idea

February 2, 2021

1 Dam of Candies

Geek wants to make a special space for candies on his bookshelf. Currently, it has N books of different heights and unit width. Help him select 2 books such that he can store maximum candies between them by removing all the other books from between the selected books. The task is to find out the area between 2 books that can hold the maximum candies without changing the original position of selected books.

[1]:
```
import random
```

1.1 $O(N^2)$-time, $O(1)$-space solution

An $O(N^2)$ solution is straightforward so we use that to check the correctness of a more efficient solution

[2]:
```
def damOfCandies(arr):
    candies = 0
    interval = None
    for i, a in enumerate(arr):
        for j in range(i + 2, len(arr)):
            tmp = min(a, arr[j]) * (j - i - 1)
            if tmp > candies:
                candies = tmp
                interval = [i, j]
    return candies, interval
```

1.2 Ideas for a more efficient solution

[3]:
```
import matplotlib.pyplot as plt
%matplotlib inline
plt.style.use('ggplot')
```

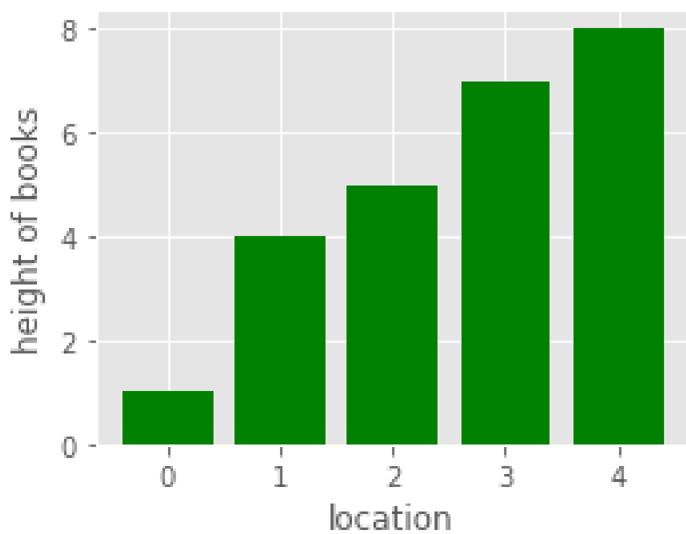
[4]:
```
def plot(heights, fname):
    x_pos = [i for i in range(len(heights))]
    plt.figure(figsize=(4, 3))
    plt.bar(x_pos, heights, color='green')
```

```
    plt.xlabel('location')
    plt.ylabel('height of books')
    plt.xticks(range(len(heights)))
    plt.savefig(fname, dpi=100, bbox_inches='tight')
```

Observation 1: We have linear solution if the heights is monotonically increasing or decreasing. For example, in the case of increasing sequences, the maximum area can be calculated by

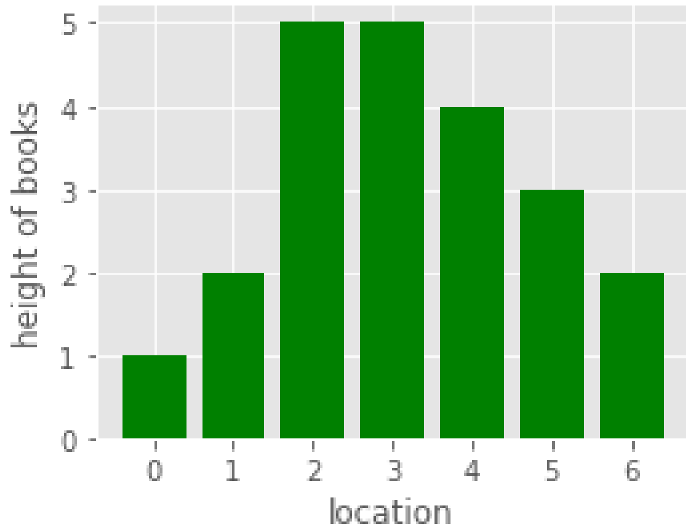
[5]:
```
heights = [1, 4, 5, 7, 8]
plot(heights, './document/plot_increasing.png')

maxArea = max([h * (len(heights) - i - 2) for i, h in enumerate(heights[:-2])])
print(f'maximum area = {maxArea}')
```

```
maximum area = 8
```



Observation 2: If the sequence is like a hill or plateau, a linear solution is also possible.

[6]:
```
heights = [1, 2, 5, 5, 4, 3, 2]
plot(heights, './document/plot_hillOrPlateau.png')
```



Let starts with $i = 0$ and $j = N-1$. The first area is $(N-2) \cdot \min(h_0, h_{n-1})$. Then, we can increase i and/or decrease j to obtain a potentially bigger area. There three cases we need to consider: 1. $h_0 < h_{n-1}$: In this case, moving j to the left only decrease the area, so we increase i by 1; 1. $h_0 > h_{n-1}$: In this case, moving i to the right only decrease the area, so we decrease j by 1; 1. $h_0 = h_{n-1}$: In this case, we need increase i by 1 and decrease j by 1 to obtain a possible increase in area.

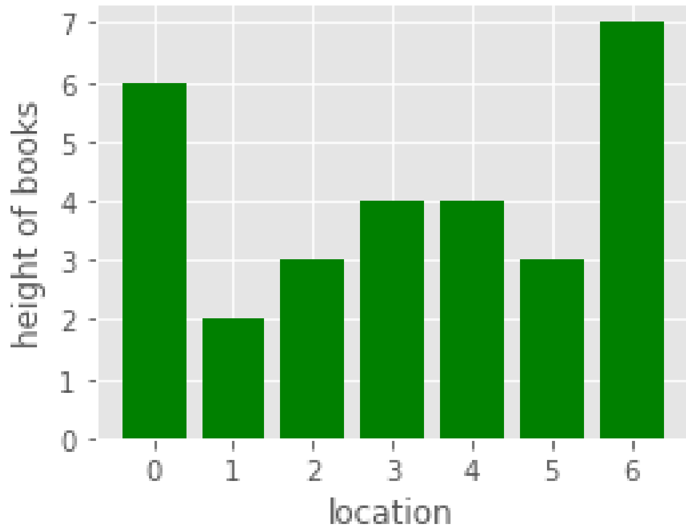
The following code can find the maximum area:

[7]:
```
i, j, maxArea = 0, len(heights) - 1, 0
while i < j:
    l, r = heights[i], heights[j]
    maxArea = max(maxArea, (j - i - 1) * min(l, r))
    i += int(l <= r)
    j -= int(l >= r)
print(f'maximum area = {maxArea}')
```

```
maximum area = 8
```

Observation 3: Dippings Doesn't count. As an example, the maximum area of following example is formed by the two boundary books while the shorter books in between don't matter.

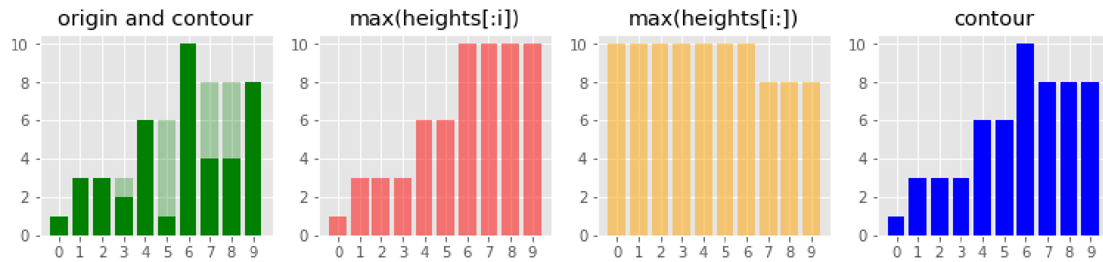
[8]:
```
heights = [6, 2, 3, 4, 4, 3, 7]
plot(heights, './document/plot_dipping.png')
```



Observation 3 implies that we only need to consider the "contour" of book determined "local maxima". See the following example.

[9]:
```python
heights = [1, 3, 3, 2, 6, 1, 10, 4, 4, 8]
# plot(heights, './document/plot_arbitray.png')

N = len(heights)
max_L, l = [], 0
max_R, r = [], 0
for i in range(N):
    l = max(l, heights[i])
    max_L.append(l)
    r = max(r, heights[N - i - 1])
    max_R.append(r)

contour = [min(max_L[i], max_R[N - i - 1]) for i in range(N)]

fig, axes = plt.subplots(1, 4, figsize=(13, 2.5))
axes[0].bar(range(N), contour, color='green', alpha=.3)
axes[0].bar(range(N), heights, color='green')
axes[0].set_title('origin and contour')
axes[1].bar(range(N), max_L, color='red', alpha=.5)
axes[1].set_title(f'max(heights[:i])')
axes[2].bar(range(N)[::-1], max_R, color='orange', alpha=.5)
axes[2].set_title(f'max(heights[i:])')
axes[3].bar(range(N), contour, color='blue')
axes[3].set_title('contour')
for ax in axes:
```

```
    ax.set_xticks(range(N))
plt.savefig('./document/plot_localMaximaAndContour.png', dpi=200,␣
 ↪transparent=False, bbox_inches='tight')
```



The contour gives the same result as the original height sequence. Since getting the contour also takes linear time, there is an $O(N)$-time solution. Note that we *don't* have to save the contour sequence, which means we can design an $O(1)$-space solution, as follows.

1.3 $O(N)$-time $O(1)$-space solution

[10]:
```
def damOfCandiesLinear(heights):
    N = len(heights)

    l, r = 0, 0
    i, j, candies = 0, 0, 0
    while (N - 2) - (i + j) > 0:
        width = (N - 2) - (i + j)
        l, r = max(l, heights[i]), max(r, heights[N - 1 - j])
        candies = max(candies, width * min(l, r))
        i += int(l <= r)
        j += int(l >= r)

    return candies
```

1.4 Tests

[11]:
```
num_tries = 20

for t in range(num_tries):
    print(f'{t}:')
    N = random.randint(5, 15)
    heights = [random.randint(1, 10) for _ in range(N)]
    print(f'\theights = {heights}')
```

```python
    candies, [i, j] = damOfCandies(heights)
    print(f'\tN-squared method = {candies}')

    candiesLinear = damOfCandiesLinear(heights)
    print(f'\tLinear method = {candies}')
```

```
0:
        heights = [4, 5, 7, 6, 7, 9, 5, 7]
        N-squared method = 28
        Linear method = 28
1:
        heights = [3, 1, 3, 9, 9, 7, 1, 8]
        N-squared method = 24
        Linear method = 24
2:
        heights = [3, 6, 10, 6, 8, 1, 3, 8, 2]
        N-squared method = 32
        Linear method = 32
3:
        heights = [2, 4, 5, 7, 6, 2, 10, 9, 3, 10, 8]
        N-squared method = 42
        Linear method = 42
4:
        heights = [5, 2, 1, 1, 4]
        N-squared method = 12
        Linear method = 12
5:
        heights = [5, 5, 5, 7, 9, 4, 4, 3, 9, 7, 9]
        N-squared method = 45
        Linear method = 45
6:
        heights = [1, 1, 7, 10, 3, 2, 1, 3, 2, 5, 2]
        N-squared method = 30
        Linear method = 30
7:
        heights = [9, 3, 5, 9, 3, 9, 10, 2, 5, 6, 5, 10, 1, 10, 3]
        N-squared method = 108
        Linear method = 108
8:
        heights = [6, 3, 7, 4, 10, 3, 1, 4, 4, 6]
        N-squared method = 48
        Linear method = 48
9:
        heights = [1, 6, 5, 8, 1, 8, 10, 7, 5, 3, 7]
        N-squared method = 48
        Linear method = 48
10:
```

```
        heights = [8, 3, 7, 5, 5, 5, 9, 8, 4, 8, 8, 8]
        N-squared method = 80
        Linear method = 80
11:
        heights = [7, 2, 5, 1, 7, 6, 2, 7]
        N-squared method = 42
        Linear method = 42
12:
        heights = [9, 7, 10, 7, 5, 8, 4, 3, 6, 7, 8]
        N-squared method = 72
        Linear method = 72
13:
        heights = [10, 3, 3, 4, 6, 9, 7, 8, 8, 4]
        N-squared method = 56
        Linear method = 56
14:
        heights = [4, 4, 5, 9, 8, 10, 5, 8, 2]
        N-squared method = 24
        Linear method = 24
15:
        heights = [6, 4, 10, 6, 8, 4, 9, 9, 7, 3, 5]
        N-squared method = 45
        Linear method = 45
16:
        heights = [4, 4, 6, 1, 2, 1, 3]
        N-squared method = 15
        Linear method = 15
17:
        heights = [10, 5, 5, 10, 10, 3, 10, 2, 9, 3, 6, 1, 1]
        N-squared method = 63
        Linear method = 63
18:
        heights = [8, 5, 8, 8, 6, 6, 5, 6, 3, 6, 3]
        N-squared method = 48
        Linear method = 48
19:
        heights = [7, 4, 10, 2, 9, 2, 3, 8, 4, 9, 10, 3, 1, 2, 5]
        N-squared method = 70
        Linear method = 70
```