

Cost function with Regularization

Simpler model

small numbers w_1, w_2, \dots, w_n, b

less likely to overfit

size x_1	bedrooms x_2	floors x_3	Age x_4	Avg income x_5	...	distance to coffee shop x_{100}	price y
						$n = 100$	

$w_1, w_2, w_3, \dots, w_{100}, b$

$$J(\vec{w}, b) = \frac{1}{2m} \sum_{i=1}^m (f_{\vec{w}, b}(\vec{x}^{(i)}) - y^{(i)})^2 + \underbrace{\frac{\lambda}{2m} \sum_{j=1}^n w_j^2}_{\text{Regularization term}} + \underbrace{\frac{\lambda}{2m} b^2}_{\text{can include or exclude } b}$$

$\lambda \rightarrow \text{lambda}$
regularization parameter $\lambda > 0$

$$\min_{\vec{w}, b} J(\vec{w}, b) = \min_{\vec{w}, b} \left[\frac{1}{2m} \sum_{i=1}^m (f_{\vec{w}, b}(\vec{x}^{(i)}) - y^{(i)})^2 + \frac{\lambda}{2m} \sum_{j=1}^n w_j^2 \right]$$

Regularized linear regression

$$\min_{\vec{w}, b} J(\vec{w}, b) = \min_{\vec{w}, b} \left[\frac{1}{2m} \sum_{i=1}^m (f_{\vec{w}, b}(\vec{x}^{(i)}) - y^{(i)})^2 + \frac{\lambda}{2m} \sum_{j=1}^n w_j^2 \right]$$

repeat {

$j = 1 \dots n$

$$w_j = w_j - \alpha \frac{\partial}{\partial w_j} J(\vec{w}, b)$$

$$b = b - \alpha \frac{\partial}{\partial b} J(\vec{w}, b)$$

} simultaneous updates

$$\frac{\partial}{\partial w_j} J(\vec{w}, b) = \frac{1}{m} \sum_{i=1}^m (f_{\vec{w}, b}(\vec{x}^{(i)}) - y^{(i)}) x_j^{(i)} + \frac{\lambda}{m} w_j$$

$$\frac{\partial}{\partial b} J(\vec{w}, b) = \frac{1}{m} \sum_{i=1}^m (f_{\vec{w}, b}(\vec{x}^{(i)}) - y^{(i)})$$

Don't have to regularize b

Regularized logistic regression

$$J(\vec{w}, b) = -\frac{1}{m} \sum_{i=1}^m [y^{(i)} \log(f_{\vec{w}, b}(\vec{x}^{(i)})) - (1 - y^{(i)}) \log(1 - f_{\vec{w}, b}(\vec{x}^{(i)}))] + \frac{\lambda}{2m} \sum_{j=1}^n w_j^2$$

$$\min_{\vec{w}, b} J(\vec{w}, b) \rightarrow w_j \downarrow$$

repeat {

$j = 1 \dots n$

$$w_j = w_j - \alpha \frac{\partial}{\partial w_j} J(\vec{w}, b)$$

$$b = b - \alpha \frac{\partial}{\partial b} J(\vec{w}, b)$$

} simultaneous updates

$$\frac{\partial}{\partial w_j} J(\vec{w}, b) = \frac{1}{m} \sum_{i=1}^m (f_{\vec{w}, b}(\vec{x}^{(i)}) - y^{(i)}) x_j^{(i)} + \frac{\lambda}{m} w_j$$

$$\frac{\partial}{\partial b} J(\vec{w}, b) = \frac{1}{m} \sum_{i=1}^m (f_{\vec{w}, b}(\vec{x}^{(i)}) - y^{(i)})$$

