

- Heap Sort
- Kth largest element
- Sort nearly sorted array
- Median of stream of integer

Ques Heap sort :-

Sort an array

arr[]  $\rightarrow$  13, 14, 7, 6, 10, 2, 5, 8, 3, 1

$\downarrow$  sort

ans[]  $\rightarrow$  \_\_\_\_\_

idea :-

array  $\rightarrow$  Build a min heap.  $\rightarrow O(n)$

$\downarrow$   
get min & delete min  $\rightarrow O(\log n)$

$\downarrow$   
ans[] array.

T.C  $\rightarrow O(n) + O(\log n) \Rightarrow O(\log n)$

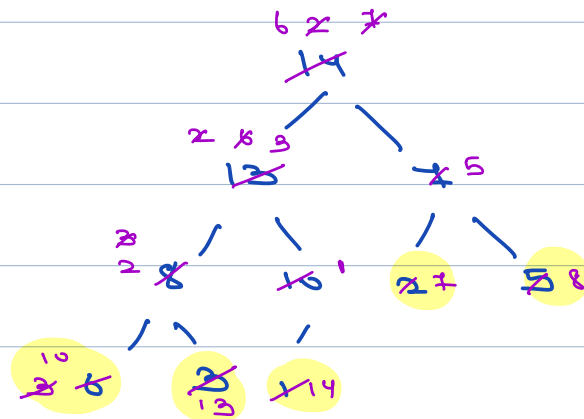
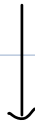
S.C  $\rightarrow O(n)$  (Extra space)

idea 2 :- Do it in place.

arr[]  $\rightarrow$  13, 14, 7, 6, 10, 2, 5, 8, 3, 1

$\downarrow$  max heap.

ans[]  $\rightarrow$  <sup>0</sup>14, <sup>1</sup>13, <sup>2</sup>7, <sup>3</sup>8, <sup>4</sup>10, <sup>5</sup>2, <sup>6</sup>5, <sup>7</sup>6, <sup>8</sup>3, <sup>9</sup>1



given array  $\rightarrow$  array

1) Build Min heap  $\rightarrow O(n)$

$j = n-1$ ,

while ( $j > 0$ )  $\rightarrow O(\log n)$

swap ( $0, j$ );

$j--$ ;

heapify ( $0, arr, j$ );

$\rightarrow$  last valid index

3

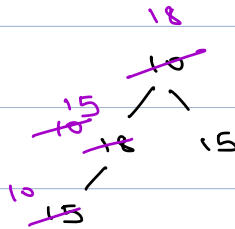
T.C  $\rightarrow O(n) + O(\log n) \rightarrow O(\log n)$

S.C  $\rightarrow O(1)$

Heap sort  $\rightarrow$  inplace  $\rightarrow$  S.C  $\rightarrow O(1)$

stable  $\rightarrow$  no.

10, 18, 15, 15



Ques :- arr[], find k<sup>th</sup> largest element.

e.g. 1)

arr[] = [8, 5, 1, 2, 4, 9, 7]

k = 3

Ans :- 7

- First largest element = 9
- Second largest element = 8
- Third largest element = 7

e.g. 2)

arr[] → 1, 2, 3, 4, 5.

k = 5, → 1.

idea 1 :- Sorting & return arr[n-k]

e.g., k=3, arr[] → [8, 5, 1, 2, 4, 9, 7]

↓ sort

<sup>0 1 2 3 4 5 6</sup>  
[1, 2, 4, 5, 7, 8, 9]

idea 2 :- Heap sort :-

Build a max heap.

↓

do the heap sort step k-1 times.

T.C →  $O(n) + O(k \log n)$

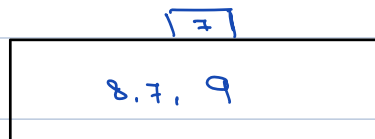
S.C →  $O(1)$ .

idea 3

Min heap :-

$k = 3$ ,

8, 5, 1, 2, 4, 9, 7, 4

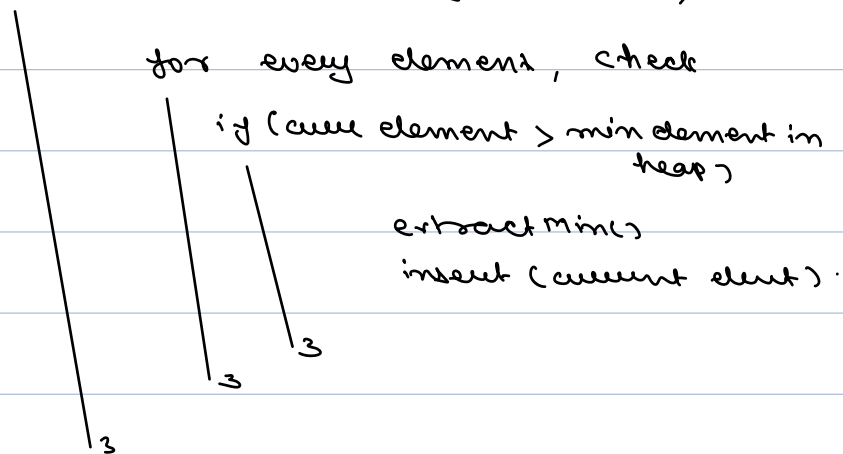


min heap

①

Store first  $k$  Elements in a minheap (eg).

Iterate on remaining elements,

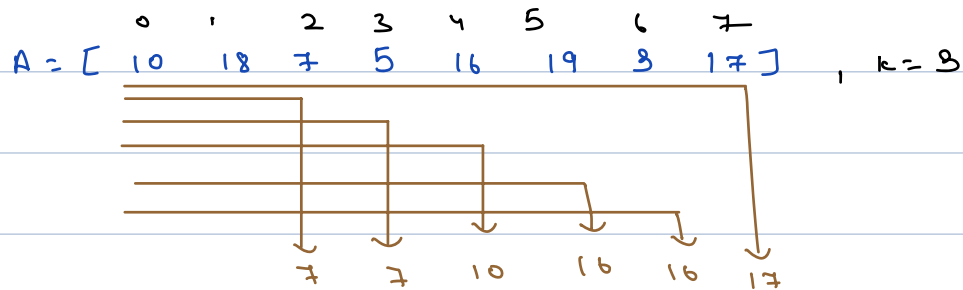


ans = getMin(),

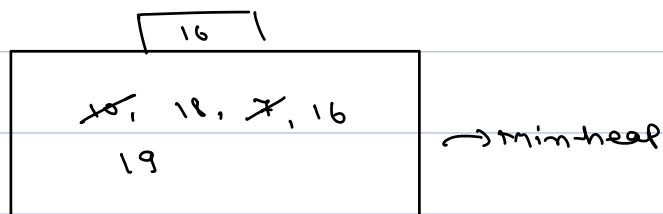
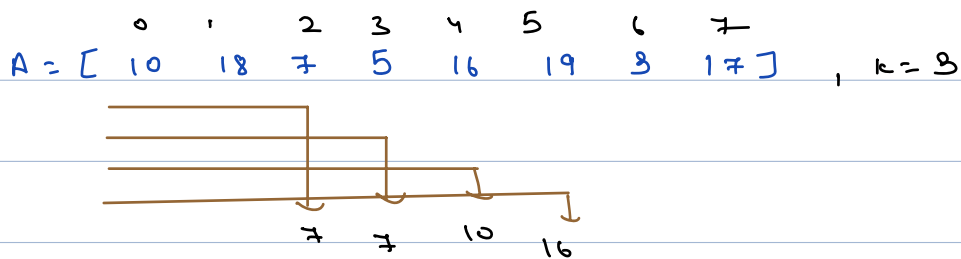
T.C  $\Rightarrow O(n \log k)$

S.C  $\Rightarrow O(k)$

Ques) Given an Integer array,  $\forall i = (k-1)$ ,  
find  $k^{\text{th}}$  largest element from  $0 \leq i$ .



Idea :-



①

Store first  $k$  Elements in a minheap (pq).  
Print (getmin());

Iterate on remaining elements,

for every element, check

if (curr element > min element in heap)

extract min()

insert (current elem).

Print (getmin());

T.C  $\rightarrow O(n \log k)$

S.C  $\rightarrow O(k)$



Ques Given an int[], & k, every element is at max k dist. away from its sorted posn, we have to sort the array.

$\rightarrow k < \leq m$

A: [ 3, 1, 5, 6, 2, 7, 4 ]

k = 3

A = [ 1, 2, 3, 4, 5, 6, 7 ]

T.C  $\rightarrow O(m \log k)$

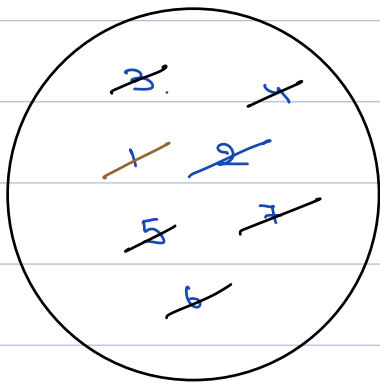
idea 1 :- Sort the array.

T.C  $\rightarrow O(m \log n)$ .

idea 2 :-

A: [ 3, 1, 5, 6, 2, 7, 4 ] k = 3.

min  
heap



0  $\rightarrow$  0 to 3

1  $\rightarrow$  0 to 4

2  $\rightarrow$  0 to 5

3  $\rightarrow$  0 to 6

4  $\rightarrow$  1 to 6

5  $\rightarrow$  2 to 6

6  $\rightarrow$  3 to 6

1)  $PQ \rightarrow \text{minheap}$ .

2) for  $i \rightarrow 0$  to  $n-1$

$PQ.add(arr[i]);$

if  $(PQ.size() > k)$  {

next  $\rightarrow PQ.getMin();$   
smaller  
element

$PQ.remove();$

}

}

when loop ends remove all rest elements one by one.

T.C  $\rightarrow O(n \log k)$

S.C  $\rightarrow O(k)$

Ques Given a running stream of Integers,  
find median for all input.

§ 5, 10, 2, 1, 43 → § 1, 2, 4, 5, 103 → 4

§ 5, 10, 2, 3, 1, 43 → § 1, 2, 3, 4, 5, 103 →  $\frac{3+4}{2} \Rightarrow 3.5$

T/P → 

<u>9</u>	<u>8</u>	<u>17</u>	<u>20</u>	<u>25</u>	<u>10</u>	<u>5</u>	<u>3</u>
9	8.5	9	13	17	13.5	10	9.5

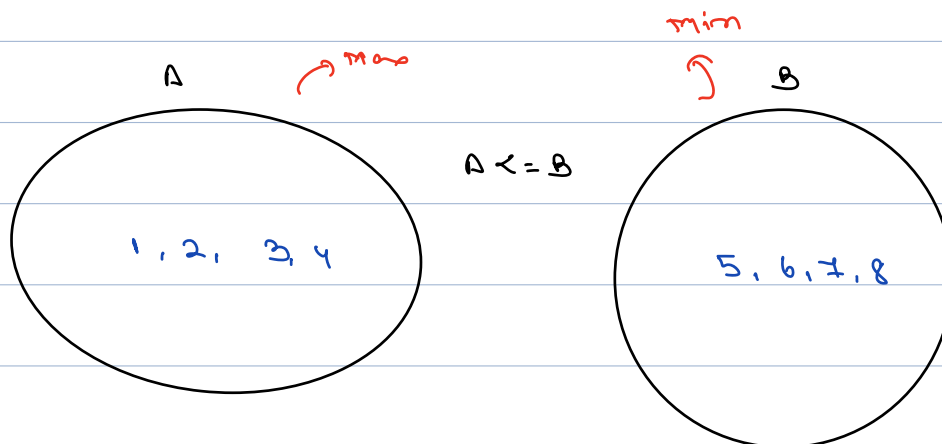
Idea 1 :- we insertion sort.

T.C →  $O(n^2)$

S.C →  $O(1)$

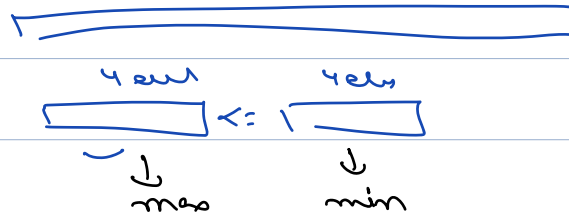
Idea 2 :- 1, 5, 7, 4, 3, 6, 2, 8

$\text{sizeof}(A) - \text{sizeof}(B) = 0$



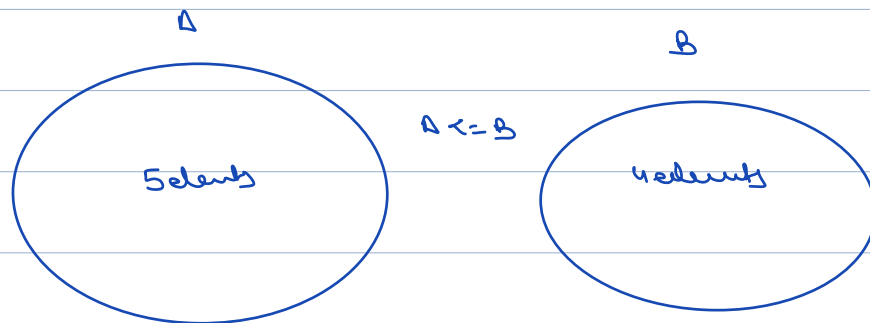
median =  $\frac{\text{Max of A} + \text{Min of B}}{2}$

8 elements



Case 2 when odd elements :-

9 elements



Median = Max of A ,  
5th element ,  
0

Size of (A) - Size of (B) = 1

$$A \leq B$$

1) Even

$$\text{Size}(A) - \text{Size}(B) = 0$$

$$\text{median} = \frac{\text{Max}(A) + \text{Min}(B)}{2}$$

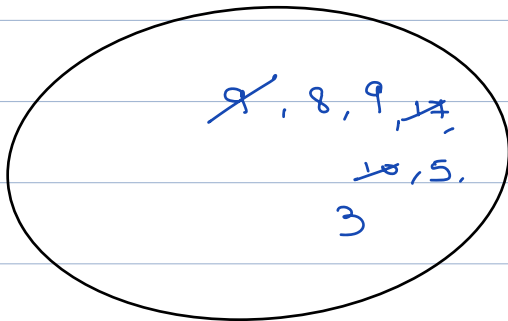
2) Odd

$$\text{Size}(A) - \text{Size}(B) = 1$$

$$\text{median} = \text{Max}(A)$$

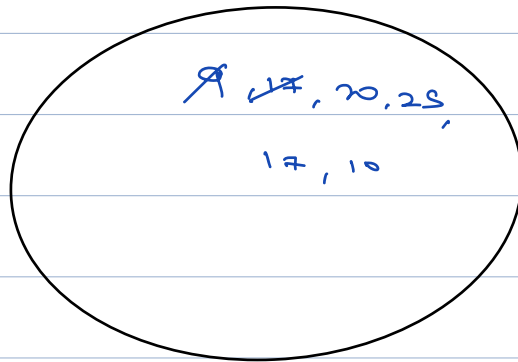
I/P →	9	8	17	20	25	10	5	3
O/P	9	8.5	9	13	17	13.5	10	9.5

A



max heap

B



min heap

```
void running_median (int arr[]) {
```

```
    Max heap <int> maxh;
```

```
    Min heap <int> minh;
```

```
    maxh.insert (arr[0]);
```

```
    print (arr[0]);
```

```
    for (i = 1; i < n; i++) {
```

```
        ele = arr[i];
```

```
        if (ele < maxh.getMax()) {
```

```
            | 3 maxh.insert (ele);
```

```
            else {
```

```
            | 1 minh.insert (ele);
```

```
        if (maxh.size() - minh.size() > 1) {
```

```
            | 3 Transfer 1 element from maxh to minh;
```

```
        if (maxh.size() - minh.size() < 0) {
```

```
            | 3 Transfer 1 element from minh to maxh;
```

```
        int s = maxh.size() + minh.size();
```

```
        if (s % 2 == 1) {
```

```
            | 3 print (maxh.getMax());
```

3

2

T.C  $\rightarrow O(n \log n)$

$$J.C \rightarrow O_{\underline{m}}.$$

4, 3, 6, 1, 10

1, 2, 4, 3, 5

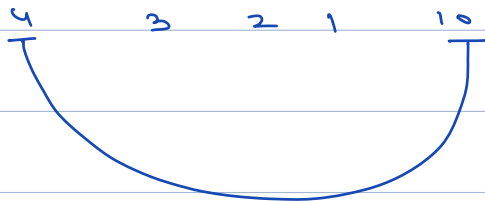
There are  $N$  poles, and the height of the poles is given by an integer array  $A$ .  
You have to connect two poles such that all the poles between them have a height **smaller** than the minimum height of the two poles.  
Find the total number of pairs of such poles.

**NOTE:**

The heights of the given poles are distinct.

i j

$$arr[i] < \min(arr[i], arr[j]) \quad (i < r < j)$$





orig  $\rightarrow$  1, 2, 3, 4

curState  $\rightarrow$  1, 2, 3, 4

$\downarrow$   
2, 2, 3, 4

$\downarrow$   
3, 2, 3, 4

$\downarrow$   
4, 2, 3, 4

