Practice3 Report

22320607 Xu Ziyang

2023/10/14 YY/MM/DD

Quick View

In today's Report, I will show following things by order:

- 1. Code that can change the question to solve displaced by picture.
- 2. Model that universal for question 1, 2, 3.
- 3. Graph and calculation corresponding to questions and $\omega.$

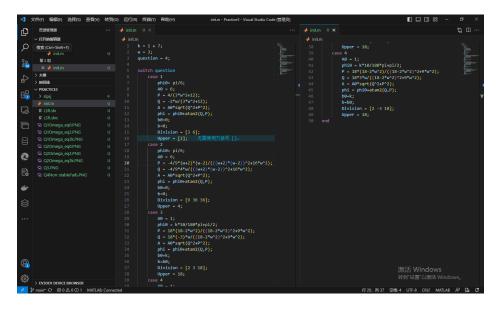
(I'm tired of using OpenDocument. Hope \LaTeX can make my format better.) CONTENTS 2

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1 CODE 3

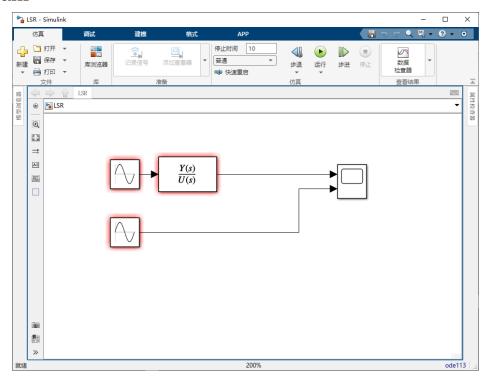
1 Code



2 MODEL 4

2 Model

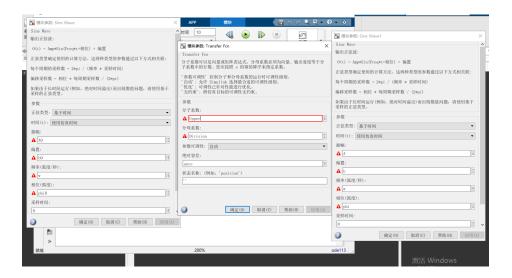
Main



Detail

Here are more information of each model and shows which variable is used.

2 MODEL 5



3 Answers

6

3.1 Question1

3.1.1 Calculation

$$3\dot{y}(t) + 6y(t) = 2u(t)$$

Do laplace transform get:

$$y(s) = \frac{2}{3s+6}u(s)$$

$$W(s) = \frac{2}{3s+6}$$
Let $s = j\omega$.
$$W(j\omega) = \frac{2}{3j\omega+6}$$

$$W(j\omega) = \frac{2(6-3j\omega)}{36+9\omega^2}$$

$$P(\omega) = Re(W(j\omega)) = \frac{12}{36+9\omega^2}$$

$$Q(\omega) = Im(W(j\omega)) = \frac{-6\omega}{36+9\omega^2}$$

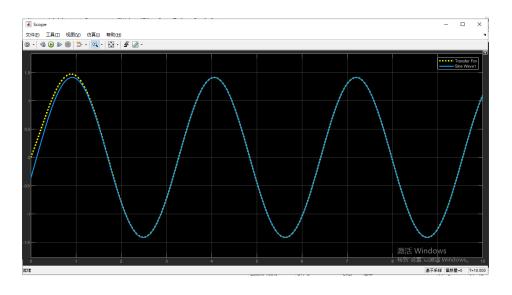
$$A(\omega) = \sqrt[2]{P(\omega)^2 + Q(\omega)^2} = \frac{\sqrt[2]{144+36\omega^2}}{36+9\omega^2}$$

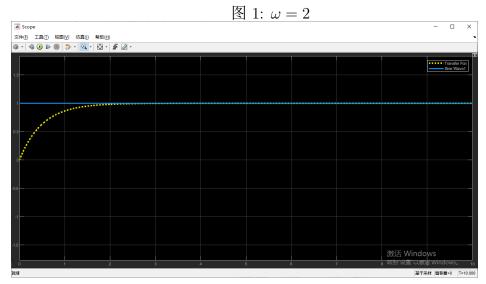
$$\phi(\omega) = atan2(Q(\omega), P(\omega))$$

If
$$u(t) = Asin(\omega t + \phi_0) + b_0$$

$$y(t) = A(\omega)A_0sin(\omega t + \phi_0 + \phi(\omega)) + b_0$$

3.1.2 Graph





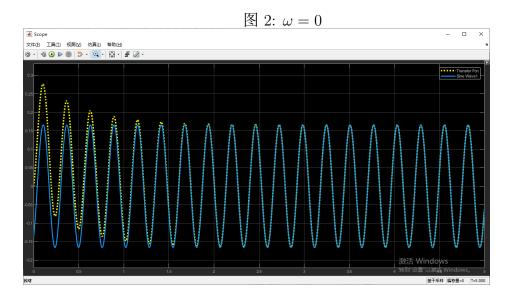


图 3: $\omega = 3k$

3.2 Question2

3.2.1 Calculation

$$9\ddot{y}(t) + 36\dot{y}(t) + 36y(t) = 4u(t)$$

Do laplace transform get:

$$y(s) = \frac{4}{9s^2 + 36s + 36}u(s)$$

$$W(s) = \frac{4}{9s^2 + 36s + 36}$$
Let $s = j\omega$.
$$W(j\omega) = \frac{4}{-9\omega^2 + 36j\omega + 36}$$

$$W(j\omega) = \frac{4(-9\omega^2 + 36 - 36j\omega)}{(36 - 9\omega^2)^2 + (36\omega)^2}$$

$$P(\omega) = Re(W(j\omega)) = \frac{4(-9\omega^2 + 36)}{(36 - 9\omega^2)^2 + (36\omega)^2}$$

$$Q(\omega) = Im(W(j\omega)) = \frac{4(-36\omega)}{(36 - 9\omega^2)^2 + (36\omega)^2}$$

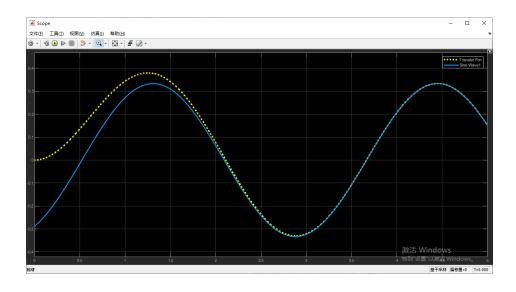
$$A(\omega) = \sqrt[3]{P(\omega)^2 + Q(\omega)^2} = \frac{4\sqrt[3]{(36 - 9\omega^2)^2 + (36\omega)^2}}{(36 - 9\omega^2)^2 + (36\omega)^2}$$

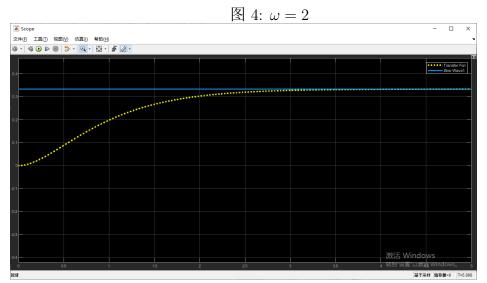
If
$$u(t) = Asin(\omega t + \phi_0) + b_0$$

$$y(t) = A(\omega)A_0sin(\omega t + \phi_0 + \phi(\omega)) + b_0$$

 $\phi(\omega) = atan2(Q(\omega), P(\omega))$

3.2.2 Graph





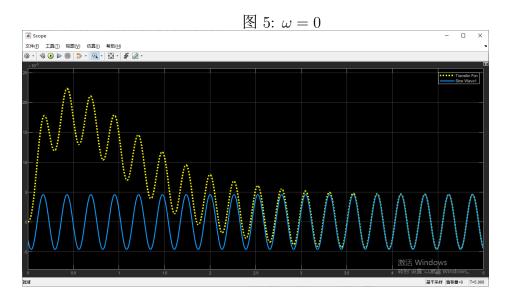


图 6: $\omega = 3k$

3.3 Question3

3.3.1 Calculation

$$2\ddot{y}(t) + 3\dot{y}(t) + 18y(t) = 18u(t)$$

Do laplace transform get:

 $u(t) = A\sin(3t + \frac{\pi}{2} + \frac{10k\pi}{180}) + k$

$$y(s) = \frac{18}{2s^2 + 3s + 18}u(s)$$

$$W(s) = \frac{18}{2s^2 + 3s + 18}$$
Let $s = j\omega$.
$$W(j\omega) = \frac{18}{-2\omega^2 + 3j\omega + 18}$$

$$W(j\omega) = \frac{18(-2\omega^2 + 18 - 3j\omega)}{(18 - 2\omega^2)^2 + (9\omega)^2}$$

$$P(\omega) = Re(W(j\omega)) = \frac{18(-2\omega^2 + 18)}{(18 - 2\omega^2)^2 + (9\omega)^2}$$

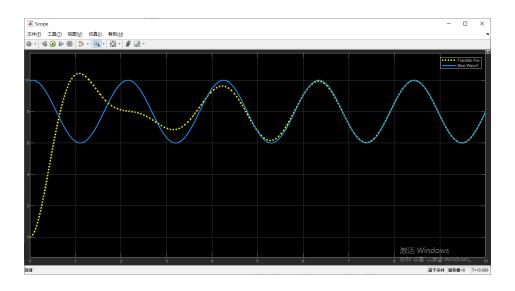
$$Q(\omega) = Im(W(j\omega)) = \frac{18(-3\omega)}{(18 - 2\omega^2)^2 + (9\omega)^2}$$

$$A(\omega) = \sqrt[2]{P(\omega)^2 + Q(\omega)^2} = \frac{18\sqrt[2]{(18 - 2\omega^2) + (9\omega^2)}}{(18 - 2\omega^2)^2 + (9\omega)^2}$$

$$\phi(\omega) = atan2(Q(\omega), P(\omega))$$

$$y(t) = A(3)\sin(3t + \frac{\pi}{2} + \frac{10k\pi}{180}) + k = 2\sin(3t + \frac{\pi}{2} + \frac{10k\pi}{180}) + k$$

3.3.2 Graph



3.4 Question4

3.4.1 Failed

Reason:

Let u(t) = 0.

Think about how to solve $2\ddot{y} - 3\dot{y} + 18 = 0$.

And then consider about equation $2\lambda^2 - 3\lambda + 18 = 0$.

Get that

$$Re(\lambda) = \frac{3}{4} > 0$$

Then the system is **unstable**. So our method **Failed**.

3.4.2 Graph

