Name:

AMATH 515   Homework Set 2

(1) Recall that

$$\text{prox}_{tf}(y) = \arg\min_x \frac{1}{2t}\|x - y\|^2 + f(x)$$

$$f_t(y) = \min_x \frac{1}{2t}\|x - y\|^2 + f(x).$$

Suppose f is convex.

(a) Prove that f_t is convex.

(b) Prove that prox_{tf} is a single-valued mapping.

(c) Compute prox_{tf} and f_t, where $f(x) = \|x\|_1$.

(d) Compute prox_{tf} and f_t for $f = \delta_{\mathbb{B}_\infty}(x)$, where $\mathbb{B}_\infty = [-1, 1]^n$.

(2) More prox identities.

(a) Suppose f is convex and let $g(x) = f(x) + \frac{1}{2}\|x - x_0\|^2$. Find formulas for prox_{tg} and g_t in terms of prox_{tf} and f_t.

(b) The elastic net penalty is used to detect groups of correlated predictors:

$$g(x) = \beta\|x\|_1 + (1 - \beta)\frac{1}{2}\|x\|^2, \quad \beta \in (0, 1).$$

Write down the formula for prox_{tg} and g_t.

(c) Let $f(x) = \frac{1}{2}\|Cx\|^2$. Write $\text{prox}_{tf}(y)$ in closed form.

(d) Let $f(x) = \|x\|_2$. Write $\text{prox}_{tf}(y)$ in closed form.

Coding Assignment