Agenda :-

- Addition and Multiplication Rule
- Permutation basics
- Combination basics and properties
- Pascal Triangle
- Find N-th column title

$^{10}C_1 * {}^{7}C_1$

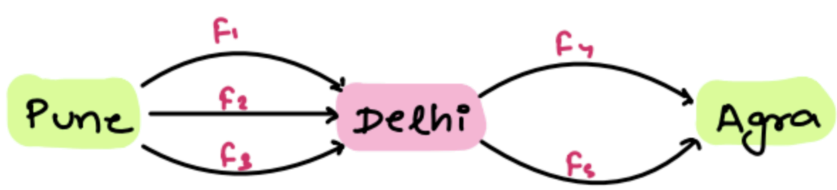
Given 10 girls and 7 boys, How many different pairs can be formed?

Note: pair = 1 boy + 1 girl

Boys	Girls
B_1	G_1
B_2	G_2
B_3	G_3
⋮	⋮
B_7	G_{10}

$7 \times 10 = 70$ ways

Ques)

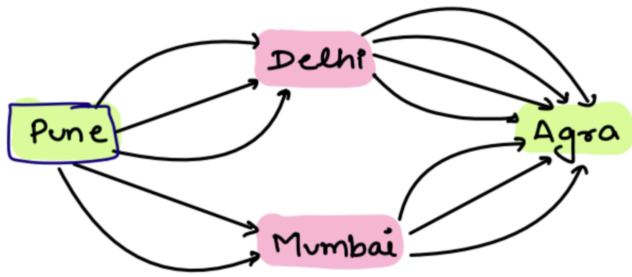


No. of ways Pune to Agra?

no. of ways to reach from Pune to delhi * ways to reach from Delhi to Agra.

3 * 2 -> 6.

Ques 7.



Pune to Agra.

Pune to Agra via delhi or pune to Agra via mumbai.

12 + 6 => 18 ways.

And → *
OR → +

-: Permutation :-

↳ arrangement of Objects.

↳ here Order matters.

$(i, j) \neq (j, i)$.

RB, BR

RBG, BGR, RGB, ...

Ques)

Given 3 distinct characters. In how many ways, we can arrange them?

s = "abc"

$\underline{3} * \underline{2} * \underline{1} = 6$ ways.

a → b → c
a → c → b

b → c → a
b → a → c

c → a → b
c → b → a

s = 'abcd'

$\underline{4} * \underline{3} * \underline{2} * \underline{1} \Rightarrow 24$.

In how many ways n distinct characters can be arranged?

$\underline{n} \times (\underline{n-1}) \times (\underline{n-2}) \times (\underline{n-3}) \times \underline{\ } \cdot \cdot \cdot \underline{\ } \quad \underline{\ } \times \cdot \cdot \underline{\ } \cdot \underline{\ }$

$\Rightarrow \underline{n!}$.

<u>Ques</u> how many ways you can arrange 2 out of 4 characters?

$^4P_2 \Rightarrow \underline{12}$.

<u>a b c d</u>

$\underline{4} * \underline{3} \Rightarrow \underline{12}$.

<u>Ques</u> 3 distinct characters, arrange 2 characters out of this, (a, b, c).

$\underline{3} \quad \underline{2}$

$^3P_2 \Rightarrow \underline{6}$

a b
b a
c a
a c
b c
c b

Ques) Given 5 distinct characters in how many ways we can arrange them in 2 places.

$\underline{5} * \underline{4} \Rightarrow 20$ ways.

$5P_2$

$\Rightarrow \frac{5!}{3!}$

$\Rightarrow \frac{5 \times 4 \times \cancel{3!}}{\cancel{3!}}$

Ques) N distinct characters, we need to arrange 3 characters.

$\underline{n} * \underline{(n-1)} * \underline{(n-2)} \Rightarrow n * (n-1) * (n-2)$

Ques) N distinct characters, we need to arrange r characters.

$\underbrace{\underline{n} * \underline{(n-1)} * \underline{(n-2)} * \underline{\quad} \ldots (\underline{n-(r-1)})}_{r \text{ diff.}}$

$$\frac{n * (n-1) * (n-2) \ldots (n-r+1) * (n-r)(n-r-1) \ldots 1}{(n-r)(n-r-1) \ldots 1}$$

$$\Rightarrow \boxed{\frac{n!}{(n-r)!} = {}^{n}P_{r}}$$ → no. of ways to arrange r places from n distinct characters.

Combination

↳ It is no. of ways of selecting something.

↳ Order doesn't matter.

$RB = BR$

$(i,j) = (j,i)$.

Given 4 players, count the number of ways of selecting 3 players.

$\{ P_1 \quad P_2 \quad P_3 \quad P_4 \}$

${}^{4}C_{3}$.

$P_1 P_2 P_3$

$P_1 P_2 P_4$

$P_1 P_3 P_4$

$P_2 P_3 P_4$

4 ways.

Ques) No. of ways to arrange 4 players in 3 slots.

__ __ __ $\{P_1, P_2, P_3, P_4\}$

${}^4P_3$

$P_1\ P_2\ P_3$
$P_1\ P_3\ P_2$
$P_2\ P_3\ P_1$
$P_2\ P_1\ P_3$
$P_3\ P_1\ P_2$
$P_3\ P_2\ P_1$

$P_1\ P_2\ P_3$	$P_1\ P_2\ P_4$	$P_1\ P_3\ P_4$	$P_2\ P_3\ P_4$
$P_1\ P_3\ P_2$	$P_1\ P_4\ P_2$	$P_1\ P_4\ P_3$	$P_2\ P_4\ P_3$
$P_2\ P_1\ P_3$	$P_2\ P_1\ P_4$	$P_3\ P_1\ P_4$	$P_3\ P_2\ P_4$
$P_2\ P_3\ P_1$	$P_2\ P_4\ P_1$	$P_3\ P_4\ P_1$	$P_3\ P_4\ P_2$
$P_3\ P_1\ P_2$	$P_4\ P_1\ P_2$	$P_4\ P_1\ P_3$	$P_4\ P_2\ P_3$
$P_3\ P_2\ P_1$	$P_4\ P_2\ P_1$	$P_4\ P_3\ P_1$	$P_4\ P_3\ P_2$
$\{P_1\ P_2\ P_3\}$	$\{P_1\ P_2\ P_4\}$	$\{P_1\ P_3\ P_4\}$	$\{P_2\ P_3\ P_4\}$

for every selection = 6 arrangements.

Total no. of selection * no. of arrangement of each section = Total no. of arrangements.

$x * 6 = 24$

$\boxed{x = 4}$

B_1 B_2 B_3

Permutation

$B_1 B_2$
$B_2 B_1$

$B_2 B_3$
$B_3 B_2$

$B_1 B_3$
$B_3 B_1$

Combination.

$B_1 B_2$
$B_1 B_3$
$B_2 B_3$

Ques) Given n elements how many ways we can arrange r items out of that?

$\downarrow$

^nP_r.

arrange r items $\rightarrow$ $r!$

arrangement — selection

$r!$ — 1

^nP_r — x.

$r! * x = {}^nP_r$

$x = \frac{^nP_r}{r!}$

$$x = \frac{n!}{(n-r)! * r!}$$

$$x = {}^nC_r$$ → no. of ways of selecting r things out of n things.

$$\boxed{{}^nC_r = \frac{n!}{(n-r)! * r!}}$$

$$ {}^nC_r = \frac{{}^nP_r}{r!}$$

$$\boxed{{}^nP_r = {}^nC_r * r!}$$

${}^nP_r$ → arranging r things out of n things

${}^nC_r$ → selecting r things out of n things.

Break 8:10 Am - 8:20 Am

Property - 1 :-

$n \rightarrow n$

$\downarrow$

${}^nC_n \rightarrow 1$.

↳ selecting n things out of n things.

a b c d

$\downarrow$

a, b, c, d.

Selecting 0 items from n items :-

↳ ${}^nC_0 \rightarrow 1$ way.

Property - 2

4 boys (B₁ B₂ B₃ B₄)

↳ 3 boys.

$${}^nC_r = {}^nC_{n-r}$$

${}^4C_3$

selecting		not seching
B_1, B_2, B_3	$\rightarrow$	B_4
B_1, B_2, B_4	$\rightarrow$	B_3
B_2, B_3, B_4	$\rightarrow$	B_1
B_1, B_3, B_4	$\rightarrow$	B_2

Property :- given n distinct elements, select r items.

$\underbrace{\_\_\ \_\_\ \_\_\ \_\_}_{r \text{ places}}$ __

→ nth item

→ not select nth item.

$$^{n}C_{r} = {}^{n-1}C_{r-1} + {}^{n-1}C_{r}$$

$B_1 \; B_2 \; B_3 \; B_4 \; B_5$

$$^{5}C_{3} = {}^{4}C_{2} + {}^{4}C_{3}$$

$$\frac{(n-1)!}{(n-r)! \times (r-1)!} + \frac{(n-1)!}{r! \times (n-1-r)!}$$

$$\frac{(N-1)!}{(R-1)! \times (N-R) \times (N-R-1)!} + \frac{(N-1)!}{R \times (R-1)! \times (N-R-1)!}$$

$$\Rightarrow \frac{(N-1)!}{(R-1)! \times (N-R-1)!} \left[\frac{1}{N-R} + \frac{1}{R} \right]$$

$$\frac{(N-1)!}{(R-1)! \times (N-R-1)!} \left[\frac{\cancel{R} + N - \cancel{R}}{R \times N-R} \right]$$

$$\Rightarrow \frac{N \, (N-1)!}{R \times (R-1)! \times (N-R) \, (N-R-1)!} \Rightarrow \frac{N!}{R! \times (N-R)!}$$

$$\Rightarrow {}^{N}C_{R}.$$

Asked in Interviews ?

Ques Pascal's Δ.

$n = 6$,

$${}^nC_r = \frac{n!}{r! \, (n-r)!}$$

${}^0C_0$							$\Rightarrow$	1
${}^1C_0$	${}^1C_1$						$\Rightarrow$	1 1
${}^2C_0$	${}^2C_1$	${}^2C_2$					$\Rightarrow$	1 2 1
${}^3C_0$	${}^3C_1$	${}^3C_2$	${}^3C_3$				$\Rightarrow$	1 3 3 1
${}^4C_0$	${}^4C_1$	${}^4C_2$	${}^4C_3$	${}^4C_4$			$\Rightarrow$	1 4 6 4 1
${}^5C_0$	${}^5C_1$	${}^5C_2$	${}^5C_3$	${}^5C_4$	${}^5C_5$			
${}^6C_0$	${}^6C_1$	${}^6C_2$	${}^6C_3$	${}^6C_4$	${}^6C_5$	${}^6C_6$		

Brute force :-

run 2 for loops, calculate the value of, ${}^nC_r$ for every place and print it.

$^0C_0$ 1

$^1C_0$ 1 $\quad$ $^1C_1$ 1

$^2C_0$ 1 $\quad$ $^2C_1$ 2 $\quad$ $^2C_2$ 1

$^3C_0$ $\quad$ $^3C_1$ 3 $\quad$ $^3C_2$ 3 $\quad$ $^3C_3$ 1

$^4C_0$ 1 $\quad$ $^4C_1$ 4 $\quad$ $^4C_2$ 6 $\quad$ $^4C_3$ 4 $\quad$ $^4C_4$ 1

$$^nC_r = \frac{n!}{r! \times (n-r)!}$$

$n = 4$

5×5

	0	1	2	3	4
0					
1					
2					
3	1			1	
4					

$$^nC_r = {}^{n-1}C_{r-1} + {}^{n-1}C_r$$

$$^3C_1 = {}^2C_0 + {}^2C_1$$

$$^4C_2 = {}^3C_1 + {}^3C_2$$

$n = 4$

```
PascalsTriangle(n) {
    nCR[n+1][n+1] = {0}
    for (i = 0; i <= n; i++) {
        nCR[i][0] = 1;
        nCR[i][i] = 1;
        for (j = 1; j < i; j++) {
            nCR[i][j] = nCR[i-1][j] + nCR[i-1][j-1]
        }
    }
}
```

T.C → $O(n^2)$

S.C → $O(n^2)$

Ques) Excel Column Title :-

1	2	3	4	... 26	27	28		52	53
A	B	C	D	Z	AA	AB	...	AZ	BA ....

N = 30 → AD , N = 50 → AX , N = 78 .

Base (2)	Base 8	Base-26
0	0	A
1	1	B
10	2	C
11	3	⋮
100	4	Z
	5	AA
	6	AB
	7	⋮
	10	AZ
	11	BA
	12	BB
	⋮	⋮
	17	BZ
	20	CA
	21	⋮
	⋮	

1 - 26 (A - Z)

27 - 52 (AA - AZ)

53 - 78 (BA - BZ)

79 - 104 (CA - CZ)

N = 1000

26	1000	12	→ L
26	38	12	→ L
26	1	1	→ A
	0		

→ ALL ←

N = 78

26	78	0	→ X
26	3	3	→ C
	0		

whenever we get a remainder 0, divide by 1 less quotient, so you don't get 0 remainder, since we don't have equivalent of it.

26	78	26	Z
26	2	2	B
	0		

→ BZ ←

idea 2

(A - Z) $\rightarrow$ (1 - 26)

(A - Z) $\rightarrow$ (0 - 25)

change the mapping

26	78 − 1 = 77	25	Z
26	2 − 1 => 1	1	B
	0		

BZ

26	1000 − 1 = 999	11	L
26	38 − 1 = 37	11	L
26	1 − 1 = 0	0	A
	0		

ALL

```
void columnTitle(int n) {
    ans = "";
    while(n > 0) {
        ans = (char) ((n - 1) % 26 + 'A') + ans; // char + string
        n = (n - 1) / 26
    }
    return ans
}
```

→ Rain water Trapping O(1)

→ Subarrays OR

→ Strange Inequality