

Find the largest i such that

$$n - \frac{(1+i) \cdot i}{2} \leq \frac{(2k - (b-i) + 1)(b-i)}{2}$$

which means

$$(2k - 2b)i \leq 2kb - b^2 + b - 2n$$

So if $k = b$, just test whether $(1+k)k/2 = n$. And in case $b < k$, we have

$$i \leq \frac{2kb - b^2 + b - 2n}{2(k - b)}.$$

Without loss of generality, we can assume that $i = 0$. In the algorithm, we can do simply trick to fix if $i > 0$. Now, we want split b into two halves of length $b_l = \lfloor (b-1)/2 \rfloor$ and $b_r = \lfloor b/2 \rfloor$, respectively. And then, we find the length l of the left half of k , such that it is possible to split n into two parts n_l and n_r so that we can find b_l distinct numbers in $\{1, \dots, l\}$ whoses sum is n_l , and b_r distinct numbers in $\{l+1, \dots, k\}$ whose sum is n_r . Finally, the first task can be solved by calling `bonetrousle(n_l, k, b_l)` and the second task by modifying the output of `bonetrousle($n_l - b_r l, k - l, b_r$)`.