Dual Problem of Linearly Separable SVM (Hard Margin)

$$① \quad L(\alpha) = \sum_{n=1}^{N} \alpha_n - \frac{1}{2} \sum_{m=1}^{N} \sum_{n=1}^{N} \alpha_m \alpha_n y_m y_n \mathbf{k}(\mathbf{x}_m, \mathbf{x}_n)$$

$$\text{subject to} \quad \alpha_n \geq 0, \quad \sum_{n=1}^{N} \alpha_n y_n = 0$$

Equation 5. Dual problem for SVM

Dual Problem of Linearly Non-Separable SVM (Soft Margin)

$$② \quad L(\alpha) = \sum_{n=1}^{N} \alpha_n - \frac{1}{2} \sum_{m=1}^{N} \sum_{n=1}^{N} \alpha_m \alpha_n y_m y_n \mathbf{k}(\mathbf{x}_m, \mathbf{x}_n)$$

$$\text{subject to} \quad 0 \leq \alpha_n \leq C, \quad \sum_{n=1}^{N} \alpha_n y_n = 0$$

Equation 7. Dual problem for the soft-margin SVM

① C가 ∞가 되면 ②는 ① 번 과 동일해짐.

↳ $0 \leq \alpha_n \leq \infty$ = $0 \leq \alpha_n$

⇒ C가 ∞인 soft margin은 hard margin 과 같다.

② C가 0이 되면 ⇒ 불가능!!

Original Problem

$$\text{minimize} \ \frac{1}{2}\|w\|_2^2 + C \sum_{i=1}^{n} \xi_i$$

$$\text{subject to} \ y_i(w^T x_i + b) \geq 1 - \xi_i, i = 1,2, \ldots \ldots, n$$

$$L(\alpha) = \sum_{n=1}^{N} \alpha_n - \frac{1}{2} \sum_{m=1}^{N} \sum_{n=1}^{N} \alpha_m \alpha_n y_m y_n \mathbf{k}(\mathbf{x}_m, \mathbf{x}_n)$$

$$\text{subject to} \quad 0 \leq \alpha_n \leq C, \quad \sum_{n=1}^{N} \alpha_n y_n = 0$$

Equation 7. Dual problem for the soft-margin SVM

$0 \le \alpha_n \le 0$ 이면 $\alpha_n = 0$, $\therefore \cancel{\sum_{n=1}^{N} \alpha_n y_n = 0}$

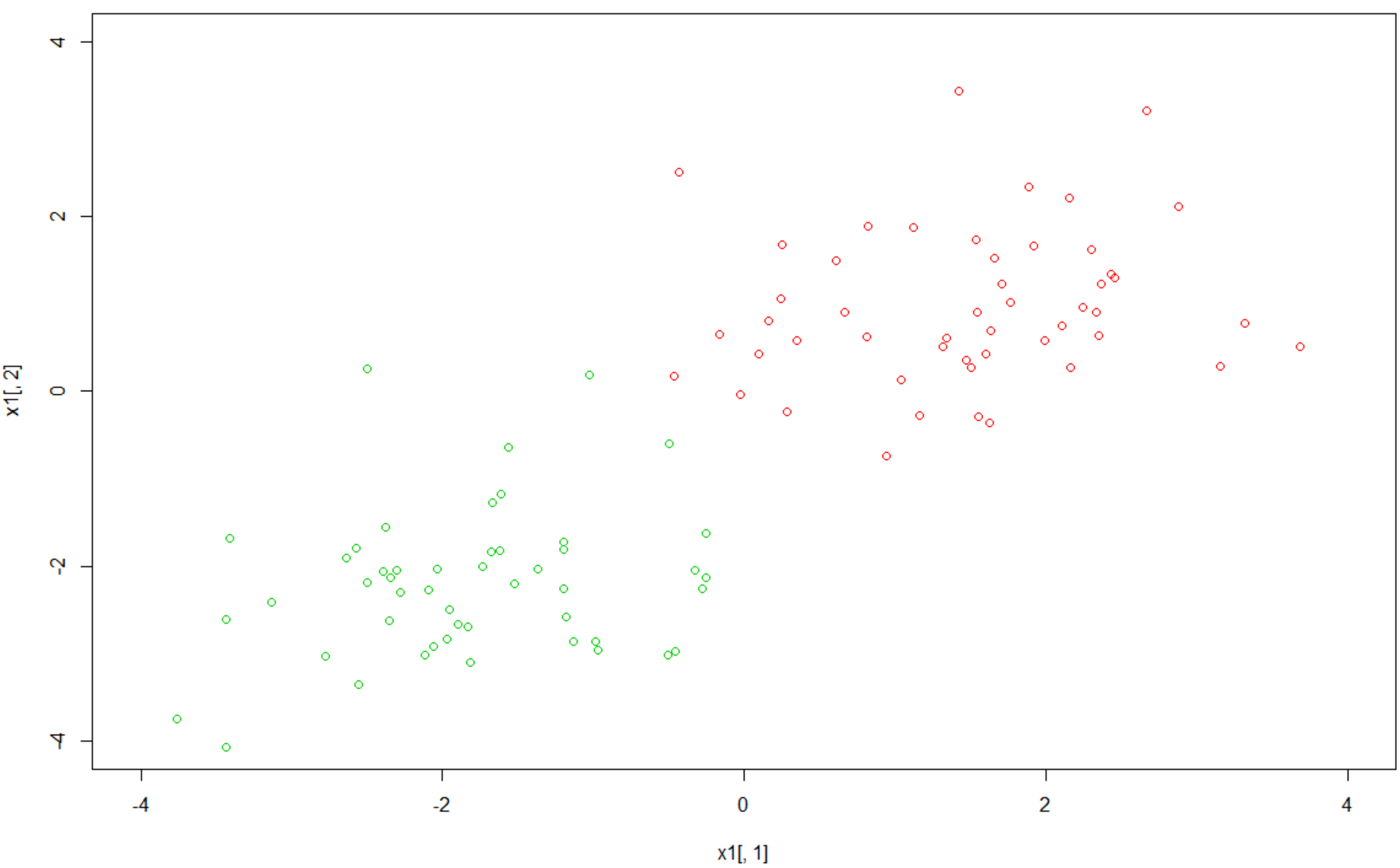
$$\Rightarrow \quad L(\alpha) = \cancel{\sum_{n=1}^{N} \alpha_n} - \frac{1}{2} \cancel{\sum_{m=1}^{N} \sum_{n=1}^{N}} \alpha_m \alpha_n y_m y_n \mathbf{k}(\cancel{\mathbf{x}_m, \mathbf{x}_n})$$

$\Rightarrow \quad L(\alpha) = 0$

⇒ 모든 제약이 사라짐 (simply $w = 0$, for all w_i)

<실험①>

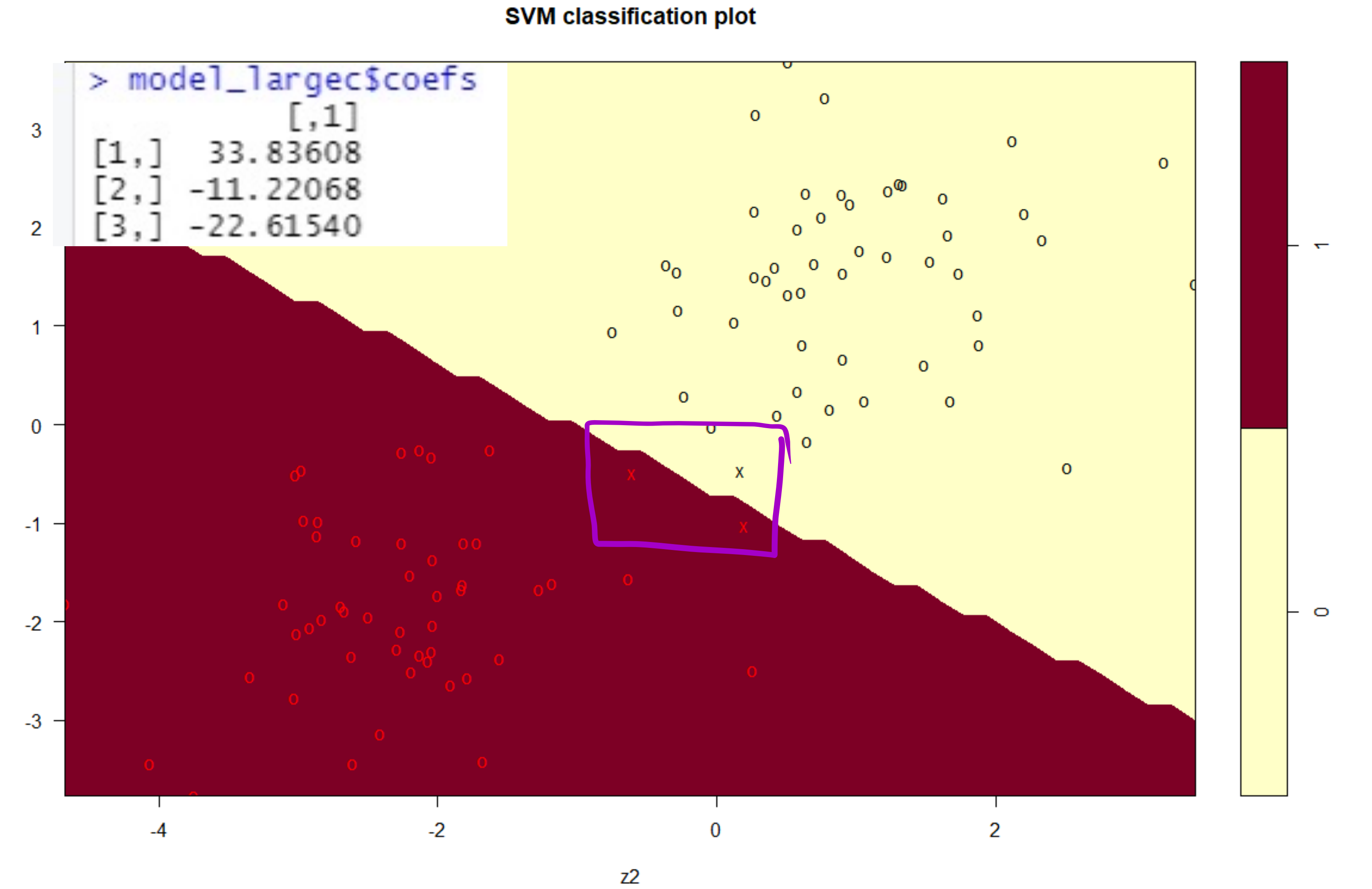
Linearly Separable 한 데이터



① C가 10^{15}로 매우 큰 경우

⇒ Hard Margin의 support vector 3개 존재

⇒ 이 때의 w값!!



② C가 10^{-15} 로 매우 작은경우

⇒ 모든 점이 Support Vector가 됨 ⇒ ε(slack variable) 고려X

⇒ W가 전부 '0'

```
> model_smallc$coefs
      [,1]
 [1,] 1e-14
 [2,] 1e-14
 [3,] 1e-14
 [4,] 1e-14
 [5,] 1e-14
 [6,] 1e-14
 [7,] 1e-14
 [8,] 1e-14
 [9,] 1e-14
[10,] 1e-14
[11,] 1e-14
[12,] 1e-14
[13,] 1e-14
[14,] 1e-14
[15,] 1e-14
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[43,] 1e-14
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[45,] 1e-14
[46,] 1e-14
[47,] 1e-14
[48,] 1e-14
```

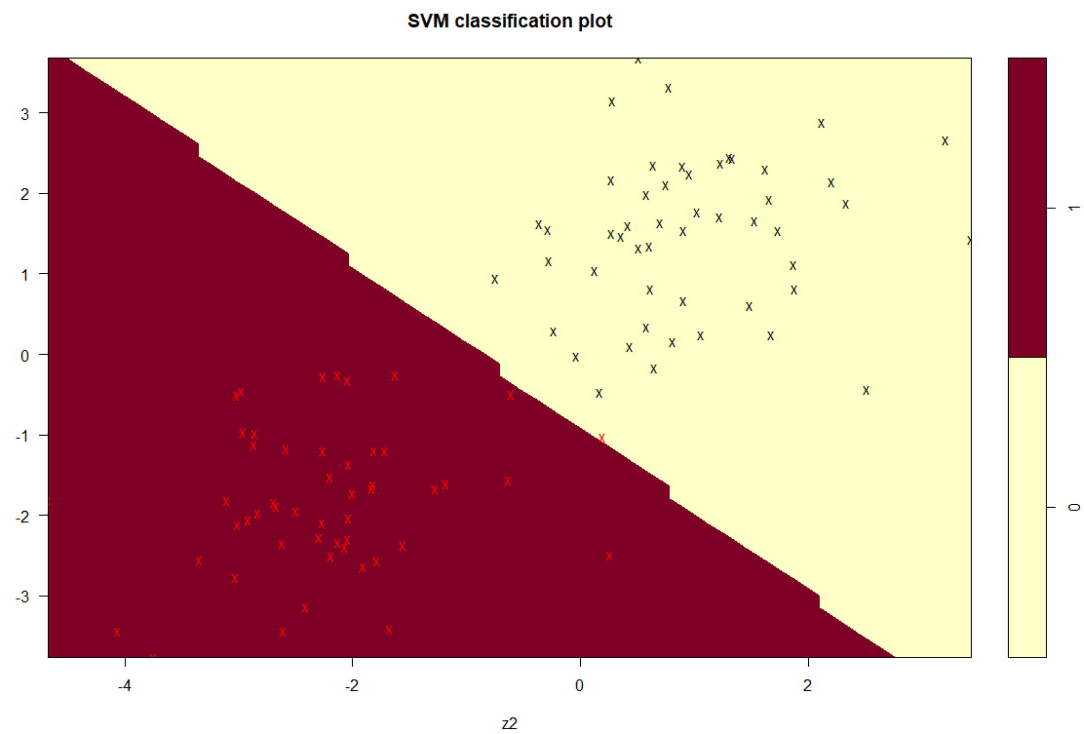

(결론)

* soft margin에서는 'C' 값이 ∞가 되면 hard margin과 같아진다.

* C가 0이 되는 것은 ε (slack variable)을 최적화 과정에서 고려하지 않기 때문에 가능하지 않다.