Markov Chain Monte Carlo ctd

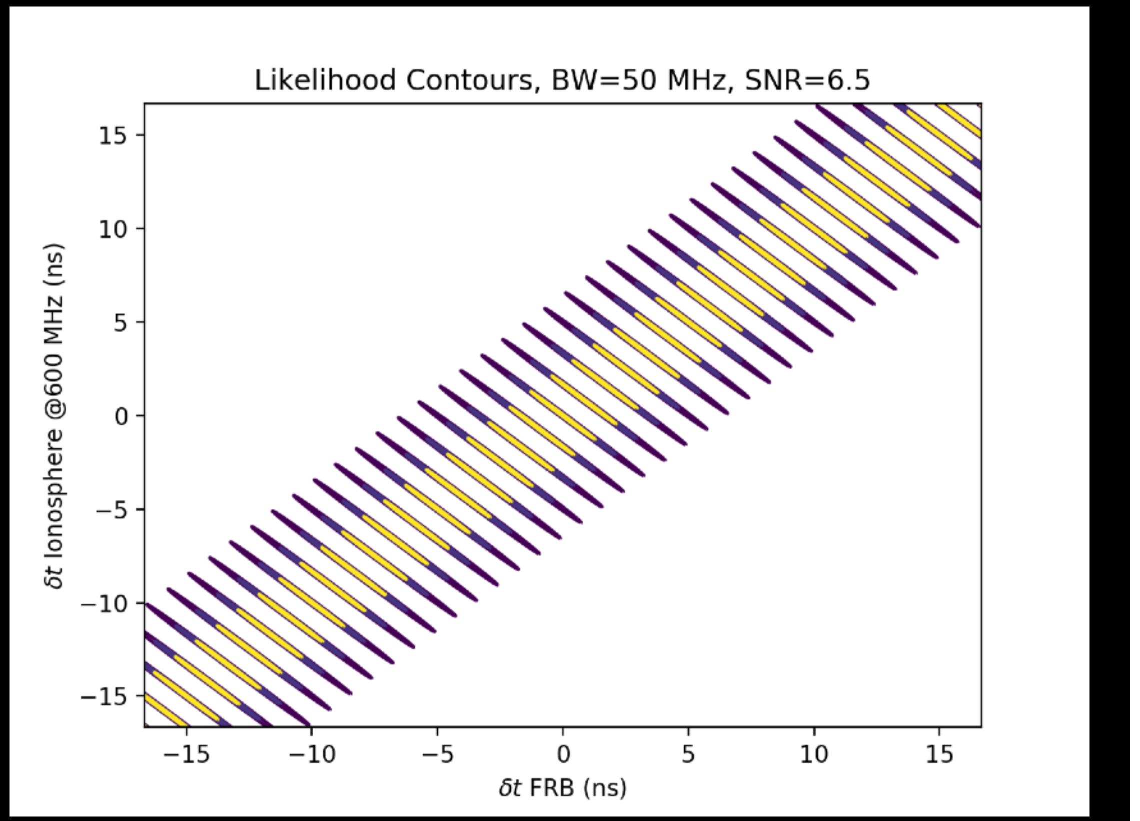
Reminder

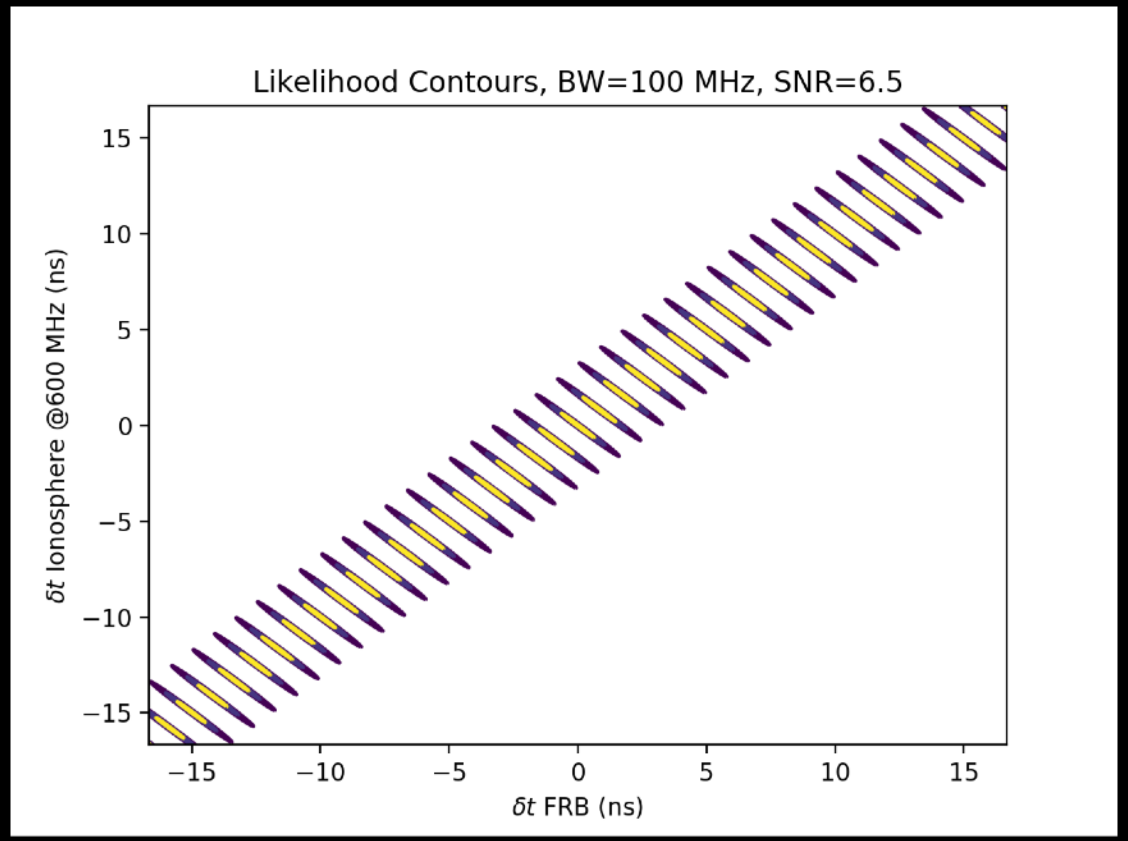
- Often faced with problems not well described by Taylor expansion of χ^2.
- MCMC lets us map out parameter space in a way that doesn't care about this.
- Chains give us a random sampling of parameters space - any statistical quantity we'd want to get by integrating over parameter space, we get by integrating over chain.
- Chains robust to taking steps, but we must make sure chain is converged before we use it.
- Gelman-Rubin (many chains) or power spectrum (one chain) are tools to check convergence, # of independent samples.

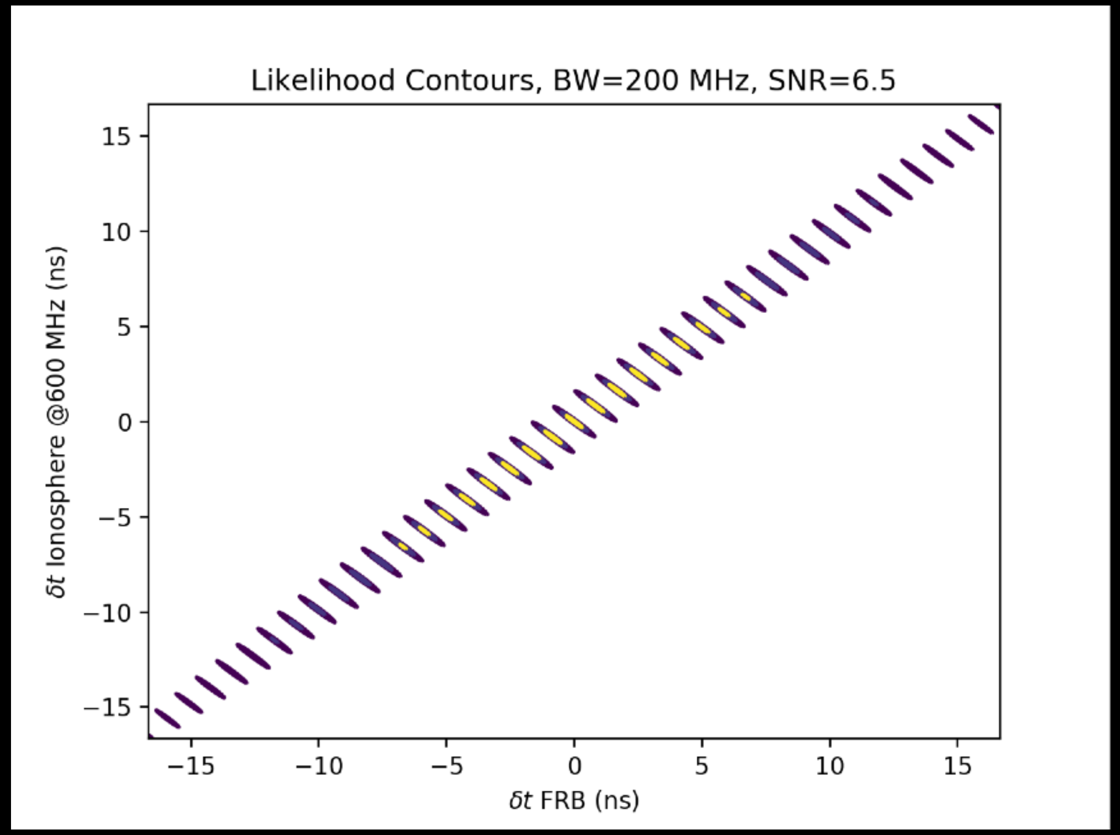
Example - Arrival time of FRBs

- Fast radio bursts are astronomical enigma, largely because we don't know where they are.
- With CHIME, working to localize to sub-arcsecond accuracy (down from many arcminutes).
- Do this by measuring arrival time of electric fields at two different places.
- Problem - ionosphere introduces unknown, frequency-dependent delay.
- Physical delay (the one we care about) has phase proportional to frequency. Ionosphere has phase proportional to $1/\nu$. If we add ionosphere to physical such that phase moves by 2π, we can't tell the difference.
- For modest SNR, there are *many* possible delays shifted by 2π, unless bandwidth is extremely broad.
- Looking at likelihood curvature tells us everything about one possible 2π shift, but completely misses the others.

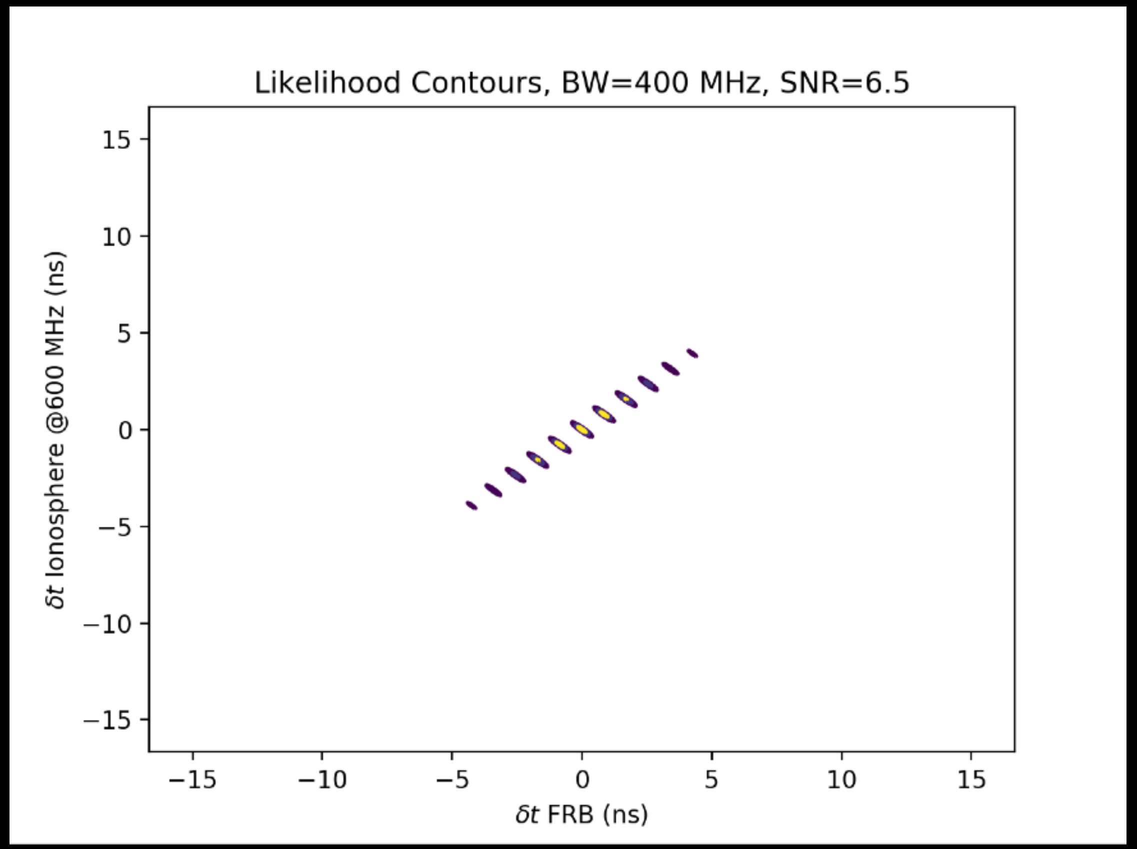
FRB Arrival Times, SNR=6.5







Likelihood contours for SNR=6.5, for fractional bandwidth of 1/8, 1/4, 1/2, and 1. Looking at 1 peak gives completely wrong picture of uncertainty.



Importance Sampling

- MCMC gives us samples drawn randomly from underlying probability distribution/phase space.
- Common situation - we have done a lot of work to get a chain. Now we want to tweak the likelihood (say, see how much an external prior matters). Do we have to start over?
- No! The new phase space density is the old density times the new likelihood. We can *reweight* our chain. If our region of phase space hasn’t moved too much, we can use the weighted chain. We’ll still need to check convergence, naturally.
- This technique is called “importance sampling”.

High Temp

- Often we care about limits for rare events (can we really rule out something at 5σ?).
- Hard to do normlly, since 5σ is very rare. Importance sampling to the rescue!
- Run a chain with a high temperature - Like -> $\exp(-\delta\chi^2/2T)$. If we make T large, the chain will explore previously unlikely regions.
- The true likelihood should be $\exp(-\delta\chi^2/2)$, so we need to weight each sample by $\exp(-\delta\chi^2/2)/\exp(-\delta\chi^2/2T) = \exp(-\delta\chi^2/2(1-1/T))$
- We can use this to find limits, since high T chain spends much of its time in far reaches of probability.
- We will probably need a longer chain to converge…

Priors/Edges

- Say a region of parameter space is excluded. How would you handle with MCMC?
- Set χ^2 to a large number. Chain will never go there…
- When you look at your errors afterwards, you should pay attention to this. Very often the likelihood hasn't gone to zero, so the lower limit should be the prior.
- This is relatively straightforward for single-parameter limits, but can be harder for more complicated excluded regions. Try not to get fooled…

Cosmic Microwave Background

- CMB is our current best tool in cosmology.
- Comes from looking at acoustic waves in early universe.
- Waves start in-phase at big bang, we see a snapshot when the universe becomes neutral at 380,000 years.
- Angular size tells us distance to this surface, peak heights tell us about physics (e.g. dark matter vs. baryons).
- CAMB will tell you what the curve looks like given model of universe. You can use WMAP data to see how well that model fits.
- Basic parameters are "distance" to CMB, regular matter density, dark matter, initial slope from inflation, initial amplitude, turn-on of first stars.
- MCMC can tell you what the universe looks like!
  NB - you won't get a good answer from WMAP on the first stars (optical depth to reionization).