Voting

MATH1210

16 April 2018

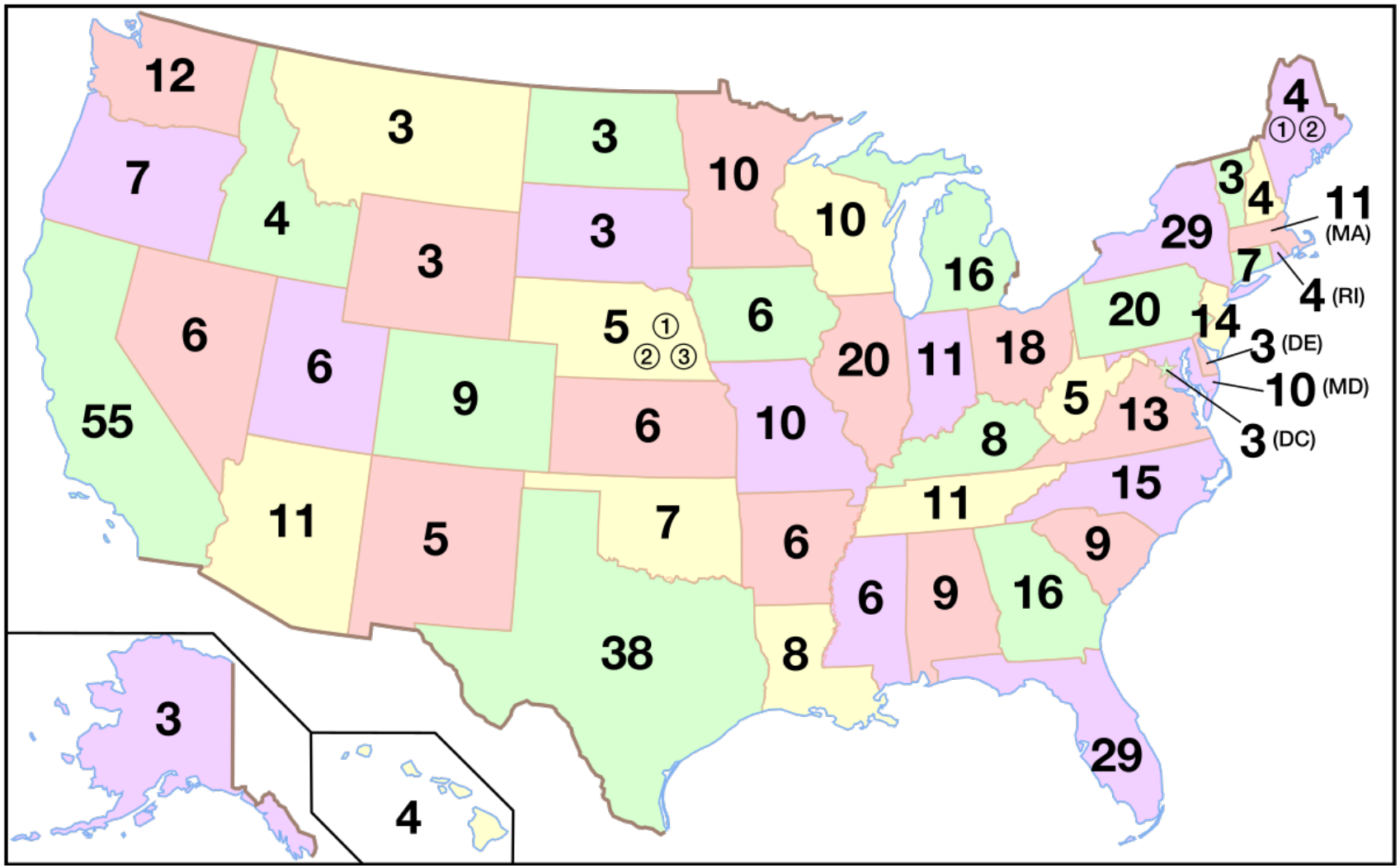
US elections

Around 138.8 millions votes were cast in November 2016.

- Hilary Clinton (D) got 65,853,516 votes (48.18%)
- Donald Trump (R) got 62,984,825 votes (46.09%)

Donald Trump was elected, with 304 electoral college votes, to Hilary Clinton's 227.

US electoral votes



538 electors in total — you need 270 to become president

US electoral votes

		Hillary Clinton Democratic			Donald Trump Republican		
State or district	Electoral method	#	%	Electoral votes	#	%	Electoral votes
Alabama	WTA	729,547	34.36%	–	1,318,255	62.08%	9
Alaska	WTA	116,454	36.55%	–	163,387	51.28%	3
Arizona	WTA	1,161,167	45.13%	–	1,252,401	48.67%	11
Arkansas	WTA	380,494	33.65%	–	684,872	60.57%	6
California	WTA	8,753,788	61.73%	55	4,483,810	31.62%	–
Colorado	WTA	1,338,870	48.16%	9	1,202,484	43.25%	–
Connecticut	WTA	897,572	54.57%	7	673,215	40.93%	–
Delaware	WTA	235,603	53.18%	3	185,127	41.79%	–
District of Columbia	WTA	282,830	90.48%	3	12,723	4.07%	–

US electoral votes

		Clinton		Trump		Votes needed
California	55	8,753,788	61.73%	4,483,810	31.62%	6,618,799
Texas	38	3,877,868	43.24%	4,685,047	52.23%	4,281,458
Florida	29	4,504,975	47.82%	4,617,886	49.02%	4,561,431
New York	29	4,547,562	58.40%	2,814,589	36.15%	3,681,076
Illinois	20	3,090,729	55.83%	2,146,015	38.76%	2,618,372
Pennsylvania	20	2,926,441	47.85%	2,970,733	48.58%	2,948,587
Ohio	18	2,394,164	43.56%	2,841,005	51.69%	2,617,585
Georgia	16	1,877,963	45.64%	2,089,104	50.77%	1,983,534
Michigan	16	2,268,839	47.27%	2,279,543	47.50%	2,274,191
North Carolina	15	2,189,316	46.17%	2,362,631	49.83%	2,275,974
New Jersey	14	2,148,278	54.99%	1,601,933	41.00%	1,875,106
	270					35,736,110
Could win with	25.74648	percent of the popular vote!				

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"The United States is the only country that elects a politically powerful president via an electoral college and the only one in which a candidate can become president without having obtained the highest number of votes in the sole or final round of popular voting." – George C. Edwards, 2011'

A toy election

We have 3 magnificent candidates. They will tell you in one sentence why they should win. Now rank the three candidates in one of the six possible orders:

A, B, C
A, C, B
B, A, C
B, C, A
C, A, B
C, B, A

Who should win?

Different voting systems

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- Condorcet
  - winner is one who beats all others in head-to-heads

A particular example

Suppose there are 30 voters, choosing between candidates A, B and C. They vote:
12: A, B, C
10: C, B, A
8: B, C, A

- FPTP: A wins, with 12 votes. But $\frac{18}{30} = 60\%$ wanted A least of all!

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- Borda: A has $12 \times 3 + 10 \times 1 + 8 \times 1 = 54$
  B has $12 \times 2 + 1 - \times 2 + 8 \times 3 = 68$
  C has $12 \times 1 + 10 \times 3 + 8 \times 2 = 58$
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- Condorcet: Ignore C: $AvB \to A = 12, B = 18$
  Ignore A: $BvC \to B = 20, B = 10$
  Ignore B: $AvC \to A = 12, C = 18$
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A reasonable system in which voters rank all candidates would require:

1. *No dictators*: no single person should determine the outcome
2. *Unanimity*: if everyone prefers A to B, then the outcome should rank A above B
3. *Independence of Irrelevant Alternatives*: outcome's relative ranking of A and B shouldn't change if voters change ranking of other candidates, but not A and B.

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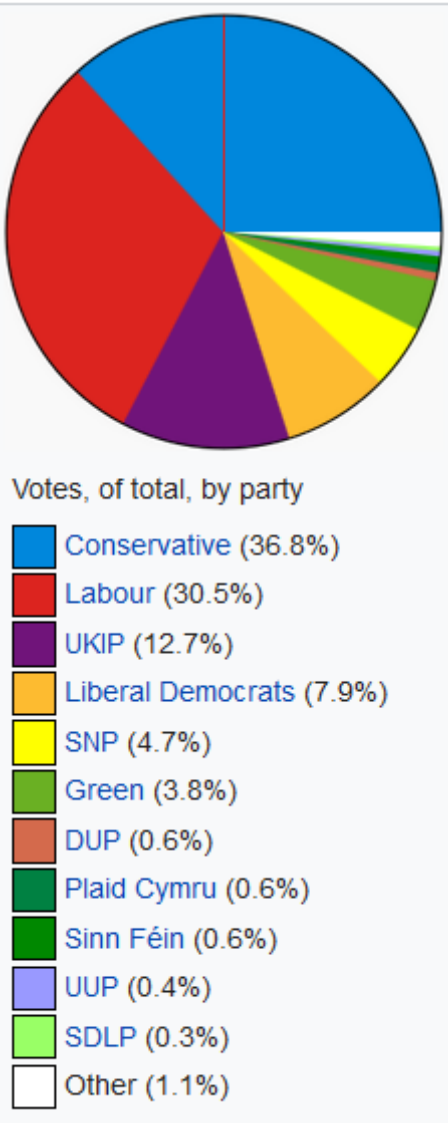
Theorem (Arrow's Impossibility Theorem)

If there are three or more candidates, there is no voting system that satisfies these requirements.

$\Longrightarrow$ 1972 Nobel for Economics.

Problems — FPTP 1



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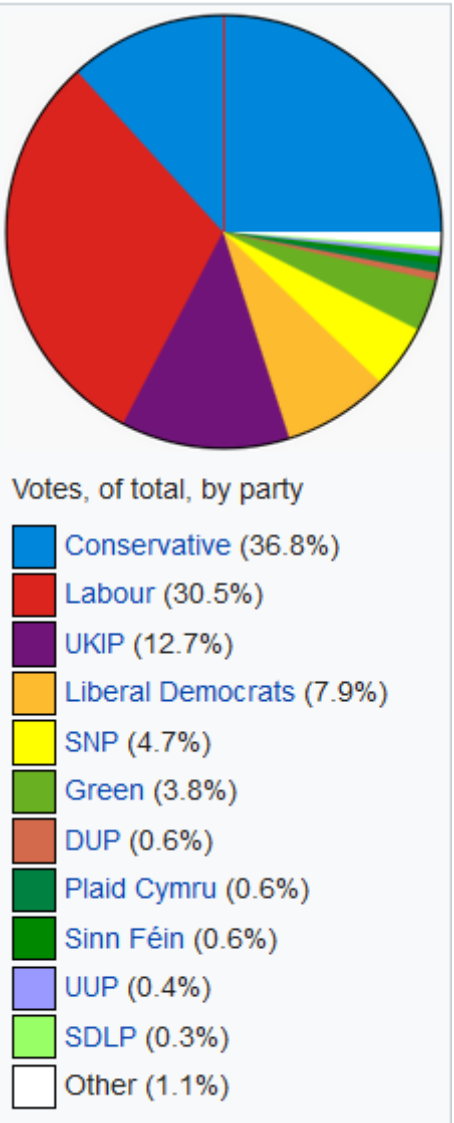

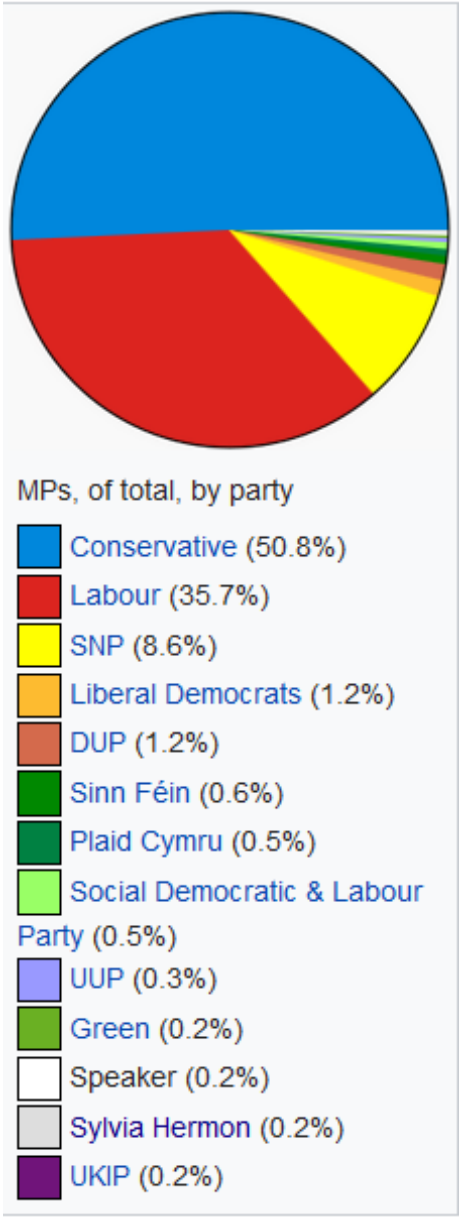

Splitting the vote

Suppose we have 3 candidates. Two, A and B, are moderates and share many common values, and the third C is an extremist who alienates supporters of the other two.

Suppose votes are cast as:

30% : A

30% : B

40% : C

Then C wins, in spite of the fact that over half the voters hate this choice. Should A and B have joined forces? Should they have had to?

Problems — Condorcet 1

Suppose 30 voters, choosing between A, B and C.
10: A, B, C
10: B, C, A
10: C, A, B
Then in AvB we have $A = 20$, $B = 10$. In BvC we have $B = 20$, $C = 10$, and in AvC we have $A = 10$, $C = 20$. We have no winner!

Problems — Condorcet 2

Suppose there are 3 candidates: A is strongly liberal, C is strongly conservative, and B is moderate. The electorate are very polarised, and 45% support A, and 45% support C. We will likely have votes:
45: A, B, C
10: B, A, C or B, C, A
45: C, B, A
Then B will win the Condorcet vote by virtue of the second places. (It makes no difference which order the B supporters put A and C.) Condorcet favours moderates, but when the electorate clearly favours something away from the centre, should the centrist win? Here the existence of B gives both A and C a zero chance of winning!

Problems — Borda

Very susceptible to tactical voting. Suppose we have 5 voters, voting for A, B, C, who vote
3: A, B, C
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Then B wins.

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But if A voters recognise the danger, and change to
3: A, C, B
2: B, C, A
then A wins. Does this disenfranchise B's supporters?

Problems — Borda 2

Suppose we have 7 voters, voting for A, B, C, who vote
3: C, B, A
2: A, C, B
2: B, A, C
Then C wins (with B second and A last).

Problems — Borda 2

Suppose we have 7 voters, voting for A, B, C, who vote
3: C, B, A
2: A, C, B
2: B, A, C
Then C wins (with B second and A last).
But if new candidate X enters the race, and we have
3: C, B, A, X
2: A, X, C, B
2: B, A, X, C
then even though X was useless and came last, now A wins, with B second and C third. So the entrance of a hopeless candidate reversed the result.

Problems — Single transferable vote

Suppose there are 21 voters, voting for A, B, C, D, who rank:
7: A, B, C, D
6: B, A, C, D
5: C, B, A, D
3: D, C, B, A
A wins.

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But now suppose the three voters who favoured D change their mind and opt for A (nothing else changes):
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Now B wins! So A has lost out by attracting more votes. Should A stop canvassing in some areas?

Problems — Single transferable vote 2

Suppose there are 26 voters, voting for A, B, C, who rank:
9: A, B, C
8: B, C, A
9: C, B, A
C wins.

Problems — Single transferable vote 2

Suppose there are 26 voters, voting for A, B, C, who rank:
9: A, B, C
8: B, C, A
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C wins.
But now suppose we split the constituency into two parts, who vote:
6: A, B, C
4: B, C, A
3: C, B, A
and
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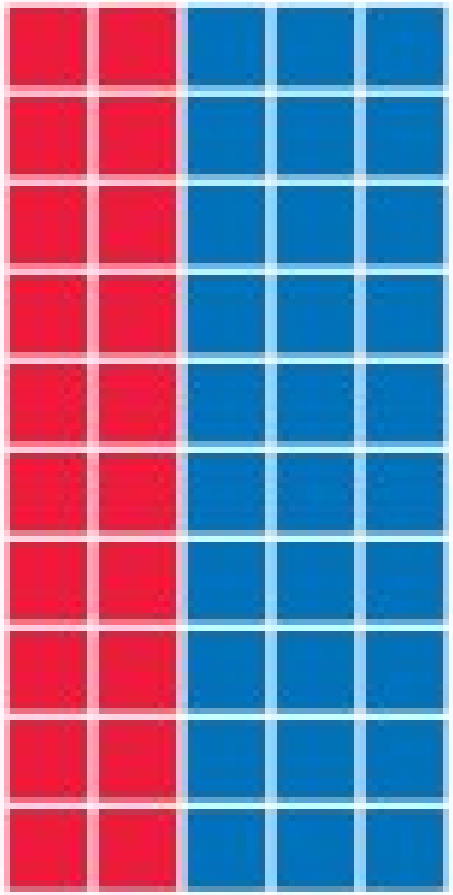
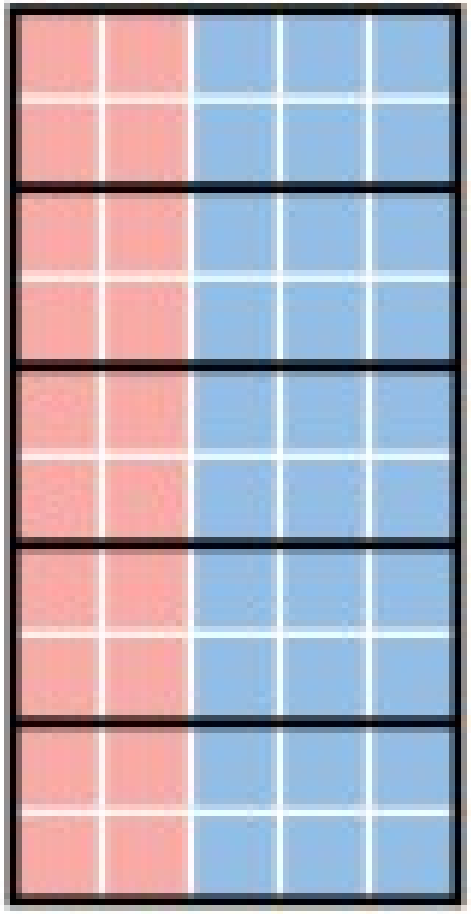
This is exactly the same distribution of votes, but in each part, B wins!
So by dividing up the voters, we change the winner.

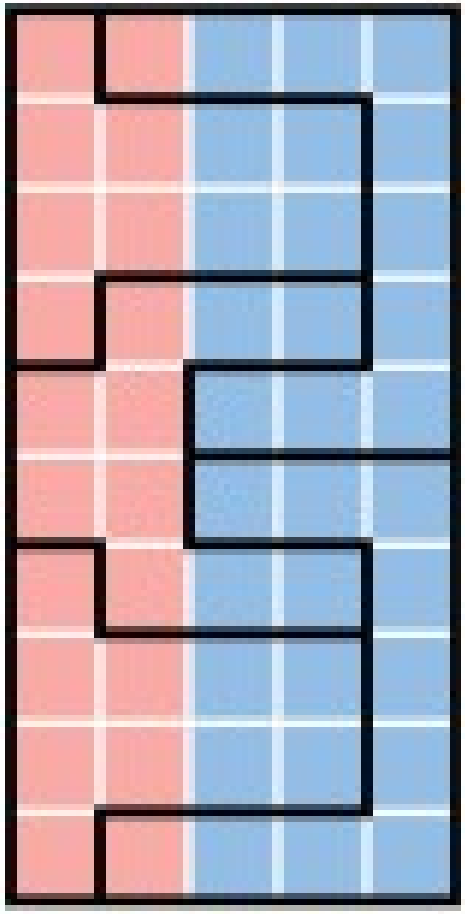
Gerrymandering



50 PRECINCTS
60% BLUE
40% RED



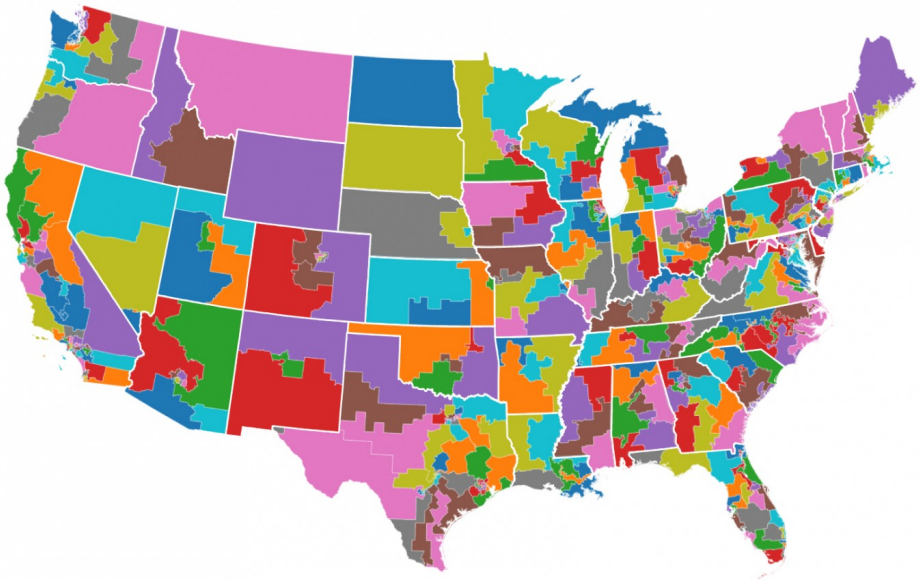
5 DISTRICTS
5 BLUE
0 RED
BLUE WINS



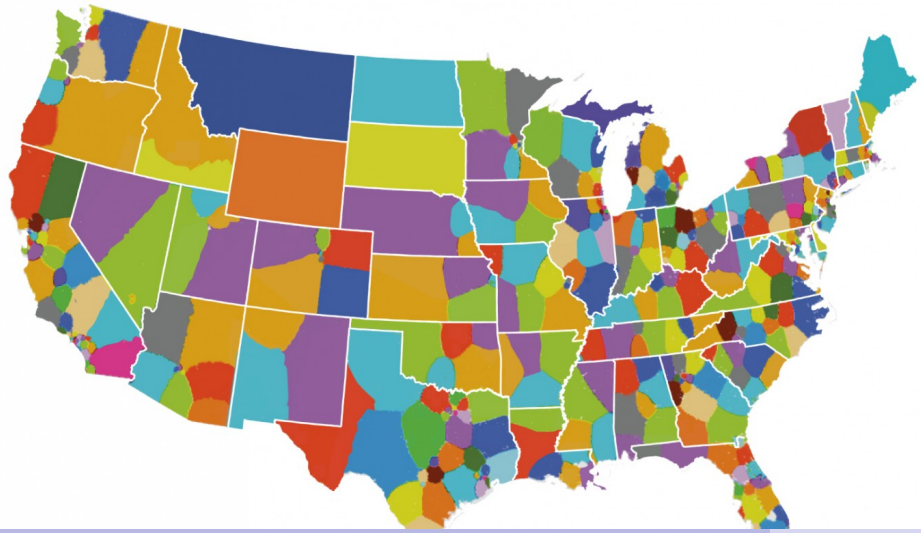
5 DISTRICTS
3 RED
2 BLUE
RED WINS

Gerrymandering in US elections

Current congressional district map



Computer-drawn map to optimize compactness



The Prisoner's dilemma

Two prisoners, A and B are being questioned about a serious crime.

- If both stay silent, both will get convicted of a lesser crime (1 year in jail).
- If A blames B, and B remains silent, A will be rewarded by being set free, and B will be jailed for 3 years (and vice versa).
- If both blame each other, both get 2 years.

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	B says nothing	B blames A
A says nothing	A & B both get 1 year	A gets 3 years; B is free
A blames B	A is free; B gets 3 years	A & B both get 2 years

"Mutual cooperation is better than mutual defection but is not rational"

The Prisoner's dilemma in the EU refererendum

Basic principle of the Prisoner's dilemma:

- Everyone has a choice of C or D
- It's better for everyone if everyone chooses C
- Every individual gains by switching from C to D if nobody else switches

What does this have to do with the EU referendum?