Coding Theory: Problems 3

1. You showed in problem set 2 that E_n, the set of even weight binary vectors of length n, is a subspace of $\mathbb{Z}_2^n$. Hence E_n is a binary linear code. What are the parameters $[n, k, d]$ of E_n? Write down a generator matrix for E_n in standard form.

2. Let C be a binary linear $[8, k, 3]$-code. Given that $A_2(8, 3) = 20$, find an upper bound on k, the dimension of C. Construct a code which attains this upper bound.

3. Let C be the 3-ary linear code generated by

$$G = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 2 \end{bmatrix}.$$

   What is $|C|$? List the codewords of C and hence compute its minimum distance. Is C perfect?

4. Let C be the binary linear code generated by

$$G = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \end{bmatrix}.$$

   Find a generator matrix in standard form for **the same code** C.

5. Construct standard arrays for the binary linear codes C_1, C_2 generated by

$$G_1 = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}, \qquad G_2 = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 \end{bmatrix}.$$

   (a) Using C_2 with coset decoding,
      i. decode received vectors 11111 and 01011,
      ii. give an example of 2 errors occuring and being corrected,
      iii. give an example of 2 errors occuring and **not** being corrected.

   (b) Assume each code is transmitted down a binary symmetric channel with symbol error probability $p = 0.01$. Since both these codes are linear, the word error probability is independent of the codeword transmitted. Compute $P_{\text{err}}(C_1)$ and $P_{\text{err}}(C_2)$.

   (c) If C_2 is used only for error **detection**, what is the probabilty P_{undetec} that any transmitted codeword will be received with undetected errors, under the assumption of part (b)?

6. Let C be a **perfect** binary linear $[7, 4, d]$-code. Deduce what d is. Assume C is transmitted down a binary symmetric channel with symbol error probability $p = 0.01$. Calculate the word error probability of the code $P_{\text{err}}(C)$.