AMATH 515 Homework 2

Due Date: 02/19/2020

Homework Instruction: Please follow order of this notebook and fill in the codes where commented as `TODO` .

In [1]:
```
UW_ID = "1772371"
FIRST_NAME = "Philip"
LAST_NAME = "Pham"
```

In [2]:
```
import numpy as np
import scipy.io as sio
import matplotlib.pyplot as plt
```

Please complete the solvers in `solver.py`

In [3]:
```
import sys
sys.path.append('./')
from solvers import *
```

Problem 3: Compressive Sensing

Consier the optimization problem,

$$\min_x \ \frac{1}{2}\|Ax - b\|^2 + \lambda\|x\|_1$$

In the following, please specify the f and g and use the proximal gradient descent solver to obtain the solution.

In [4]:
```
# create the data
np.random.seed(123)
m = 100  # number of measurements
n = 500  # number of variables
k = 10   # number of nonzero variables
s = 0.05 # measurements noise level
#
A_cs = np.random.randn(m, n)
x_cs = np.zeros(n)
x_cs[np.random.choice(range(n), k, replace=False)] = np.random.choice([-1.0, 1.0], k)
b_cs = A_cs.dot(x_cs) + s*np.random.randn(m)
#
lam_cs = 0.1*norm(A_cs.T.dot(b_cs), np.inf)
```

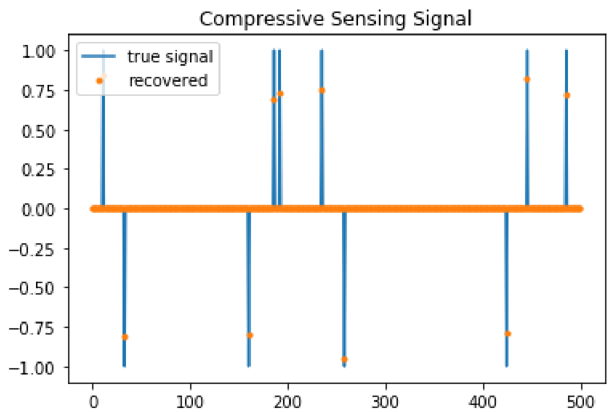
In [5]:
```
# define the function, prox and the beta constant
def func_f_cs(x):
    return np.sum(np.square(np.matmul(A_cs, x) - b_cs)) / 2.

def func_g_cs(x):
    return lam_cs * np.sum(np.abs(x))

def grad_f_cs(x):
    return np.matmul(A_cs.T, np.matmul(A_cs, x) - b_cs)

def prox_g_cs(x, t):
    delta = t * lam_cs
    return np.where(np.abs(x) >= delta, x - np.sign(x) * delta, 0.)
# Largest eigenvalue of Gramian matrix.
beta_f_cs = sorted(np.abs(np.linalg.eigvals(np.matmul(A_cs.T, A_cs))))[-1]
```

Proximal gradient descent on compressive sensing

In [6]:
```
# apply the proximal gradient descent solver
x0_cs_pgd = np.zeros(x_cs.size)
x_cs_pgd, obj_his_cs_pgd, err_his_cs_pgd, exit_flag_cs_pgd = \
    optimizeWithPGD(x0_cs_pgd, func_f_cs, func_g_cs, grad_f_cs, prox_g_cs, beta_f_cs)
```

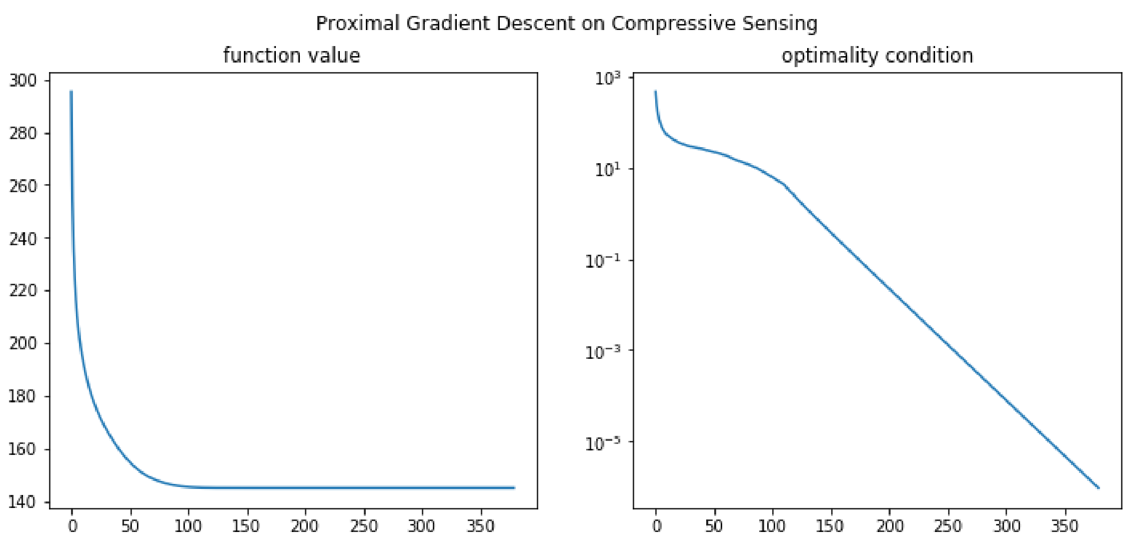
In [7]:
```
# plot signal result
plt.plot(x_cs)
plt.plot(x_cs_pgd, '.')
plt.legend(['true signal', 'recovered'])
plt.title('Compressive Sensing Signal')
```

Out[7]:
```
Text(0.5, 1.0, 'Compressive Sensing Signal')
```



In [8]:
```
# plot result
fig, ax = plt.subplots(1, 2, figsize=(12,5))
ax[0].plot(obj_his_cs_pgd)
ax[0].set_title('function value')
ax[1].semilogy(err_his_cs_pgd)
ax[1].set_title('optimality condition')
fig.suptitle('Proximal Gradient Descent on Compressive Sensing')
```

Out[8]:
```
Text(0.5, 0.98, 'Proximal Gradient Descent on Compressive Sensing')
```

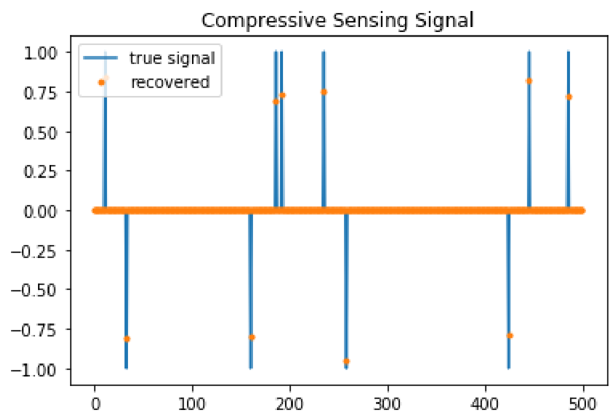


Accelerate proximal gradient descent on compressive sensing

In [9]:
```python
# apply the proximal gradient descent solver
x0_cs_apgd = np.zeros(x_cs.size)
x_cs_apgd, obj_his_cs_apgd, err_his_cs_apgd, exit_flag_cs_apgd = \
    optimizeWithAPGD(x0_cs_apgd, func_f_cs, func_g_cs, grad_f_cs, prox_g_cs, beta_f_cs)
```

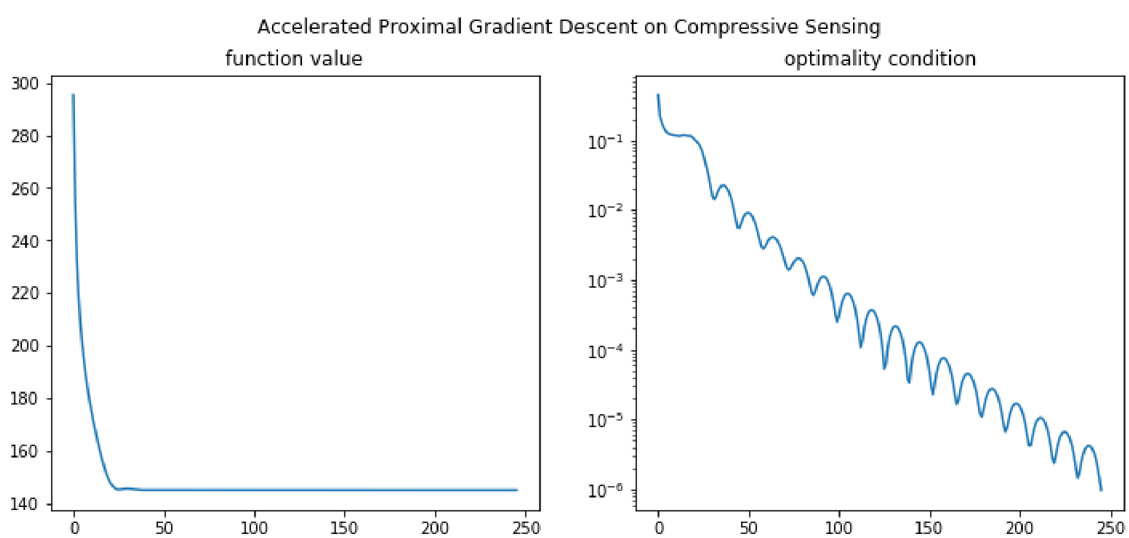
In [10]:
```python
# plot signal result
plt.plot(x_cs)
plt.plot(x_cs_apgd, '.')
plt.legend(['true signal', 'recovered'])
plt.title('Compressive Sensing Signal')
```

Out[10]: `Text(0.5, 1.0, 'Compressive Sensing Signal')`



In [11]:
```python
# plot result
fig, ax = plt.subplots(1, 2, figsize=(12,5))
ax[0].plot(obj_his_cs_apgd)
ax[0].set_title('function value')
ax[1].semilogy(err_his_cs_apgd)
ax[1].set_title('optimality condition')
fig.suptitle('Accelerated Proximal Gradient Descent on Compressive Sensing')
```

Out[11]: `Text(0.5, 0.98, 'Accelerated Proximal Gradient Descent on Compressive Sensing')`



Problem 4: Logistic Regression on MINST Data

Now let's play with some real data, recall the logistic regression problem,

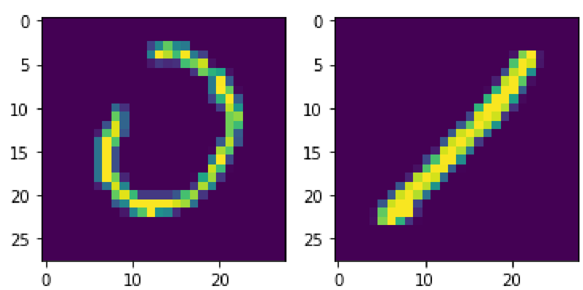
$$\min_x \sum_{i=1}^m \{\log(1 + \exp(\langle a_i, x\rangle)) - b_i\langle a_i, x\rangle\} + \frac{\lambda}{2}\|x\|^2.$$

Here our data pair $\{a_i, b_i\}$, a_i is the image and b_i is the label. In this homework problem, let's consider the binary classification problem, where $b_i \in \{0, 1\}$.

In [12]:
```
# import data
mnist_data = np.load('mnist01.npy', allow_pickle=True)
#
A_lgt = mnist_data[0]
b_lgt = mnist_data[1]
A_lgt_test = mnist_data[2]
b_lgt_test = mnist_data[3]
#
# set regularizer parameter
lam_lgt = 0.1
#
# beta constant of the function
beta_lgt = 0.25*norm(A_lgt, 2)**2 + lam_lgt
```

In [13]:
```
# plot the images
fig, ax = plt.subplots(1, 2)
ax[0].imshow(A_lgt[0].reshape(28,28))
ax[1].imshow(A_lgt[7].reshape(28,28))
```

Out[13]: <matplotlib.image.AxesImage at 0x11f3ad9b0>



In [14]:
```
# define function, gradient and Hessian
def lgt_func(x):
    logits = np.matmul(A_lgt, x)
    return np.sum(np.log(1 + np.exp(logits)) - b_lgt * logits) + 0.5 * lam_lgt * np.sum(np.squ
are(x))
#
def lgt_grad(x):
    logits = np.matmul(A_lgt, x)
    exp_logits = np.exp(logits)
    return np.sum(A_lgt.T * exp_logits / (1 + exp_logits), axis=1) - np.matmul(A_lgt.T, b_lgt)
+ lam_lgt * x
#
def lgt_hess(x):
    n_lgt = A_lgt.shape[-1]
    logits = np.matmul(A_lgt, x)
    exp_logits = np.exp(logits)
    rescaled_A_T = A_lgt.T * np.sqrt(exp_logits) / (1 + exp_logits)
    return np.matmul(rescaled_A_T, rescaled_A_T.T) + np.eye(n_lgt) * lam_lgt
```

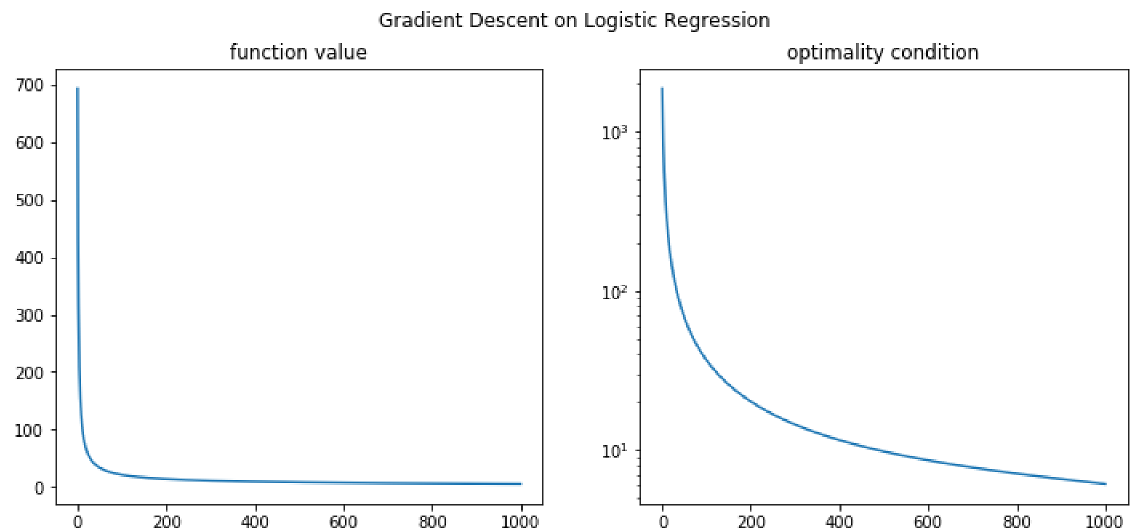
Gradient descent on logistic regression

In [15]:
```
# apply the gradient descent
x0_lgt_gd = np.zeros(A_lgt.shape[1])
x_lgt_gd, obj_his_lgt_gd, err_his_lgt_gd, exit_flag_lgt_gd = \
    optimizeWithGD(x0_lgt_gd, lgt_func, lgt_grad, beta_lgt)
```

```
Gradient descent reach maximum number of iteration.
```

In [16]:
```
# plot result
fig, ax = plt.subplots(1, 2, figsize=(12,5))
ax[0].plot(obj_his_lgt_gd)
ax[0].set_title('function value')
ax[1].semilogy(err_his_lgt_gd)
ax[1].set_title('optimality condition')
fig.suptitle('Gradient Descent on Logistic Regression')
```

Out[16]:
```
Text(0.5, 0.98, 'Gradient Descent on Logistic Regression')
```



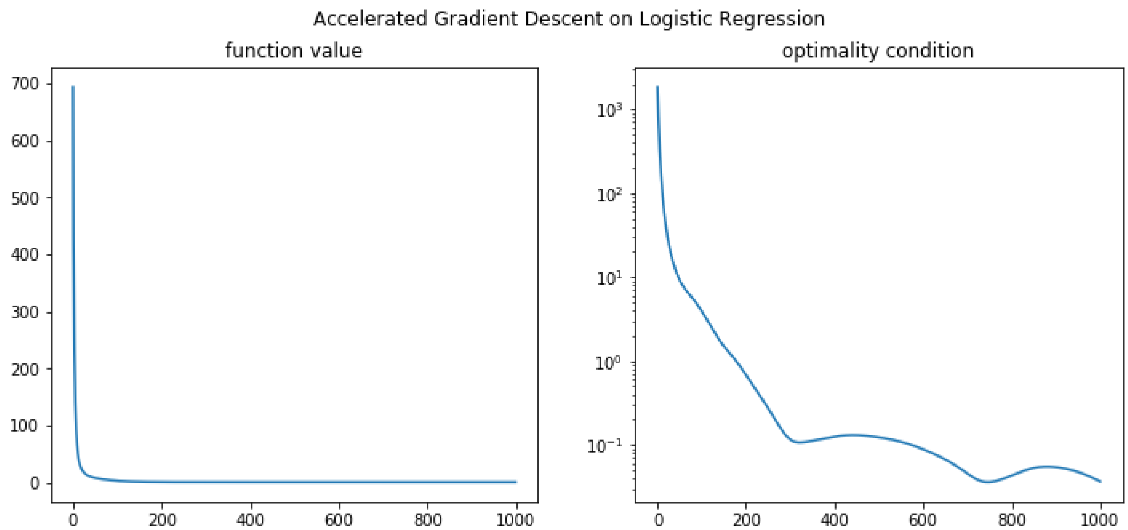
Accelerate Gradient descent on logistic regression

In [17]:
```
# apply the accelerated gradient descent
x0_lgt_agd = np.zeros(A_lgt.shape[1])
x_lgt_agd, obj_his_lgt_agd, err_his_lgt_agd, exit_flag_lgt_agd = \
    optimizeWithAGD(x0_lgt_agd, lgt_func, lgt_grad, beta_lgt)
```

```
Accelerated gradient descent reach maximum of iteration
```

In [18]:
```python
# plot result
fig, ax = plt.subplots(1, 2, figsize=(12,5))
ax[0].plot(obj_his_lgt_agd)
ax[0].set_title('function value')
ax[1].semilogy(err_his_lgt_agd)
ax[1].set_title('optimality condition')
fig.suptitle('Accelerated Gradient Descent on Logistic Regression')
```

Out[18]:
```
Text(0.5, 0.98, 'Accelerated Gradient Descent on Logistic Regression')
```

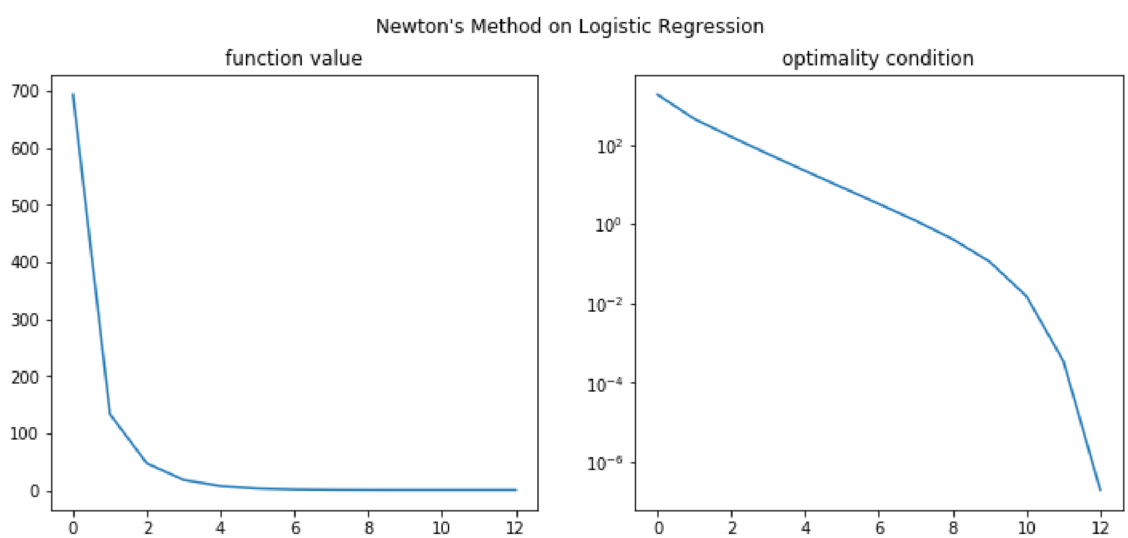


Newton's Method on logistic regression

In [19]:
```python
# apply the accelerated gradient descent
x0_lgt_nt = np.zeros(A_lgt.shape[1])
x_lgt_nt, obj_his_lgt_nt, err_his_lgt_nt, exit_flag_lgt_nt = \
    optimizeWithNT(x0_lgt_nt, lgt_func, lgt_grad, lgt_hess)
```

In [20]:
```python
# plot result
fig, ax = plt.subplots(1, 2, figsize=(12,5))
ax[0].plot(obj_his_lgt_nt)
ax[0].set_title('function value')
ax[1].semilogy(err_his_lgt_nt)
ax[1].set_title('optimality condition')
fig.suptitle('Newton\'s Method on Logistic Regression')
```

Out[20]:
```
Text(0.5, 0.98, "Newton's Method on Logistic Regression")
```



Test Logistic Regression

In [21]:
```
# define accuracy function
def accuracy(x, A_test, b_test):
    r = A_test.dot(x)
    b_test[b_test == 0.0] = -1.0
    correct_count = np.sum((r*b_test) > 0.0)
    return correct_count/b_test.size
```

In [22]:
```
print('accuracy of the result is %0.3f' % accuracy(x_lgt_nt, A_lgt_test, b_lgt_test))
```

```
accuracy of the result is 1.000
```