1. (a) By expanding and grouping terms of the same order in x and y, we can write z as the product of a row matrix in X and y with a column matrix of fit parameters:

$$
\begin{aligned}
z - z_0 &= a\Big((x-x_0)^2 + (y-y_0)^2\Big) \\
z &= z_0 + a(x^2 - 2xx_0 + x_0^2 + y^2 - 2yy_0 + y_0^2) \\
z &= (z_0 + ax_0^2 + ay_0^2) + (-2ax_0)(x) + (-2ay_0)(y) + (a)(x^2+y^2) \\
z &= \begin{pmatrix} 1 & x & y & x^2+y^2 \end{pmatrix} \begin{pmatrix} z_0 + ax_0^2 + ay_0^2 \\ -2ax_0 \\ -2ay_0 \\ a \end{pmatrix} \\
z &= \begin{pmatrix} 1 & x & y & x^2+y^2 \end{pmatrix} \begin{pmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \end{pmatrix}
\end{aligned}
$$

Where in the last line I redefined new fit parameters b_0, b_1, b_2, b_3 equal to the four terms in a, x_0, y_0, z_0 in the line above. For many data points, x, y, and z become column vectors, and the row vector becomes a matrix, A, satisfying $\mathbf{z} = \mathbf{Ab}$.

(b) To solve for the fit parameters satisfying $\mathbf{z} = \mathbf{Ab}$, we want to calculate $\mathbf{b} = \mathbf{A}^{-1}\mathbf{z}$. Since A isn't square, hw3prob1.py uses np.linalg.pinv to find the pseudo inverse.

(c)

2. $\chi^2 = \sum_i \frac{(x_i-\mu_i)^2}{\sigma_i^2} = 1588.2376532931526$

3. The best fit parameters were

$$
\begin{aligned}
H_0 &= 69 \pm 2, \\
\omega_b h^2 &= 0.0224 \pm 0.001, \\
\omega_c h^2 &= 0.116 \pm 0.005, \\
A_s &= (2.05 \pm 0.04) \cdot 10^{-9}, \\
slope &= 0.97 \pm 0.01,
\end{aligned}
$$

as seen in Figure 1. This gives $\chi^2 = 1229.439$. If we assume uncorrelated errors as before, fitting for tau wouldn't change these values much, but

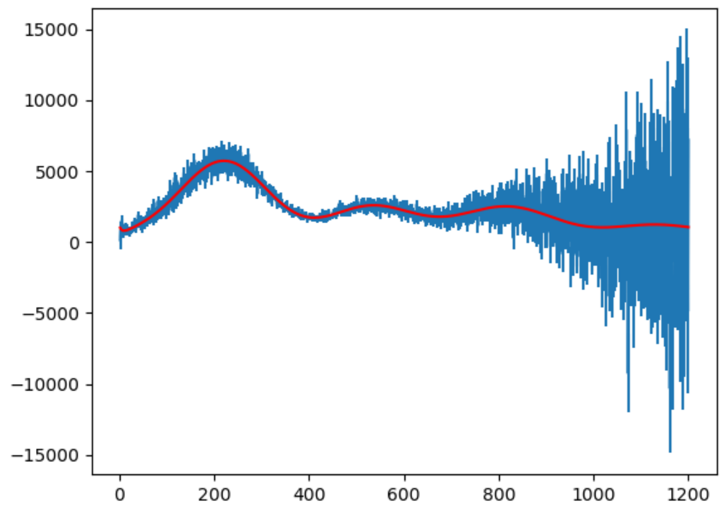

Figure 1: Data from wmap with fit from Levenberg-Marquardt

I estimated the derivative with the two-sided numerical derivative $f'(x) = \frac{f(x+h)-f(x-h)}{2h}$. Below is a plot of the gradient with respect to the first parameter H_0, for a few values of h. The results are almost identical. On close inspection, the central value (dividing the initial parameters by 70) gave the lowest minimum.

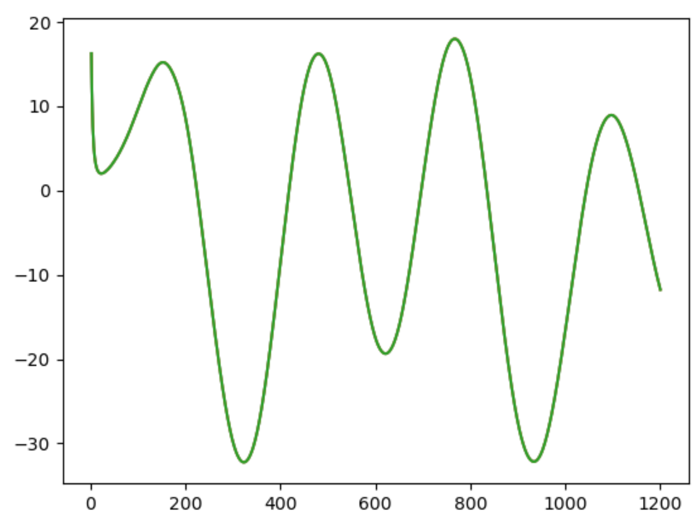

Figure 2: Derivative w.r.t. H_0 for 3 different step sizes, where the lines are almost indistinguishable.

4. The parameters from the MCMC were:

$$\begin{aligned}
H_0 &= 70.4 \pm 2, \\
\omega_b h^2 &= 0.112 \pm 0.005, \\
\omega_c h^2 &= 0.116 \pm 0.005, \\
A_s &= (2.06 \pm 0.07) \cdot 10^{-9}, \\
slope &= 0.977 \pm 0.01, \\
\tau &= 0.0563 \pm 0.02.
\end{aligned}$$

The Fourier transforms were not flat, indicating that the chains did not converge.

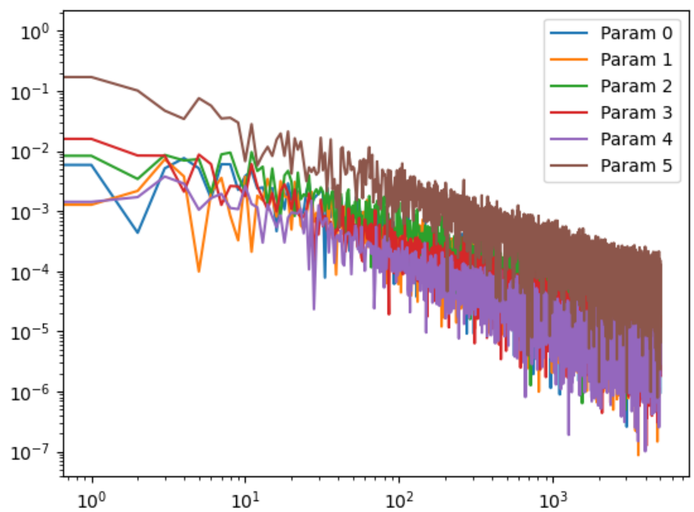

Figure 3: Fourier transform of first MCMC chains

5. And then my poor little computer gave out and so did my brain. I'll update tomorrow if I can.