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The Study of Chaotic Neural Network and its Applications in Associative Memory*

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Abstract. Based on an analysis of current principal chaotic neural network models and their applications in information processing, we propose a one-dimensional, two-way coupled map network and a modified definition of an auto-associative matrix. The two-way coupled map network overcomes the weakness of the globally coupled map network and has the same abilities in pattern classification. Numerical simulation experiments have shown that the associative success rate and recall speed of our modified definition of the auto-associative matrix are an improvement them over existing methods. Moreover, the associative recall process of the network is analyzed in detail and explanations of improvement are given, basd on our theoretical analysis.

Key words: associative memory, chaos, chaotic neural networks

1. Introduction

Recently, chaotic dynamical behavior has attracted a great deal of attention in many research fields, and a great deal of progress in chaotic study has been made. Many people are certain that chaotic dynamic behavior plays an important role in real neurons and neural networks. Many researchers have attempted to model artificial neural networks with chaotic dynamics on the basis of deterministic differential equations or stochastic models. Aihara et al. [1] have proposed deterministic difference equations which describe an artificial neural network model composed of chaotic neurons. This model has advantages in terms of computational time and memory for numerical analyses due to the spatiotemporal complex dynamics of the neurons. Ishii et al. [2] have proposed a modified symmetric global coupled map (S–GCM) and applied the model to information processing applications, such as associative memory. Moreover, Inoue et al. [3] have presented a chaotic neuro-computer in which a neuron is composed of a pair of coupled oscillators. The neuro-computer runs on a deterministic rule, but it is capable of stochastic searching and solving difficult optimization problems.

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Ishii's modified global coupled chaotic system [2] and Inoue's chaotic neuro-computer [3] are two main current chaotic neural networks used for pattern recognition and associative memory. Based on an analysis of these chaotic neural network models and their applications in information processing, we propose a one-dimensional, two-way coupled map network and a modified method of defining the auto-associative matrix. Section 2 of this paper, reviews the globally coupled map chaotic model and its construction for associative memory. In order to overcome a fundamental weakness in the integration of hardware for the globally coupled map network, the one dimensional, two-way coupled map network is proposed. It is demonstrated that the two-way coupled map network has the same functionality as the globally coupled map network and can be extended to many applications in information processing. In Section 3, the self-association covariance matrix of the chaotic neuro-computer is improved, and a new definition of self-association covariance matrix is presented, based on a modification of Hebb's rule.

2. Associative Memory Chaotic Neuron Systems

2.1. SYMMETRIC GLOBALLY COUPLED MAP

A symmetric globally coupled map (S–GCM) is defined as

$$x_i(n+1) = (1-\varepsilon) f(x_i(n)) + \frac{\varepsilon}{N} \sum_{i=1}^{N} f(x_i(n)), \tag{1}$$

$$f(x) = \alpha x^3 - \alpha x + x, \quad x \in [-1, 1]. \tag{2}$$

In Equation (1), $x_i(n)$ denotes the ith unit's value at time n, and N is the number of units. Each unit's dynamic is almost entirely given by the asymmetric cubic map described by Equation (2). The asymmetric cubic map is a cubic function and has two extrema in its range when $\alpha > 2$. Therefore, the asymmetric cubic map has at most two attracting periodic orbits. Figure 1 shows the function shape of the asymmetric cubic map. From the paper by Shin Ishii [2], Figure 1(b) shows the symmetric bifurcation diagram of the asymmetric cubic map over parameter α. From the figure, we can see that the neuron is in a chaotic state when $\alpha = 3.4$. The portion described as a sum in Equation (1) is defined as feedback from the average of all of the units. The S–GCM has two parameters, α and ε. When these parameters are set to specific values, the system falls into attractors called 'cluster frozen attractors'. When the system is attracted into one of the cluster frozen attractors, the units belonging to the same cluster come to adopt an identical orbit.

The general strategy of constructing association memory by S–GCM is as follows: An N-dimensional binary coding function C, which converts a state vector $\mathbf{x} \in [-1, 1]^N$ to a binary vector $\mathbf{C}(\mathbf{x}) \in \{-1, 1\}^N$ is defined as:

$$\mathbf{C}(\mathbf{x})_i = \begin{cases} 1, & x_i \geq x^*, \\ -1, & \text{otherwise,} \end{cases}$$

where x^* denotes the stationary point of the S–GCM, which is equal to 0. Using this binary coding function $\mathbf{C}$, an S–GCM state can be translated into an N-bit binary representation.

The N-dimensional function $\mathbf{V}$ converts a binary vector $\mathbf{I} \in \{-1, +1\}^N$ into a state vector $\mathbf{V}(\mathbf{I}) \in [-1, 1]^N$. Function $\mathbf{V}$ is defined as follows:

$$\mathbf{V}(\mathbf{I})_i = \begin{cases} x^+ + \text{rand}, & \mathbf{I}_i = 1, \\ x^- + \text{rand}, & \mathbf{I}_i = -1, \end{cases}$$

where x^+ and x^- denote the two, two-cycle periodic solutions of the asymmetric cubic map, namely $f(x^+) = x^-$ and $f(x^-) = x^+$. The notation rand represents a small random value.

When the S–GCM is regarded as an associative memory system which processes an N-dimensional binary vector $\mathbf{I} \in \{-1, +1\}^N$ to an N-dimensional binary output vector $\mathbf{O} \in \{-1, +1\}^N$, the general strategy of S–GCM for associative memory is

$$\mathbf{I} \xrightarrow{V} x(0) \xrightarrow{\text{S–GCM}} x(T) \xrightarrow{C} O.$$

S–GCM has two working modes, namely unit map and random evolution, or the preserving mode and the destroying mode. The two modes are all global. If the parameters of each neuron could be controlled, the two modes can switch partially.

Let $\left\{X^1, X^2, \cdots, X^T | X^p \in \{-1, +1\}^N\right\}$ be a set of memorized patterns, where X_i^p denotes the ith element value in the pth memorized pattern and T the number of memorized patterns. A covariance matrix of the set of memorized patterns is defined by

$$\sigma_{ij} = \frac{1}{N} \sum_{p=1}^{T} X_i^p X_j^p. \tag{3}$$

The basic idea of designing associative memory systems with S–GCM is that the memory process of memorized patterns converts into an evolutionary process of the coupled system:

$$x_i(t+1) = (1-\varepsilon) f_i\left(x_i(t)\right) + \frac{\varepsilon}{N} \sum_{j=1}^{N} f_i\left(x_j(t)\right), \tag{4}$$

$$f_u(x) = \alpha_i x^3 - \alpha_i x + x, \tag{5}$$

The evolution of α_i is defined as

$$\alpha_i - \alpha_i + (\alpha_i - \alpha_{\min}) \tanh (\beta E_i), \tag{6}$$

$$E_i = -x_i \sum_{j=1}^{N} \sigma_{ij} x_j, \tag{7}$$

where $\alpha_{\min}$, β, ε are constant parameters. In the experiments below, they are set as $\alpha_{\min} = 3.4$, $\beta = 2.0$ and $\varepsilon = 0.1$. In Equation (6), each α_i is controlled to lie between $\alpha_{\min}$ and $\alpha_{\max} = 4.0$. E_i is the conventional energy function, denoting the ith partial energy. If E_i is high and positive, which means the ith unit value does not suit the covariance matrix, α_i changes according to Equation (6), and the unit becomes disturbed. During the course of this disturbance, the processing mode is changed from the preserving mode to the destroying mode. The unit is disturbed enough to make a chaotic motion, which enables the unit to search for the proper state within the range. In some sense, Equation (6) affects x_i through α_i in indirect mode.

2.2. TWO-WAY COUPLED MAP CHAOTIC NEURAL NETWORK

The S–GCM has substantial power. When it increases nonlinearly, the system can successfully transform between coherent phase, ordered phase, intermediate phase and turbulent phase. In the phase space of the intermediate state, an attractor arises that has a dynamical tree structure. The character of the S–GCM has played an important role in information storage and associative memory.

The S–GCM has a fatal weakness, however, which is the global coupling of the network, which implies a connection between any two neurons, and this connection results in a rapid increase of network connecting lines as the number of neurons increases. This is a major abstracte to the integration of network hardware. For this reason, based on an analysis of S–GCM, we have made a few modifications and proposeed a nearest-neighbour, two-way coupled map (TCM) network. Its mathematical evolution equations are described as follows:

$$\begin{aligned} &x_i(n+1) = (1 - r_i - r_2) f\,(x_i(n)) + r_1 f\,(x_{x-1}(n)) + r_2 f\,(x_{i+1}(n))\,, \\ &i = 2, 3, \cdots, N-1. \end{aligned} \tag{8}$$

The asymmetric cubic map function is chosen to express each neuron's dynamics,

$$f(x) = \alpha x^3 - \alpha x + x, \quad x \in [-1, 1]. \tag{9}$$

The parameter α is chosen within the chaotic region; the open boundary condition is chosen as boundary condition

$$x_1(n+1) = (1 - r_2) f\,(x_1(n)) + r_2 f\,(x_2(n))\,, \tag{10}$$

$$x_N(n+1) = (1 - r_1) f(x_N(n)) + r_1 f(x_2(n)) . \quad (11)$$

When r_1 and r_2 are within a definite range, the system will show a stable coherent chaotic state, that is, each neuron of the system runs in chaotic dynamics, but because of a coupling controlling effect among the neurons, the final movement of the system is uniform in space and periodic in time.

Compared to S–GCM, TCM demands more modifications in integration of hardware. As the number of neurons increases, the number of connecting lines decrease greatly. For a coupling system of N neurons, the S–GCM-to-TCM ratio (in terms of number of connecting lines) is $S_{GCM}/S_{TCM} = (N-1)/2$. Besides this, it should be pointed out out that, with the increase of the number of neurons in the S–GCM network, the length of connecting lines between remote neurons increases, which results in time delay and a parasitic capacity effect. Researchers have proved from theoretical analyses that, although the value of parasitic electric capacity is very small, it can completely change the dynamic behavior of the time-delay S–GCM network, causing difficulties in its implementation and use. By contrast, within the TCM network, because each neuron is only connected with neighboring neurons, the harmful effects mentioned above are much less pronounced.

The spatiotemporal features of the TCM attractors are mainly determined by the parameters values; α is the bifurcation parameter of the asymmetric cubic map function and indicates the strength of each neuron's chaos, ε indicates the strength of the couplings. This model has two conflicting aspects; the chaotic instability in each neuron makes coherence disappear, but mutual couplings make coherence recur once again. Therefore, roughly speaking, as α becomes large, the TCM becomes chaotic, and as ε becomes large, the TCM becomes coherent, or stable. When studying the dynamics of TCM, we have discovered the following four phases:

1. Coherent Phase. When α is small and ε is large, all the neurons fall into the same orbit.
2. Ordered Phase. In this phase the system falls into cluster frozen attractors. The number of clusters primarily increases as the series 2, 4, 8, 16, ... as α increases or ε decreases.
3. Partial Ordered Phase. Attractors fall into a large number of clusters in some cases, and fall into a small number of clusters in other cases. Namely, the TCM attractors vary depending on their initial states (coexistence of many-cluster and few-cluster attractors).
4. Turbulent Phase. When α is large and ε is small, 'chaos' is stronger than 'order', and each neuron follows its own chaotic orbit.

The TCM model is defined in Equations (6)–(9). When these parameters are set to specific values and their initial values are randomly set within the range of –1 to 1, the system falls into one of several stable states called 'cluster frozen attractors' after some map iterations of the network. When the system is attracted into one of the cluster frozen attractors, the units belonging to the same cluster come to adopt

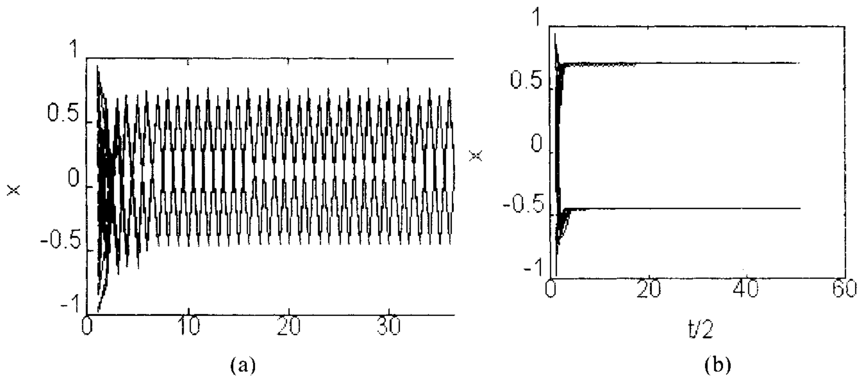

Figure 1. Two-cluster frozen attracto ($\alpha = 3.4$, $r_1 = 0.75$, $r_2 = 0.06$). (a) The time series of all the neurons are plotted when their initial values are randomly set. (b) The same time series as in (a) are plotted for every two steps ($t = 2, 4, 6, \ldots$). Note that the number of neurons is 40, and there are 40 lines drawn in each figure.

an identical orbit. Here, each 'identical orbit' is typically a two-cycle periodic orbit like those shown in Figure 1, and sometimes four-cycle, eight-cycle, and so on, depending on the parameter values and initial state. It can also be a chaotic orbit.

Figure 1(a) shows the time series of all the units when their initial values are randomly set within the range –1 to 1, where $\alpha = 3.4$, $r_1 = 0.75$, $r_2 = 0.06$, and the number of neurons $N = 40$. Figure 1(b) shows the same time series for every two steps ($t = 2, 4, 6, \ldots$). Even if all the neurons initially vary in their values, they come to split into two clusters and neurons belonging to the same cluster come to adopt an identical orbit, i.e., a two-cycle periodic orbit in this case.

The basic controlling mechanism is that the uniform state of systematic final movement is reached by coupling control between neighboring neuron coherence. Zhang et al. [7] simply proved that, although the TCM network's opening border condition is chaotic in time, each neuron is uniform in space, namely, its unitary solution is as follows,

$$x_i(n) = x^*, \qquad x(n+1) = f(x^*).$$

The classification function of the TCM network plays an important role in information processing. Classification is a key technology in many fields, such as pattern recognition and database retrieval. Zhang et al. [7] extend the concept of memorizing information by controlling chaos with the TCM network, and further develop the application of the TCM network in the field of data encoding for secure communication.

We also construct an associative memory system by TCM, using a general strategy of constructing associative memory, achieving associative memory for partial information to some extent. However, compared to the S–GCM and the Hopfield network, the associative ability still needs to be improved. We shall not deal with this aspect further in the present paper. However, continuous, thorough study, will expand and improve the associative memory and other information processing applications using chaotic dynamics, demonstrating its superiority over the Hopfield network. The associative memory concept shows that an efficient search is achieved with strong chaos at an early stage of association. However, chaos becomes suppressed at the end of the association. This 'annealing'-like mechanism seems to match our intuition concerning memory retrieval.

3. Associative Memory Chaotic Neuron Systems

3.1. INOUE'S CHAOS NEURO-COMPUTER

Inoue's chaos neuro-computer network is composed of several processing units (neurons), each processing element (neuron) being composed of two chaos oscillators which are coupled to each other. The equation of motion of the coupled oscillator can be expressed by a discrete one-dimensional map function, viz., $f(x)$ (the first map) and $g(x)$ (the second map), their coupling relations and iteration evolution process, as follows:

$$x_i(n+1) = \frac{((1 + D_i(n)) * f(x_i(n)) + D_i(n) * g(y_i(n))}{(2 * D_i(n) + 1)}, \tag{12}$$

$$y_i(n+1) = \frac{((1 + D_i(n)) * g(y_i(n)) + D_i(n) * f(x_i(n))}{(2 * D_i(n) + 1)}, \tag{13}$$

where $D_i(n)$ is the coupling coefficient between two oscillators in the ith neuron at time n, and $x_i(n)$ and $y_i(n)$ are the variables of the first and the second oscillators in the ith neuron at time n, respectively.

For simplicity, the first and the second map adopt the same map, namely $f(x) = g(x)$, and coupling coefficient $D_i(n)$ is a constant. The logistic map is chosen for $f(x)$ and $g(x)$, as expressed by Equation (3) and Equation (4) of [3], respectively.

The state of the ith neuron at time n, which is denoted by $u_i(n)$, is defined by

$$u_i(n) = 1 \text{ (excitation), if } |x_i(n) - y_i(n)| < \varepsilon, \tag{14}$$

$$\quad = 0 \text{ (inhibition), otherwise}, \tag{15}$$

where ε is the criterion parameter of the synchronization. When $\varepsilon = 0$ and additionally, the parameters of the two maps are set the same, the excitor $u_i(n) = 1$ shows that the two oscillators completely synchronize with each other. Once complete synchronization occurs, the neuron is always in the same state, even when the coupling coefficient $D_i(n)$ becomes small. The parameters of the two oscillators are chosen to have somewhat different values, in order to avoid complete synchronization.

All neurons are connected to each other with coupling constant w_{ij}. The state of the neuron has an influence on $D_i(n)$ through the medium of the connection. The relation between w_{ij} and $D_i(n)$ plays an important role in chaos neuro-computer, and it is expressed by

$$DD_i(n) = \sum_i w_{ji} u_j(n) + s_i + \theta_i, \tag{16}$$

$$\begin{aligned} D_i(n) &= DD_i(n), \text{ if } DD_i(n) > 0, \\ &= 0, \text{otherwise}, \end{aligned} \tag{17}$$

where s_i is the external input and θ_i is the threshold value.

Let $\{X^1, X^2, \cdots, X^T | X^p \in \{-1, +1\}^N\}$ be a set of memorized patterns, where X_i^p denotes the ith element value in the pth memorized pattern and T the number of memorized patterns. A covariance matrix of the set of memorized patterns in Inoue's chaos neuro-computer is defined by

$$w_{ij} = \sum_{p=1}^{T} (2 * X_i^P - 1)(2 * X_j^P - 1), \tag{18}$$

but with $w_{ii} = 0$.

3.2. MODIFIED CHAOS NEURO-COMPUTER

As the reader may known, Hopfield et al. [4, 5] showed that discrete Hopfield neural networks can be used for associative memory and difficult optimization problems, the archetype being the 'traveling salesman problem'. Therefore, in our work, a chaotic neuron system similar to discrete Hopfield neural networks (in terms of topological structure) has been devised for the associative memory application. It has also been studied and developed for intelligent information processing.

In analyzing Ishii's coupled chaotic system, we found that its main problem is the slower recall speed of associative memorizing. On the other hand, the success rate of associative memory is lower for Inoue's chaos neuro-computer. For this reason, we merge Ishii's coupling chaotic system with Inoue's chaos neuro-computer. Furthermore, we improve the definition of the self-association covariance matrix, and propose a new definition for it.

A chaotic neural network system can be denoted by a weighed nondirectional graph. Each neuron is composed of two chaotic oscillators which are coupled to each other. The equation of motion of the coupled oscillator can be expressed by a discrete one-dimensional map function, indicated as $f(x)$ (the first map) and $g(x)$ (the second map), respectively. Their coupling relations and iterative evolution process are the same as in Equations (12) and (13). The state of the ith neuron at time n, which is denoted by $u_i(n)$, is also defined by Equations (14) and (15). All neurons are connected to each other with coupling constant w_{ij}. The relations

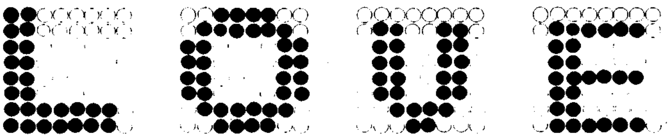

Figure 2. The stored patterns L, O, V, E.

between w_{ij} and $D_i(n)$ are the same as in Equations (16) and (17). However, the first map $f(x)$ and the second map $g(x)$ all use an asymmetric cubic map.

Associative memory systems must be able to output the target memorized pattern from an input that is relatively close to the target. Our associative memory systems employ the following covariance matrix, namely, the learning method is similar to the conventional covariance type. Our auto-associative covariance is a modification of Hebb's rule. Our definitions of the auto-associative covariance matrix follow.

Let $\overline{x^p}$ denote the spatial average value of the pth pattern. The symmetric self-associative covariance matrix which memorizes T patterns is then defined as

$$w_{ij} = \sum_{p=1}^{T} \left(x_i^p - \overline{x^p}\right)\left(x_j^p - \overline{x^p}\right), \tag{19}$$

with $w_{ii} = 0$.

In the following simulation experiments, it is shown that our proposed system has improved the associative memory success rate compared with Inoue's chaos neuro-computer. On the other hand, it has increased associative memory recall speed compared with Ishii's chaotic element network.

4. Simulating Experiments

The letters L, O, V, E are stored in an 8×8 square lattice, as shown in Figure 2, where the black and white points correspond to $u_i(n) = 1$ and $u_i(n) = 0$, respectively.

In Inoue's chaos neuro-computer, the coupled oscillators of the neurons adopt a logistic map, and their parameters are set as $a = 4.0$, $b = 3.995$ and $\varepsilon = 0.01$, where a and b are parameter r values of the first oscillator and the second oscillators corresponding to Equations (3) and (4) of [3], respectively. In our chaotic neural network, the coupled oscillators of the neuron adopt as asymmetric cubic map, and their parameters are chosen as $a = 3.4$, $b = 3.399$ and $\varepsilon = 0.02$, where a and b are parameter r values of the first oscillator and the corresponding second oscillator (see Equation (2)). Under the condition of the above parameters settings, these oscillators are all in chaotic states. Our system's symmetric auto-associative covariance matrix which memorizes T patterns is defined as Equation (19). One-step self-association of V and O from sufficient partial information is shown in

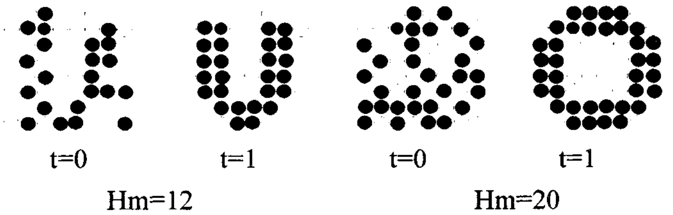

Figure 3. The association of V and O from partial self-information.

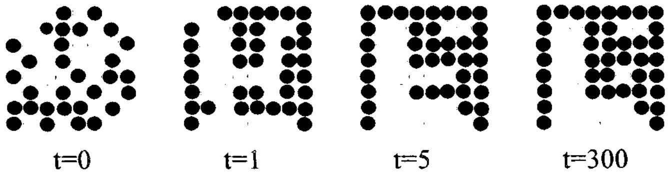

Figure 4. Inou's computer association of O from partial self-information (Hm = 20).

Figure 3, where Hm denotes the Hamming distance between partial information and complete information. Similar associations have been obtained for L and E. Figure 4 shows the associative process of O from the same partial information as Figure 3 when Inoue's chaos neuro-computer is adopted. The results of the experiments show that complete information cannot be recovered using Inoue's method. Next, Ishii's chaotic elements network is inspected, under the same conditions. Figure 5 shows self-association of O in inverse pattern from sufficient partial information in 250 steps. These experimental results have shown that although Ishii's chaotic elements network can recover complete information with anti-color, it needs many steps. However, our modified system can correctly recover complete information with only one step. It has been demonstated adequately that our chaotic system is a great improvement over exiting methods, in terms of associative success rate and recall speed. This implies that our system dynamic is superior to that of current chaotic neuron networks. Accordingly, in our systems, an efficient search is achieved with strong chaos at an early stage of association. However, chaos is suppressed at the end of the association.

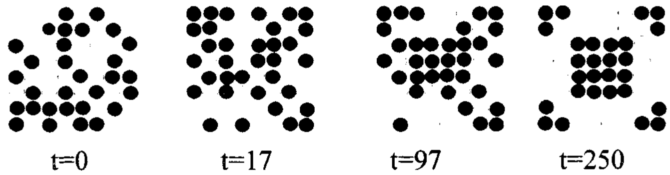

Figure 5. Ishii's network association of O from partial self-information (Hm = 20).

5. Discussion

Why does our system work well, in terms of success rate and recal speed? An interpretation of the mechanism is that chaotic dynamics and modification of the auto-covariance matrix play important roles in the rapid and unbiased access to stored patterns. Next, we analyze the evolution process of our system from the point of view of the energy function.

The ith neuron partial energy E_i is defined as follows:

$$E_i = -\frac{1}{2}u_i \sum_{j=1}^{N} w_{ij}u_j + \theta_i u_i \tag{20}$$

Since $E = \sum_{i=1}^{N} E_i$ is equivalent to the conventional energy function, we call E_i the ith partial energy. Our system searches for a local minimum of the energy function by making each partial energy small and negative, as follows: if E_i is high and positive, which means the ith neuron value does not suit the covariance matrix $[w_{ij}]$, it will cause a change in the coupled coefficient $D_i(n)$ of the ith neuron. The changing law is as described below.

According to Equations (16) and (19), the following equation can be derived:

$$\begin{aligned} DD_i(n) &= \sum_j w_{ji}u_j(n) + s_i + \theta_i \\ &= \sum_j \sum_{p=1}^{T} \left(x_i^p - \overline{x^p}\right)\left(x_j^p - \overline{x^p}\right) u_j(n) + s_i + \theta_i . \end{aligned} \tag{21}$$

If the value of the ith neuron in correctly the recalling pattern is 1, for the neuron of wrong value, from Equations (20) and (21), it could be shown that $DD_i(n)$ must be increased if the partial energy decreases, and then the $D_i(n)$ is greater than 0. From Equations (12) and (13), the following results can be derived: $|x_i(n+1) - y_i(n+1)| = |x_i(n) - y_i(n)|/(2 * D_i(n) + 1)$. From above equation, it can be seen that $|x_i(n) - y_i(n)|$ will continue to decrease with increasing numbers of steps n. When $|x_i(n) - y_i(n)| < \varepsilon$, the value of the ith neuron will change or stay at 1, according to Equation (14).

By contrast, if the value of the ith neuron in correctly recalling the pattern is 0, for the neuron of wrong value, from Equations (20) and (21), it could be shown that $DD_i(n)$ must be decreased if the partial energy decreases, and then $D_i(n)$ is less than 0 after some steps. From Equations (12) and (13), once $D_i(n)$ is smaller than 0, the value of $D_i(n)$ is set to 0. At this time, two oscillators of the neuron change according to the respective chaotic map. Because the initial values of two oscillators for all neurons are set a little differently, and, moreover, chaotic dynamics are sensitive to the initial value, therefore $|x_i(n) - y_i(n)|$ will become greater than ε after some steps. When $|x_i(n) - y_i(n) > \varepsilon$, the value of the ith neuron will change or stay at 0 according to Equation (14).

In brief, our system represents the unit-wise processing mode for all neurons; once the ith neuron value does not suit the covariance matrix $[w_{ij}]$, the neuron is changed from the preserving mode to the destroying mode, and the neuron is disturbed enough to undergo a chaotic motion, which enables the neuron to search for the proper state within the range. When the neuron suits the covariance matrix, E_i becomes small and negative, and in this case, the unit-wise processing mode is changed from the destroying to the preserving mode. When every neuron's partial energy becomes small and negative, all the $D_i(n)$ keep the same values. Thereby the output is equal to the memorized pattern required.

Why is the associative success rate of our system better than that of Inoue's? The main reason is modification of covariance matrix $[w_{ij}]$. For the definition of covariance matrix $[w_{ij}]$ by Inoue, as the spatially averaged value $\bar{x}$ of stored patterns is nearly equal to 0.5, Equation (18) is equivalent to $w_{ij} = 4\sum\left(x_i^p - \bar{x}\right)\left(x_j^p - \bar{x}\right)$. But, the spatially averaged value $\bar{x}$ of practical stored patterns is not equal to 0.5. For this reason, we have modified the covariance matrix $[w_{ij}]$ by extending it to take account of the spatially averaged value $\overline{x^p}$ of every practical stored pattern. Although Ishii's chaotic neuron network is unit-wise processing mode, from evolution Equation (4) it can be seen that the next iteration value depends on the current average value of whole system. However, the next iteration value of each neuron in our system is only affected by the neuron itself. Hence, compared with our system, the recall speed is much slower.

6. Conclusion

Based on an analysis of current principal chaotic neural network models and their applications in information processing, we propose a one-dimensional, two-way coupled map network and a modified definition method of the auto-associative matrix. The two-way coupled map network overcome the weakness of the globally coupled map network and has the same abilities in pattern classification. From numerical simulation experiments, we have proved that the associative success rate and recall speed of our modified definition of the auto-associative matrix are improvement over existing methods. Moreover, the associative recall process of the network is analyzed in detail, and explanations of improvement are given based on our theoretical analysis.

In conventional self-associative neural network, the retrieval results are mainly described by stable balancing states, namely, whether a stable state corresponds to a stored pattern. However, a chaotic neurons network is a dynamic system, and obviously represents nonperiodic associative dynamic behavior, and its recovering process from initial partial information may be regarded as a dynamic search process for complete information. This search process could be explained as follows, since the auto-associative matrix definition in our system is a modification of Hebb's rule, which conventional auto-associative neural networks have used; therefore, the feedback inputs from the constituent neurons through the recurrent interconnections have the effect of making the network converge to a stored pattern. Hence, it can be conjectured that the interconnecting stable dynamic behavior which auto-associative matrix presents, and the unstable behavior of coupled chaotic oscillators, produce a whole dynamic memory searching process, and this dynamic memory searching process can be used as associative memory in nonlinear dynamic evolution.

A hypothesis on the possible roles of chaos in odour recognition and discrimination in the olfactory system of rabbits has been proposed by Freeman et al. [6]. They hypothesize (1) that chaotic neural activity serves as a basal state with a global attractor, which ensures rapid and unbiased access to previously learned sensory patterns; (2) that when stimulated by a known input, the system is constrained to a limit cycle attractor; and (3) that when stimulated by a novel input, the system falls into a high-level chaotic state which provides the driving activity essential for learning novel inputs. The chaotic neuron networks are similar phenomena in terms of these two points. Future problems to study will be that the chaotic dynamics of a chaotic neural network may play functional roles, not only in rapid and unbiased access to learned patterns but also in classifying and learning novel patterns. Another future problem is to discuss the relationship of the associative dynamics with other nonlinear phenomena, such as chaotic itinerancy, chaotic stochasticity and self-organized criticality.

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