

CHAPTER 1 - BASIC CONCEPTS

List of topics for this chapter :

Systems and Units

Charge and Current

Voltage, Power, and Energy

Circuit Elements

SYSTEMS AND UNITS

Problem 1.1 Simplify the following units. Try to express each as a single unit.

(a) $\frac{\text{volt} \cdot \text{coulomb}}{\text{second}}$

(b) $\text{volt} \cdot \text{ampere} \cdot \text{second}$

(c) $\frac{\text{second}}{\text{volt} \cdot \text{coulomb}}$

(a) **watt**

(b) **joule**

(c) **1 / watt**

CHARGE AND CURRENT

Charge is an electrical property of the atomic particles of which matter consists, measured in coulombs (C).

Electric current is the time rate of change of charge, measured in amperes (A). A direct current (dc) is a current that remains constant with time. An alternating current (ac) is a current that varies sinusoidally with time.

Problem 1.2 The charge transferred in time is given in Figure 1.1. Determine the current, $i(t)$, flowing through the wire.

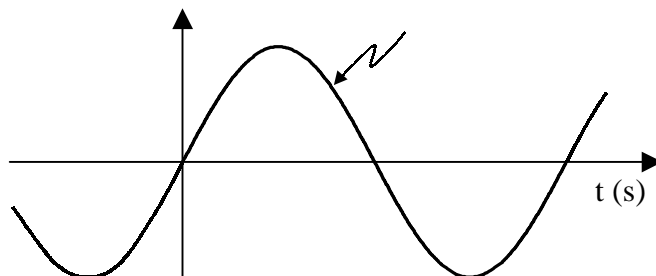


Figure 1.1

The relationship between current, charge, and time is

$$i(t) = dq(t)/dt$$

Therefore,

$$i(t) = \frac{d}{dt}(10\sin(2t))$$

$$i(t) = \underline{\underline{20\cos(2t) \text{ amps}}}$$

Problem 1.3 [1.5]* Determine the total charge flowing through an element for $0 < t < 2$ seconds when the current entering its positive terminal is $i(t) = e^{-2t}$ mA.

$$q(t) = \int i(t) dt$$

$$q = \int_0^2 e^{-2t} dt = \left. -\frac{1}{2}e^{-2t} \right|_0^2 = \left(-\frac{1}{2}e^{-4} - \left(-\frac{1}{2}e^0 \right) \right) = \frac{-1}{2}(e^{-4} - 1)$$

$$q = \underline{\underline{0.4908 \text{ mC}}}$$

Problem 1.4 The current flowing through a wire is given by Figure 1.1. Determine the net charge moving through the wire if $q(0) = 0$.

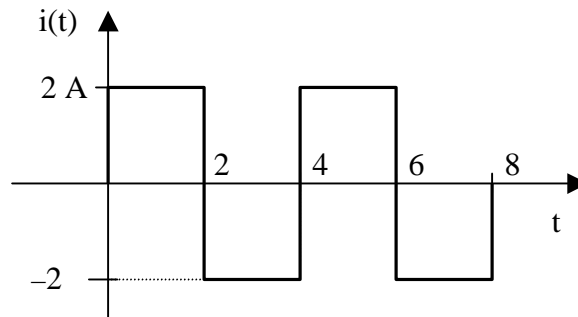
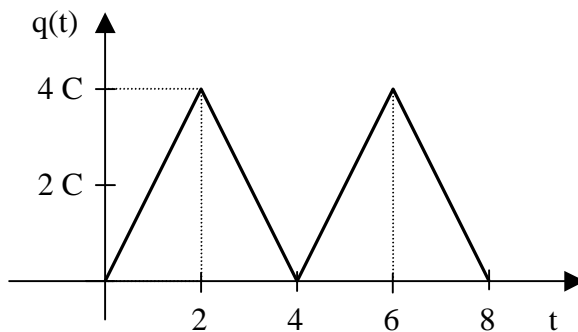


Figure 1.1

The net charge moving through the wire is



$$q(t) = \begin{cases} 2t \text{ C} & 0 < t < 2 \\ -2t + 8 \text{ C} & 2 < t < 4 \\ 2t - 8 \text{ C} & 4 < t < 6 \\ -2t + 16 \text{ C} & 6 < t < 8 \end{cases}$$

* This indicates a problem from Fundamentals of Electric Circuits by Alexander and Sadiku.

VOLTAGE, POWER, AND ENERGY

Voltage (or potential difference) is the energy required to move a unit charge through an element, measured in volts (V). Power is the time rate of expending or absorbing energy, measured in watts (W). Energy is the capacity to do work, measured in joules (J).

Passive sign convention is satisfied when the current enters through the positive terminal of an element and $p = +vi$. If the current enters through the negative terminal, $p = -vi$.

Problem 1.5 Given $v = 10$ volts and $i(t)$ as shown in Figure 1.1, sketch the power and energy.

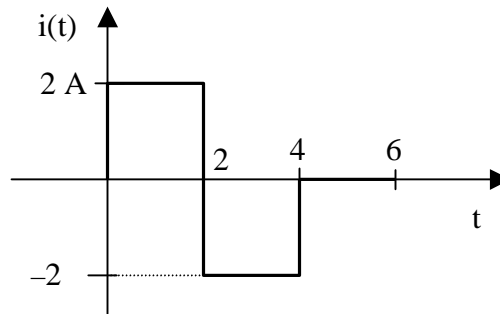


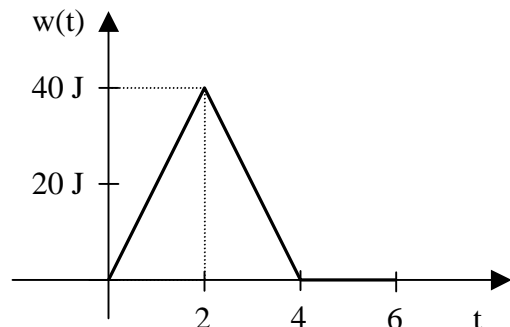
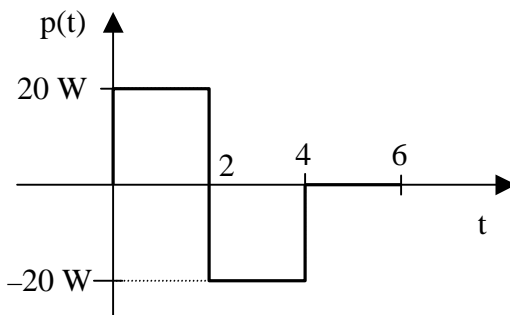
Figure 1.1

$$i(t) = \begin{cases} 2 \text{ A} & 0 < t < 2 \\ -2 \text{ A} & 2 < t < 4 \\ 0 \text{ A} & 4 < t < 6 \end{cases}$$

Therefore,

$$p(t) = v(t)i(t) = \begin{cases} 20 \text{ W} & 0 < t < 2 \\ -20 \text{ W} & 2 < t < 4 \\ 0 \text{ W} & 4 < t < 6 \end{cases} \quad w(t) = \int p(t) dt = \begin{cases} 20t \text{ J} & 0 < t < 2 \\ 80 - 20t \text{ J} & 2 < t < 4 \\ 0 \text{ J} & 4 < t < 6 \end{cases}$$

Sketches of the power and energy are shown in the figures below.



Problem 1.6 [1.13] Figure 1.1 shows the current through and the voltage across a device. Find the total energy absorbed by the device for the period $0 < t < 4$ seconds.

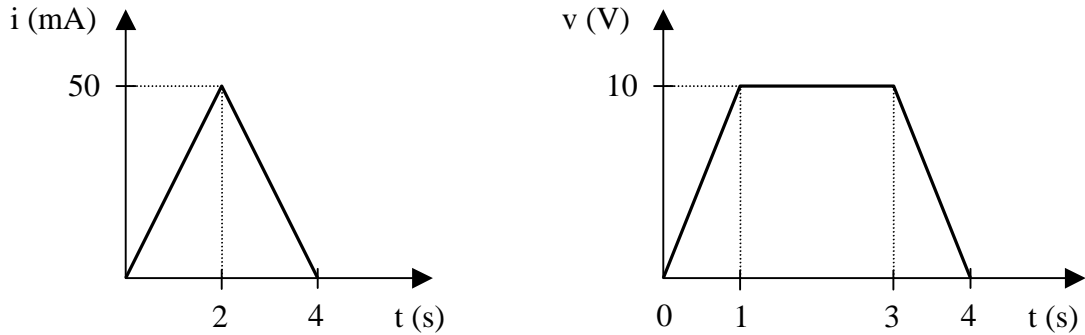


Figure 1.1

The curves in Figure 1.1 are represented by

$$i(t) = \begin{cases} 25t \text{ mA} & 0 < t < 2 \\ 100 - 25t \text{ mA} & 2 < t < 4 \end{cases} \quad v(t) = \begin{cases} 10t \text{ V} & 0 < t < 1 \\ 10 \text{ V} & 1 < t < 3 \\ 40 - 10t \text{ V} & 3 < t < 4 \end{cases}$$

So,

$$\begin{aligned} w(t) &= \int v(t)i(t) dt \\ w &= \int_0^1 (10t)(25t) dt + \int_1^2 (10)(25t) dt + \int_2^3 (10)(100 - 25t) dt + \int_3^4 (40 - 10t)(100 - 25t) dt \\ w &= 250 \left[\int_0^1 t^2 dt + \int_1^2 t dt + \int_2^3 (4 - t) dt + \int_3^4 (16 - 8t + t^2) dt \right] \\ w &= 250 \left[\frac{t^3}{3} \Big|_0^1 + \frac{t^2}{2} \Big|_1^2 + \left(4t - \frac{t^2}{2} \right) \Big|_2^3 + \left(16t - 4t^2 + \frac{t^3}{3} \right) \Big|_3^4 \right] \\ w &= 250 \left[\left(\frac{1}{3} - 0 \right) + \left(\frac{4}{2} - \frac{1}{2} \right) + \left(12 - \frac{9}{2} - 8 + \frac{4}{2} \right) + \left(64 - 64 + \frac{64}{3} - 48 + 36 - \frac{27}{3} \right) \right] \\ w &= 250 [0.3333 + 1.5 + 1.5 + 0.3333] \\ w &= \underline{\underline{916.7 \text{ mJ}}} \end{aligned}$$

Problem 1.7 Given that the power absorbed by an element is $p(t) = 10\cos^2(4t)$ watts and the current through the element is $i(t) = 20\cos(4t)$ mA, find the voltage across the element.

$$v(t) = \underline{\underline{500\cos(4t) \text{ volts}}}$$

CIRCUIT ELEMENTS

An electric circuit is an interconnection of electrical elements.

An ideal independent source is an active element that provides a specified voltage or current that is completely independent of other circuit variables. An ideal dependent (or controlled) source is an active element in which the source quantity is controlled by another voltage or current.

Problem 1.8 Given the circuit in Figure 1.1, find V_o .

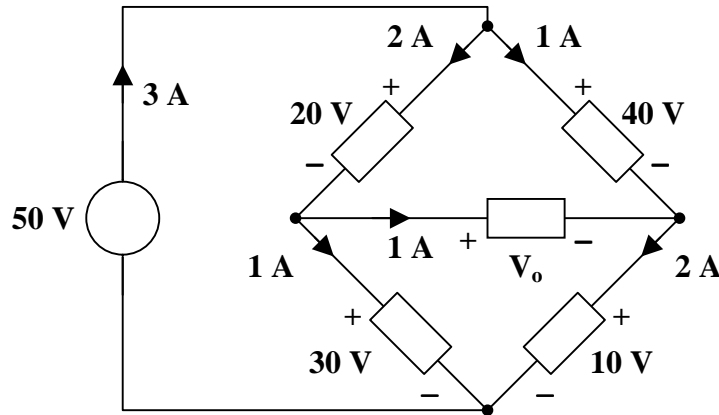


Figure 1.1

This circuit, like any other electric circuit, must obey the law of conservation of energy. Hence,

$$\sum p = 0$$

The power absorbed by the voltage source is

$$p_{\text{abs}_{50\text{V}}} = -(50)(3) = -150 \text{ watts}$$

The power absorbed by each circuit element in the bridge is

$$p_{\text{abs}_{20\text{V}}} = (20)(2) = 40 \text{ watts}$$

$$p_{\text{abs}_{40\text{V}}} = (40)(1) = 40 \text{ watts}$$

$$p_{\text{abs}_{V_o}} = (V_o)(1) = V_o$$

$$p_{\text{abs}_{30\text{V}}} = (30)(1) = 30 \text{ watts}$$

$$p_{\text{abs}_{10\text{V}}} = (10)(2) = 20 \text{ watts}$$

Thus,

$$\sum p = 0 = -150 + 40 + 40 + p_{\text{abs}_{V_o}} + 30 + 20$$

$$p_{\text{abs}_{V_o}} = 150 - 40 - 40 - 30 - 20$$

$$p_{\text{abs}_{V_o}} = 20 \text{ watts}$$

but

$$p_{\text{abs}_{V_o}} = (V_o)(1) = V_o$$

Therefore,

$$V_o = \underline{\underline{20 \text{ volts}}}$$

Problem 1.9 [1.17] Find V_o in the circuit of Figure 1.1.

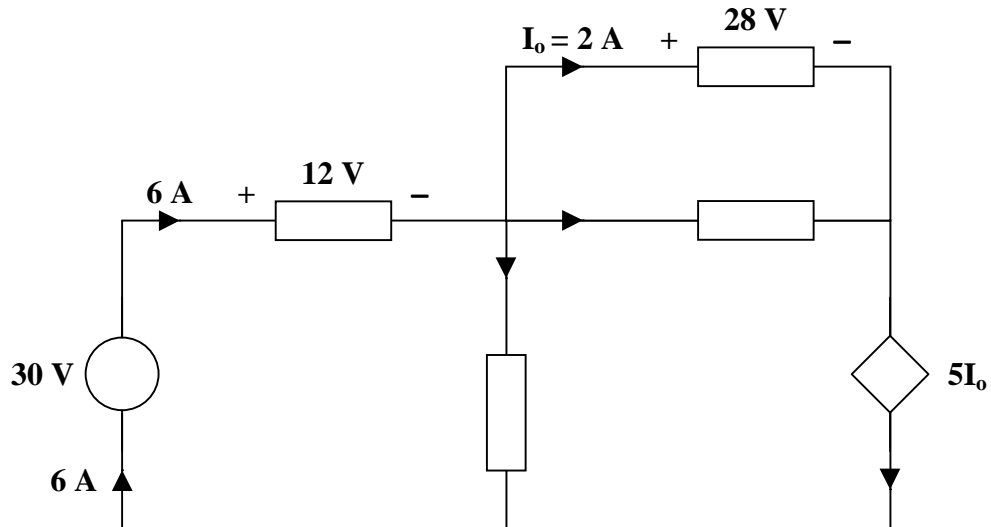


Figure 1.1

Since $\sum p = 0$,

$$-(30)(6) + (12)(6) + 3V_o + (28)(I_o) + (28)(1) - (5I_o)(3) = 0$$

where $I_o = 2$.

$$-180 + 72 + 3V_o + (28)(2) + 28 - (5)(2)(3) = 0$$

$$3V_o + 156 = 210$$

$$3V_o = 54$$

$$V_o = \underline{\underline{18 \text{ volts}}}$$