Combinatorial Representation Theory: Algebra and its interfaces with geometry, topology and combinatorics

The project is on very central topics in algebra. Its purpose is to foster basic research in combinatorial representation theory. The main research topics this proposal will cover are *frieze patterns*, *diagram algebras* and *singularity theory*. Its aim is to promote interdisciplinary research within the school and the university. The chosen topics have great potential for promoting the discipline of mathematics as they naturally offers links to outreach activities, e.g. through public audience talks. An excellent opportunity to enhance the visibility of the department of pure mathematics. We plan to work on a suite of related research activities, with one common theme (combinatorial representation theory).

The team. The group in Leeds has a very broad expertise in algebra with research in representation theory, cluster algebras, cluster categories, mathematical physics, algebraic geometry and algebraic groups. Group members Baur, Faber, Faria Martins, Marsh, Martin and Parker are all key player in these areas. Our combined background makes us uniquely qualified to work on this project. The group also has a strong tradition of research with international partners and will involve these to create an outstanding environment for the proposed work. The programme grant will facilitate communication across the different topics, in Leeds, within the UK and with international partners. To support our research and give cohesion to the group, we apply for funding of 4 post-doctoral researchers working in the group, with different background. Two major workshops at the beginning and towards the end will bring cohesion to the project and consolidate progress made.

Members of the team have obtained and hold prestigious awards. Baur held an SNSF professor position at ETH and is a Royal Society Wolfson Fellow (2018-2023). Faber has held a Schrödinger Fellowship of the Austrian Science Fund and is a Marie Curie Fellow (2018-2020). Marsh was awarded a Whitehead Prize by the LMS and held a Leadership Fellowship. He is one of the founders of cluster categories. Faria Martins and Marsh obtained Leverhulme Fellowships. The combined grant income of the group is over 5.5 Mio GBP so far.

The team has ample experience mentoring graduate students (around 35 students so far) and post-doctoral researchers (over 20). Baur and Marsh have been mentors for Marie Curie Fellows.

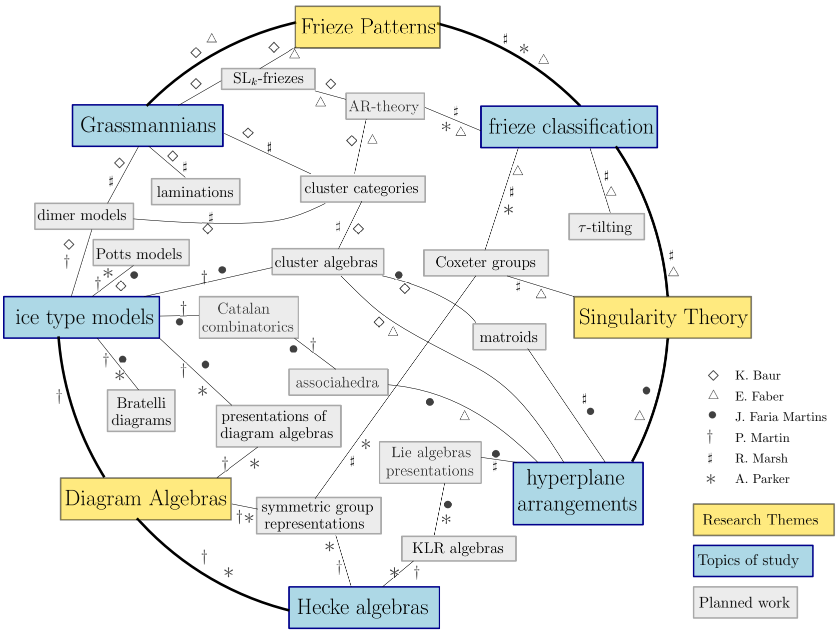
The group is involved in organisation of numerous international meetings. Baur and Marsh are organisers of a programme on Cluster Algebras and Representation Theory at the Isaac Newton Institute, Cambridge.

Topics.
Frieze patterns (Baur, Faber, Marsh, Parker) Friezes are arrays of integers in the plane satisfying a determinantal rule. They were first studied by Coxeter and Conway in the 1970s, motivated by Gauss's study of the pentagramma mirificum in spherical geometry. In the early 2000s interest in friezes was renewed by the discovery of cluster algebras by Fomin-Zelevinsky. This led to intense study of frieze patterns and their generalisations, including links to difference equations, combinatorial and Poisson geometry, Auslander-Reiten theory, Grassmannians and moduli spaces. As examples, in 2018, Baur, Faber et al. obtained a categorical interpretation of higher friezes, Lee et al. connected friezes with algebraic varieties, Ovsienko gave an interpretation of friezes as solutions of matrix equations in $\mathrm{SL}(2,\mathbb{Z})$.
Diagram algebras (Faria Martins, Martin, Parker) From their origins in computational physics, with its natural geometric setting, diagram categories are a rich source of combinatorial and geometric representation theory. The topic embraces topological quantum field theory, with applications in quantum computing, and ice

type models and Potts models in statistical mechanics, as well as classical symmetric group, Brauer algebra and partition algebra problems, Lie theory and formal (higher) category theory. The combinatorial theory links to frieze patterns, and the cobording formalism offers the prospect of links to singularity theory. Indeed diagram categories represent a synthesis of algebra and geometry, which is one of the recurring themes in all three main topics.

Singularity theory (Faber, Faria Martins, Marsh) Singularity theory studies classification problems of non-generic situations of maps and spaces. Recently, there has been intense research activity in connections to representation theory: A highlight is Van den Bergh's introduction of noncommutative crepant resolutions of singularities (NCCRs) in order to interpret Bridgeland's solution to Bondal and Orlov's conjecture about the derived invariance of flops. Since then, noncommutative desingularisations have been investigated more deeply and also reveal insights into the so-called singularity categories of algebraic varieties and other categorical invariants. Other highlights are Iyama-Wemyss' minimal modification algebras and mutations of NCCR's (2014+) and Wemyss' homological minimal model program (2018). Buchweitz, Faber, and Ingalls recently established a McKay correspondence for reflection groups, through a surprisingly canonical construction of an NCR.

Research - links between the topics. The following diagram presents the main topics and the research problems we will address with this.



Links between singularity and cluster theory have emerged in recent years: Certain singularity categories have a cluster structure and cluster tilting objects and mutations appear for example as central objects Wemyss' homological minimal model programme (2018). Diagram algebras and cluster structures are linked via triangulations of surfaces.

- TL basis diagrams can be cut in two to give basis diagrams for standard modules. Such half-diagrams can be constructed combinatorially in an inductive way (Bratelli diagrams). Work of Marsh-Martin in progress (since 2004!) should show that clusters have a similar construction (also sequel of Baur-Martin?) using labelled clusters. Combinatorics of labelled clusters looks interesting (half-Catalan combinatorics). Relate to rational Catalan combinatorics?

- Cluster algebras give new presentations of Coxeter groups (cf Barot-Marsh and subsequent papers). Do they also give presentations of TL algebras and related algebras?
- As in the above, there could be presentations of complex reflection groups arising from cluster algebras, and corresponding presentations of diagram algebras related to complex reflection groups (cf an earlier write-up of Marsh-Martin).

Other links? Use diagram. Use Grassmannian. Ideas??

Impact , outcome. This project combines basic research in a variety of areas and will establish new links between many areas, drawing on the expertise the group has. Classification... some examples...

Eleonore:
we want of course to strengthen algebra group and work closer together and blabla like that, but is there a particular problem we want to solve or some common goal? Maybe we want to mention training aspect (for the post docs)?

(1) K. Baur, E. Faber, S. Gratz, K. Serhiyenko, G. Todorov, *Friezes satisfying higher SL_k-determinants*, arXiv:1810.10562

(2) M. Cuntz, T. Holm, *Frieze patterns over integers and other subsets of the complex numbers*. J. Comb. Algebra 3 (2019), no. 2, 153–188.

(3) V. Ovsienko, *Partitions of unity in $SL(2,\mathbb{Z})$, negative continued fractions, and dissections of polygons*, Res. Math. Sci. 5 (2018), no. 2, Paper No. 21, 25 pp.

(4) K. Lee, L. Li, M. Mills, R.Schiffler, A. Seceleanu, *Frieze varieties : A characterization of the finite-tame-wild trichotomy for acyclic quivers*, arXiv:1803.08459