

Avg PSP 74 → 77 → 75 → 80



Monday Goal

Decimal Number System 0-9 (base 10)

$$\begin{array}{r} 2 \ 1 \ 0 \\ 3 \ 4 \ 2 \end{array} = 300 + 40 + 2$$
$$= 3 \times 10^2 + 4 \times 10^1 + 2 \times 10^0$$

$$\begin{array}{r} 3 \ 2 \ 1 \ 0 \\ 2 \ 5 \ 6 \ 3 \end{array} = 2000 + 500 + 60 + 3$$
$$= 2 \times 10^3 + 5 \times 10^2 + 6 \times 10^1 + 3 \times 10^0$$

Binary Number System

0-1 (base 2)

high voltage, on, true
low voltage, off, false

$$\begin{array}{r} 2 \ 1 \ 0 \\ 1 \ 1 \ 0 \end{array} = 1 \times 2^2 + 1 \times 2^1 + 0 \times 2^0$$
$$= 4 + 2 + 0 = \underline{6}$$

$$\begin{array}{r} 3 \ 2 \ 1 \ 0 \\ 1 \ 0 \ 1 \ 1 \end{array} = 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0$$
$$= 8 + 0 + 2 + 1 = \underline{11}$$

Binary to Decimal

$$\begin{array}{r} 4 \ 3 \ 2 \ 1 \ 0 \\ 1 \ 0 \ 1 \ 0 \ 1 \\ \downarrow \downarrow \downarrow \\ 2^4 + 2^2 + 2^0 = 16 + 4 + 1 = \underline{21} \end{array}$$

$$\begin{array}{r} 6 \ 5 \ 4 \ 3 \ 2 \ 1 \ 0 \\ 1 \ 0 \ 1 \ 1 \ 0 \ 1 \ 0 \\ 2^6 + 2^4 + 2^3 + 2^1 = 64 + 16 + 8 + 2 = \underline{90} \end{array}$$

Decimal to Binary

int x = 5; ^{decimal}
store in binary

remainders

2	20	0
2	10	0
2	5	1
2	2	0
2	1	1
	0	

4	3	2	1	0
1	0	1	0	0

$$2^4 + 2^2 = 16 + 4 = \underline{20}$$

2	45	1
2	22	0
2	11	1
2	5	1
2	2	0
2	1	1
	0	

101101 (Ans)

Addition of Decimal Numbers

	1	1	
	3	6	8
+	4	5	3
	8	2	1

→ carry

Addition of Binary Numbers

	1	1	1		1	
	1	0	1	0	1	→ 21
+		1	1	0	1	→ 13
	1	0	0	0	1	0 → 34

$2^5 + 2^1$

$1 + 1 = 2 \rightarrow 10$ ^{carry}

$$\begin{array}{r}
 110101 \rightarrow 53 \\
 + 100110 \rightarrow 38 \\
 \hline
 1011011 \rightarrow 91
 \end{array}$$

$$\begin{array}{r}
 10110 \rightarrow 22 \\
 + 00111 \rightarrow 7 \\
 \hline
 11101 \rightarrow 29
 \end{array}$$

Bitwise operators

		AND	OR	XOR
A	B	A & B	A B	A ^ B
0	0	0	0	0
0	1	0	1	1
1	0	0	1	1
1	1	1	1	0

↗ Addition without carry

↙
All 1's \Rightarrow 1
else \Rightarrow 0

↙
All 0's \Rightarrow 0
else \Rightarrow 1

↙ Same Same
puppy shame



A	~A (Not)
0	1
1	0

↳ 5 & 6

$$\begin{array}{r}
 5 \rightarrow 101 \\
 6 \rightarrow 110 \\
 \hline
 100 \rightarrow 4 \text{ (Ans)}
 \end{array}$$

2) $20 \mid 45$

$$\begin{array}{r}
 20 \rightarrow \begin{array}{|c|c|c|c|c|} \hline 1 & 0 & 1 & 0 & 0 \\ \hline \end{array} \\
 \text{or} \\
 45 \rightarrow \begin{array}{|c|c|c|c|c|} \hline 1 & 0 & 1 & 1 & 0 & 1 \\ \hline \end{array} \\
 \hline
 \begin{array}{|c|c|c|c|c|c|} \hline 1 & 1 & 1 & 1 & 0 & 1 \\ \hline \end{array} \\
 \begin{array}{c} 5 \quad 4 \quad 3 \quad 2 \quad 1 \quad 0 \end{array}
 \end{array}$$

$$\begin{aligned}
 2^5 + 2^4 + 2^3 + 2^2 + 2^0 &= 32 + 16 + 8 + 4 + 1 \\
 &= \underline{61} \text{ (Ans)}
 \end{aligned}$$

3) $20 \wedge 45$

$$\begin{array}{r}
 20 \rightarrow \begin{array}{|c|c|c|c|c|} \hline 1 & 0 & 1 & 0 & 0 \\ \hline \end{array} \\
 \wedge \\
 45 \rightarrow \begin{array}{|c|c|c|c|c|} \hline 1 & 0 & 1 & 1 & 0 & 1 \\ \hline \end{array} \\
 \hline
 \begin{array}{|c|c|c|c|c|c|} \hline 1 & 1 & 1 & 0 & 0 & 1 \\ \hline \end{array} \\
 \begin{array}{c} 5 \quad 4 \quad 3 \quad 2 \quad 1 \quad 0 \end{array}
 \end{array}$$

$$\rightarrow 32 + 16 + 8 + 1 = \underline{57} \text{ (Ans)}$$

Binary Representation of Negative Numbers

$45 \rightarrow 1 \ 0 \ 1 \ 1 \ 0 \ 1$

int $x = 45;$

\rightarrow 4 Bytes \rightarrow 32 bits

$00 \dots 0101101$

8 bit system (our understanding)

$$\begin{array}{c}
 7 \quad 6 \quad 5 \quad 4 \quad 3 \quad 2 \quad 1 \quad 0 \\
 45 \rightarrow \begin{array}{|c|c|c|c|c|c|c|c|} \hline 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ \hline \end{array}
 \end{array}$$

flip bits \rightarrow $\begin{array}{|c|c|c|c|c|c|c|c|} \hline 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ \hline \end{array} \rightarrow$ 1's complement

Add 1 $\rightarrow + \begin{array}{|c|c|c|c|c|c|c|c|} \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ \hline \end{array}$

-ve bit $\textcircled{1}$ $\begin{array}{|c|c|c|c|c|c|c|c|} \hline 1 & 1 & 0 & 1 & 0 & 0 & 1 & 1 \\ \hline \end{array} \rightarrow$ 2's complement

$$-2^7 + 2^6 + 2^4 + 2^1 + 2^0$$

$$= -128 + 64 + 16 + 2 + 1 = \underline{-45}$$

Q → what is binary for -3 in 8 bit system.

3 → $\begin{matrix} 7 & 6 & 5 & 4 & 3 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{matrix}$

flip bits → $\begin{matrix} 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \end{matrix}$

Add 1 → $\begin{matrix} + & & & & & & & 1 \\ \hline \end{matrix}$

$\begin{matrix} 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 \\ \hline 7 & 6 & 5 & 4 & 3 & 2 & 1 & 0 \end{matrix}$

$$\begin{aligned} & -2^7 + 2^6 + 2^5 + 2^4 + 2^3 + 2^2 + 2^0 \\ & = -128 + 64 + 32 + 16 + 8 + 4 + 1 = \underline{-3} \end{aligned}$$

Q → what is binary for -10 in 8 bit system.

10 → $\begin{matrix} 7 & 6 & 5 & 4 & 3 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \end{matrix}$

flip → $\begin{matrix} 1 & 1 & 1 & 1 & 0 & 1 & 0 & 1 \end{matrix}$

Add 1 → $\begin{matrix} \hline \end{matrix}$

$\begin{matrix} 1 & 1 & 1 & 1 & 0 & 1 & 1 & 0 \end{matrix} \rightarrow \underline{-10}$

Range of Datatypes

8 bit system →

$\begin{matrix} \text{---} \\ \text{-ve} \end{matrix}$

min → $\begin{matrix} 7 & 6 & 5 & 4 & 3 & 2 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{matrix}$

$$-2^7 = \underline{-128}$$

max → $\begin{matrix} 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{matrix}$

$$2^6 + 2^5 + 2^4 + 2^3 + 2^2 + 2^1 + 2^0$$

$$= 2^7 - 1 = \underline{127}$$

Range $\rightarrow [-128 \quad 127]$

integers \rightarrow 32 bit system

min $\rightarrow 1 \ 0 \ 0 \dots 0 \rightarrow -2^{31} = -2147483648$
 -2×10^9 (approx.)

max $\rightarrow 0 \ 1 \ 1 \dots 1 \rightarrow 2^{30} + 2^{29} + \dots + 2^1 + 2^0$
 $= 2^{31} - 1 = 2147483647$
 2×10^9 (approx.)

Range $\rightarrow [-2^{31} \quad (2^{31}-1)]$
 $\approx [-2 \times 10^9 \quad 2 \times 10^9]$

long \rightarrow 8 bytes \rightarrow 64 bits

min $\rightarrow -2^{63} \approx -9 \times 10^{18}$

max $\rightarrow 2^{62} + 2^{61} + 2^{60} + \dots + 2^0 = 2^{63} - 1 \approx 9 \times 10^{18}$

Range $\rightarrow [-2^{63} \quad (2^{63}-1)] \approx [-9 \times 10^{18} \quad 9 \times 10^{18}]$

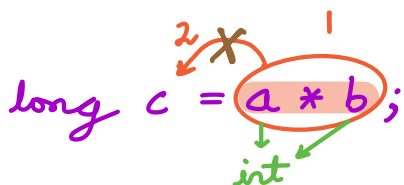
Constraints

int $a = 10^5, b = 10^6;$

int $c = a * b;$

$10^5 * 10^6 = 10^{11}$ Integer Overflow

long $c = a * b;$



10^{11} Integer Overflow

$\text{long } c = \text{long } (a * b);$
 3 2 1
 int
 10^{11} Integer overflow

$\text{long } c = \text{long } (a) * \text{long } (b);$
 4 3 2 1
 $\text{long} * \text{long} = \text{long} \checkmark$

$\text{long } c = \text{long } (a) * b$
 3 2
 $\text{long} * \text{int} = \text{long} \checkmark$

Q → Given an integer array, find sum of elements.

$\text{int } \text{sum} = 0$
 for $i \rightarrow 0$ to $(N-1)$ {
 $\text{sum} += A[i]$
 } return sum
~~// Partially successful~~ ✓

Constraints

$$1 \leq N \leq 10^5$$

$$1 \leq A[i] \leq 10^6$$

max

$$N = 10^5 \quad \forall i, A[i] = 10^6$$

$$\text{sum} = 10^6 + 10^6 + \dots + 10^6$$

$$= 10^6 * 10^5 = 10^{11}$$

$10^{11} > 2 * 10^9 \rightarrow$ Integer overflow

$$15 \% 10 = 5 \text{ (remainder)}$$

popular ($10^9 + 7$)

$$0 \leq A \% M < M$$