

## CHAPTER 4 - CIRCUIT THEOREMS

List of topics for this chapter :

- Linearity Property
- Principle of Superposition
- Source Transformations
- Thevenin's Theorem
- Norton's Theorem
- Maximum Power Transfer
- Verifying Circuit Theorems with PSpice

### LINEARITY PROPERTY

Linearity is the condition in which the change in value of one quantity is directly proportional to that of another quantity. A linear circuit is one whose output is linearly related (or directly proportional) to its input.

**Problem 4.1** If all the independent sources are multiplied by a value,  $K$ , and all the currents and voltages of the circuit increase by the same value, then a circuit is linear. Show that the circuit in Figure 4.1 is linear by solving for  $I_x$  and  $V_x$ .

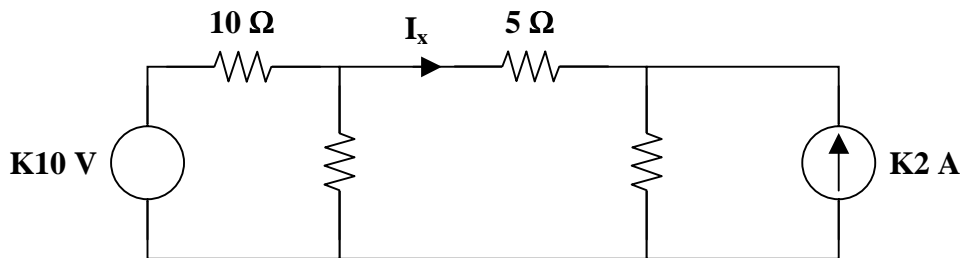
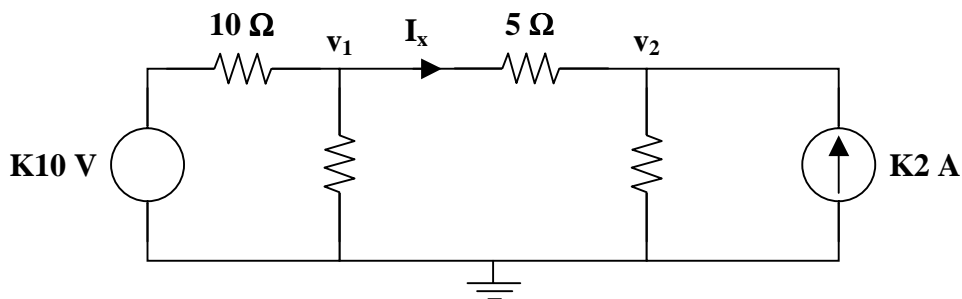


Figure 4.1



Use nodal analysis to find  $v_1$  and  $v_2$ .

At node 1 :

$$\frac{v_1 - K10}{10} + \frac{v_1 - 0}{10} + \frac{v_1 - v_2}{5} = 0$$

At node 2 :

$$\frac{v_2 - v_1}{5} + \frac{v_2 - 0}{5} = K2$$

Simplifying,

$$\begin{aligned} v_1 - K10 + v_1 + (2)(v_1 - v_2) &= 0 \\ 4v_1 - 2v_2 &= K10 \end{aligned}$$

$$\begin{aligned} v_2 - v_1 + v_2 &= K10 \\ -v_1 + 2v_2 &= K10 \end{aligned}$$

The system of simultaneous equations is

$$\begin{bmatrix} 4 & -2 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} K10 \\ K10 \end{bmatrix}$$

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \frac{1}{8-2} \begin{bmatrix} 2 & 2 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} K10 \\ K10 \end{bmatrix} = \frac{1}{6} \begin{bmatrix} K20 + K20 \\ K10 + K40 \end{bmatrix} = \begin{bmatrix} (40/6) K \\ (50/6) K \end{bmatrix} = \begin{bmatrix} (20/3) K \\ (25/3) K \end{bmatrix}$$

Clearly,  $V_x = v_1 - v_2 = (20/3) K - (25/3) K = (-5/3) K$

and  $I_x = V_x / 5 = [(-5/3) K](1/5) = (-1/3) K$

The voltage source and current source are multiples of K. The voltage  $V_x$  and the current  $I_x$  are multiples of K. Therefore, **the circuit is linear**.

#### Problem 4.2 [4.3]

- In the circuit in Figure 4.1, calculate  $v_o$  and  $i_o$  when  $v_s = 1 \text{ V}$ .
- Find  $v_o$  and  $i_o$  when  $v_s = 10 \text{ V}$ .
- What are  $v_o$  and  $i_o$  when each of the  $1\text{-}\Omega$  resistors is replaced by a  $10\text{-}\Omega$  resistor and  $v_s = 10 \text{ V}$ ?

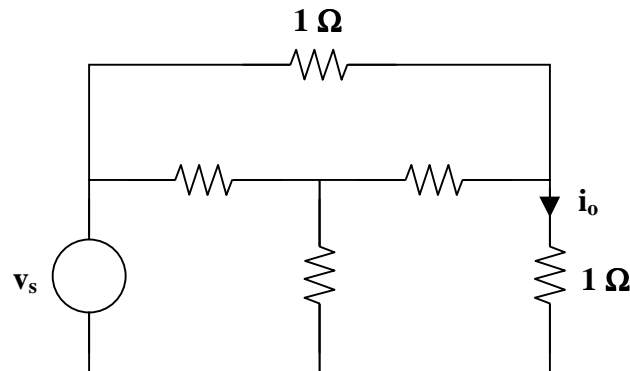
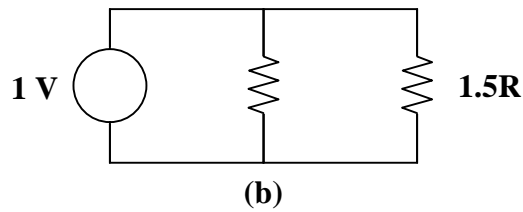
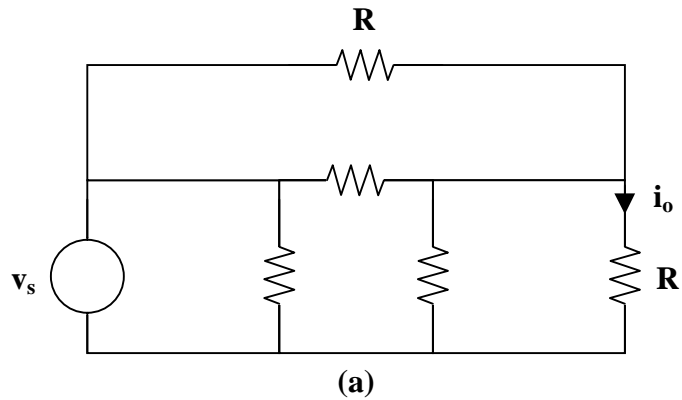


Figure 4.1

First, transform the Y (or T) subcircuit to its  $\Delta$  (or  $\Pi$ ) equivalent.



$$R \parallel 3R = \frac{3R^2}{4R} = \frac{3}{4}R$$

$$\frac{3}{4}R + \frac{3}{4}R = \frac{3}{2}R$$

$$v_o = v_s/2, \quad \text{independent of } R$$

and

$$i_o = v_o/R$$

(a) When  $v_s = 1 \text{ V}$  and  $R = 1 \Omega$ ,

$$v_o = \underline{\underline{0.5 \text{ V}}} \quad i_o = \underline{\underline{0.5 \text{ A}}}$$

(b) When  $v_s = 10 \text{ V}$  and  $R = 1 \Omega$ ,

$$v_o = \underline{\underline{5 \text{ V}}} \quad i_o = \underline{\underline{5 \text{ A}}}$$

(c) When  $v_s = 10 \text{ V}$  and  $R = 10 \Omega$ ,

$$v_o = \underline{\underline{5 \text{ V}}} \quad i_o = \underline{\underline{0.5 \text{ A}}}$$

## PRINCIPLE OF SUPERPOSITION

The superposition principle states that the voltage across (or current through) an element in a linear circuit is the algebraic sum of the voltages across (or currents through) that element due to each independent source acting alone.

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**Problem 4.3** Solve for  $I_x$  in Figure 4.1 using superposition.

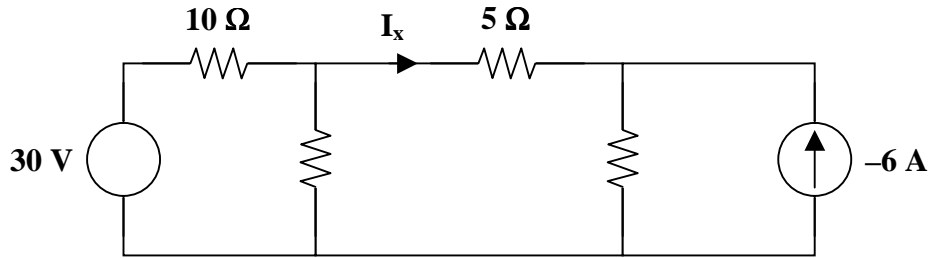


Figure 4.1

➤ **Carefully DEFINE the problem.**

Each component is labeled completely. The problem is clear.

➤ **PRESENT everything you know about the problem.**

Using the principle of superposition, we will need to find the desired current when the current source is turned off, or set equal to zero. This implies the replacement of the current source with an open circuit. Let this value be equal to  $I_x'$ . We will also need to find the desired current when the voltage source is turned off. This implies the replacement of the voltage source with a short circuit. Let this value be equal to  $I_x''$ . Then, the desired current is the sum of these two currents, i.e.,

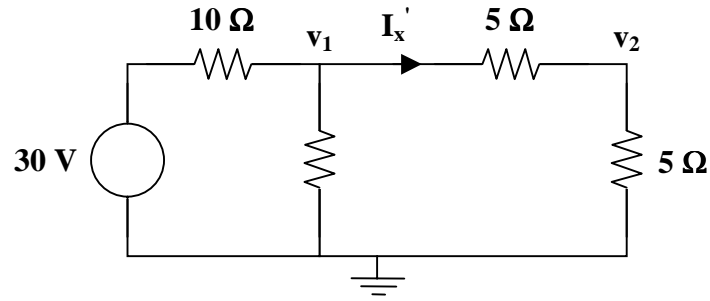
$$I_x = I_x' + I_x''$$

➤ **Establish a set of ALTERNATIVE solutions and determine the one that promises the greatest likelihood of success.**

This circuit, having two independent sources, can be analyzed using nodal or mesh analysis, as seen in Chapter 3. The problem statement requires the use of superposition. The principle of superposition can be very useful, especially when there are several independent sources. More work may be required to find the desired voltage or current, but the analysis is performed using simpler circuits.

➤ **ATTEMPT a problem solution.**

Setting the current source to zero, the circuit becomes



Use nodal analysis to find  $v_1$  and  $v_2$ .

At node 1 :

$$\frac{v_1 - 30}{10} + \frac{v_1 - 0}{10} + \frac{v_1 - v_2}{5} = 0$$

At node 2 :

$$\frac{v_2 - v_1}{5} + \frac{v_2 - 0}{5} = 0$$

Simplifying,

$$v_1 - 30 + v_1 + (2)(v_1 - v_2) = 0$$

$$v_2 - v_1 + v_2 = 0$$

$$4v_1 - 2v_2 = 30$$

$$-v_1 + 2v_2 = 0$$

$$2v_1 - v_2 = 15$$

The system of simultaneous equations is

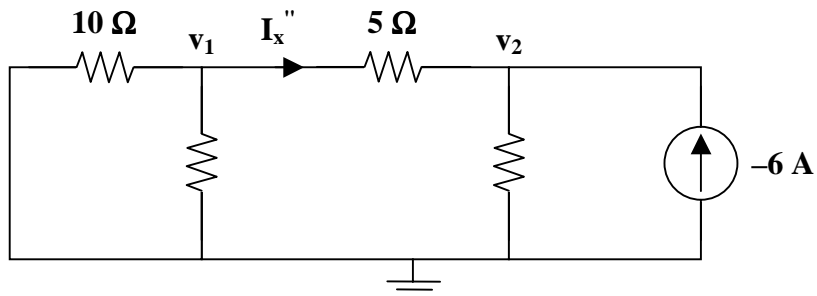
$$\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 15 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \frac{1}{4-1} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 15 \\ 0 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 30 \\ 15 \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$$

Clearly,  $V_x' = v_1 - v_2 = 10 - 5 = 5$  volts.

and  $I_x' = \frac{V_x'}{5} = \frac{5}{5} = 1$  amp.

Setting the voltage source to zero, the circuit becomes



Use nodal analysis to find  $v_1$  and  $v_2$ .

At node 1 :

$$\frac{v_1 - 0}{10} + \frac{v_1 - 0}{10} + \frac{v_1 - v_2}{5} = 0$$

At node 2 :

$$\frac{v_2 - v_1}{5} + \frac{v_2 - 0}{5} = -6$$

Simplifying,

$$v_1 + v_1 + (2)(v_1 - v_2) = 0$$

$$v_2 - v_1 + v_2 = -30$$

$$4v_1 - 2v_2 = 0$$

$$-v_1 + 2v_2 = -30$$

$$2v_1 - v_2 = 0$$

The system of simultaneous equations is

$$\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ -30 \end{bmatrix}$$

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \frac{1}{4-1} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ -30 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} -30 \\ -60 \end{bmatrix} = \begin{bmatrix} -10 \\ -20 \end{bmatrix}$$

Clearly,  $V_x'' = v_1 - v_2 = -10 - (-20) = 10$  volts.

and  $I_x'' = \frac{V_x''}{5} = \frac{10}{5} = 2$  amps.

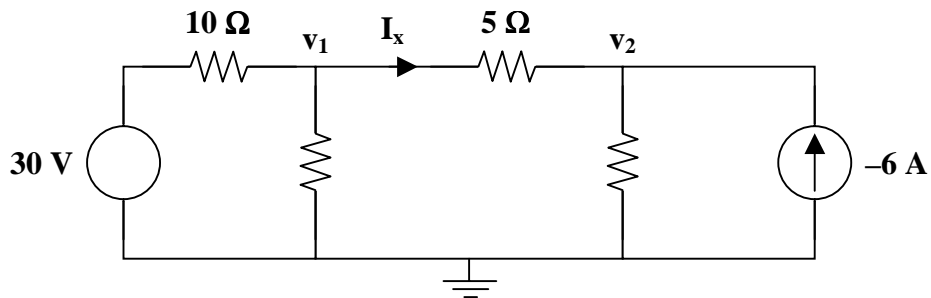
The sum of the currents,  $I_x'$  and  $I_x''$ , due to the two independent sources acting alone is the current,  $I_x$ , of the circuit due to both sources.

$$I_x = I_x' + I_x''$$

$$I_x = 1 + 2 = 3 \text{ amps.}$$

➤ **EVALUATE the solution and check for accuracy.**

Find  $I_x$  using both sources and nodal analysis. The circuit is as follows.



At node 1 :

$$\frac{v_1 - 30}{10} + \frac{v_1 - 0}{10} + \frac{v_1 - v_2}{5} = 0$$

At node 2 :

$$\frac{v_2 - v_1}{5} + \frac{v_2 - 0}{5} = -6$$

Simplifying,

$$v_1 - 30 + v_1 + (2)(v_1 - v_2) = 0$$

$$4v_1 - 2v_2 = 30$$

$$2v_1 - v_2 = 15$$

$$v_2 - v_1 + v_2 = -30$$

$$-v_1 + 2v_2 = -30$$

The system of simultaneous equations is

$$\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 15 \\ -30 \end{bmatrix}$$

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \frac{1}{4-1} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 15 \\ -30 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 30-30 \\ 15-60 \end{bmatrix} = \begin{bmatrix} 0 \\ -15 \end{bmatrix}$$

Clearly,  $V_x = v_1 - v_2 = 0 - (-15) = 15$  volts.

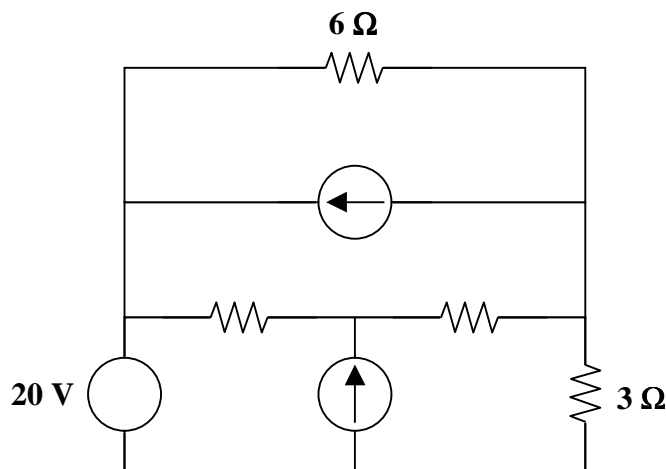
and  $I_x = \frac{V_x}{5} = \frac{15}{5} = 3$  amps.

This answer is the same as the answer obtained using the principle of superposition. Our check for accuracy was successful.

- **Has the problem been solved SATISFACTORILY? If so, present the solution; if not, then return to “ALTERNATIVE solutions” and continue through the process again.**  
This problem has been solved satisfactorily.

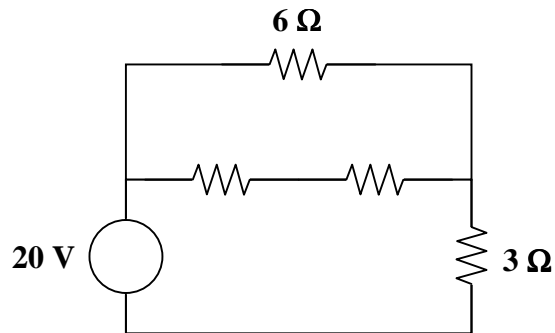
$$I_x = \underline{3 \text{ A}}$$

**Problem 4.4** [4.11] Apply the superposition principle to find  $v_o$  in the circuit of Figure 4.1.



**Figure 4.1**

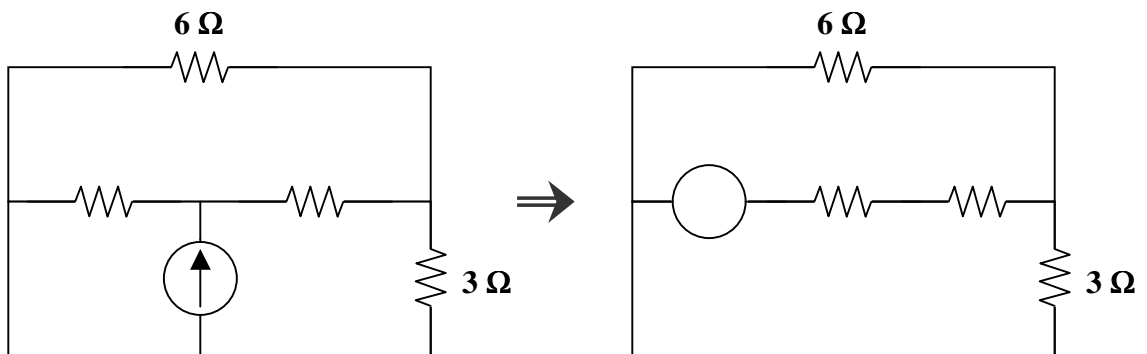
Let  $v_o = v_{o1} + v_{o2} + v_{o3}$ , where  $v_{o1}$ ,  $v_{o2}$ , and  $v_{o3}$  are due to the 20-V, 1-A, and 2-A sources, respectively. For  $v_{o1}$ , consider the circuit below.



$$6 \parallel (4 + 2) = 3 \, \Omega$$

$$v_{o1} = (1/2)(20) = 10 \, \text{V}$$

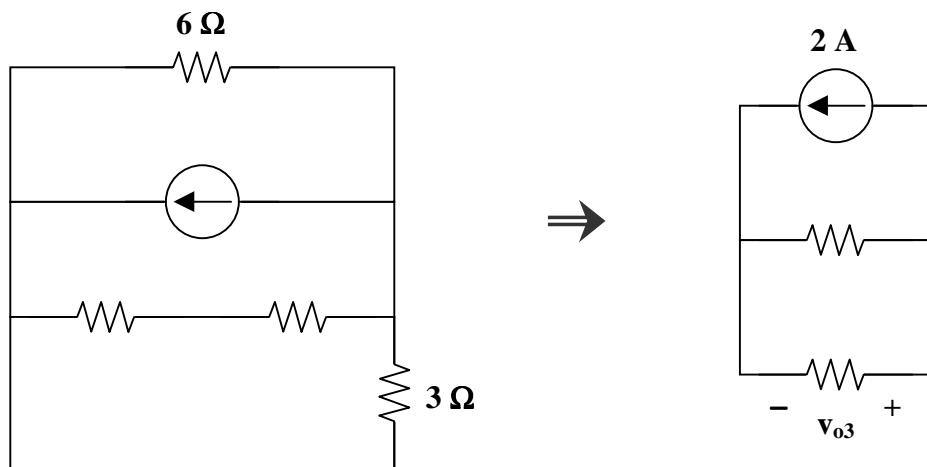
For  $v_{o2}$ , consider the circuit below.



$$3 \parallel 6 = 2 \, \Omega$$

$$v_{o2} = \left( \frac{2}{4 + 2 + 2} \right) (4) = 1 \, \text{V}$$

For  $v_{o3}$ , consider the circuit below.





$$6 \parallel (4 + 2) = 3 \, \Omega$$

$$v_{o3} = (-1)(3) = -3$$

Thus,

$$v_o = 10 + 1 - 3 = \underline{\underline{8 \, \text{V}}}$$

**Problem 4.5** A linear circuit has the following independent sources:  $V_1$ ,  $V_2$ , and  $I_s$  along with a current,  $I_R$ , through a resistor. It cannot be assumed that these are the only elements (resistors and dependent sources) in the circuit. Complete the following table.

Circuit number	$V_1$ (volts)	$V_2$ (volts)	$I_s$ (amps)	$I_R$ (amps)
Condition #1	10	0	0	2
Condition #2	0	5	0	-1
Condition #3	0	0	3	1
Condition #4	10	5	3	
Condition #5	10	-20	3	
Condition #6				6

This problem is based on linearity and the principle of superposition.

From the Conditions #1, #2, and #3, we know

- $I_R = 2$  amps when the only contributing independent source is  $V_1 = 10$  volts.
- $I_R = -1$  amp when the only contributing independent source is  $V_2 = 5$  volts.
- $I_R = 1$  amp when the only contributing independent source is  $I_s = 3$  amps.

Condition #4 :

$$I_R = \left(\frac{10}{10}\right)(2) + \left(\frac{5}{5}\right)(-1) + \left(\frac{3}{3}\right)(1) = 2 - 1 + 1 = \underline{\underline{2 \, \text{A}}}$$

Condition #5 :

$$I_R = \left(\frac{10}{10}\right)(2) + \left(\frac{-20}{5}\right)(-1) + \left(\frac{3}{3}\right)(1) = 2 + 4 + 1 = \underline{\underline{7 \, \text{A}}}$$

Condition #6 : The following are only three of an infinite set of solutions.

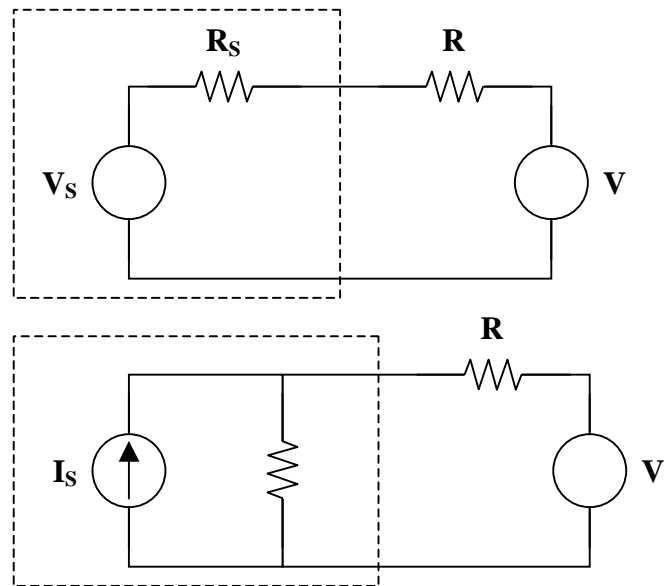
If only one source is not equal to zero

then	$V_1 = ?$	$V_2 = \underline{0}$	$I_s = \underline{0}$	$I_R = 6$	$6 = \left(\frac{V_1}{10}\right)(2)$	$V_1 = \underline{\underline{30}}$
or	$V_1 = \underline{0}$	$V_2 = ?$	$I_s = \underline{0}$	$I_R = 6$	$6 = \left(\frac{V_2}{5}\right)(-1)$	$V_2 = \underline{\underline{-30}}$
or	$V_1 = \underline{0}$	$V_2 = \underline{0}$	$I_s = ?$	$I_R = 6$	$6 = \left(\frac{I_s}{3}\right)(1)$	$I_s = \underline{\underline{18}}$

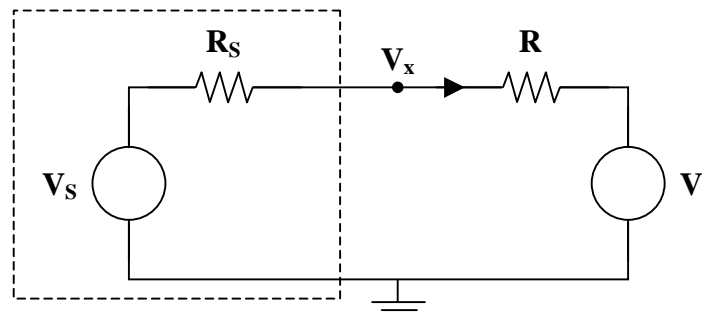
## SOURCE TRANSFORMATIONS

A source transformation is the process of replacing a voltage source  $V_s$  in series with a resistor  $R$  with a current source  $I_s$  in parallel with a resistor  $R$  or vice versa.

**Problem 4.6** Given that the following circuits are linear, prove that the two circuits are equivalent, where  $R_s = V_s / I_s$ .



Two circuits are said to be equivalent if they have the same voltage-current relationship at their terminals. Begin by finding  $V_x$  and  $I_x$  in the following circuit.



Clearly,

$$I_x = \frac{V_s - V_x}{R_s}$$

or

$$V_x = V_s - R_s I_x$$

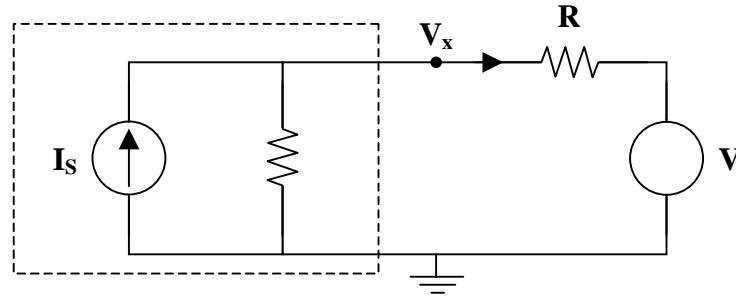
Using KVL,

$$-V_s + (R_s + R)I_x + V = 0$$

or

$$I_x = \frac{V_s - V}{R_s + R}$$

Now, find  $V_x$  and  $I_x$  in this circuit.



Using KCL,

$$I_s = I_x + \frac{V_x}{R_s}$$

or

$$V_x = R_s I_s - R_s I_x$$

We know that  $R_s = \frac{V_s}{I_s}$ . So,

$$V_x = V_s - R_s I_x.$$

Using mesh analysis,

$$\begin{aligned} R I_x + V + R_s (I_x - I_s) \\ (R_s + R)I_x = R_s I_s - V \\ I_x = \frac{R_s I_s - V}{R_s + R} \end{aligned}$$

We know that  $R_s = \frac{V_s}{I_s}$ . So,

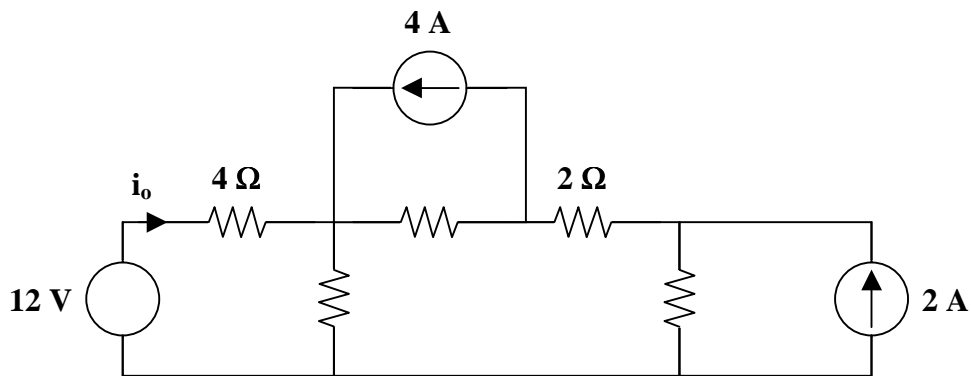
$$I_x = \frac{V_s - V}{R_s + R}.$$

In both circuits,  $V_x = V_s - R_s I_x$  and  $I_x = \frac{V_s - V}{R_s + R}$ .

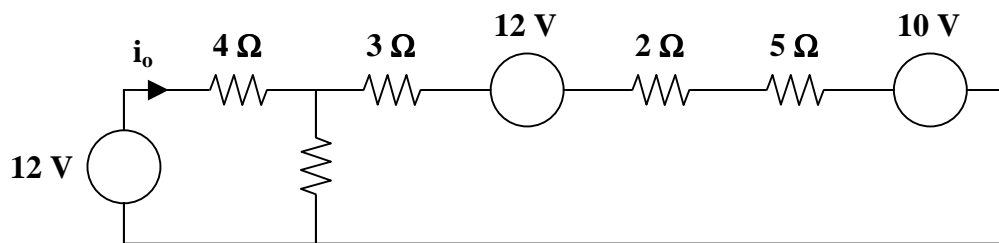
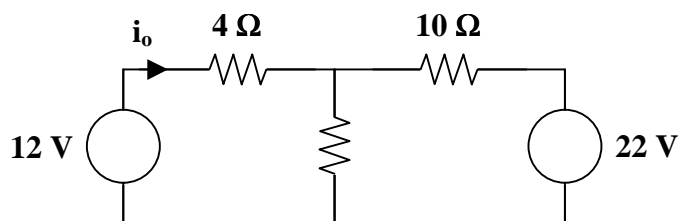
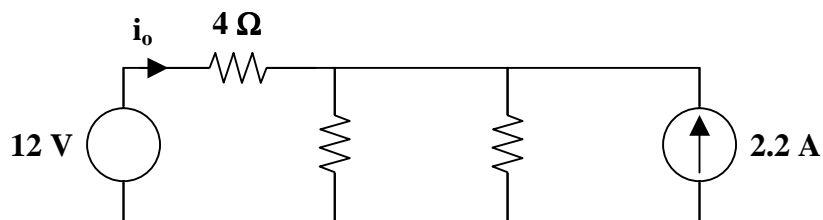
Therefore, **the two circuits are equivalent.**

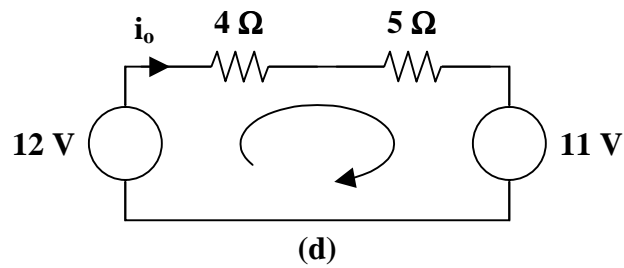
**Problem 4.7***[4.23]*

Given the circuit in Figure 4.1, use source transformation to find  $i_o$ .

**Figure 4.1**

Transforming only the current sources leads to Fig. (a). Continuing with source transformations finally produces the circuit in Fig. (d).

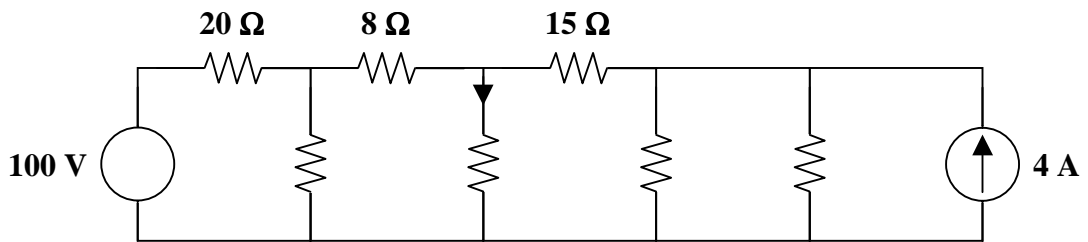
**(a)****(b)****(c)**



Applying KVL to the loop in Fig. (d),

$$-12 + 9i_o + 11 = 0 \longrightarrow i_o = 1/9 = \underline{\underline{111.11 \text{ mA}}}$$

**Problem 4.8** Using source transformations, solve for  $I_o$  in Figure 4.1.

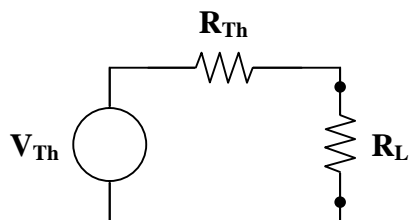


**Figure 4.1**

$$I_o = \underline{\underline{2 \text{ A}}}$$

## THEVENIN'S THEOREM

Thevenin's theorem states that a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a voltage source  $V_{Th}$  in series with a resistor  $R_{Th}$ , where  $V_{Th}$  is the open-circuit voltage at the terminals and  $R_{Th}$  is the input or equivalent resistance at the terminals when the independent sources are zero. An alternative way to find  $R_{Th}$  is to find the short-circuit current between the terminals,  $I_{sc}$ ; then,

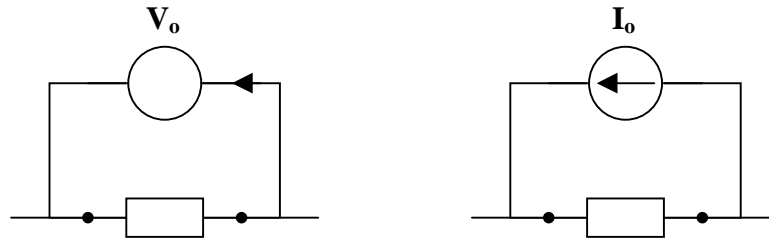


$$V_{Th} = V_{oc}$$

$$R_{Th} = \frac{V_{oc}}{I_{sc}}$$

Note that finding the equivalent resistance between the terminals using this approach is a valid method only if there are no dependent sources. Calculating  $R_{Th}$  using  $V_{oc}$  and  $I_{sc}$  is a valid method with independent and dependent sources.

Finally, if  $V_{oc} = I_{sc} = 0$ , then the circuit must be excited with a voltage source or a current source at the terminals, as shown below, to find  $R_{Th}$ .



Find  $I_o$  if given a voltage source  $V_o = 1$  V. Find  $V_o$  if given a current source  $I_o = 1$  A.

Now,  $R_{Th} = 1/I_o$  or  $R_{Th} = V_o/1$ .

**Problem 4.9** Solve for the Thevenin equivalent circuit as seen by the 10- $\Omega$  resistor with the current  $I_o$  flowing through it in Figure 4.1.

➤ **Carefully DEFINE the problem.**

Each component is labeled completely. The problem is clear.

➤ **PRESENT everything you know about the problem.**

To find the Thevenin equivalent circuit, we need to find the open-circuit voltage ( $V_{oc}$ ) across the terminals of the 10- $\Omega$  resistor with the current  $I_o$  flowing through it. We also need to find the short-circuit current ( $I_{sc}$ ) through these terminals. Then,

$$V_{Th} = V_{oc} \quad \text{and} \quad R_{Th} = V_{oc} / I_{sc}$$

Because the circuit has no dependent sources, there is an alternative way to find the Thevenin resistance.  $R_{Th}$  is the input resistance at the terminals when the independent sources are set equal to zero. This method will not be used in the initial attempt to find a solution. It is better to find the short-circuit current and calculate the Thevenin resistance because it will work for any circuit. However, we can find the input resistance at the terminals to check our initial solution.

➤ **Establish a set of ALTERNATIVE solutions and determine the one that promises the greatest likelihood of success.**

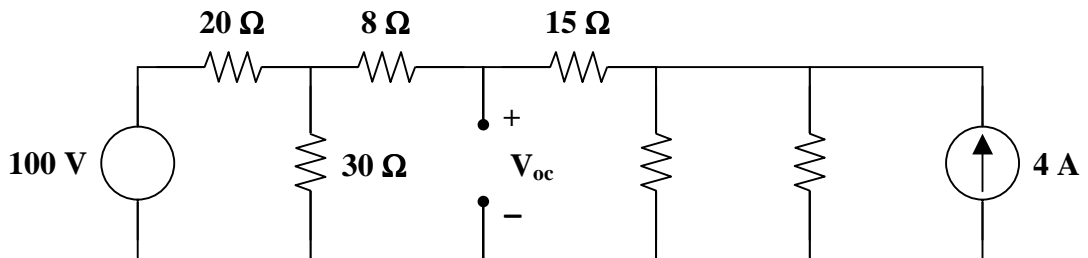
To find the open-circuit voltage, the obvious techniques are nodal analysis, mesh analysis, or source transformations along with KVL. Because we want to find a voltage, let's eliminate mesh analysis as a promising technique. It is clear that nodal analysis produces a set of three equations and three unknowns. Simplifying the circuit using source transformations yields a single loop from which we can easily find the open-circuit voltage.

To find the short-circuit current, a similar argument can be made. So, let's eliminate nodal analysis because we are looking for a current rather than a voltage. Mesh analysis gives a set of equations to be solved. Lastly, source transformations allows the circuit to be reduced to a single loop from which we can easily find the short-circuit current.

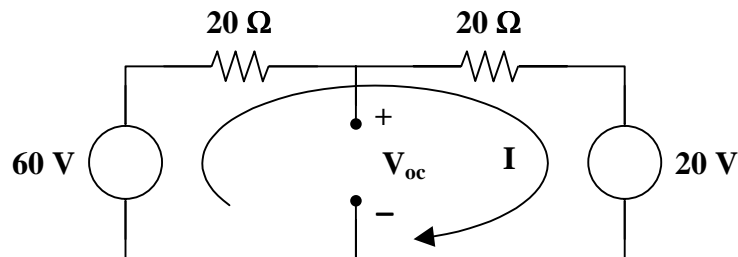
With the open-circuit voltage and the short-circuit current at the terminals, the Thevenin equivalent circuit is found as stated above.

➤ **ATTEMPT a problem solution.**

Begin by finding the open-circuit voltage as seen by the element.



After a couple of source transformations, as seen in Problem 4.8, the circuit becomes



Using KVL, find  $V_{oc}$ .

$$-60 + 20I + 20I + 20 = 0$$

$$40I = 40 \longrightarrow I = 1 \text{ amp}$$

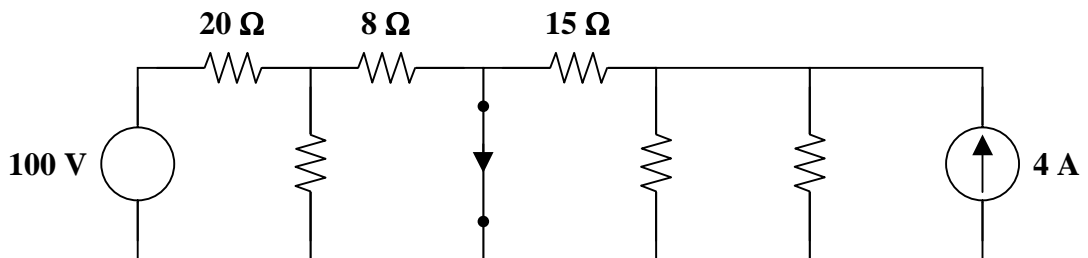
where

$$I = \frac{60 - V_{oc}}{20} = \frac{V_{oc} - 20}{20} = 1$$

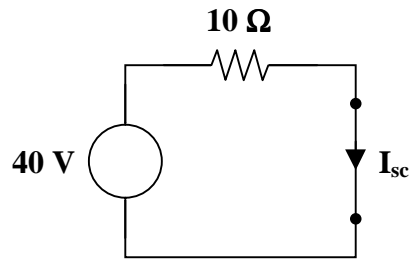
$$60 - V_{oc} = V_{oc} - 20$$

$$V_{oc} = 40 \text{ volts}$$

Now, find the short-circuit current through the element.



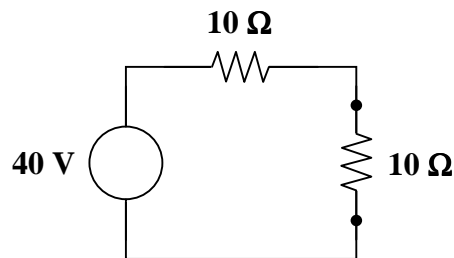
Again using source transformations, the circuit can be reduced as shown below.



Clearly, 
$$I_{sc} = \frac{40}{10} = 4 \text{ amps.}$$

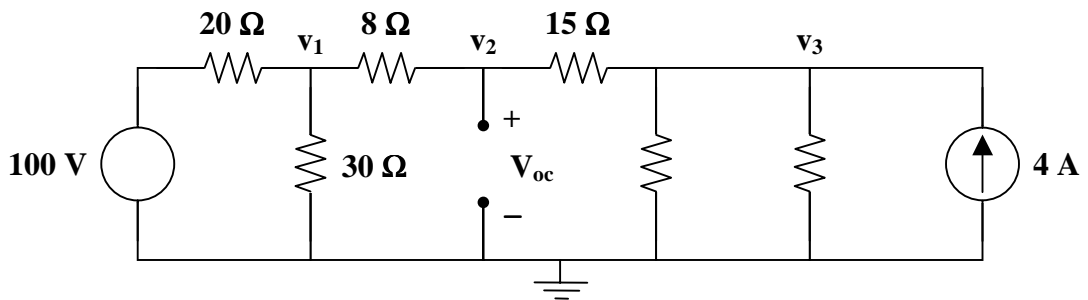
Then, 
$$R_{Th} = \frac{40}{10} = 4 \text{ ohms.}$$

Therefore, the Thevenin equivalent circuit is



➤ **EVALUATE the solution and check for accuracy.**

To check the open-circuit voltage, perform nodal analysis using the circuit below.



For node 1, 
$$\frac{v_1 - 100}{20} + \frac{v_1}{30} + \frac{v_1 - v_2}{8} = 0$$

For node 2, 
$$\frac{v_2 - v_1}{8} + \frac{v_2 - v_3}{15} = 0$$

For node 3, 
$$\frac{v_3 - v_2}{15} + \frac{v_3}{10} + \frac{v_3}{10} - 4 = 0$$



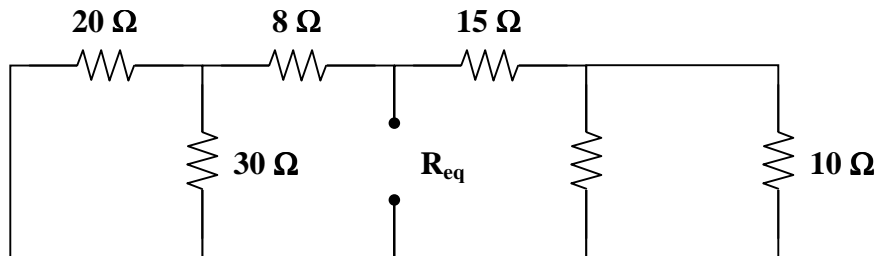
Simplifying these equations and putting them into matrix form results in

$$\begin{bmatrix} 50 & -30 & 0 \\ -15 & 23 & -8 \\ 0 & -2 & 8 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 1200 \\ 0 \\ 120 \end{bmatrix}$$

which yields  $v_1 = 48$  volts  $v_2 = 40$  volts  $v_3 = 25$  volts

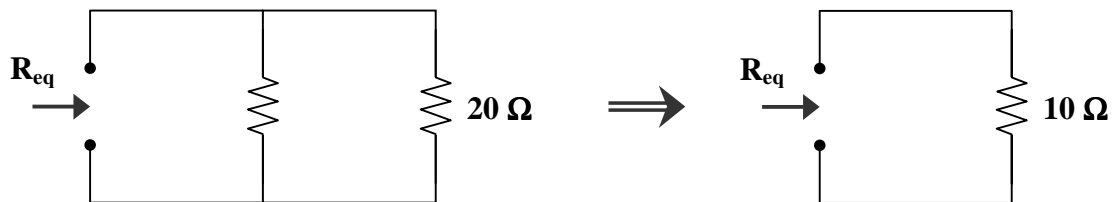
Hence,  $V_{Th} = V_{oc} = v_2 = 40$  volts

Since there are only independent sources, an alternative way to find  $R_{Th}$  is to set the sources to zero and calculate the resistance of the modified network. After replacing the voltage source with a short circuit and the current source with an open circuit, the network becomes



Combine the resistors to the left of the open circuit.  $(20 \Omega \parallel 30 \Omega) + 8 \Omega = 20 \Omega$

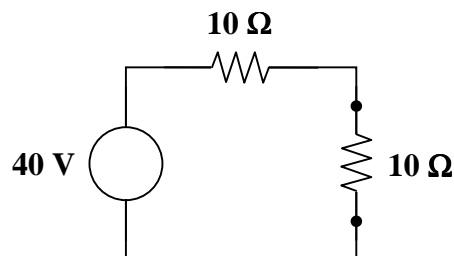
Combine the resistors to the right of the open circuit.  $(10 \Omega \parallel 10 \Omega) + 15 \Omega = 20 \Omega$



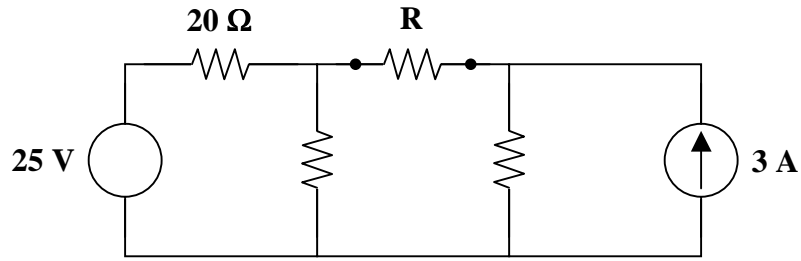
An equivalent resistance of  $10 \Omega$  matches the value that was calculated using  $V_{oc}$  and  $I_{sc}$ .

Our check for accuracy was successful.

- **Has the problem been solved SATISFACTORILY? If so, present the solution; if not, then return to “ALTERNATIVE solutions” and continue through the process again.**  
This problem has been solved satisfactorily. **The Thevenin equivalent circuit is as follows.**

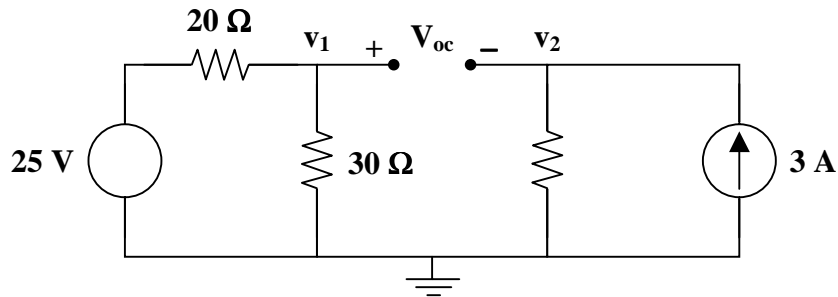


**Problem 4.10** Find the Thevenin equivalent as seen by R in Figure 4.1.



**Figure 4.1**

First, find the open-circuit voltage, where  $V_{Th} = V_{oc}$ .



Using nodal analysis,

At node 1 :

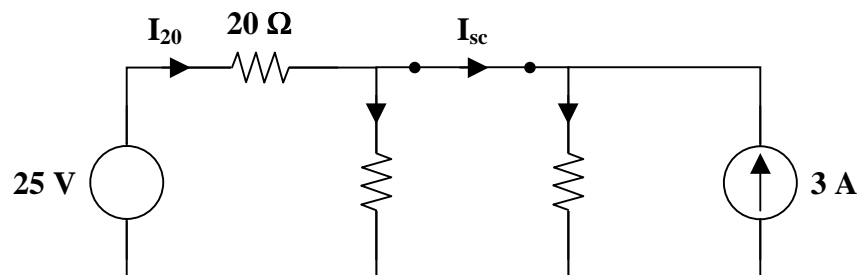
$$\begin{aligned}\frac{v_1 - 25}{20} + \frac{v_1 - 0}{30} &= 0 \\ 3v_1 + 2v_1 &= 75 \\ v_1 &= 15\end{aligned}$$

At node 2 :

$$\begin{aligned}\frac{v_2 - 0}{5} &= 3 \\ v_2 &= 15\end{aligned}$$

Clearly,  $V_{oc} = v_1 - v_2 = 15 - 15 = 0$  volts.

Now, find the short-circuit current.

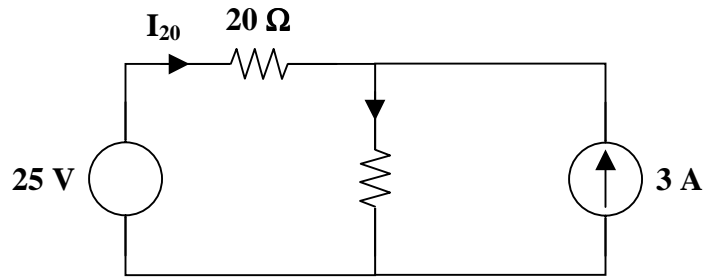


Clearly,  $I_{sc} = I_{20} - I_{30}$ .

Perform KVL using the left loop.

$$-25 + 20I_{20} + 30I_{30} = 0$$

Begin by finding  $I_{20}$  in the following modified circuit.



$$-25 + 20I_{20} + \frac{30}{7}I_x = 0 \quad \text{where } I_x = I_{20} + 3$$

$$140I_{20} + (30)(I_{20} + 3) = 175$$

$$170I_{20} = 85$$

$$I_{20} = 0.5 \text{ amps}$$

Now,

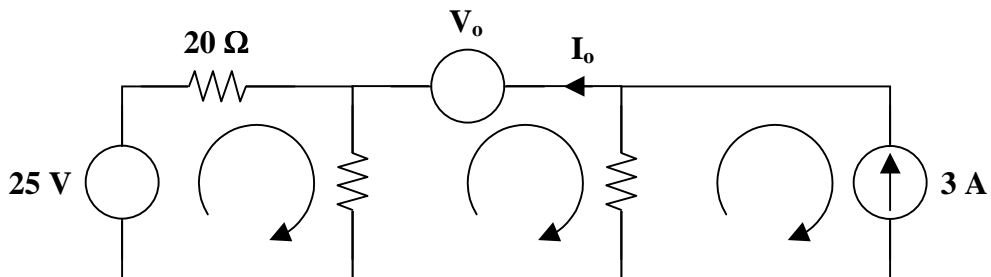
$$-25 + (20)(0.5) + 30I_{30} = 0$$

$$I_{30} = 15/30 = 0.5 \text{ amps}$$

Finally,

$$I_{sc} = I_{20} - I_{30} = 0.5 - 0.5 = 0 \text{ amps.}$$

Since 0/0 is undefined, we need to excite the circuit with a 1-V voltage source at the terminals of R in order to find  $R_{Th}$ .



Use mesh analysis to find  $I_o$ . Then,  $R_{Th} = V_o / I_o$ .

$$\text{For loop 1 : } -25 + 20i_1 + (30)(i_1 - i_2) = 0$$

$$\text{For loop 2 : } (30)(i_2 - i_1) + V_o + (5)(i_2 - i_3) = 0$$

$$\text{For loop 3 : } i_3 = -3$$

$$\text{where } V_o = 1 \text{ volt}$$

This is the constraint equation.

Simplifying,

$$50i_1 - 30i_2 = 25$$

$$-30i_1 + 35i_2 = -16$$

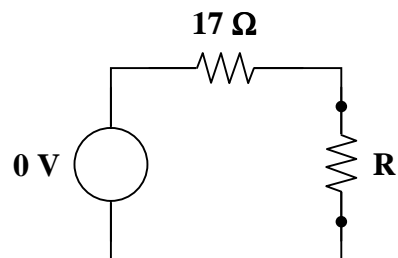
In matrix form,

$$\begin{bmatrix} 50 & -30 \\ -30 & 35 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 25 \\ -16 \end{bmatrix}$$

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 0.4647 \\ -0.0588 \end{bmatrix}$$

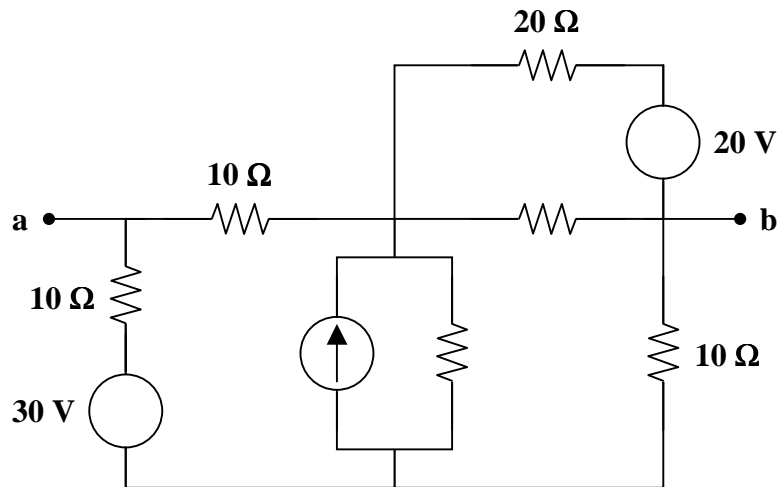
Now,  $I_o = -i_2 = 0.0588$  and  $R_{Th} = \frac{V_o}{I_o} = \frac{1}{0.0588} = 17 \text{ ohms}$ .

Therefore, **the Thevenin equivalent circuit is as follows.**



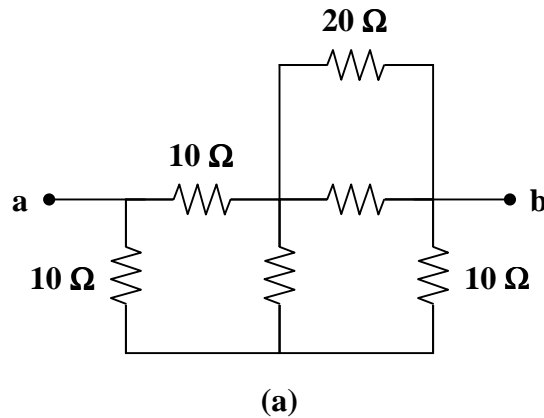
**Problem 4.11** [4.33]

For the circuit in Figure 4.1, find the Thevenin equivalent between terminals a and b.



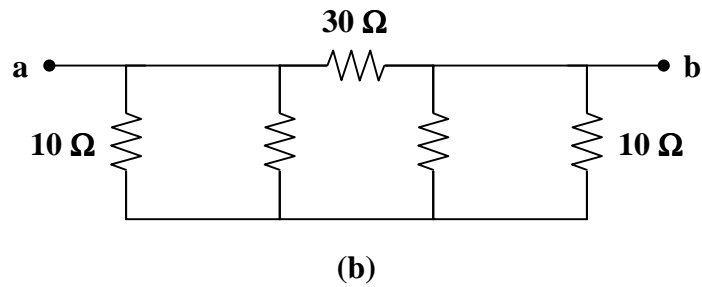
**Figure 4.1**

To find  $R_{Th}$ , consider the circuit in Fig. (a).



where  $20 \parallel 20 = 10 \Omega$ .

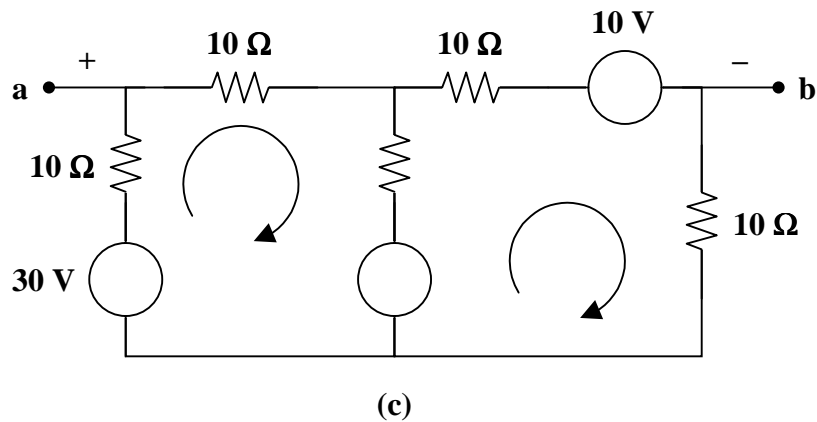
Now, transform the wye subnetwork to a delta as shown in Fig. (b).



where  $10 \parallel 30 = 7.5 \Omega$ .

Hence,  $R_{Th} = R_{ab} = 30 \parallel (7.5 + 7.5) = 30 \parallel 15 = \underline{10 \Omega}$

To find  $V_{Th}$ , we transform the 20-V and the 5-V sources to obtain the circuit shown in Fig. (c).



For loop 1,

$$\begin{aligned} -30 + 50 + 30i_1 - 10i_2 &= 0 \\ -2 &= 3i_1 - i_2 \end{aligned} \quad (1)$$

For loop 2,

$$\begin{aligned} -50 - 10 + 30i_2 - 10i_1 &= 0 \\ 6 &= -i_1 + 3i_2 \end{aligned} \quad (2)$$

Solving (1) and (2),

$$i_1 = 0 \text{ A} \quad \text{and} \quad i_2 = 2 \text{ A}$$

Applying KVL to the output loop,

$$\begin{aligned} -V_{ab} - 10i_1 + 30 - 10i_2 &= 0 \\ V_{ab} &= 10 \text{ V} \end{aligned}$$

$$V_{Th} = V_{ab} = \underline{\underline{10 \text{ V}}}$$

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**Problem 4.12** Find the Thevenin equivalent as seen by R in Figure 4.1.

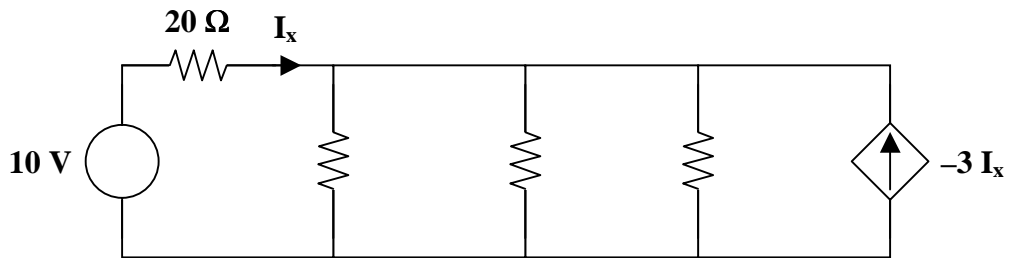
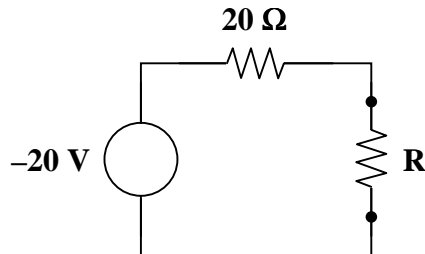


Figure 4.1

Therefore, the Thevenin equivalent circuit is as follows.



## NORTON'S THEOREM

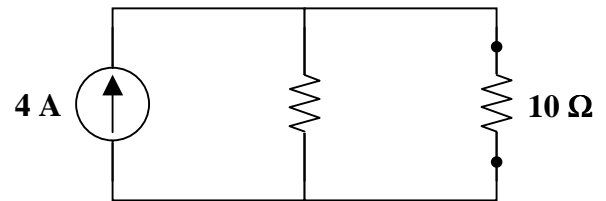
Norton's theorem states that a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a current source  $I_N$  in parallel with a resistor  $R_N$ , where  $I_N$  is the short-circuit current at the terminals and  $R_N$  is the input or equivalent resistance at the terminals when the independent sources are turned off.

**Problem 4.13** Find the Norton equivalent of the circuit in Figure 4.1.

In Problem 4.9,  $V_{oc} = 40$  volts,  $I_{sc} = 4$  amps, and  $R_{Th} = 10$  ohms.

Recall that  $R_N = R_{Th}$ ; so  $R_N = 10$  ohms.

Therefore, the Norton equivalent circuit is as shown.



**Problem 4.14** [4.41] Given the circuit in Figure 4.1, obtain the Norton equivalent as viewed from terminals:

(a) a-b

(b) c-d

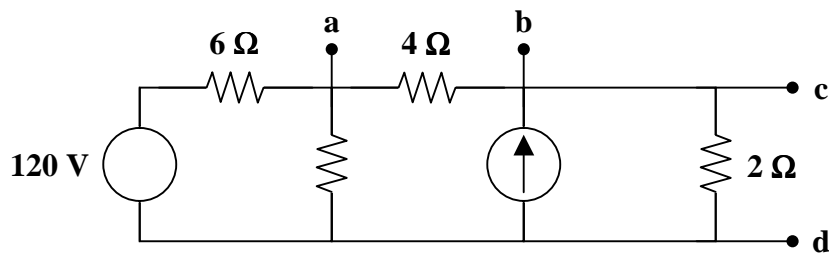
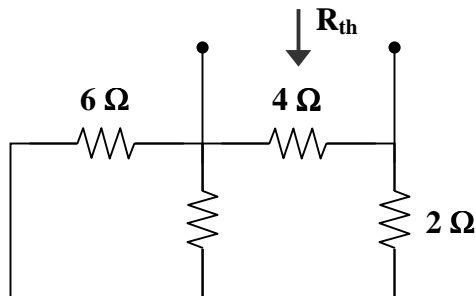


Figure 4.1

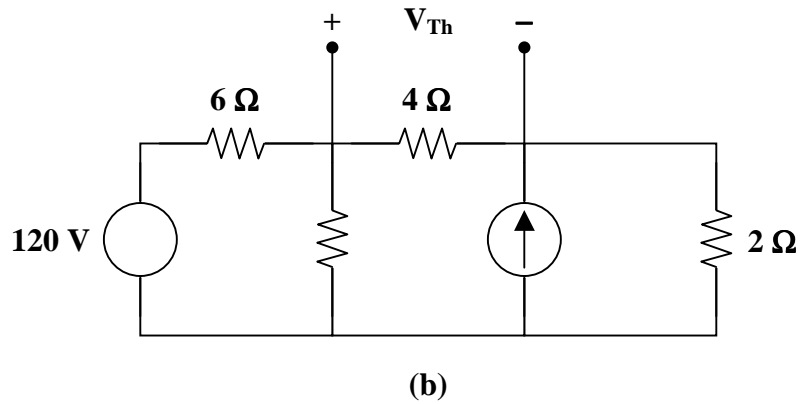
(a) From the circuit in Fig. (a),



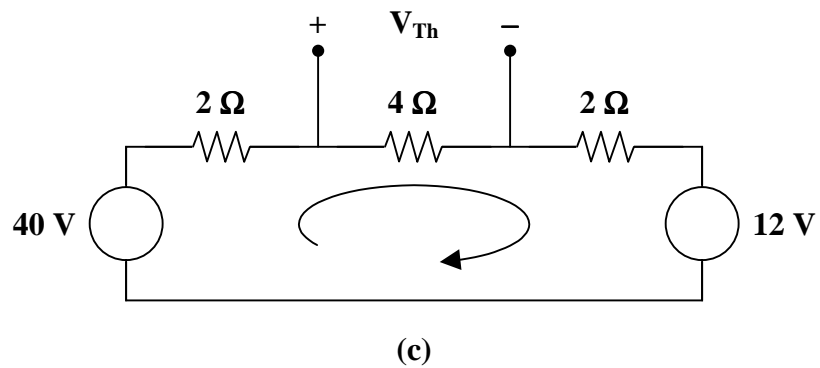
(a)

$$R_N = 4 \parallel (2 + 6 \parallel 3) = 4 \parallel 4 = \underline{2 \Omega}$$

For  $I_N$  or  $V_{Th}$ , consider the circuit in Fig. (b).



After some source transformations, the circuit becomes that shown in Fig. (c).



Applying KVL to the circuit in Fig. (c),

$$-40 + 8i + 12 = 0 \longrightarrow i = 7/2$$

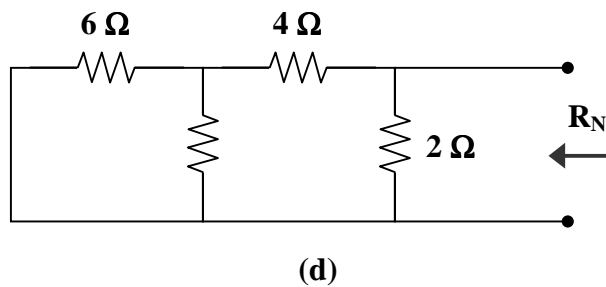
Hence,

$$V_{Th} = 4i = 14 \text{ V}$$

and

$$I_N = V_{Th} / R_N = 14/2 = \underline{7 \text{ A}}$$

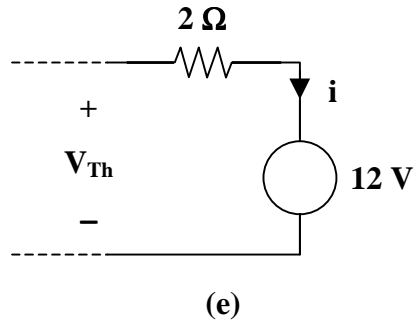
(b) To get  $R_N$ , consider the circuit in Fig. (d).





$$R_N = 2 \parallel (4 + (6 \parallel 3)) = 2 \parallel 6 = \underline{1.5 \Omega}$$

To get  $I_N$ , the circuit in Fig. (c) applies except that it needs slight modification as in Fig. (e).



$$i = (40 - 12) / (2 + 4 + 2) = 7/2 \quad \text{and} \quad V_{Th} = 12 + 2i = 19$$

$$I_N = V_{Th} / R_N = 19 / 1.5 = \underline{12.667 \text{ A}}$$

**Problem 4.15** Find the Norton equivalent of the circuit in Figure 4.1.

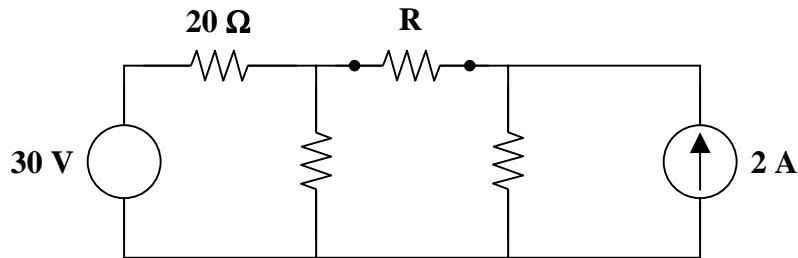
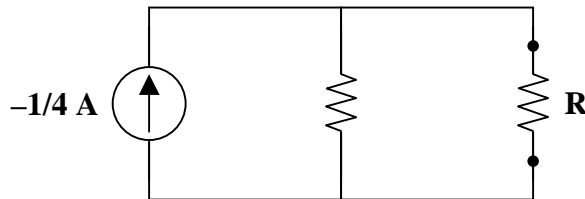


Figure 4.1

Therefore, the Norton equivalent circuit is as follows.

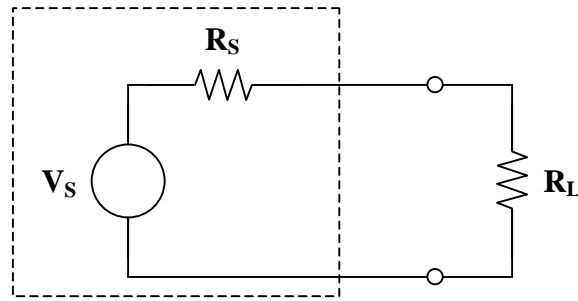


## MAXIMUM POWER TRANSFER

Maximum power is transferred to the load when the load resistance equals the Thevenin resistance as seen by the load ( $R_L = R_{Th}$ ).

**Problem 4.16**

Given the circuit in Figure 4.1, complete the following table.

**Figure 4.1**

	$V_s$ (volts)	$R_s$ (ohms)	$R_L$ (ohms)	Power of $R_L$ (watts)
Condition #1	20	10		
Condition #2	$V$	10		
Condition #3	20		10	
Condition #4	20			

Let  $p$  be the power of the load resistor,  $R_L$ . Then,  $p = \left( \frac{V_s}{R_s + R_L} \right)^2 R_L$ .

We want to find the maximum power transferred to the load. The maximum power is transferred to the load when the load resistance is equivalent to the Thevenin resistance as seen by the load.

In this case,  $R_L = R_s$  and  $p = \frac{V_s^2}{4R_L}$

$$\text{Condition \#1 : } R_L = R_s = \underline{10 \, \Omega} \qquad p = \frac{(20)^2}{(4)(10)} = \frac{400}{40} = \underline{10 \text{ watts}}$$

$$\text{Condition \#2 : } R_L = R_s = \underline{10 \, \Omega} \qquad p = \frac{V^2}{(4)(10)} = \frac{V^2}{40} \text{ watts}$$

$$\text{Condition \#3 : } p = \left( \frac{20}{R_s + 10} \right)^2 (10)$$

Clearly, for maximum power,  $R_s = \underline{0 \, \Omega}$ . *{with  $R_L$  fixed, making  $R_s$  as small as possible maximizes the power to  $R_L$ }*

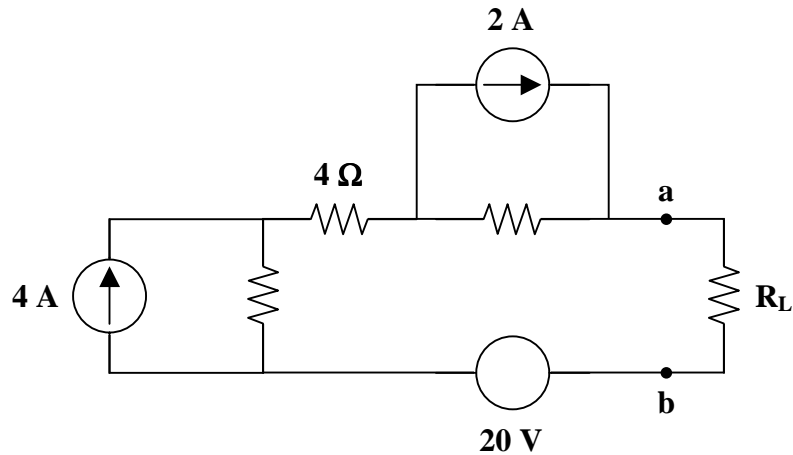
$$\text{Then, } p = \left( \frac{20}{10} \right)^2 (10) = (2^2)(10) = \underline{40 \text{ watts}}.$$

Condition #4 : For maximum power,  $R_s = \underline{0 \, \Omega}$  and  $R_L = \underline{0^+ \, \Omega}$ .

Then,  $p = \frac{20^2}{(4)(0^+)} = \underline{\infty^- \text{ watts}}.$

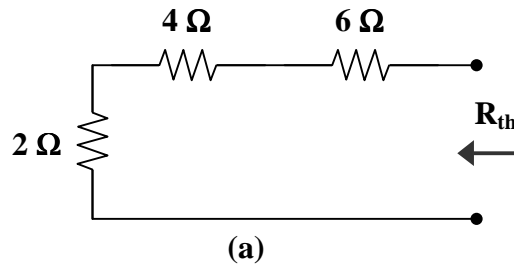
**Problem 4.17** [4.59]

- For the circuit in Figure 4.1, obtain the Thevenin equivalent at terminals a-b.
- Calculate the current in  $R_L = 8 \Omega$ .
- Find  $R_L$  for maximum power deliverable to  $R_L$ .
- Determine that maximum power.

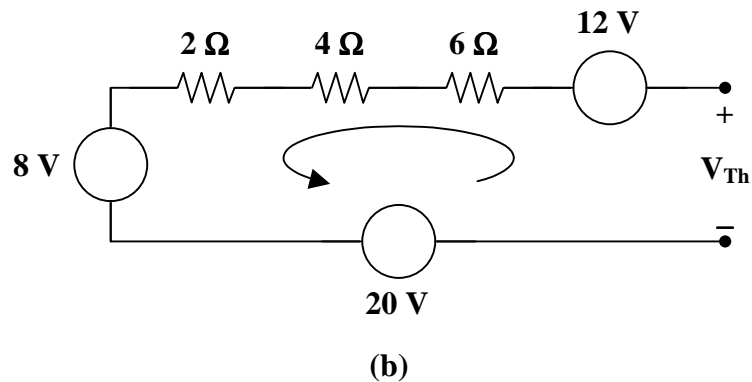


**Figure 4.1**

- $R_{Th}$  and  $V_{Th}$  are calculated using the circuits in Fig. (a) and (b) respectively.



From Fig. (a),  $R_{Th} = 2 + 4 + 6 = \underline{12 \Omega}$



From Fig. (b),

$$-V_{Th} + 12 + 8 + 20 = 0$$

$$V_{Th} = \underline{\underline{40 \text{ V}}}$$

$$(b) \quad i = \frac{V_{Th}}{R_{Th} + R} = \frac{40}{12 + 8} = \underline{\underline{2 \text{ A}}}$$

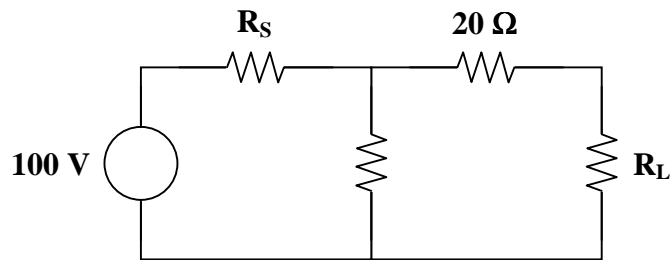
(c) For maximum power transfer,

$$R_L = R_{Th} = \underline{\underline{12 \Omega}}$$

$$(d) \quad p = \frac{V_{Th}^2}{4 R_{Th}} = \frac{(40)^2}{(4)(12)} = \underline{\underline{33.33 \text{ watts}}}$$

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**Problem 4.18** Using Figure 4.1, what values of  $R_S$  and  $R_L$  result in maximum power delivered to the load? What is the power absorbed by the load?



**Figure 4.1**

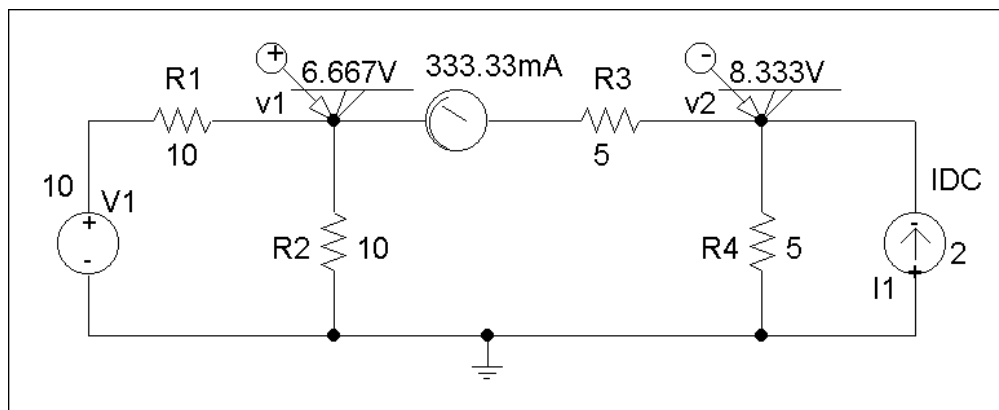
$$R_S = \underline{\underline{0 \Omega}}$$

$$R_L = \underline{\underline{20 \Omega}}$$

$$p_{RL} = \underline{\underline{125 \text{ watts}}}$$

## VERIFYING CIRCUIT THEOREMS WITH PSpICE

**Problem 4.19** Solve Problem 4.1 using PSpice.



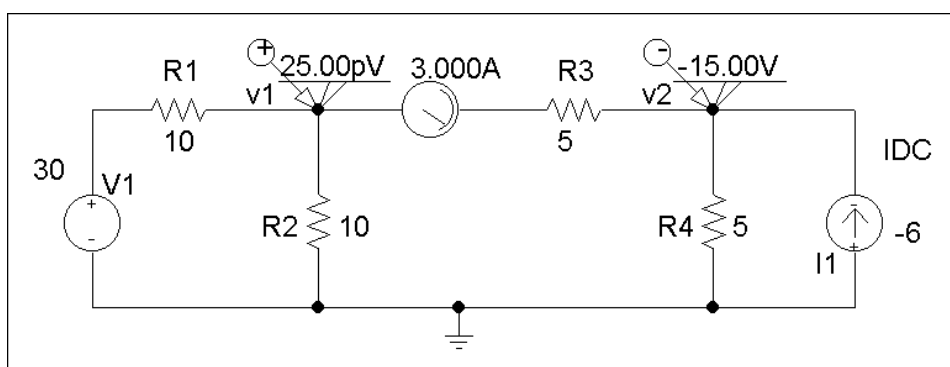
A voltage source in PSpice needs a value to simulate rather than performing a symbolic simulation. So, let  $K = 1$ . Then,  $V_x = -5/3$  volts and  $I_x = -1/3$  amps. Clearly,  $V_x$  is the voltage across  $R3$  and  $I_x$  is the current flowing through  $R3$  from left to right. Thus, for  **$K = 1$** ,

$$V_x = v_1 - v_2 = 6.667 - 8.333 = -1.667 = \underline{-5/3 \text{ V}}$$

and  $I_x = V_x/5 = -1/3$  amps or  $I_x = \underline{-333.333 \text{ mA}}$ .

These answers agree with the answers obtained in Problem 4.1 given that  $K = 1$ .

**Problem 4.20** Solve Problem 4.3 using PSpice.



Clearly,  $V_x$  is the voltage across  $R3$  and  $I_x$  is the current flowing through  $R3$  from left to right. Thus,  $V_x = v_1 - v_2 = (25 \times 10^{-12}) - (-15) = 15 \text{ V}$  and  $I_x = V_x/5 = \underline{3 \text{ A}}$ .

This answer agrees with the answer obtained in Problem 4.3.