-half of that dissipated at the resonant frequency. The geometric mean between the half-power frequencies is the resonant frequency, or

$$\omega_0 = \sqrt{\omega_1 \omega_2}$$

8. The bandwidth is the frequency band between half-power frequencies:

$$B = \omega_2 - \omega_1$$

9. The quality factor is a measure of the sharpness of the resonance peak. It is the ratio of the resonant (angular) frequency to the bandwidth,

$$Q = \frac{\omega_0}{B}$$

10. A filter is a circuit designed to pass a band of frequencies and reject others. Passive filters are constructed with resistors, capacitors, and inductors. Active filters are constructed with resistors, capacitors, and an active device, usually an op amp.
11. Four common types of filters are lowpass, highpass, bandpass, and bandstop. A lowpass filter passes only signals whose frequencies are below the cutoff frequency  $\omega_c$ . A highpass filter passes only signals whose frequencies are above the cutoff frequency  $\omega_c$ . A bandpass filter passes only signals whose frequencies are within a prescribed range ( $\omega_1 < \omega < \omega_2$ ). A bandstop filter passes only signals whose frequencies are outside a prescribed range ( $\omega_1 > \omega > \omega_2$ ).
12. Scaling is the process whereby unrealistic element values are magnitude-scaled by a factor  $K_m$  and/or frequency-scaled by a factor  $K_f$  to produce realistic values.

$$R' = K_m R, \quad L' = \frac{K_m}{K_f} L, \quad C' = \frac{1}{K_m K_f} C$$

13. *PSpice* can be used to obtain the frequency response of a circuit if a frequency range for the response and the desired number of points within the range are specified in the AC Sweep.
14. The radio receiver—one practical application of resonant circuits—employs a bandpass resonant circuit to tune in one frequency among all the broadcast signals picked up by the antenna.

15. The Touch-Tone telephone and the crossover network are two typical applications of filters. The Touch-Tone telephone system employs filters to separate tones of different frequencies to activate electronic switches. The crossover network separates signals in different frequency ranges so that they can be delivered to different devices such as tweeters and woofers in a loudspeaker system.

## REVIEW QUESTIONS

- 14.1 A zero of the transfer function

$$H(s) = \frac{10(s+1)}{(s+2)(s+3)}$$

is at

- (a) 10      (b)  $-1$       (c)  $-2$       (d)  $-3$
- 14.2 On the Bode magnitude plot, the slope of the pole  $1/(5+j\omega)^2$  is
- (a) 20 dB/decade      (b) 40 dB/decade  
(c)  $-40$  dB/decade      (d)  $-20$  dB/decade
- 14.3 On the Bode phase plot, the slope of  $[1 + j10\omega - \omega^2/25]^2$  is
- (a)  $45^\circ$ /decade      (b)  $90^\circ$ /decade  
(c)  $135^\circ$ /decade      (d)  $180^\circ$ /decade
- 14.4 How much inductance is needed to resonate at 5 kHz with a capacitance of 12 nF?
- (a) 2652 H      (b) 11.844 H  
(c) 3.333 H      (d) 84.43 mH
- 14.5 The difference between the half-power frequencies is called the:
- (a) quality factor      (b) resonant frequency  
(c) bandwidth      (d) cutoff frequency
- 14.6 In a series  $RLC$  circuit, which of these quality factors has the steepest curve at resonance?
- (a)  $Q = 20$       (b)  $Q = 12$   
(c)  $Q = 8$       (d)  $Q = 4$

- 14.7 In a parallel  $RLC$  circuit, the bandwidth  $B$  is directly proportional to  $R$ .

(a) True      (b) False

- 14.8 When the elements of an  $RLC$  circuit are both magnitude-scaled and frequency-scaled, which quality is unaffected?

(a) resistor      (b) resonant frequency  
(c) bandwidth      (d) quality factor

- 14.9 What kind of filter can be used to select a signal of one particular radio station?

(a) lowpass      (b) highpass  
(c) bandpass      (d) bandstop

- 14.10 A voltage source supplies a signal of constant amplitude, from 0 to 40 kHz, to an RC lowpass filter. The load resistor experiences the maximum voltage at:

(a) dc      (b) 10 kHz  
(c) 20 kHz      (d) 40 kHz

Answers: 14.1b, 14.2c, 14.3d, 14.4d, 14.5c, 14.6a, 14.7b, 14.8d, 14.9c, 14.10a.

## PROBLEMS

### Section 14.2 Transfer Function

- 14.1 Find the transfer function  $\mathbf{V}_o/\mathbf{V}_i$  of the  $RC$  circuit in Fig. 14.65.

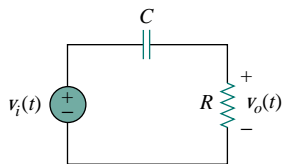


Figure 14.65 For Prob. 14.1.

- 14.2 Obtain the transfer function  $\mathbf{V}_o/\mathbf{V}_i$  of the  $RL$  circuit of Fig. 14.66.

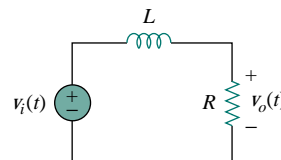


Figure 14.66 For Probs. 14.2 and 14.36.

- 14.3** (a) Given the circuit in Fig. 14.67, determine the transfer function  $\mathbf{H}(s) = \mathbf{V}_o/\mathbf{V}_i$ .  
 (b) If  $R = 40 \text{ k}\Omega$  and  $C = 2 \mu\text{F}$ , specify the locations of the poles and zeros of  $\mathbf{H}(s)$ .

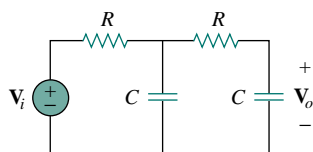
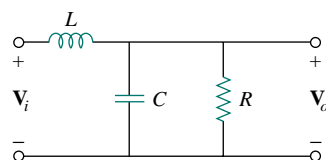
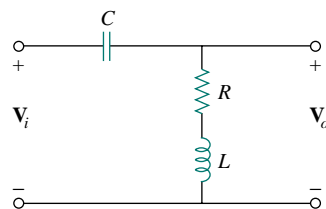


Figure 14.67 For Prob. 14.3.

- 14.4** Find the transfer function  $\mathbf{H}(j\omega) = \mathbf{V}_o/\mathbf{V}_i$  of the circuits shown in Fig. 14.68.



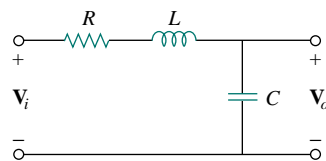
(a)



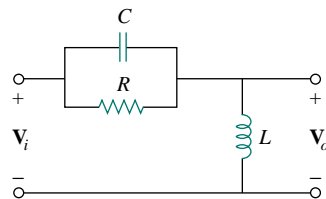
(b)

Figure 14.68 For Prob. 14.4.

- 14.5** Repeat Prob. 14.4 for the circuits in Fig. 14.69.



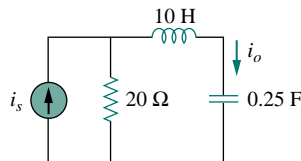
(a)



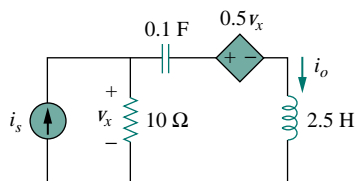
(b)

Figure 14.69 For Prob. 14.5.

- 14.6** Obtain the transfer function  $\mathbf{H}(j\omega) = \mathbf{I}_o/\mathbf{I}_s$  of the circuits shown in Fig. 14.70.



(a)



(b)

Figure 14.70 For Prob. 14.6.

### Section 14.3 The Decibel Scale

- 14.7** Calculate  $|\mathbf{H}(j\omega)|$  if  $H_{\text{dB}}$  equals  
 (a) 0.05 dB (b)  $-6.2 \text{ dB}$  (c) 104.7 dB
- 14.8** Determine the magnitude (in dB) and the phase (in degrees) of  $\mathbf{H}(j\omega)$  at  $\omega = 1$  if  $\mathbf{H}(j\omega)$  equals  
 (a) 0.05 (b) 125  
 (c)  $\frac{10j\omega}{2 + j\omega}$  (d)  $\frac{3}{1 + j\omega} + \frac{6}{2 + j\omega}$

### Section 14.4 Bode Plots

- 14.9** A ladder network has a voltage gain of



$$\mathbf{H}(j\omega) = \frac{10}{(1 + j\omega)(10 + j\omega)}$$

Sketch the Bode plots for the gain.

- 14.10** Sketch the Bode plots for

$$\mathbf{H}(j\omega) = \frac{10 + j\omega}{j\omega(2 + j\omega)}$$

- 14.11** Construct the Bode plots for

$$G(s) = \frac{s + 1}{s^2(s + 10)}, \quad s = j\omega$$

- 14.12** Draw the Bode plots for

$$\mathbf{H}(j\omega) = \frac{50(j\omega + 1)}{j\omega(-\omega^2 + 10j\omega + 25)}$$

- 14.13** Construct the Bode magnitude and phase plots for



$$H(s) = \frac{40(s + 1)}{(s + 2)(s + 10)}, \quad s = j\omega$$

- 14.14** Sketch the Bode plots for

$$G(s) = \frac{s}{(s + 2)^2(s + 1)}, \quad s = j\omega$$

- 14.15** Draw Bode plots for

$$G(s) = \frac{(s+2)^2}{s(s+5)(s+10)}, \quad s = j\omega$$

- 14.16** A filter has

$$H(s) = \frac{s}{s^2 + 10s + 100}$$

Sketch the filter's Bode magnitude and phase plots.

- 14.17** Sketch Bode magnitude and phase plots for



$$N(s) = \frac{100(s^2 + s + 1)}{(s+1)(s+10)}, \quad s = j\omega$$

Construct the straight-line approximate plots and the exact plots.

- 14.18** Construct Bode plots for

$$\mathbf{T}(\omega) = \frac{10j\omega(1+j\omega)}{(10+j\omega)(100+10j\omega-\omega^2)}$$

- 14.19** Find the transfer function  $\mathbf{H}(\omega)$  with the Bode magnitude plot shown in Fig. 14.71.

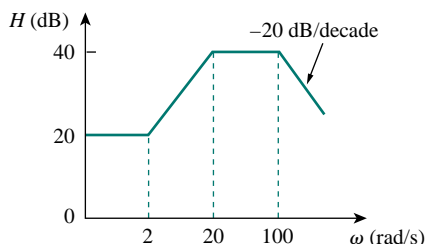


Figure 14.71 For Prob. 14.19.

- 14.20** The Bode magnitude plot of  $\mathbf{H}(\omega)$  is shown in Fig. 14.72. Find  $\mathbf{H}(\omega)$ .

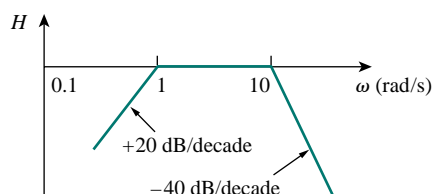


Figure 14.72 For Prob. 14.20.

- 14.21** The Bode phase plot of  $\mathbf{G}(\omega)$  of a network is depicted in Fig. 14.73. Find  $\mathbf{G}(\omega)$ .

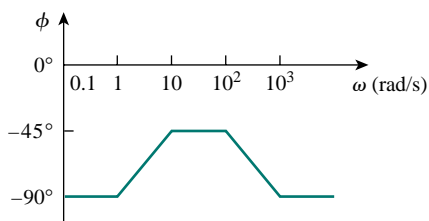


Figure 14.73 For Prob. 14.21.

## Section 14.5 Series Resonance

- 14.22** A series  $RLC$  network has  $R = 2 \text{ k}\Omega$ ,  $L = 40 \text{ mH}$ , and  $C = 1 \text{ }\mu\text{F}$ . Calculate the impedance at resonance and at one-fourth, one-half, twice, and four times the resonant frequency.

- 14.23** Design a series  $RLC$  circuit that will have an impedance of  $10 \text{ }\Omega$  at the resonant frequency of  $\omega_0 = 50 \text{ rad/s}$  and a quality factor of 80. Find the bandwidth.

- 14.24** Design a series  $RLC$  circuit with  $B = 20 \text{ rad/s}$  and  $\omega_0 = 1000 \text{ rad/s}$ . Find the circuit's  $Q$ .

- 14.25** For the circuit in Fig. 14.74, find the frequency  $\omega$  for which  $v(t)$  and  $i(t)$  are in phase.

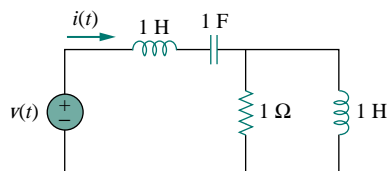


Figure 14.74 For Prob. 14.25.

## Section 14.6 Parallel Resonance

- 14.26** Design a parallel resonant  $RLC$  circuit with  $\omega_0 = 10 \text{ rad/s}$  and  $Q = 20$ . Calculate the bandwidth of the circuit.

- 14.27** A parallel resonant circuit with quality factor 120 has a resonant frequency of  $6 \times 10^6 \text{ rad/s}$ . Calculate the bandwidth and half-power frequencies.

- 14.28** It is expected that a parallel  $RLC$  resonant circuit has a midband admittance of  $25 \times 10^3 \text{ S}$ , quality factor of 80, and a resonant frequency of  $200 \text{ krad/s}$ . Calculate the values of  $R$ ,  $L$ , and  $C$ . Find the bandwidth and the half-power frequencies.

- 14.29** Rework Prob. 14.22 if the elements are connected in parallel.

- 14.30** For the “tank” circuit in Fig. 14.75, find the resonant frequency.

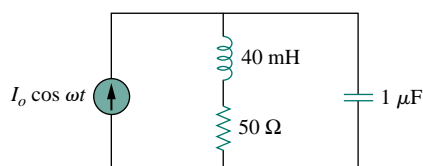


Figure 14.75 For Probs. 14.30 and 14.71.

- 14.31** For the circuits in Fig. 14.76, find the resonant frequency  $\omega_0$ , the quality factor  $Q$ , and the bandwidth  $B$ .

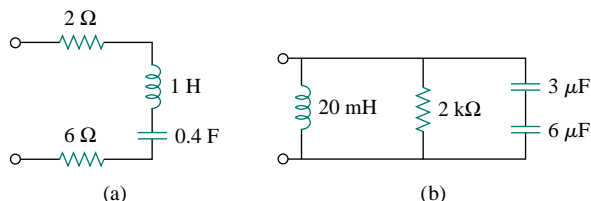


Figure 14.76 For Prob. 14.31.

- 14.32** Calculate the resonant frequency of each of the circuits in Fig. 14.77.

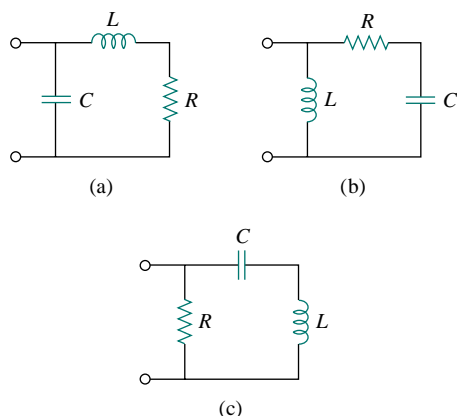


Figure 14.77 For Prob. 14.32.

- \*14.33** For the circuit in Fig. 14.78, find:



- (a) the resonant frequency  $\omega_0$   
(b)  $\mathbf{Z}_{in}(\omega)$

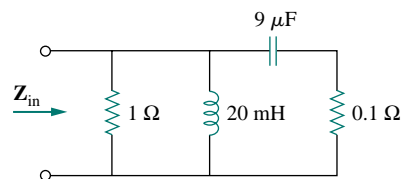


Figure 14.78 For Prob. 14.33.

- 14.34** In the circuit of Fig. 14.79,  $i(t) = 10 \sin t$ . Calculate the value of  $C$  such that  $v(t) = V_o \sin t$  V. Find  $V_o$ .

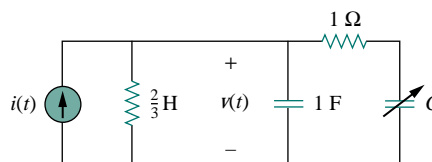


Figure 14.79 For Prob. 14.34.

- 14.35** For the network illustrated in Fig. 14.80, find  
(a) the transfer function  $\mathbf{H}(\omega) = \mathbf{V}_o(\omega)/\mathbf{I}(\omega)$ ,  
(b) the magnitude of  $\mathbf{H}$  at  $\omega_0 = 1$  rad/s.

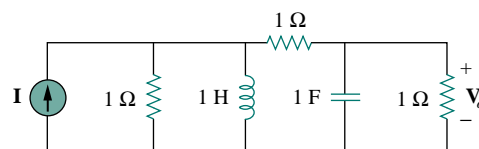


Figure 14.80 For Probs. 14.35, 14.61, and 14.72.

## Section 14.7 Passive Filters

- 14.36** Show that the circuit in Fig. 14.66 is a lowpass filter. Calculate the corner frequency  $f_c$  if  $L = 2$  mH and  $R = 10$  k $\Omega$ .

- 14.37** Find the transfer function  $\mathbf{V}_o/\mathbf{V}_s$  of the circuit in Fig. 14.81. Show that the circuit is a lowpass filter.

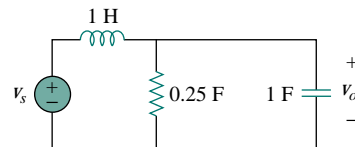


Figure 14.81 For Prob. 14.37.

- 14.38** Determine the cutoff frequency of the lowpass filter described by

$$\mathbf{H}(\omega) = \frac{4}{2 + j\omega 10}$$

Find the gain in dB and phase of  $\mathbf{H}(\omega)$  at  $\omega = 2$  rad/s.

- 14.39** Determine what type of filter is in Fig. 14.82. Calculate the corner frequency  $f_c$ .

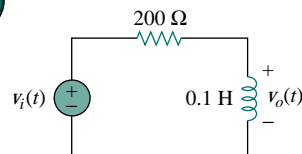


Figure 14.82 For Prob. 14.39.

\*An asterisk indicates a challenging problem.

- 14.40** Obtain the transfer function of a highpass filter with a passband gain of 10 and a cutoff frequency of 50 rad/s.
- 14.41** In a highpass  $RL$  filter with a cutoff frequency of 100 kHz,  $L = 40$  mH. Find  $R$ .
- 14.42** Design a series  $RLC$  type bandpass filter with cutoff frequencies of 10 kHz and 11 kHz. Assuming  $C = 80$  pF, find  $R$ ,  $L$ , and  $Q$ .
- 14.43** Determine the range of frequencies that will be passed by a series  $RLC$  bandpass filter with  $R = 10\ \Omega$ ,  $L = 25$  mH, and  $C = 0.4\ \mu\text{F}$ . Find the quality factor.
- 14.44** (a) Show that for a bandpass filter,

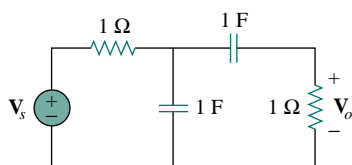
$$\mathbf{H}(s) = \frac{sB}{s^2 + sB + \omega_0^2}$$

where  $B$  = bandwidth of the filter and  $\omega_0$  is the center frequency.

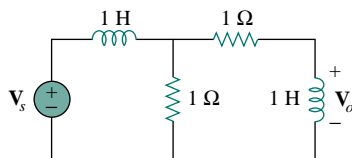
- (b) Similarly, show that for a bandstop filter,

$$\mathbf{H}(s) = \frac{s^2 + \omega_0^2}{s^2 + sB + \omega_0^2}$$

- 14.45** Determine the center frequency and bandwidth of the bandpass filters in Fig. 14.83.



(a)



(b)

Figure 14.83 For Prob. 14.45.

- 14.46** The circuit parameters for a series  $RLC$  bandstop filter are  $R = 2\ \text{k}\Omega$ ,  $L = 0.1$  H,  $C = 40$  pF. Calculate:
- the center frequency
  - the half-power frequencies
  - the quality factor
- 14.47** Find the bandwidth and center frequency of the bandstop filter of Fig. 14.84.

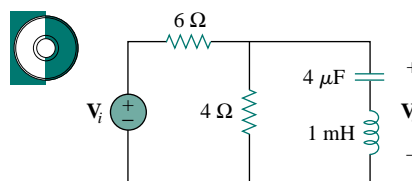
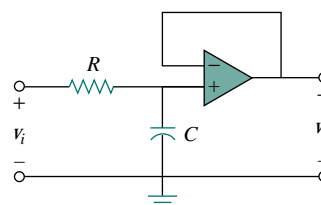


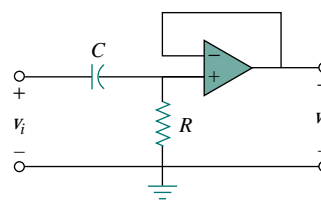
Figure 14.84 For Prob. 14.47.

## Section 14.8 Active Filters

- 14.48** Find the transfer function for each of the active filters in Fig. 14.85.



(a)



(b)

Figure 14.85 For Probs. 14.48 and 14.49.

- 14.49** The filter in Fig. 14.85(b) has a 3-dB cutoff frequency at 1 kHz. If its input is connected to a 120-mV variable frequency signal, find the output voltage at:
- 200 Hz
  - 2 kHz
  - 10 kHz
- 14.50** Obtain the transfer function of the active filter in Fig. 14.86. What kind of filter is it?

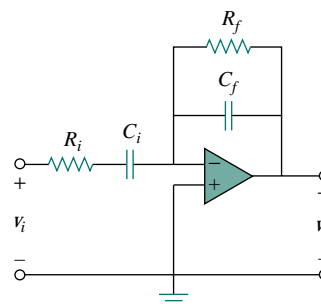


Figure 14.86 For Prob. 14.50.

- 14.51** A highpass filter is shown in Fig. 14.87. Show that the transfer function is

$$\mathbf{H}(\omega) = \left(1 + \frac{R_f}{R_i}\right) \frac{j\omega RC}{1 + j\omega RC}$$

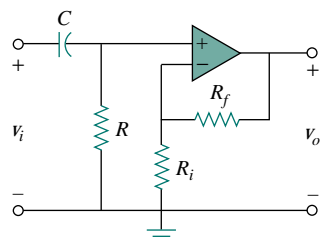


Figure 14.87 For Prob. 14.51.

- 14.52** A “general” first-order filter is shown in Fig. 14.88. (a) Show that the transfer function is

$$\mathbf{H}(s) = \frac{R_4}{R_3 + R_4} \times \frac{s + (1/R_1 C)[R_1/R_2 - R_3/R_4]}{s + 1/R_2 C},$$

$$s = j\omega$$

- (b) What condition must be satisfied for the circuit to operate as a highpass filter?  
 (c) What condition must be satisfied for the circuit to operate as a lowpass filter?

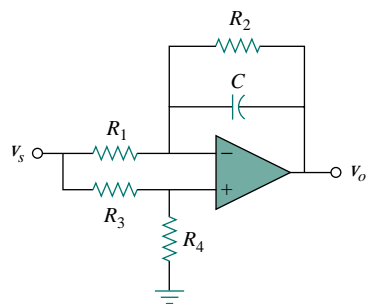


Figure 14.88 For Prob. 14.52.

- 14.53** Design an active lowpass filter with dc gain of 0.25 and a corner frequency of 500 Hz.  
**14.54** Design an active highpass filter with a high-frequency gain of 5 and a corner frequency of 200 Hz.  
**14.55** Design the filter in Fig. 14.89 to meet the following requirements:  
 (a) It must attenuate a signal at 2 kHz by 3 dB compared with its value at 10 MHz.

- (b) It must provide a steady-state output of  $v_o(t) = 10 \sin(2\pi \times 10^8 t + 180^\circ)$  V for an input  $v_s(t) = 4 \sin(2\pi \times 10^8 t)$  V.

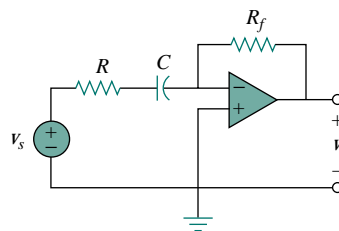


Figure 14.89 For Prob. 14.55.

- \*14.56** A second-order active filter known as a Butterworth filter is shown in Fig. 14.90.

- (a) Find the transfer function  $\mathbf{V}_o/\mathbf{V}_i$ .  
 (b) Show that it is a lowpass filter.

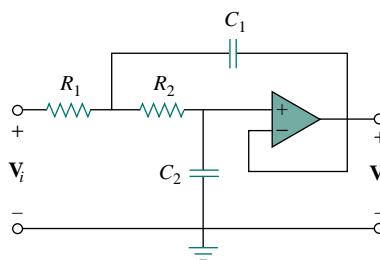


Figure 14.90 For Prob. 14.56.

## Section 14.9 Scaling

- 14.57** Use magnitude and frequency scaling on the circuit of Fig. 14.75 to obtain an equivalent circuit in which the inductor and capacitor have magnitude 1 H and 1 C respectively.  
**14.58** What values of  $K_m$  and  $K_f$  will scale a 4-mH inductor and a 20-μF capacitor to 1 H and 2 F respectively?  
**14.59** Calculate the values of  $R$ ,  $L$ , and  $C$  that will result in  $R = 12$  kΩ,  $L = 40$  μH, and  $C = 300$  nF respectively when magnitude-scaled by 800 and frequency-scaled by 1000.  
**14.60** A series  $RLC$  circuit has  $R = 10$  Ω,  $\omega_0 = 40$  rad/s, and  $B = 5$  rad/s. Find  $L$  and  $C$  when the circuit is scaled:  
 (a) in magnitude by a factor of 600,  
 (b) in frequency by a factor of 1000,  
 (c) in magnitude by a factor of 400 and in frequency by a factor of  $10^5$ .  
**14.61** Redesign the circuit in Fig. 14.80 so that all resistive elements are scaled by a factor of 1000 and all



frequency-sensitive elements are frequency-scaled by a factor of  $10^4$ .

- \*14.62** Refer to the network in Fig. 14.91.



- (a) Find  $Z_{in}(s)$ .  
 (b) Scale the elements by  $K_m = 10$  and  $K_f = 100$ . Find  $Z_{in}(s)$  and  $\omega_0$ .

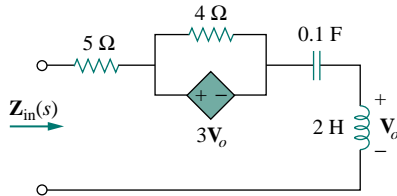


Figure 14.91 For Prob. 14.62.

- 14.63** (a) For the circuit in Fig. 14.92, draw the new circuit after it has been scaled by  $K_m = 200$  and  $K_f = 10^4$ .  
 (b) Obtain the Thevenin equivalent impedance at terminals  $a$ - $b$  of the scaled circuit at  $\omega = 10^4$  rad/s.

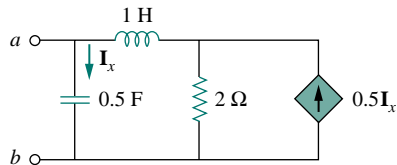


Figure 14.92 For Prob. 14.63.

- 14.64** Scale the lowpass active filter in Fig. 14.93 so that its corner frequency increases from 1 rad/s to 200 rad/s. Use a 1- $\mu$ F capacitor.

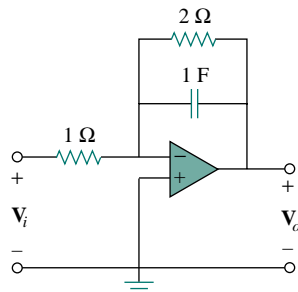


Figure 14.93 For Prob. 14.64.

## Section 14.10 Frequency Response Using PSpice

- 14.65** Obtain the frequency response of the circuit in Fig. 14.94 using PSpice.

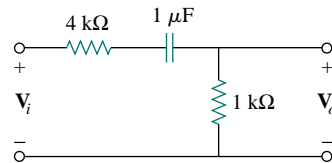


Figure 14.94 For Prob. 14.65.

- 14.66** Use PSpice to provide the frequency response (magnitude and phase of  $i$ ) of the circuit in Fig. 14.95. Use linear frequency sweep from 1 to 10,000 Hz.

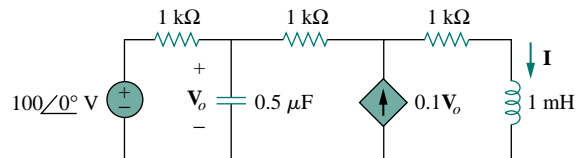


Figure 14.95 For Prob. 14.66.

- 14.67** In the interval  $0.1 < f < 100$  Hz, plot the response of the network in Fig. 14.96. Classify this filter and obtain  $\omega_0$ .

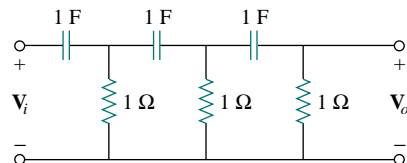


Figure 14.96 For Prob. 14.67.

- 14.68** Use PSpice to generate the magnitude and phase Bode plots of  $V_o$  in the circuit of Fig. 14.97.

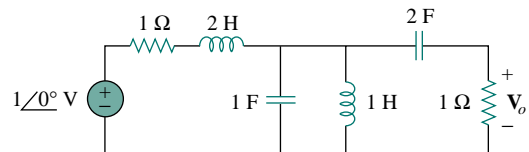


Figure 14.97 For Prob. 14.68.

- 14.69** Obtain the magnitude plot of the response  $V_o$  in the network of Fig. 14.98 for the frequency interval  $100 < f < 1000$  Hz.

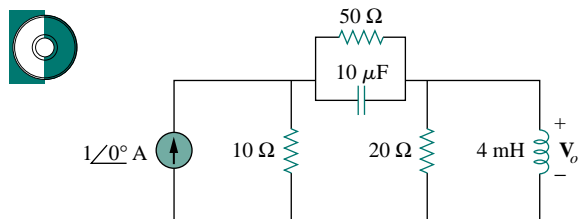


Figure 14.98 For Prob. 14.69.

- 14.70** Obtain the frequency response of the circuit in Fig. 14.40 (see Practice Problem 14.10). Take  $R_1 = R_2 = 100\ \Omega$ ,  $L = 2\text{ mH}$ . Use  $1 < f < 100,000\text{ Hz}$ .
- 14.71** For the “tank” circuit in Fig. 14.75, obtain the frequency response (voltage across the capacitor) using *PSpice*. Determine the resonant frequency of the circuit.
- 14.72** Using *PSpice*, plot the magnitude of the frequency response of the circuit in Fig. 14.80.

### Section 14.11 Applications

- 14.73** The resonant circuit for a radio broadcast consists of a 120-pF capacitor in parallel with a 240-μH inductor. If the inductor has an internal resistance of 400 Ω, what is the resonant frequency of the circuit? What would be the resonant frequency if the inductor resistance were reduced to 40 Ω?
- 14.74** A series-tuned antenna circuit consists of a variable capacitor (40 pF to 360 pF) and a 240-μH antenna coil which has a dc resistance of 12 Ω.
- Find the frequency range of radio signals to which the radio is tunable.
  - Determine the value of  $Q$  at each end of the frequency range.

- 14.75** The crossover circuit in Fig. 14.99 is a lowpass filter that is connected to a woofer. Find the transfer function  $\mathbf{H}(\omega) = \mathbf{V}_o(\omega)/\mathbf{V}_i(\omega)$ .

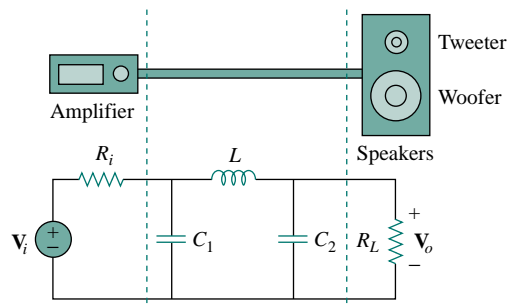


Figure 14.99 For Prob. 14.75.

- 14.76** The crossover circuit in Fig. 14.100 is a highpass filter that is connected to a tweeter. Determine the transfer function  $\mathbf{H}(\omega) = \mathbf{V}_o(\omega)/\mathbf{V}_i(\omega)$ .

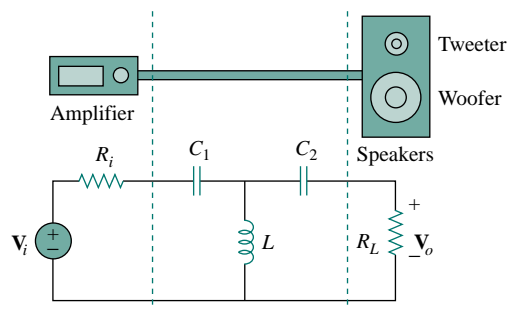


Figure 14.100 For Prob. 14.76.

## COMPREHENSIVE PROBLEMS

- 14.77** A certain electronic test circuit produced a resonant curve with half-power points at 432 Hz and 454 Hz. If  $Q = 20$ , what is the resonant frequency of the circuit?
- 14.78** In an electronic device, a series circuit is employed that has a resistance of 100 Ω, a capacitive reactance of 5 kΩ, and an inductive reactance of 300 Ω when used at 2 MHz. Find the resonant frequency and bandwidth of the circuit.
- 14.79** In a certain application, a simple  $RC$  lowpass filter is designed to reduce high frequency noise. If the desired corner frequency is 20 kHz and  $C = 0.5\ \mu\text{F}$ , find the value of  $R$ .
- 14.80** In an amplifier circuit, a simple  $RC$  highpass filter is needed to block the dc component while passing the time-varying component. If the desired rolloff frequency is 15 Hz and  $C = 10\ \mu\text{F}$ , find the value of  $R$ .
- 14.81** Practical  $RC$  filter design should allow for source and load resistances as shown in Fig. 14.101. Let  $R = 4\text{ k}\Omega$  and  $C = 40\text{ nF}$ . Obtain the cutoff frequency when:
- $R_s = 0$ ,  $R_L = \infty$ ,
  - $R_s = 1\text{ k}\Omega$ ,  $R_L = 5\text{ k}\Omega$ .

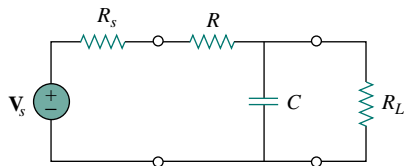


Figure 14.101 For Prob. 14.81.

- 14.82** The  $RC$  circuit in Fig. 14.102 is used for a lead compensator in a system design. Obtain the transfer function of the circuit.

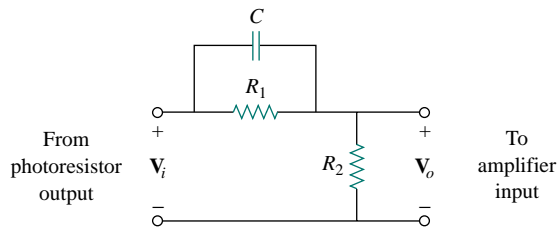


Figure 14.102 For Prob. 14.82.

- 14.83** A low-quality factor, double-tuned bandpass filter is shown in Fig. 14.103. Use *PSpice* to generate the magnitude plot of  $V_o(\omega)$ .

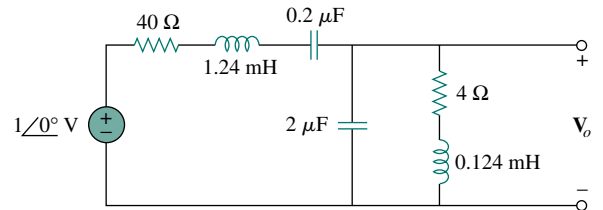


Figure 14.103 For Prob. 14.83.