Simulated Annealing Strategy in Chaotic Neural Network with Legendre Function

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Abstract: As to the monotonous activation funcation of chaotic neural network, a new transient chaotic neuron model is constructed by combining multiple Legendre polynomials and sigmoid functions as a new non-monotonic excitation function, and the dynamic characteristics of chaotic neurons are analyzed. A new transient chaotic neuron network model is constructed by using this model. This model improves the ability of the network to fall into local minima by using the idea of piecewise simulated annealing. The effectiveness of this model has been verified by the application of multi-user detectors in nonlinear function optimization, traveling salesman problem (TSP) and direct sequence-code division multiple access (DS-CDMA) on multi-user detector.

Key Words: Chaotic neural network, Legendre function, Code division multiple access, Traveling salesman problem

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1 INTRODUCTION

Hopfield introduces the energy function into the neural network, and the Hopfield neural network (HNN) constructed can solve the combinatorial optimization problem effectively. However, the Hopfield neural network using gradient descent method makes it easy to fall into a local minimum solution in the process of combinatorial optimization. In recent years, many scholars[1] are inspired by the internal randomness and global ergodicity of chaotic motion, and combine the characteristics of chaotic dynamics with Hopfield neural network to construct a new neural network to overcome the tendency of Hopfield neural network to fall into the problem of local minimum solutions. Based on the Aihara model, Hsu et al. proposed a chaotic network model with piecewise linear PWL function as the activation function[2]. He et al. proposed a new chaotic neural network model by introducing transient chaos and time-varying gain. Xu et al. and others proposed the wavelet[3] Hopfield neural network using the Morlet wavelet function as the activation function .The model has high accuracy in the optimization of nonlinear functions. Ye et al. combined the cubic polynomial of the Legendre function and the sigmoid function as a new activation function, and proposed a new chaotic neural network model[4] .Ye et al. used the combination of Bessel function and Sigmoid function as a new activation function, and proposed a new chaotic neural network model. Xiu et al. proposed an activation function composed of Gauss function and Sigmoid function, and proposed a new transient chaotic neural network model.This paper proposes a new transient neural network by introducing the high-order polynomial of the Legendre function, and adopts the optimization of non-linear function[5], traveling salesman problem[6] and direct spread sequence code division multiple access multi-user detector[6], to verify the effectiveness of the model.

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2 LEGENDRE FUNCTION CHAOTIC NEUR AL NETWORK MODEL

2.1 Legendre Function Chaotic Neuron

Multiple high-order Legendre function polynomials are combined with Sigmoid functions as a new transient chaotic neuron model, and this model is called Sigmoid-Legendre-Multiple-Function model, referred to as SMLF model, the single neuron model can be described as follows:

$$x(t) = f(y(t)) \tag{1}$$

$$y(t+1) = ky(t) - z(t)(x(t) - I_0) \tag{2}$$

$$z(t+1) = (1-\beta)z(t) \tag{3}$$

$$f(u) = \begin{cases} \sum_{i=1}^{n} \frac{\lambda_i}{2}(P_{2i+1}(u)+1) + (1-\sum_{i=1}^{n}\lambda_i)S(u), -1 \le u \le 1 \\ S(u), otherwise \end{cases} \tag{4}$$

$$s(u) = 1/(1+\exp(-u/\varepsilon_0)) \tag{5}$$

$$P_n(x) = \frac{1}{2^n \cdot n!} \cdot \frac{d^n}{dx^n}(x^2-1)^n \quad n = 0,1,2,\cdots \tag{6}$$

Where $x(t)$ is the activation function, that is, the output of the neuron at time; $y(t)$ is the internal state of the neuron at time t ; the value range of k is $0 \le k \le 1$,indicating the ability of the neuron to retain its internal state; $f(u)$ is the activation function of the model, which is composed of the polynomial of the Legendre function and the Sigmoid function; ε_0 is the steepness coefficient of the activation function; β is the simulated annealing parameter; $z(t)$ is the self-feedback Connection item; I_0 is a positive parameter; λ_i is a combination parameter, and the value range is $0 \le \lambda_i \le 1$.

The transient chaotic dynamics characteristics of neurons can be described by the inverted bifurcation diagram and the maximum Lyapunov exponent time evolution diagram. Since chaotic motion is extremely sensitive to the initial value conditions, the orbits produced by two similar initial values will evolve over time. It is separated exponentially, and the Lyapunov exponent is a quantitative description of the strength of the phenomenon. The calculation formula of Lyapunov exponent is as follows:

$$\lambda_L = \lim_{n\to\infty} \frac{1}{n} \sum_{n}^{n-1} \ln \left| \frac{dy(t+1)}{dy(t)} \right| \tag{7}$$

For the SMLF model, set i to 1 and 2, the chaotic neuron model has the following equations:

$$\frac{dy(t+1)}{dy(t)} = k - z(t)\frac{dx(t)}{dy(t)} \tag{8}$$

$$\begin{aligned}\frac{dx(t)}{dy(t)} = \frac{\lambda_1}{2}\frac{dP_3(y(t))}{dy(t)} + \frac{\lambda_2}{2}\frac{dP_5(y(t))}{dy(t)} + \\ \frac{1-\lambda_1-\lambda_2}{\varepsilon_0}(s(y(t)))(1-s(y(t)))\end{aligned} \tag{9}$$

$$\frac{dP_3(y(t))}{dy(t)} = \frac{1}{2}(15y(t)\wedge 2-3) \tag{10}$$

$$\frac{dP_5(y(t))}{dy(t)} = \frac{1}{8}(315y(t)\wedge 4-210y(t)\wedge 2+15) \tag{11}$$

The Lyapunov exponent can reflect the chaotic power of the model, and then reflect the strength of the global optimization ability of the network. In one-dimensional mapping, $\lambda_L > 0$ indicates that the model is in a state of chaos. The larger the value, the stronger the degree of chaos. $\lambda_L = 0$ means that the model is in a stable boundary state. $\lambda_L < 0$ indicates that the motion is stable and the model does not have a chaotic state.

In order to make the neuron show chaotic characteristics, set the parameters as follows:

$\varepsilon_0 = 0.02$, $y(1) = 0.1$, $z(1) = 0.98$, $k = 1$, $I_0 = 0.56$, $\beta = 0.001$, $\lambda_1 = 1/3$, $\lambda_2 = 1/100$

The time evolution graph of the maximum Lyapunov exponent time evolution graph and the inverted bifurcation graph of the neuron are shown in the Fig.1~Fig.2.

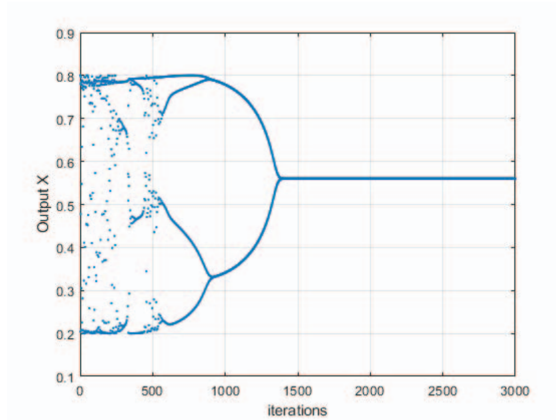

Fig 1. Inverted bifurcation diagram of neuron at time

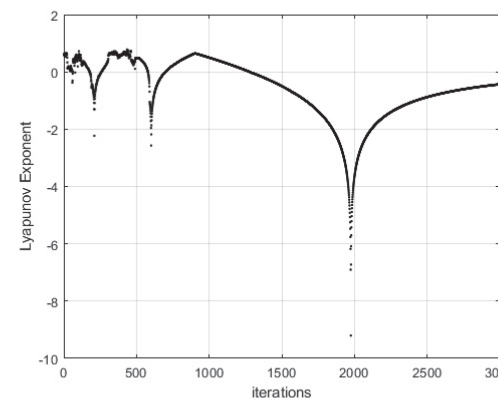

Fig 2. Time evolution diagram of the maximum Lyapunov exponent when $\beta = 0.001$ $\lambda_1 = 1/3$ $\lambda_2 = 1/100$

When the selected parameters are $\varepsilon_0 = 0.02$, $y(1) = 0.1$, $z(1) = 0.98$, $k = 1$, $I_0 = 0.56$, $\beta = 0.001$, $\lambda_1 = 1/3$, $\lambda_2 = 5/100$ the time evolution diagram and the inverted bifurcation diagram of the maximum Lyapunov exponent of the neuron are shown in Fig.3~Fig.4.

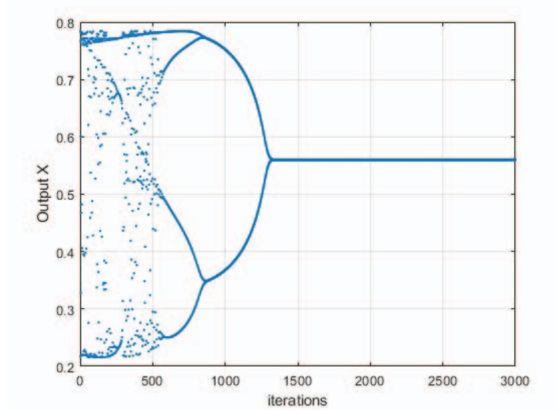

Fig 3.Inverted bifurcation diagram of neuron at time

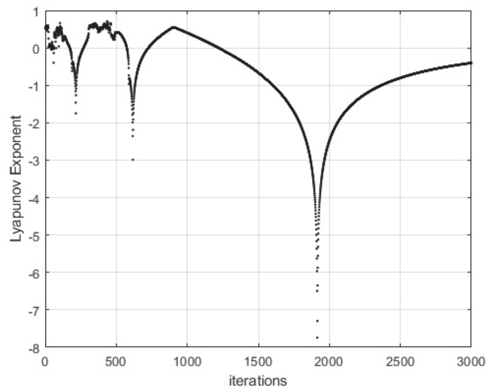

Fig 4. Time evolution diagram of the maximum Lyapunov exponent when $\beta = 0.001$ $\lambda_1 = 1/3$ $\lambda_2 = 5/100$

The time evolution diagram and the inverted bifurcation diagram of the maximum Lyapunov exponent of the neuron when the selected parameters $\varepsilon_0 = 0.02$, $y(1) = 0.1$, $z(1) = 0.98$, $k = 1$, $I_0 = 0.56$, $\beta = 0.001$, $\lambda_1 = 1/3$, $\lambda_2 = 1/10$ as shown in Fig.5~Fig.6.

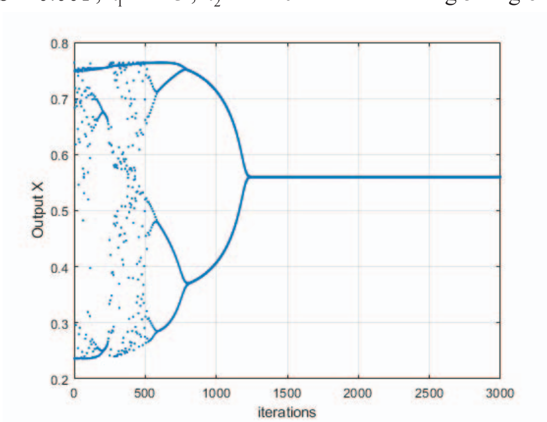

Fig 5.Inverted bifurcation diagram of neuron at time

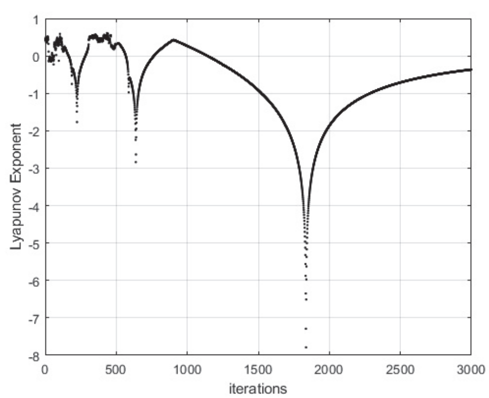

Fig 6. Time evolution diagram of the maximum Lyapunov exponent when $\beta = 0.001$ $\lambda_1 = 1/3$ $\lambda_2 = 1/10$

According to the above simulation experiment results, it can be found that this neuron model has transient chaotic dynamics behavior; λ_1 at a certain time, the value of λ_2 affects the chaotic state of the network. The larger the value, the easier the network will exit the chaotic state. When $\lambda_2 = 0$, the model degenerates to the SLF model in[4]; the self-feedback connection term $z(t)$ of chaotic neurons is controlled by the annealing parameter β, and the size of β is affects the decay rate of $z(t)$. The larger the value of β, the faster $z(t)$ decay. When $z(t) = 0$, The system reaches a stable state; therefore, the value of β will affect the chaotic state of the network.

2.2 SMLF Transient Chaotic Neural Network Model with Piecewise Linear Strategy

According to the above transient chaotic neuron model, the following transient chaotic neural network model is established:

$$x_i(t) = f(y_i(t)) \tag{12}$$

$$y_i(t+1) = ky_i(t) + \gamma\left[\sum_{\substack{i=1\\ j\neq i}}^{n} w_{ij}x_j(t) + I_i\right] - z_i(t)(x_i(t) - I_0) \tag{13}$$

$$z_i(t+1) = \begin{cases}(1-\beta_1)z_i(t), z_i(t) > \dfrac{3}{10}z(0)\\ (1-\beta_2)z_i(t), otherwise\end{cases} \tag{14}$$

$$f(u) = \begin{cases}\displaystyle\sum_{i=1}^{n}\frac{\lambda_i}{2}(P_{2i+1}(u)+1) + (1-\sum_{i=1}^{n}\lambda_i)S(u), -1 \le u \le 1\\ S(u), otherwise\end{cases} \tag{15}$$

$$s(u) = 1/(1+\exp(-u/\varepsilon_0)) \tag{16}$$

$$P_n(x) = \frac{1}{2^n \cdot n!}\cdot\frac{d^n}{dx^n}(x^2-1)^n \quad n = 0,1,2,\cdots \tag{17}$$

In this model,the term $y_i(t)$ is the internal state of the i-th neuron at time t, $x_i(t)$ is the activation function, that is, the output of the first neuron at time t, $f(u)$ is the activation function of the network; β_1, β_2 are Segmented simulated annealing parameter; w_{ij} is the weight of neuron j and i, where $w_{ij} = w_{ji}$, $w_{ii} = 0$; λ is the combined parameter, and the value range is $0 \le \lambda \le 1$; ε_0 is the steepness coefficient of the activation function; I_i is the input deviation of the neuron; k is the damping factor of the neural diaphragm, $0 \le k \le 1$, which reflects the ability of the network to retain memory or forget the internal state; γ is the input positive scale parameter representing the energy function of the chaotic dynamics The influence of characteristics; I_0 is a positive parameter.

3 APPLICATION OF SMLF TRANSIENT CH AOTIC NEURAL NETWORK MODEL IN OPTIMIZATION PROBLEMS

3.1 Application in Function Optimization

Choose the following optimization function [5]:

$$\begin{aligned}f(x_1,x_2) = {} & (x_1-0.7)^2\left[(x_2+0.6)^2+0.1\right] + \\ & (x_2-0.5)^2\left[(x_1+0.4)^2+0.15\right]\end{aligned} \tag{18}$$

The minimum value of the above optimization function is 0; the minimum point is (0.7,0.5); the local minimum point is (0.6,0.4) and (0.6,0.5). We set parameters as follows: $\varepsilon_0 = 0.02$, $k = 1$, $\gamma = 0.1$, $I_0 = 0.56$, $\beta_1 = 0.001$, $\beta_2 = 0.003$, $y(1) = 0.383$, $y(2) = 0.283$, $z_1 = z_2 = 0.98$, $\lambda_1 = 1/3$, $\lambda_2 = 1/100$

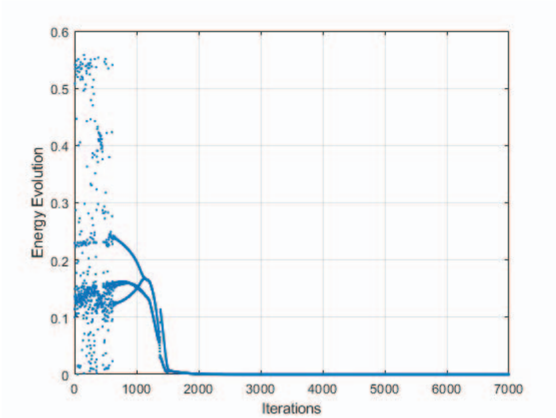

Fig 7. The time evolution diagram of the time energy function

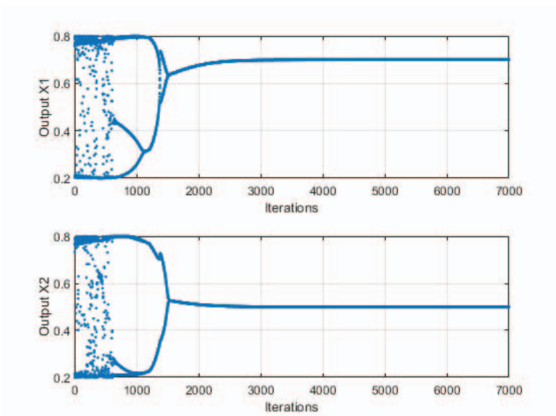

Fig 8. The optimal solution of x_1 , x_2 when $\lambda_2 = 1/100$

The energy function[5] evolution diagram of the neural network model to solve the optimization function is shown in Fig.7, and the time evolution diagram of x_1 and x_2 is shown in Fig.8.

Set $\lambda_2 = 5/100$ and $\lambda_2 = 1/10$,other parameters remain unchanged, and study the influence of λ_2 on the network, and the time evolution diagram of x_1 and x_2 is shown in Fig.9~Fig.10.

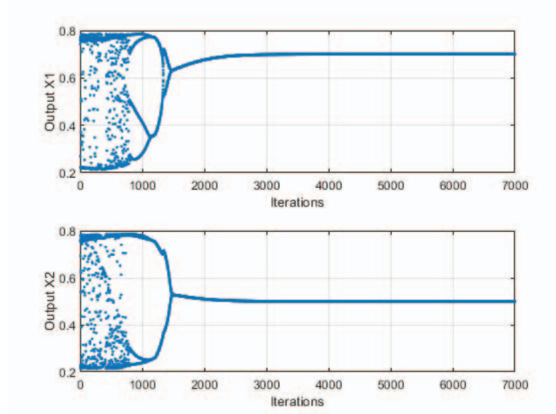

Fig 9. The optimal solution of x_1 , x_2 when $\lambda_2 = 5/100$

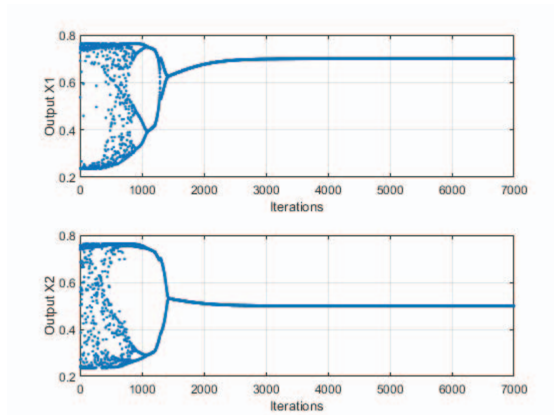

Fig 10. The optimal solution of x_1 , x_2 when $\lambda_2 = 1/10$

From Fig.8~Fig.10, the value of λ_2 affects the time for the neuron to exit the chaotic state. The larger the value of λ_2 , the easier the neuron to exit the chaotic state.When $\lambda_2 = 1/100$, x_1 , x_2 only needs to run about 1500 times to exit the chaotic state. The influence of the value of λ_2 on the network is consistent with the influence of the value of λ in the SLF model [4].

3.2 Application to 20-City TSP

In the field of combinatorial optimization, TSP problem[8] is a very typical problem. Due to the difficulty of its solution, many scholars have been looking for an efficient and practical method to solve this problem. The TSP problem can be simply described as "finding a shortest loop containing all n cities, where each city must be visited only once". $(n-1)!/2$ is the number of possible paths for TSP problems[9] in n cities.

In this paper, the SMLF transient chaotic neural network model is applied to solve the TSP problem[10] of 20 cities. The simulation results show that this model has a good ability to solve the TSP problem. Take the energy function[4] as:

$$E=\frac{A}{2}\sum_{i=1}^{n}(\sum_{i=1}^{n}x_{ij}-1)^2+\frac{B}{2}\sum_{j=1}^{n}(\sum_{i=1}^{n}x_{ij}-1)^2 +\frac{D}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n}d_{ik}x_{ij}(x_{k,j+1}+x_{k,j-1}) \tag{19}$$

Among them, x_{ij} refers to the visit of city i in order j, the parameter $A=B$, d_{ik} represents the distance between city i and city k, and the shortest effective path is equivalent to a global minimum energy value. In order to facilitate comparison, the normalized 20 city coordinates in document [4]are selected as follows:

(0.41,0.94); (0.37, 0.84), (0.54, 0.67); (0.25, 0.62); (0.07, 0.64); (0.02, 0.99); (0.68, 0.58); 0.71,0.44); (0.54, 0.62); (0.83, 0.69); (0.64, 0.60); (0.18, 0.54); (0.22, 0.60); (0.83, 0.46); (0.91,0.38); (0.25, 0.38); (0.24, 0.42); (0.58,0.69); (0.71,0.71); (0.74,0.78);

In the SMLF chaotic neural network model, the parameters are set as follows:

$A=B=1$, $D=2$, $\gamma=0.4$, $k=1$, $I_0=0.5$, $\varepsilon=0.02$, $z(1)=0.8$, $\beta_1=0.01$, $\beta_2=0.03$, $\lambda_1=2/3$, $\lambda_2=1/100$

The program runs a total of 5 times,and 100 different initial conditions of are generated randomly in the interval [-0.1, 0.1] .The results are summarized in table 1.

Table1. Results of SMLF Model Solving TSP Problem in 20 Cities

Legal path	Optimal path	Legal ratio(%)	Optimal ratio(%)
80	74	80	74
82	80	82	80
76	73	76	73
80	77	80	77
79	77	79	77

The results show that the solution ability of this model is better than the STL chaotic neural network model in solving the 20-city TSP problem.

In the STL chaotic neural network model, take the parameters as follow:

$A=B=1$, $D=2$, $\gamma=0.4$, $k=1$, $I_0=0.5$, $\varepsilon=0.02$, $z(1)=0.8$, $\beta=0.002$, $\lambda=1/3$

The program runs 5 times, each time the initial value is randomly assigned 100 times in the interval (-0.1, 0.1). The simulated results are shown in Table 2.

Table2. Results of SLF Model Solving TSP Problem in 20 Cities

Legal path	Optimal path	Legal ratio(%)	Optimal ratio(%)
64	62	64	62
61	59	61	59
65	63	65	63
57	56	57	56
72	69	72	69

The simulation experiment shows that when solving the TSP problem of 20 cities, the solution ability of the SMLF neural network model is better than that of the SLF neural network model.

The shortest distance of the TSP problem in 20 cities is 3.1934, as shown in Fig.11.

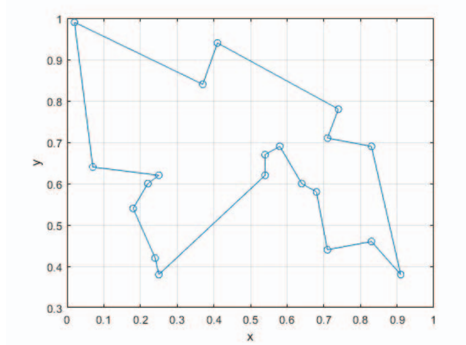

Fig 11. The shortest path of 20 cities TSP normalized coordinates

4 APPLICATION ON DS-CDMA MULTI-USE R DETECTOR

At present, the most widely used code division multiple access technology is Direct Sequence Code Division Multiple Access (DS-CDMA)[11], which uses pseudo-random sequences to spread the transmitted information to a wider frequency band, and the receiving end uses the same pseudo-random sequence as the transmitter despreads the received information and restore the original information. The principle of the multi-user detector is to demodulate the information of a specific user from the interfering digital information at the receiving end. In recent years, many scholars have used the idea of converting CDMA[12] problems into combinatorial optiization problems to propose new multi-user detectors[13] for chaotic neural networks. In this paper, a new transient neural network model constructed in this paper is used to conduct a simulation experiment comparing with HNN and HCNN multi-user detectors in a synchronous transmission CDMA[14] system.

Verdue transforms the multi-user detection problem into a linear programming problem, and proposes that the best multi-user detector can be obtained by the maximum likelihood (ML) criterion under the condition of equal probability of the transmitted signal prior. The detector will find the best one among 2^k possible transmission messages sent by k users. In the synchronous transmission CDMA system, it can be described as follow:

$$\mathbf{d}=\arg\min_{b\in(-1,1)^k}\left[-2\mathbf{y}^\top\mathbf{Ab}+\mathbf{b}^\top\mathbf{ARAb}\right] \tag{20}$$

$$r(t)=\sum_{m=1}^{M}A_m-b_mc_m(t)+n(t) \tag{21}$$

Where $\mathbf{R}$ is the cross-correlation matrix of the system, $\mathbf{R}_{ij}=\frac{1}{T}\int_0^T c_i(t)c_j(t)dt$; $c_i(t)$ is the energy-normalized spread spectrum signal of user i ; $n(t)$ is the additive white Gaussian noise with unit power spectral density; $r(t)$ is the signal received by the receiver during the synchronization system transmission; and A_m is the m-th The amplitude of the signal received by the user; M is the number of users;

b_m represents the information transmitted by the m-th user; $b \in (-1,1)$ is the information bits sent by the m-th user.

The SMLF chaotic neural network model is used to realize the DS-CDMA multi-user detector, in which the weight matrix of the network is set to $-\mathbf{ARA} - \mathbf{AR}_{diag}\mathbf{A}$, where $\mathbf{R}_{diag}$ is the diagonal matrix $\mathbf{R}$; $\mathbf{y}$ is set to the bias vector of the network. The number of users is $M = 20$, The Gold pseudo-random sequence with a spreading gain of 31 is used for spreading, and the transmitted information vector $\mathbf{b}$ is randomly generated. The average bit error rate refers to the percentage of error information in the total amount of information in a data transmission process. The signal-to-noise ratio refers to the ratio of signal to noise in an electronic system.The parameters of SMLF are as follows:

$a(0) = b(0) = 0.2$, $\varepsilon = 0.04$, $c(0) = 0.05$, $u_0 = 0.65$, $k = 0.7$, $\gamma = 0.001$, $\beta = 0.003$, $\beta_1 = 0.001$, $\beta_2 = 0.06$

The signal-to-noise ratio gain varies from 10db to 50db, and 200 simulation experiments are carried out. The results of the signal-to-noise ratio and average bit error rate of each algorithm are shown in Fig.12.

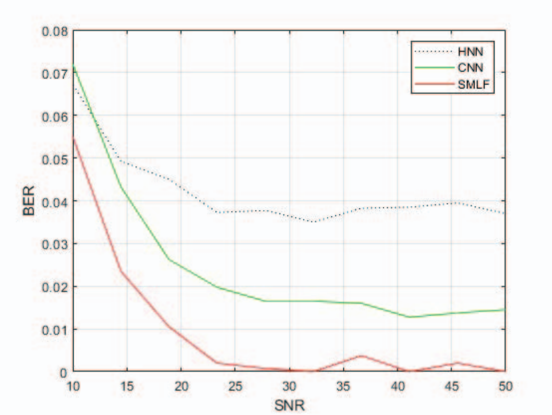

Fig 12. Simulation results of the anti-interference ability of each algorithm

The simulation result shows that the SMLF network model has strong practicability and has strong anti-noise interference ability in the DS-CDMA multi-user detector.

5 CONCLUSION

In this paper, the higher-order polynomial of Legendre basis function is introduced into Chen's transient chaotic neural network, and the linear combination of the higher-order polynomial of Legendre basis function and Sigmoid function is used to form a new activation function to construct a new transient chaotic neural network. This model is applied to solve optimization functions, traveling salesman problems, and DS-CDMA multi-user detectors, and the influence of parameters on the network is also discussed. The simulation results show that this model has a strong ability to optimize network performance.

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