Adaptive Landweber Method to Deblur Images

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***Abstract*—In this letter, we present an adaptive Landweber method (ALM) to reconstruct an image from a blurred observation. The standard Landweber method (LM) is an iterative method to solve "ill-posed" problems encountered in image restoration. The standard LM uses a constant update parameter. It has the disadvantage of slow convergence. Instead of using a constant update parameter, the adaptive method computes the update parameter at each iteration. In the ALM, the adaptive update parameter is calculated as the ratio of the L_2 norm of the first-order derivatives of the restored images at current and previous iterations. The adaptive LM emphasizes speed at the beginning stages and stability at late stages of iteration. The ALM has a higher convergence rate and lower MSE and mean absolute error than the standard LM. We use examples to demonstrate the performance of the ALM.**

***Index Terms*—Adaptive method, image debluring, image restoration, Landweber method.**

I. Introduction

In image enhancement, a blurred image can be recovered by a direct inverse method such as Tikhonov regularization [1] or by iterative solutions such as the Landweber method [2] and the Richardson–Lucy method [3], [4]. The Landweber method is the simplest of the iterative methods [5]. The method is also referred to as Bially or Van Cittert iteration method, presumably because it has been independently discovered by different researchers [5], [6]. To begin, consider the problem of an image, of size $M \times N$, blurred by a Gaussian point spreading function and corrupted by measurement noise. In matrix and vector form, we can write the problem as

$$\mathbf{g} = \mathbf{H}\mathbf{f} + \mathbf{w} \tag{1}$$

where $\mathbf{g}$ and $\mathbf{f}$ are lexicographically stacked images of size $MN \times 1$; $\mathbf{H}$ represents the blurring operator of size $MN \times MN$; and $\mathbf{w}$ is the measurement noise. The Landweber method looks for the solution $\mathbf{f}_{\mathrm{LM}}$ by

$$\mathbf{f}_{\mathrm{LM}}^{(0)} = \mathbf{0} \tag{2}$$

$$\mathbf{f}_{\mathrm{LM}}^{(k+1)} = \mathbf{f}_{\mathrm{LM}}^{(k)} + \beta \mathbf{H}^T \left(\mathbf{g} - \mathbf{H}\mathbf{f}_{\mathrm{LM}}^{(k)}\right) \tag{3}$$

where T denotes transpose; superscript (k) denotes the kth iteration; subscript "LM" stands for the *Landweber method*; and β is a predetermined parameter such that

$$0 < \beta < \frac{2}{\sigma_1^2} \tag{4}$$

Manuscript received February 27, 2002; revised September 13, 2002. The associate editor coordinating the review of this manuscript and approving it for publication was Prof. Gaetano Scarano.

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Digital Object Identifier 10.1109/LSP.2003.810012

1. Initialize $\beta > 0$ and $\mathbf{f}_A^{(0)} = \mathbf{0}$
2. $\mathbf{f}_A^{(1)} = \mathbf{f}_A^{(0)} + \beta \mathbf{H}^T[\mathbf{g} - \mathbf{H}\mathbf{f}_A^{(0)}]$
3. $\mathbf{f}_A^{(2)} = \mathbf{f}_A^{(1)} + \beta \mathbf{H}^T[\mathbf{g} - \mathbf{H}\mathbf{f}_A^{(1)}]$
4. IF $||\nabla \mathbf{f}_A^{(1)}||_2 \neq 0$
   $\alpha^{(2)} = \frac{||\nabla \mathbf{f}_A^{(2)}||_2}{||\nabla \mathbf{f}_A^{(1)}||_2}$
   ELSE
   $\alpha^{(2)} = \beta$
   ENDIF
5. FOR $k = 3 : K$
   (a) $\mathbf{f}_A^{(k)} = \mathbf{f}_A^{(k-1)} + \max(\beta, \alpha^{(k-1)}) \mathbf{H}^T \left[\mathbf{g} - \mathbf{H}\mathbf{f}_A^{(k-1)}\right]$
   (b) IF $||\nabla \mathbf{f}_A^{(k-1)}||_2 \neq 0$
   $\alpha^{(k)} = \frac{||\nabla \mathbf{f}_A^{(k)}||_2}{||\nabla \mathbf{f}_A^{(k-1)}||_2}$
   ELSE
   $\alpha^{(k)} = \beta$
   ENDIF
   ENDFOR

Fig. 1. Adaptive Landweber method.

where σ_1 is the largest singular value of the matrix $\mathbf{H}$ [7]. A large β ensures a quick convergence to the "optimal" image, but increases the risk of instability. A small β converges slowly to the original image. In either case, it is important to have a criterion for stopping the iterations at an "optimal" point. A widely used method is computing the discrepancy error $e_{\mathrm{LM}}^{(k)} = ||\mathbf{H}\mathbf{f}_{\mathrm{LM}}^{(k)} - \mathbf{g}||_2^2$ and stopping the iterations when $e_{\mathrm{LM}}^{(k)}$ is less than a predetermined threshold [2]. One disadvantage of the Landweber method is slow convergence [2] and the difficulty to choose proper update parameter β. Recently, many algorithms have been developed that converges faster than the Landweber method, such as the conjugate gradient algorithm [8], the preconditioned conjugate gradient algorithm, and the generalized minimal residual method [9]. Nonetheless, the simplicity of the Landweber method makes it a proper choice in some cases.

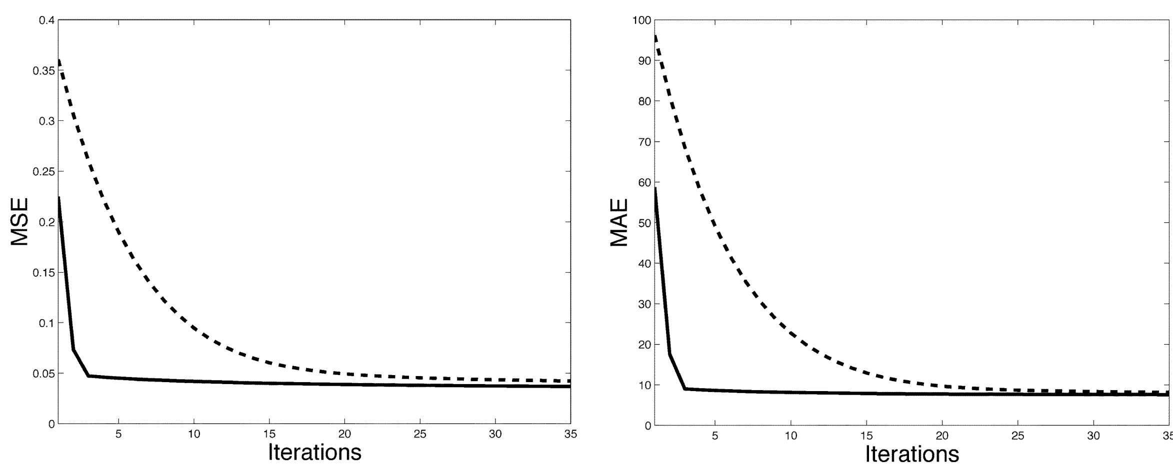
In this letter, e propose an adaptive method that has a better result and faster convergence than the standard LM. Instead of using a constant update parameter β, the adaptive Landweber method (ALM) computes the update parameter at each iteration and chooses the maximum of the computed parameter and the preset constant parameter to use in the next iteration. From experiments, it is shown that the ALM has lower MSE and mean absolute error (MAE) and produces sharper images. Section II discusses the standard LM and the ALM. Examples are given in Section III to demonstrate the performance of the ALM and compare it with the LM. Section IV gives discussion and direction of future work.

(a) (b)

Fig. 2. (a) Original image $\mathbf{f}$, $\|\nabla \mathbf{f}\|_2 = 1109$. (b) Blurred image $\mathbf{g}$, $\|\nabla \mathbf{g}\|_2 = 549$.

(a) (b)

Fig. 3. At iteration 35, $\beta = 0.3$. (a) Deblurred image $\mathbf{f}_{\mathrm{LM}}$ by the LM, $\|\nabla \mathbf{f}_{\mathrm{LM}}\|_2 = 686$. (b) Deblurred image $\mathbf{f}_A$ by the ALM, $\|\nabla \mathbf{f}_A\|_2 = 837$.



(a) (b)

Fig. 4. "Lena" image. (a) MSE of the LM (dashed line). MSE of the ALM (solid line). (b) MAE of the LM (dashed line). MAE of the ALM (solid line).

II. ALM

For a given problem, convergence speed of the LM is decided by β. Using a small β can be safe in the sense of ensuring iterative stability but convergence is slow. On the other hand, choosing a large β can speed up convergence but may cause instability in iterations. While (4) sets the range of possible choice of β between zero and $2/\sigma_1^2$, computing singular value decomposition of a large matrix is very time consuming and computer memory demanding. Therefore, it is beneficial to develop a fast algorithm that uses small β and converges quickly. We propose a modified Landweber method that updates β adaptively as the iteration increases. At the $(k+1)$th iteration, we compute the ratio of the L_2 norms of the first-order derivatives of $\mathbf{f}^{(k)}$ and $\mathbf{f}^{(k-1)}$ such as

$$\alpha^{(k)} = \frac{\left\|\nabla \mathbf{f}^{(k)}\right\|_2}{\left\|\nabla \mathbf{f}^{(k-1)}\right\|_2} \tag{5}$$

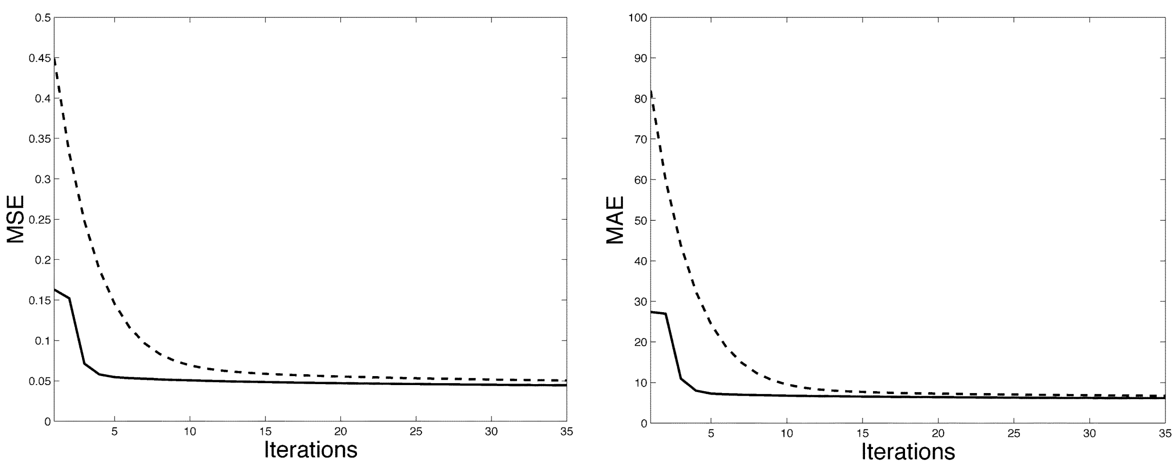
where $\nabla \mathbf{f}^{(k)}$ stands for taking the first-order derivative of $\mathbf{f}^{(k)}$. Steps of the ALM of K iterations are shown in Fig. 1; subscript A stands for *adaptive Landweber method*. In the algorithm, the

(a) (b)

Fig. 5. The “boat.” (a) Original image $\mathbf{f}$, $\|\nabla \mathbf{f}\|_2 = 1775$. (b) Blurred image $\mathbf{g}$, $\|\nabla \mathbf{g}\|_2 = 692$.

(a) (b)

Fig. 6. Restored “boat” image at iteration 35. (a) By the LM, $\|\nabla \mathbf{f}_{\mathrm{LM}}\|_2 = 1162$. (b) By the ALM, $\|\nabla \mathbf{f}_A\|_2 = 1375$.



(a) (b)

Fig. 7. “Boat” image. (a) MSE of the LM (dashed line). MSE of the ALM (solid line). (b) MAE of the LM (dashed line). MAE of the ALM (solid line).

first two iterations are computed the same as in the standard LM in order to initialize the adaptive method. The motivation of using the ratio of $\|\nabla \mathbf{f}_A^{(k)}\|_2$ to $\|\nabla \mathbf{f}_A^{(k-1)}\|_2$ is to take into account the sharpness of the restored images at iteration k and $k-1$ where $\|\nabla \mathbf{f}_A^{(k)}\|_2$ can be considered as an approximation to the sharpness of image $f_A^{(k)}$. Defining the update parameter as the ratio of $\|\nabla \mathbf{f}_A^{(k)}\|_2$ to $\|\nabla \mathbf{f}_A^{(k-1)}\|_2$ awards the adaptive algorithm by a large update parameter and makes it converge faster. In the late iterations, when stability is more important, $\alpha^{(k)}$ is close to one because there is not too much difference between $\mathbf{f}_A^{(k)}$ and $\mathbf{f}_A^{(k-1)}$, and an α close to one gives more weight to stability than to convergence speed. At the early state of iteration, the method has a higher convergence rate than in the late stage. Hence, speed and stability are emphasized accordingly at different stages of iteration. In general, better results are obtained by the ALM. To prevent divergence caused by a large $\max(\beta, \alpha^{(k-1)})$, we can calculate the change in discrepancy error $e_A^{(k)}$ such that $e_A^{(k)}$ shall monotonically decrease. The change of $e_A^{(k)}$ can also be used as a stopping criterion to terminate the iteration when $|e_A^{(k)} - e_A^{(k-1)}|$ is less than a small positive quantity, say $\eta > 0$.

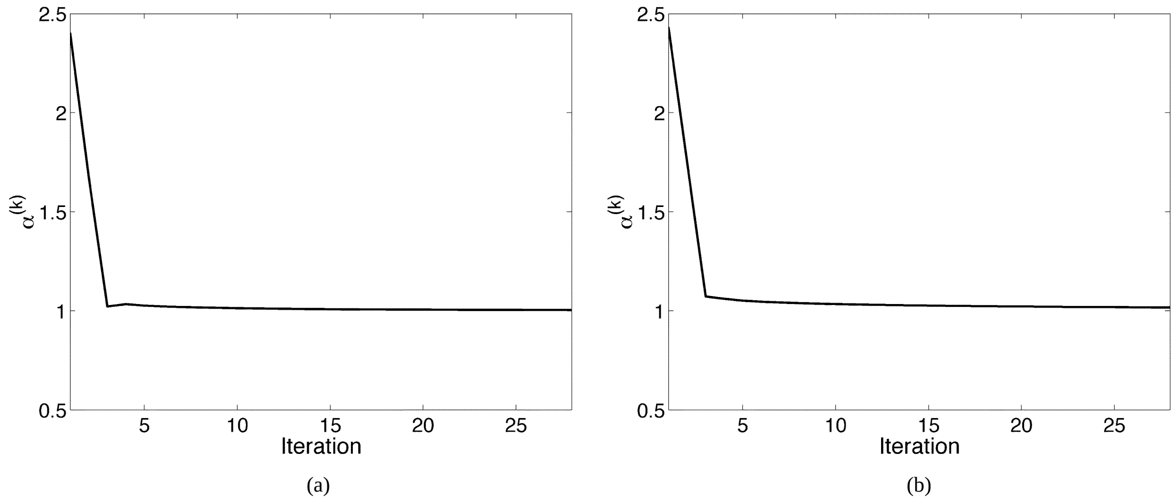


Fig. 8. Behavior of α. (a) Example 1: the "Lena" image. (b) Example 2: the "boat" image.

III. EXAMPLES

In this section, we use some examples to demonstrate the performance of the ALM and compare it with that of the standard LM. Fig. 2 shows an original image and the image blurred by a Gaussian point spreading function. For this example, the measurement noise is set to zero, i.e., $\mathbf{g} = \mathbf{Hf}$. Note the original image has a larger L_2 norm of the first-order derivative than the blurred version, which confirms our intuition that the norm of the first-order derivative can be used as an approximation to measure sharpness (blurriness) of an image. Fig. 3 compares the results of the LM and ALM at iteration 35. It is noted that the ALM produces a sharper image than the standard LM. To quantitatively compare the ALM with the standard LM, we compute the MSE and MAE of the two methods. The MSE and MAE of the restored $\mathbf{f}_A^{(k)}$ are defined as follows:

$$\text{MSE} = \frac{\left\|\mathbf{f}_A^{(k)} - \mathbf{f}\right\|_2}{MN} \quad \text{MAE} = \frac{\left|\mathbf{f}_A^{(k)} - \mathbf{f}\right|}{MN}. \tag{6}$$

The MSE and MAE of $\mathbf{f}_{\text{LM}}^{(k)}$ are defined similarly. Fig. 4 plots the MSE and MAE of restored "Lena" by the standard LM and the ALM starting at iteration 3 (because the first two iterations are the same for both the LM and ALM). It is observed that the ALM has a faster decrease of the MSE and MAE than the standard LM. Also, the MSE of the ALM are lower than those of the standard LM. The behavior of α is plotted in Fig. 8(a). It is seen that α quickly approaches one, indicating that the algorithm converges quickly to a stable point. The second example is given in Fig. 5, which shows an original "boat" image and its blurred noisy observation. In this example, the noise is nonzero, and the image has an SNR of 40 dB where SNR is defined as

$$\text{SNR} = 20 \log \frac{\|\mathbf{f}\|_2}{MN\sigma} \text{ (dB)} \tag{7}$$

where σ is the standard deviation of the noise. Deblurred images from the LM and ALM are shown in Fig. 6(a) and (b). The β in both methods is 0.5. It is seen that the ALM result is sharper than the LM result. Fig. 7 compares the MSE and MAE of both methods. It is seen that the ALM consistently outperforms the LM and has smaller errors. A plot of α is shown in Fig. 8(b). It is interesting to note that in both noise-free and noisy situations, α approaches one in just a few iterations.

IV. CONCLUSION

We present an adaptive Landweber method in image deblurring. The adaptive method computes the update parameter by the ratio of the L_2 norms of the first-order derivatives of the restored images at the current and previous iteration. The adaptive update parameter aims to emphasize speed and stability at early and late stages of the iterations, respectively. From experiments, we see that the ALM obtains better results in terms of generating a sharp image with low MSE and MAE and faster convergence. The adaptive method has a simple form and can be easily implemented. In future work, the ALM will be generalized to the case when the exact point spreading function is not known *a priori*.

REFERENCES

[1] A. N. Tikhonov and V. Arsenin, *Solutions of Ill-Posed Problems*. New York: Wiley, 1977.
[2] H. W. Engl, M. Hanke, and A. Neubauer, *Regularization of Inverse Problems*. Dordrecht, The Netherlands: Kluwer, 2000.
[3] W. H. Richardson, "Bayesian-based iterative method of image restoration," *J. Opt. Soc. Amer.*, vol. 62, pp. 55–59, 1972.
[4] L. B. Lucy, "An iterative technique for the rectification of observed distributions," *Astronom. J.*, vol. 79, 1974.
[5] J. Biemond, R. L. Lagendjik, and R. M. Mersereau, "Iterative methods for image deblurring," *Proc. IEEE*, vol. 78, pp. 856–883, May 1990.
[6] J. C. Russ, *The Image Processing Handbook*. Boca Raton, FL: CRC, 1999.
[7] M. Bertero, P. Boccacci, and M. Robberto, "An inversion method for the restoration of chopped and nodded images," *Proc. SPIE*, vol. 3354, pp. 877–886, 1998.
[8] G. H. Golub and D. P. O'Leary, "History of the conjugate gradient and Lanczos methods," *SIAM Rev.*, vol. 31, pp. 50–102, 1989.
[9] Y. Saad and M. H. Schultz, "GMRES: A generalized minimal residual algorithm for solving nonsymmetric linear system," *SIAM J. Sci. Stat. Comput.*, 1986.