Contents

1 Computed values for the gradient dynamics of the Motzkin polynomial

This section presents the values of the computed matrices and function for the gradient dynamics of the Motzkin polynomial.

$$f_{11} = \begin{pmatrix} 0 \\ 0 \end{pmatrix},\ F_{12} = \begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$f_{21} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -4 \\ 0 \\ 6 \\ 0 \\ 0 \\ 0 \\ -2 \\ 0 \\ 0 \\ -4 \\ 6 \\ -2 \end{pmatrix},\ F_{22} = \begin{pmatrix}
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}$$

$$\Delta(x) = \mathrm{blkdiag}(x_1 I_{13}, x_2 I_{13})$$

$$\pi_{\mathrm{lfr}}(x) = \begin{pmatrix} 1 \\ -4x_1^3x_2^2 \\ -4x_1^2x_2^2 \\ -4x_1x_2^2 \\ 6x_1x_2^2 \\ -2x_1x_2^4 \\ -4x_1^2x_2^3 \\ -4x_1x_2^3 \\ 6x_1^2x_2 \\ 6x_1x_2 \\ -2x_1^4x_2 \\ -2x_1^3x_2 \\ -2x_1^2x_2 \\ -2x_1x_2 \\ -4x_2^2 \\ -4x_2 \\ 6x_2^2 \\ 6x_2 \\ -2x_2^4 \\ -2x_2^3 \\ -2x_2^2 \\ -2x_2 \\ -4x_2^3 \\ -4x_2^2 \\ -4x_2 \\ 6x_2 \\ -2x_2 \end{pmatrix},\ S = \begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1.5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.3333 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.3333 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1.5 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1.5 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.5 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.5 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1.5 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.5 & 0 & 0
\end{pmatrix},\ \pi(x) = \begin{pmatrix} 1 \\ -4x_1^3x_2^2 \\ -4x_1^2x_2^2 \\ -4x_1x_2^2 \\ -2x_1x_2^4 \\ -4x_1^2x_2^3 \\ -4x_1x_2^3 \\ 6x_1^2x_2 \\ 6x_1x_2 \\ -2x_1^4x_2 \\ -2x_1^3x_2 \\ -4x_2^2 \\ -4x_2 \\ -2x_2^4 \\ -2x_2^3 \end{pmatrix},\ N(x) = \begin{pmatrix}
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2x_2 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & \frac{2x_2}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & \frac{2x_2}{3} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & -\frac{x_2}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -x_2 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -x_2 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -x_1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & \frac{3x_1}{2} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -x_1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{x_1}{3} & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -x_2 & 0 & 0 \\
4x_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -x_2 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{x_2}{2} & 0 & 0 & 1 \\
0 & x_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2x_2 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & x_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2x_2 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & x_1 & 0 & 0 & 0 & \frac{2x_2}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & x_1 & -\frac{x_2}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -x_2 & 0 & 0 & 0 & x_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -x_2 & 0 & 0 & 0 & x_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{2x_2}{3} & 0 & 0 & x_1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -\frac{x_2}{2} & 0 & 0 & 0 & 0 & 0 & 0 & x_1 & 0 \\
0 & 0 & 0 & -\frac{x_2}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & x_1
\end{pmatrix}$$

$$S_{f,1} = \begin{pmatrix}
1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
0 & -0.5 & 0 & 0 & 0 \\
0 & 0.25 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & -0.5 & 0 \\
0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & -0.5 \\
0 & 0 & 0 & 0 & -1.3333 \\
0 & 0 & 0 & 0 & 0.6667 \\
0 & -0.125 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -0.1667 \\
0 & 0 & -0.5 & 0 & 0 \\
0 & 0 & 0 & 0.125 & 0
\end{pmatrix},\ \pi_{f,1}(x) = \begin{pmatrix} 1 \\ -4x_1^3x_2^2 \\ -2x_1x_2^4 \\ -4x_1^2x_2^3 \\ 6x_1^2x_2 \end{pmatrix},\ N_{f,1}(x) = \begin{pmatrix}
\frac{x_1}{2}+1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & -\frac{4x_2}{3} \\
0 & 0 & 1 & \frac{x_2}{4} & 0 \\
0 & \frac{x_2}{2} & 0 & 1 & 0 \\
-24x_2 & 0 & 0 & 0 & 1 \\
0 & x_1 & 0 & 0 & \frac{8x_2}{3} \\
0 & 0 & x_1 & -\frac{x_2}{2} & 0 \\
0 & -x_2 & 0 & x_1 & 0 \\
48x_2 & 0 & 0 & 0 & x_1
\end{pmatrix} \tag{1}$$

$$S_{f,2} = \begin{pmatrix}
1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
0 & 0.5 & 0 & 0 & 0 \\
0 & 0.25 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0.5 & 0 \\
0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0.5 \\
0 & 0 & 0 & 0 & -1.3333 \\
0 & 0 & 0 & 0 & -0.6667 \\
0 & 0.125 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -0.1667 \\
0 & 0 & 0.5 & 0 & 0 \\
0 & 0 & 0 & 0.125 & 0
\end{pmatrix},\ \pi_{f,2}(x) = \begin{pmatrix} 1 \\ -4x_1^3x_2^2 \\ -2x_1x_2^4 \\ -4x_1^2x_2^3 \\ 6x_1^2x_2 \end{pmatrix},\ N_{f,2}(x) = \begin{pmatrix}
1-\frac{x_1}{2} & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & \frac{4x_2}{3} \\
0 & 0 & 1 & -\frac{x_2}{4} & 0 \\
0 & -\frac{x_2}{2} & 0 & 1 & 0 \\
-24x_2 & 0 & 0 & 0 & 1 \\
0 & x_1 & 0 & 0 & \frac{8x_2}{3} \\
0 & 0 & x_1 & -\frac{x_2}{2} & 0 \\
0 & -x_2 & 0 & x_1 & 0 \\
-48x_2 & 0 & 0 & 0 & x_1
\end{pmatrix} \tag{2}$$

$$S_{f,3} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & -2 & 0 & 0 \\ 0 & 0 & 0 & -2 & 0 \\ 0 & 0 & 0.75 & 0 & 0 \\ 0 & 0 & 0 & 0.75 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & -0.25 & 0 & 0 & 0 \\ -16 & 0 & 0 & 0 & 0 \\ 8 & 0 & 0 & 0 & 0 \\ -32 & 0 & 0 & 0 & 0 \\ 16 & 0 & 0 & 0 & 0 \end{pmatrix}, \ \pi_{f,3}(x) = \begin{pmatrix} 1 \\ -4x_1^3x_2^2 \\ -4x_1^2x_2^2 \\ -4x_1x_2^2 \\ -2x_1^4x_2 \end{pmatrix}, \ N_{f,3}(x) = \begin{pmatrix} \frac{x_2}{2}+1 & 0 & 0 & 0 & 0 \\ 0 & \frac{x_2}{2}+1 & 0 & 0 & 0 \\ 0 & 0 & \frac{x_2}{2}+1 & 0 & 0 \\ 0 & 0 & 0 & \frac{x_2}{2}+1 & 0 \\ 0 & 0 & 0 & 0 & \frac{x_2}{2}+1 \\ x_1 & 0 & 0 & -\frac{x_2}{32} & 0 \\ 0 & x_1 & 0 & 0 & -2x_2 \\ 0 & \frac{x_2}{2} & x_1 & 0 & 0 \\ 0 & 0 & \frac{x_2}{2} & x_1 & 0 \end{pmatrix} \tag{3}$$

$$S_{f,4} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & -0.75 & 0 & 0 \\ 0 & 0 & 0 & -0.75 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0.25 & 0 & 0 & 0 \\ -16 & 0 & 0 & 0 & 0 \\ -8 & 0 & 0 & 0 & 0 \\ -32 & 0 & 0 & 0 & 0 \\ -16 & 0 & 0 & 0 & 0 \end{pmatrix}, \ \pi_{f,4}(x) = \begin{pmatrix} 1 \\ -4x_1^3x_2^2 \\ -4x_1^2x_2^2 \\ -4x_1x_2^2 \\ -2x_1^4x_2 \end{pmatrix}, \ N_{f,4}(x) = \begin{pmatrix} 1-\frac{x_2}{2} & 0 & 0 & 0 & 0 \\ 0 & 1-\frac{x_2}{2} & 0 & 0 & 0 \\ 0 & 0 & 1-\frac{x_2}{2} & 0 & 0 \\ 0 & 0 & 0 & 1-\frac{x_2}{2} & 0 \\ 0 & 0 & 0 & 0 & 1-\frac{x_2}{2} \\ x_1 & 0 & 0 & \frac{x_2}{32} & 0 \\ 0 & x_1 & 0 & 0 & -2x_2 \\ 0 & -\frac{x_2}{2} & x_1 & 0 & 0 \\ 0 & 0 & -\frac{x_2}{2} & x_1 & 0 \end{pmatrix} \tag{4}$$

$$S_{f,5} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \ \pi_{f,5}(x) = \begin{pmatrix} 1 \\ -4x_2^2 \\ -4x_2 \\ -2x_2^4 \\ -2x_2^3 \end{pmatrix}, \ N_{f,5}(x) = \begin{pmatrix} 0 & 1 & -x_2 & 0 & 0 \\ 4x_2 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -x_2 \\ 0 & -\frac{x_2}{2} & 0 & 0 & 1 \\ x_1 & 0 & 0 & 0 & 0 \\ 0 & x_1 & 0 & 0 & 0 \\ 0 & 0 & x_1 & 0 & 0 \\ 0 & 0 & 0 & x_1 & 0 \\ 0 & 0 & 0 & 0 & x_1 \end{pmatrix} \tag{5}$$

$$S_{f,6} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \ \pi_{f,6}(x) = 1, \ N_{f,6}(x) = x_2 \tag{6}$$

$$P = \begin{pmatrix} 1.0205 & 0.0198 & -0.0409 & 0.1611 & 0.1048 & -0.1341 & 0.1372 & 0 & 0 & 0 & 0 & -0.0707 & 0 & -0.0315 & -0.0186 \\ 0.0198 & -0.0643 & -0.0067 & -0.0009 & -0.0007 & -0.0065 & 0.0042 & -0.0023 & -0.0042 & 0 & 0.0191 & 0.0007 & -0.0519 & 0.006 & 0.0257 \\ -0.0409 & -0.0067 & -0.0035 & 0.005 & -0.0051 & -0.0012 & -0.0061 & -0.0055 & -0.0179 & -0.0191 & 0.0087 & -0.0101 & -0.0604 & -0.0133 & 0.0165 \\ 0.1611 & -0.0009 & 0.005 & 0.0031 & 0.0094 & -0.0051 & 0.0018 & -0.014 & -0.0022 & -0.0155 & 0.0079 & 0.0103 & -0.0031 & -0.0007 & 0.0035 \\ 0.1048 & -0.0007 & -0.0051 & 0.0094 & -0.001 & 0 & -0.0005 & -0.0002 & 0.0016 & 0.0262 & -0.0267 & 0.0023 & 0.0127 & 0 & 0.0017 \\ -0.1341 & -0.0065 & -0.0012 & -0.0051 & 0 & 0.0042 & -0.0004 & -0.002 & -0.0003 & 0.0669 & -0.0169 & 0.0284 & 0.0522 & 0.0005 & 0.0022 \\ 0.1372 & 0.0042 & -0.0061 & 0.0018 & -0.0005 & -0.0004 & -0.0091 & 0.0024 & -0.011 & 0.0304 & -0.0792 & 0.0041 & -0.0181 & -0.0008 & -0.0016 \\ 0 & -0.0023 & -0.0055 & -0.014 & -0.0002 & -0.002 & 0.0024 & 0.0809 & 0.0176 & -0.514 & -0.0013 & -0.0157 & 0.0234 & 0.0496 & 0.0199 \\ 0 & -0.0042 & -0.0179 & -0.0022 & 0.0016 & -0.0003 & -0.011 & 0.0176 & -0.0609 & 0.0013 & -0.092 & -0.0249 & -0.0269 & 0.0163 & -0.028 \\ 0 & 0 & -0.0191 & -0.0155 & 0.0262 & 0.0669 & 0.0304 & -0.514 & 0.0013 & 0.0042 & 0 & -0.0369 & -0.2118 & 0.0145 & 0.0834 \\ 0 & 0.0191 & 0.0087 & 0.0079 & -0.0267 & -0.0169 & -0.0792 & -0.0013 & -0.092 & 0 & -3.1572 & 0.0083 & -0.0693 & -0.0071 & -0.0103 \\ -0.0707 & 0.0007 & -0.0101 & 0.0103 & 0.0023 & 0.0284 & 0.0041 & -0.0157 & -0.0249 & -0.0369 & 0.0083 & 0.0039 & -0.0023 & -0.0035 & 0.0007 \\ 0 & -0.0519 & -0.0604 & -0.0031 & 0.0127 & 0.0522 & -0.0181 & 0.0234 & -0.0269 & -0.2118 & -0.0693 & -0.0023 & -0.0433 & -0.0007 & 0.0049 \\ -0.0315 & 0.006 & -0.0133 & -0.0007 & 0 & 0.0005 & -0.0008 & 0.0496 & 0.0163 & 0.0145 & -0.0071 & -0.0035 & -0.0007 & 0.005 & 0 \\ -0.0186 & 0.0257 & 0.0165 & 0.0035 & 0.0017 & 0.0022 & -0.0016 & 0.0199 & -0.028 & 0.0834 & -0.0103 & 0.0007 & 0.0049 & 0 & -0.0195 \end{pmatrix} \tag{7}$$

2 Computed values for the genetic toggle switch in Escherichia coli

This section presents the values of the computed matrices and function for the genetic toggle switch dynamics.

$$f_{11} = \begin{pmatrix} 1.3 \\ 1 \end{pmatrix}, \; F_{12} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1.3 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$f_{21} = \begin{pmatrix} -1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 1 \\ -1 \end{pmatrix}, \; F_{22} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\Delta = \mathrm{blkdiag}(x_1 I_{11}, x_2 I_4)$$

$$\pi_{\mathrm{lfr}}(x) = \pi(x) = \begin{pmatrix} 1 \\ -x_1 \\ \frac{{x_1}^{10}}{{x_1}^{10}+1} \\ \frac{x_1^9}{{x_1}^{10}+1} \\ \frac{x_1^8}{{x_1}^{10}+1} \\ \frac{x_1^7}{{x_1}^{10}+1} \\ \frac{x_1^6}{{x_1}^{10}+1} \\ \frac{x_1^5}{{x_1}^{10}+1} \\ \frac{x_1^4}{{x_1}^{10}+1} \\ \frac{x_1^3}{{x_1}^{10}+1} \\ \frac{x_1^2}{{x_1}^{10}+1} \\ \frac{x_1}{{x_1}^{10}+1} \\ \frac{x_2^3}{x_2^3+1} \\ \frac{x_2^2}{x_2^3+1} \\ \frac{x_2}{x_2^3+1} \\ -x_2 \end{pmatrix}, \; S = I_{16}$$

$$S_{f,1} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ -0.5 & 0 & 0 & 0 & 0 \\ 0.001 & 0 & 0 & 0 & 0 \\ 0.002 & 0 & 0 & 0 & 0 \\ 0.0039 & 0 & 0 & 0 & 0 \\ 0.0078 & 0 & 0 & 0 & 0 \\ 0.0156 & 0 & 0 & 0 & 0 \\ 0.0312 & 0 & 0 & 0 & 0 \\ 0.0624 & 0 & 0 & 0 & 0 \\ 0.1249 & 0 & 0 & 0 & 0 \\ 0.2498 & 0 & 0 & 0 & 0 \\ 0.4995 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \; \pi_{f,1} = \begin{pmatrix} 1 \\ \frac{x_2^3}{x_2^3+1} \\ \frac{x_2^2}{x_2^3+1} \\ \frac{x_2}{x_2^3+1} \\ -x_2 \end{pmatrix}, \; N_{f,1} = \begin{pmatrix} 1-2x_1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -x_2 & 0 & 0 \\ 0 & 0 & 1 & -x_2 & 0 \\ -x_2 & x_2 & 0 & 1 & 0 \\ x_2 & 0 & 0 & 0 & 1 \\ 0 & x_1 & -\frac{x_2}{2} & 0 & 0 \\ 0 & 0 & x_1 & -\frac{x_2}{2} & 0 \\ -\frac{x_2}{2} & \frac{x_2}{2} & 0 & x_1 & 0 \\ \frac{x_2}{2} & 0 & 0 & 0 & x_1 \end{pmatrix}$$

$$S_{f,2} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ -1.5 & 0 & 0 & 0 & 0 \\ 0.983 & 0 & 0 & 0 & 0 \\ 0.6553 & 0 & 0 & 0 & 0 \\ 0.4369 & 0 & 0 & 0 & 0 \\ 0.2912 & 0 & 0 & 0 & 0 \\ 0.1942 & 0 & 0 & 0 & 0 \\ 0.1294 & 0 & 0 & 0 & 0 \\ 0.0863 & 0 & 0 & 0 & 0 \\ 0.0575 & 0 & 0 & 0 & 0 \\ 0.0384 & 0 & 0 & 0 & 0 \\ 0.0256 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \; \pi_{f,2} = \begin{pmatrix} 1 \\ \frac{x_2^3}{x_2^3+1} \\ \frac{x_2^2}{x_2^3+1} \\ \frac{x_2}{x_2^3+1} \\ -x_2 \end{pmatrix}, \; N_{f,2} = \begin{pmatrix} 1-\frac{2x_1}{3} & 0 & 0 & 0 & 0 \\ 0 & 1 & -x_2 & 0 & 0 \\ 0 & 0 & 1 & -x_2 & 0 \\ -x_2 & x_2 & 0 & 1 & 0 \\ x_2 & 0 & 0 & 0 & 1 \\ 0 & x_1 & -\frac{3x_2}{2} & 0 & 0 \\ 0 & 0 & x_1 & -\frac{3x_2}{2} & 0 \\ -\frac{3x_2}{2} & \frac{3x_2}{2} & 0 & x_1 & 0 \\ \frac{3x_2}{2} & 0 & 0 & 0 & x_1 \end{pmatrix}$$

$$S_{f,3} = \begin{pmatrix} 1&0&0&0&0&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0&0&0&0&0\\ 0&0&0&1&0&0&0&0&0&0&0&0\\ 0&0&0&0&1&0&0&0&0&0&0&0\\ 0&0&0&0&0&1&0&0&0&0&0&0\\ 0&0&0&0&0&0&1&0&0&0&0&0\\ 0&0&0&0&0&0&0&1&0&0&0&0\\ 0&0&0&0&0&0&0&0&1&0&0&0\\ 0&0&0&0&0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&0&0&0&0&1\\ -0.0059&0&0&0&0&0&0&0&0&0&0&0\\ 0.0326&0&0&0&0&0&0&0&0&0&0&0\\ -0.1811&0&0&0&0&0&0&0&0&0&0&0\\ 0.18&0&0&0&0&0&0&0&0&0&0&0 \end{pmatrix},\ \pi_{f,3} = \begin{pmatrix} 1\\ -x_1\\ \frac{{x_1}^{10}}{{x_1}^{10}+1}\\ \frac{x_1^9}{{x_1}^{10}+1}\\ \frac{x_1^8}{{x_1}^{10}+1}\\ \frac{x_1^7}{{x_1}^{10}+1}\\ \frac{x_1^6}{{x_1}^{10}+1}\\ \frac{x_1^5}{{x_1}^{10}+1}\\ \frac{x_1^4}{{x_1}^{10}+1}\\ \frac{x_1^3}{{x_1}^{10}+1}\\ \frac{x_1^2}{{x_1}^{10}+1}\\ \frac{x_1}{{x_1}^{10}+1} \end{pmatrix},\ N_{f,3} = \begin{pmatrix} \frac{50x_2}{9}+1&0&0&0&0&0&0&0&0&0&0&0\\ 0&\frac{50x_2}{9}+1&0&0&0&0&0&0&0&0&0&0\\ 0&0&\frac{50x_2}{9}+1&0&0&0&0&0&0&0&0&0\\ 0&0&0&\frac{50x_2}{9}+1&0&0&0&0&0&0&0&0\\ 0&0&0&0&\frac{50x_2}{9}+1&0&0&0&0&0&0&0\\ 0&0&0&0&0&\frac{50x_2}{9}+1&0&0&0&0&0&0\\ 0&0&0&0&0&0&\frac{50x_2}{9}+1&0&0&0&0&0\\ 0&0&0&0&0&0&0&\frac{50x_2}{9}+1&0&0&0&0\\ 0&0&0&0&0&0&0&0&\frac{50x_2}{9}+1&0&0&0\\ 0&0&0&0&0&0&0&0&0&\frac{50x_2}{9}+1&0&0\\ 0&0&0&0&0&0&0&0&0&0&\frac{50x_2}{9}+1&0\\ 0&0&0&0&0&0&0&0&0&0&0&\frac{50x_2}{9}+1\\ x_1&-\frac{50x_2}{9}&0&0&0&0&0&0&0&0&0&0\\ 0&-\frac{50x_2}{9}&x_1&0&0&0&0&0&0&0&0&-\frac{50x_2}{9}\\ 0&0&\frac{50x_2}{9}&x_1&0&0&0&0&0&0&0&0\\ 0&0&0&\frac{50x_2}{9}&x_1&0&0&0&0&0&0&0\\ 0&0&0&0&\frac{50x_2}{9}&x_1&0&0&0&0&0&0\\ 0&0&0&0&0&\frac{50x_2}{9}&x_1&0&0&0&0&0\\ 0&0&0&0&0&0&\frac{50x_2}{9}&x_1&0&0&0&0\\ 0&0&0&0&0&0&0&\frac{50x_2}{9}&x_1&0&0&0\\ 0&0&0&0&0&0&0&0&\frac{50x_2}{9}&x_1&0&0\\ 0&0&0&0&0&0&0&0&0&\frac{50x_2}{9}&x_1&0\\ 0&0&0&0&0&0&0&0&0&0&\frac{50x_2}{9}&x_1 \end{pmatrix}$$

$$S_{f,4} = \begin{pmatrix} 1&0&0&0&0&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0&0&0&0&0\\ 0&0&0&1&0&0&0&0&0&0&0&0\\ 0&0&0&0&1&0&0&0&0&0&0&0\\ 0&0&0&0&0&1&0&0&0&0&0&0\\ 0&0&0&0&0&0&1&0&0&0&0&0\\ 0&0&0&0&0&0&0&1&0&0&0&0\\ 0&0&0&0&0&0&0&0&1&0&0&0\\ 0&0&0&0&0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&0&0&0&0&1\\ 0.6334&0&0&0&0&0&0&0&0&0&0&0\\ 0.5279&0&0&0&0&0&0&0&0&0&0&0\\ 0.4399&0&0&0&0&0&0&0&0&0&0&0\\ -1.2&0&0&0&0&0&0&0&0&0&0&0 \end{pmatrix},\ \pi_{f,4} = \begin{pmatrix} 1\\ -x_1\\ \frac{{x_1}^{10}}{{x_1}^{10}+1}\\ \frac{x_1^9}{{x_1}^{10}+1}\\ \frac{x_1^8}{{x_1}^{10}+1}\\ \frac{x_1^7}{{x_1}^{10}+1}\\ \frac{x_1^6}{{x_1}^{10}+1}\\ \frac{x_1^5}{{x_1}^{10}+1}\\ \frac{x_1^4}{{x_1}^{10}+1}\\ \frac{x_1^3}{{x_1}^{10}+1}\\ \frac{x_1^2}{{x_1}^{10}+1}\\ \frac{x_1}{{x_1}^{10}+1} \end{pmatrix},\ N_{f,4} = \begin{pmatrix} 1-\frac{5x_2}{6}&0&0&0&0&0&0&0&0&0&0&0\\ 0&1-\frac{5x_2}{6}&0&0&0&0&0&0&0&0&0&0\\ 0&0&1-\frac{5x_2}{6}&0&0&0&0&0&0&0&0&0\\ 0&0&0&1-\frac{5x_2}{6}&0&0&0&0&0&0&0&0\\ 0&0&0&0&1-\frac{5x_2}{6}&0&0&0&0&0&0&0\\ 0&0&0&0&0&1-\frac{5x_2}{6}&0&0&0&0&0&0\\ 0&0&0&0&0&0&1-\frac{5x_2}{6}&0&0&0&0&0\\ 0&0&0&0&0&0&0&1-\frac{5x_2}{6}&0&0&0&0\\ 0&0&0&0&0&0&0&0&1-\frac{5x_2}{6}&0&0&0\\ 0&0&0&0&0&0&0&0&0&1-\frac{5x_2}{6}&0&0\\ 0&0&0&0&0&0&0&0&0&0&1-\frac{5x_2}{6}&0\\ 0&0&0&0&0&0&0&0&0&0&0&1-\frac{5x_2}{6}\\ x_1&\frac{5x_2}{6}&0&0&0&0&0&0&0&0&0&0\\ 0&\frac{5x_2}{6}&x_1&0&0&0&0&0&0&0&0&\frac{5x_2}{6}\\ 0&0&-\frac{5x_2}{6}&x_1&0&0&0&0&0&0&0&0\\ 0&0&0&-\frac{5x_2}{6}&x_1&0&0&0&0&0&0&0\\ 0&0&0&0&-\frac{5x_2}{6}&x_1&0&0&0&0&0&0\\ 0&0&0&0&0&-\frac{5x_2}{6}&x_1&0&0&0&0&0\\ 0&0&0&0&0&0&-\frac{5x_2}{6}&x_1&0&0&0&0\\ 0&0&0&0&0&0&0&-\frac{5x_2}{6}&x_1&0&0&0\\ 0&0&0&0&0&0&0&0&-\frac{5x_2}{6}&x_1&0&0\\ 0&0&0&0&0&0&0&0&0&-\frac{5x_2}{6}&x_1&0\\ 0&0&0&0&0&0&0&0&0&0&-\frac{5x_2}{6}&x_1 \end{pmatrix}$$

$$S_{f,5} = \begin{pmatrix} 1&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&1&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&1&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&1&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&1&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&1&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&1&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&1&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&1&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&1&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&1\\ 0&0.8866&0&0&0&0&0&0&0&0&0&0&0&0&0 \end{pmatrix},\ \pi_{f,5} = \begin{pmatrix} 1\\ -x_1\\ \frac{{x_1}^{10}}{{x_1}^{10}+1}\\ \frac{x_1^9}{{x_1}^{10}+1}\\ \frac{x_1^8}{{x_1}^{10}+1}\\ \frac{x_1^7}{{x_1}^{10}+1}\\ \frac{x_1^6}{{x_1}^{10}+1}\\ \frac{x_1^5}{{x_1}^{10}+1}\\ \frac{x_1^4}{{x_1}^{10}+1}\\ \frac{x_1^3}{{x_1}^{10}+1}\\ \frac{x_1^2}{{x_1}^{10}+1}\\ \frac{x_1}{{x_1}^{10}+1}\\ \frac{x_2^3}{x_2^3+1}\\ \frac{x_2^2}{x_2^3+1}\\ \frac{x_2}{x_2^3+1} \end{pmatrix},\ N_{f,5} = \begin{pmatrix} \frac{8807x_2}{7808}&1&0&0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&1&-\frac{8807x_2}{7808}&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&1&-\frac{8807x_2}{7808}&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&1&-\frac{8807x_2}{7808}&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&1&-\frac{8807x_2}{7808}&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&1&-\frac{8807x_2}{7808}&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&1&-\frac{8807x_2}{7808}&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&1&-\frac{8807x_2}{7808}&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&1&-\frac{8807x_2}{7808}&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&1&-\frac{8807x_2}{7808}&0&0&0\\ -\frac{8807x_2}{7808}&0&\frac{8807x_2}{7808}&0&0&0&0&0&0&0&0&1&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0&1&-x_2&0\\ 0&0&0&0&0&0&0&0&0&0&0&0&0&1&-x_2\\ -x_2&0&0&0&0&0&0&0&0&0&0&0&x_2&0&1\\ x_1-\frac{8807x_2}{7808}&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\ 0&x_1-\frac{8807x_2}{7808}&0&0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&x_1-\frac{8807x_2}{7808}&0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&x_1-\frac{8807x_2}{7808}&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&x_1-\frac{8807x_2}{7808}&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&x_1-\frac{8807x_2}{7808}&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&x_1-\frac{8807x_2}{7808}&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&x_1-\frac{8807x_2}{7808}&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&x_1-\frac{8807x_2}{7808}&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&x_1-\frac{8807x_2}{7808}&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&x_1-\frac{8807x_2}{7808}&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&x_1-\frac{8807x_2}{7808}&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0&x_1-\frac{8807x_2}{7808}&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0&0&x_1-\frac{8807x_2}{7808}&0\\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&x_1-\frac{8807x_2}{7808} \end{pmatrix}$$

$$P = \begin{pmatrix} 65.1394&42.2162&-34.8158&22.3949&24.4583&-17.6864&-26.8477&-61.821&-8.0911&89.9776&96.7132&-97.611&2.0871&-22.6982&-5.2142&2.9387\\ 42.2162&27.5742&-12.3709&34.7505&-34.2573&-48.9653&-3.0919&-14.3045&25.3861&11.4539&73.8419&-63.3605&22.1027&-20.727&-36.2502&-39.6294\\ -34.8158&-12.3709&30.1875&5.4484&-24.7694&-8.2696&-9.2531&33.4806&24.7396&-16.8181&4.2453&14.997&89.8425&-30.4036&-44.7064&-76.8154\\ 22.3949&34.7505&5.4484&31.7056&-76.9501&-45.7517&-5.041&19.1884&36.1357&14.5209&-11.646&-35.864&-31.1718&2.3427&-15.0105&4.8888\\ 24.4583&-34.2573&-24.7694&-76.9501&19.7012&7.4838&53.4006&19.1714&5.3898&-29.0471&-28.7647&-17.7078&-40.6794&14.5715&7.6&33.1098\\ -17.6864&-48.9653&-8.2696&-45.7517&7.4838&41.8226&42.1296&-6.566&-62.0578&-45.6941&-18.6213&41.7698&-15.6784&9.9814&23.531&23.6955\\ -26.8477&-3.0919&-9.2531&-5.041&53.4006&42.1296&25.3456&-31.1007&-62.2103&-48.0422&16.0926&65.8572&4.5791&-0.3026&16.7549&-9.738\\ -61.821&-14.3045&33.4806&19.1884&19.1714&-6.566&-31.1007&39.7442&-12.4576&14.5291&24.7449&14.0434&42.2717&-11.0346&-4.6005&-33.1091\\ -8.0911&25.3861&24.7396&36.1357&5.3898&-62.0578&-62.2103&-12.4576&39.5729&37.2141&42.2166&-41.0033&25.2799&-5.4981&-20.3807&-20.7611\\ 89.9776&11.4539&-16.8181&14.5209&-29.0471&-45.6941&-48.0422&14.5291&37.2141&60.1508&-29.2235&-73.6347&-6.821&11.4941&-31.3459&19.8702\\ 96.7132&73.8419&4.2453&-11.646&-28.7647&-18.6213&16.0926&24.7449&42.2166&-29.2235&23.5009&-23.7131&-52.3148&-8.2372&33.8011&41.9339\\ -97.611&-63.3605&14.997&-35.864&-17.7078&41.7698&65.8572&14.0434&-41.0033&-73.6347&-23.7131&51.5343&61.8672&-7.0911&-40.4226&-72.1123\\ 2.0871&22.1027&89.8425&-31.1718&-40.6794&-15.6784&4.5791&42.2717&25.2799&-6.821&-52.3148&61.8672&9.5181&4.2156&12.9434&-11.737\\ -22.6982&-20.727&-30.4036&2.3427&14.5715&9.9814&-0.3026&-11.0346&-5.4981&11.4941&-8.2372&-7.0911&4.2156&-41.9394&7.7923&35.1646\\ -5.2142&-36.2502&-44.7064&-15.0105&7.6&23.531&16.7549&-4.6005&-20.3807&-31.3459&33.8011&-40.4226&12.9434&7.7923&12.6124&-15.7859\\ 2.9387&-39.6294&-76.8154&4.8888&33.1098&23.6955&-9.738&-33.1091&-20.7611&19.8702&41.9339&-72.1123&-11.737&35.1646&-15.7859&-1.778 \end{pmatrix}$$

3 Computed values for the periodic ring dynamics

This section presents the values of the computed matrices and function for the periodic ring dynamics.

$$N(x) = \begin{pmatrix} x_1 & -1 & 0 & 0 & 0 & 0 \\ x_2 & 0 & -1 & 0 & 0 & 0 \\ 0 & x_1 & 0 & -1 & 0 & 0 \\ 0 & x_2 & 0 & 0 & -1 & 0 \\ 0 & 0 & x_1 & 0 & -1 & 0 \\ 0 & 0 & x_2 & 0 & 0 & -1 \\ 0 & 0 & 0 & x_2 & -x_1 & 0 \\ 0 & 0 & 0 & 0 & x_2 & -x_1 \end{pmatrix}, \; N_g(x) = \begin{pmatrix} x_1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ x_2 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & x_1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & x_2 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & x_1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & x_2 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_2 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & x_1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & x_2 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & x_1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & x_2 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & x_1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & x_2 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & x_1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & x_2 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & x_1 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & x_2 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & x_1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & x_2 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & x_2 & -x_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & x_2 & -x_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & x_2 & -x_1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & x_2 & -x_1 \end{pmatrix}$$

$$S_{f,1} = \begin{pmatrix} 1 & 0 & 0 \\ -3 & 0 & 0 \\ 0 & 1 & 0 \\ 9 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \; \pi_{f,1} = \begin{pmatrix} 1 \\ x_2 \\ x_2^2 \end{pmatrix}, \; N_{f,1} = \begin{pmatrix} \frac{x_1}{3}+1 & 0 & 0 \\ -x_2 & 1 & 0 \\ 0 & -x_2 & 1 \\ 3x_2 & x_1 & 0 \\ 0 & 3x_2 & x_1 \end{pmatrix}$$

$$S_{g,1} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ -3 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 9 & 0 & 0 & 0 & 0 \\ 0 & -3 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ -27 & 0 & 0 & 0 & 0 \\ 0 & 9 & 0 & 0 & 0 \\ 0 & 0 & -3 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 81 & 0 & 0 & 0 & 0 \\ 0 & -27 & 0 & 0 & 0 \\ 0 & 0 & 9 & 0 & 0 \\ 0 & 0 & 0 & -3 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \; \pi_{g,1} = \begin{pmatrix} 1 \\ x_2 \\ x_2^2 \\ x_2^3 \\ x_2^4 \end{pmatrix}, \; N_{g,1} = \begin{pmatrix} \frac{x_1}{3}+1 & 0 & 0 & 0 & 0 \\ -x_2 & 1 & 0 & 0 & 0 \\ 0 & -x_2 & 1 & 0 & 0 \\ 0 & 0 & -x_2 & 1 & 0 \\ 0 & 0 & 0 & -x_2 & 1 \\ 3x_2 & x_1 & 0 & 0 & 0 \\ 0 & 3x_2 & x_1 & 0 & 0 \\ 0 & 0 & 3x_2 & x_1 & 0 \\ 0 & 0 & 0 & 3x_2 & x_1 \end{pmatrix}$$

$$S_{f,2} = \begin{pmatrix} 1 & 0 & 0 \\ 3 & 0 & 0 \\ 0 & 1 & 0 \\ 9 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \; \pi_{f,2} = \begin{pmatrix} 1 \\ x_2 \\ x_2^2 \end{pmatrix}, \; N_{f,2} = \begin{pmatrix} 1-\frac{x_1}{3} & 0 & 0 \\ -x_2 & 1 & 0 \\ 0 & -x_2 & 1 \\ -3x_2 & x_1 & 0 \\ 0 & -3x_2 & x_1 \end{pmatrix}$$

$$S_{g,2} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 9 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 27 & 0 & 0 & 0 & 0 \\ 0 & 9 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 81 & 0 & 0 & 0 & 0 \\ 0 & 27 & 0 & 0 & 0 \\ 0 & 0 & 9 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \; \pi_{g,2} = \begin{pmatrix} 1 \\ x_2 \\ x_2^2 \\ x_2^3 \\ x_2^4 \end{pmatrix}, \; N_{g,2} = \begin{pmatrix} 1-\frac{x_1}{3} & 0 & 0 & 0 & 0 \\ -x_2 & 1 & 0 & 0 & 0 \\ 0 & -x_2 & 1 & 0 & 0 \\ 0 & 0 & -x_2 & 1 & 0 \\ 0 & 0 & 0 & -x_2 & 1 \\ -3x_2 & x_1 & 0 & 0 & 0 \\ 0 & -3x_2 & x_1 & 0 & 0 \\ 0 & 0 & -3x_2 & x_1 & 0 \\ 0 & 0 & 0 & -3x_2 & x_1 \end{pmatrix}$$

$$S_{f,3} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -3 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -3 & 0 \\ 9 & 0 & 0 \end{pmatrix}, \; \pi_{f,3} = \begin{pmatrix} 1 \\ x_1 \\ x_1^2 \end{pmatrix}, \; N_{f,3} = \begin{pmatrix} \frac{x_2}{3}+1 & 0 & 0 \\ 0 & \frac{x_2}{3}+1 & 0 \\ 0 & 0 & \frac{x_2}{3}+1 \\ x_1 & \frac{x_2}{3} & 0 \\ 0 & x_1 & \frac{x_2}{3} \end{pmatrix}$$

$$S_{g,3} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ -3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & -3 & 0 & 0 & 0 \\ 9 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & -3 & 0 & 0 \\ 0 & 9 & 0 & 0 & 0 \\ -27 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -3 & 0 \\ 0 & 0 & 9 & 0 & 0 \\ 0 & -27 & 0 & 0 & 0 \\ 81 & 0 & 0 & 0 & 0 \end{pmatrix}, \; \pi_{g,3} = \begin{pmatrix} 1 \\ x_1 \\ x_1^2 \\ x_1^3 \\ x_1^4 \end{pmatrix}, \; N_{g,3} = \begin{pmatrix} \frac{x_2}{3}+1 & 0 & 0 & 0 & 0 \\ 0 & \frac{x_2}{3}+1 & 0 & 0 & 0 \\ 0 & 0 & \frac{x_2}{3}+1 & 0 & 0 \\ 0 & 0 & 0 & \frac{x_2}{3}+1 & 0 \\ 0 & 0 & 0 & 0 & \frac{x_2}{3}+1 \\ x_1 & \frac{x_2}{3} & 0 & 0 & 0 \\ 0 & x_1 & \frac{x_2}{3} & 0 & 0 \\ 0 & 0 & x_1 & \frac{x_2}{3} & 0 \\ 0 & 0 & 0 & x_1 & \frac{x_2}{3} \end{pmatrix}$$

$$S_{f,4} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 3 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 3 & 0 \\ 9 & 0 & 0 \end{pmatrix}, \; \pi_{f,4} = \begin{pmatrix} 1 \\ x_1 \\ x_1^2 \end{pmatrix}, \; N_{f,4} = \begin{pmatrix} 1-\frac{x_2}{3} & 0 & 0 \\ 0 & 1-\frac{x_2}{3} & 0 \\ 0 & 0 & 1-\frac{x_2}{3} \\ x_1 & -\frac{x_2}{3} & 0 \\ 0 & x_1 & -\frac{x_2}{3} \end{pmatrix}$$

$$S_{g,4} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 \\ 9 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 3 & 0 & 0 \\ 0 & 9 & 0 & 0 & 0 \\ 27 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 9 & 0 & 0 \\ 0 & 27 & 0 & 0 & 0 \\ 81 & 0 & 0 & 0 & 0 \end{pmatrix}, \; \pi_{g,4} = \begin{pmatrix} 1 \\ x_1 \\ x_1^2 \\ x_1^3 \\ x_1^4 \end{pmatrix}, \; N_{g,4} = \begin{pmatrix} 1-\frac{x_2}{3} & 0 & 0 & 0 & 0 \\ 0 & 1-\frac{x_2}{3} & 0 & 0 & 0 \\ 0 & 0 & 1-\frac{x_2}{3} & 0 & 0 \\ 0 & 0 & 0 & 1-\frac{x_2}{3} & 0 \\ 0 & 0 & 0 & 0 & 1-\frac{x_2}{3} \\ x_1 & -\frac{x_2}{3} & 0 & 0 & 0 \\ 0 & x_1 & -\frac{x_2}{3} & 0 & 0 \\ 0 & 0 & x_1 & -\frac{x_2}{3} & 0 \\ 0 & 0 & 0 & x_1 & -\frac{x_2}{3} \end{pmatrix}$$

$$P = \begin{pmatrix} 0.8934 & 0 & 0 & 0.0994 & 0.0454 & -0.1659 \\ 0 & -0.5977 & -0.0454 & 0 & -0.0033 & -0.0011 \\ 0 & -0.0454 & -0.067 & 0.0033 & 0.0011 & 0 \\ 0.0994 & 0 & 0.0033 & 0.0499 & 0 & -0.0195 \\ 0.0454 & -0.0033 & 0.0011 & 0 & 0.1386 & 0 \\ -0.1659 & -0.0011 & 0 & -0.0195 & 0 & 0.0499 \end{pmatrix}$$

4 Computed values for the Van der Pol oscillator

This section presents the values of the computed matrices and function for the Van der Pol oscillator:

$$A_g = \begin{pmatrix}
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&-1&1&0&0&0&0&-1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&0&0&2&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&0&-1&1&1&0&0&0&0&0&-1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&0&0&-2&2&0&0&0&0&0&0&-2&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&0&0&0&0&0&3&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&0&0&0&0&-1&1&2&0&0&0&0&0&0&0&-1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&0&0&0&0&0&-2&2&1&0&0&0&0&0&0&0&-2&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&0&0&0&0&0&0&-3&3&0&0&0&0&0&0&0&0&-3&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&0&0&0&0&0&0&0&0&0&4&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&0&0&0&0&0&0&0&0&-1&1&3&0&0&0&0&0&0&0&0&0&-1&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&0&0&0&0&0&0&0&0&0&-2&2&2&0&0&0&0&0&0&0&0&0&-2&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&0&0&0&0&0&0&0&0&0&0&-3&3&1&0&0&0&0&0&0&0&0&0&-3&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&0&0&0&0&0&0&0&0&0&0&0&-4&4&0&0&0&0&0&0&0&0&0&0&-4&0&0&0&0&0&0&0&0&0&0
\end{pmatrix} \tag{8}$$