Practice5 Report

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Abstract

In task1, we consider if the system state-controllable, output controllable and observable. In task2, we consider if the system can reach $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ from $\begin{bmatrix} 1 \\ 3 \end{bmatrix}$ in 1 second and construct the system in simulink.

Declare:

$$k = 7$$

All System:

$$\dot{\vec{x}} = A\vec{x} + B\vec{u}(t)$$

$$\vec{y} = C\vec{x} + D\vec{u}(t)$$

1 Task1

1.1 System1

Declare:

$$A = \begin{bmatrix} 2-k & 1 \\ 0 & 3 \end{bmatrix}$$

$$B = \begin{bmatrix} -1 \\ 2+k \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

$$D = 0$$

1.1.1 Question1:State Controllability

Caculate Q_C.

$$Q_C = \begin{bmatrix} B & AB \end{bmatrix} = \begin{bmatrix} -1 & 14 \\ 9 & -27 \end{bmatrix}$$

$$Rank(Q_c) = 2 = Rank(A)$$

So the system is controllable.

1.1.2 Question2:Output Controllability

Caculate Q_{CO}.

$$Q_{CO} = \begin{bmatrix} CB & CAB & D \end{bmatrix} = \begin{bmatrix} -1 & 14 & 0 \end{bmatrix}$$

$$Rank(Q_{CO}) = 1 = Rank(C)$$

So the system is output controllable.

1.1.3 Question3:Observability

Caculate Q_O.

$$Q_O = \begin{bmatrix} C \\ CA \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -5 & 1 \end{bmatrix}$$

$$Rank(Q_O) = 2 = Rank(A)$$

So the system is output observable.

1.2 System2

Declare:

$$A = \begin{bmatrix} 0 & 1+k \\ 2 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & -1 \end{bmatrix}$$

$$D = 0$$

1.2.1 Question1:State Controllability

Caculate Q_C.

$$Q_C = \begin{bmatrix} B & AB \end{bmatrix} = \begin{bmatrix} 0 & 8 \\ 1 & 0 \end{bmatrix}$$

$$Rank(Q_c) = 2 = Rank(A)$$

So the system is controllable.

1.2.2 Question2:Output Controllability

Caculate Q_{CO}.

$$Q_{CO} = \begin{bmatrix} CB & CAB & D \end{bmatrix} = \begin{bmatrix} -1 & 8 & 0 \end{bmatrix}$$

$$Rank(Q_{CO}) = 1 = Rank(C)$$

So the system is output controllable.

1.2.3 Question3:Observability

Caculate Q_O.

$$Q_O = \begin{bmatrix} C \\ CA \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -2 & 8 \end{bmatrix}$$

$$Rank(Q_O) = 2 = Rank(A)$$

So the system is output observable.

1.3 System3

Declare:

$$A = \begin{bmatrix} -2 & 1 \\ 1+k & 3 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 1 \\ -2 & 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 1 \end{bmatrix}$$

$$D = 0$$

1.3.1 Question1:State Controllability

Caculate Q_C.

$$Q_C = \begin{bmatrix} B & AB \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 & -1 \\ -2 & 1 & -6 & 11 \end{bmatrix}$$

$$Rank(Q_c) = 2 = Rank(A)$$

So the system is controllable.

1.3.2 Question2:Output Controllability

Caculate Q_{CO}.

$$Q_{CO} = \begin{bmatrix} CB & CAB & D \end{bmatrix} = \begin{bmatrix} -2 & 1 & -6 & 11 & 0 \end{bmatrix}$$

$$Rank(Q_{CO}) = 1 = Rank(C)$$

So the system is output controllable.

1.3.3 Question3:Observability

Caculate Q_O.

$$Q_O = \begin{bmatrix} C \\ CA \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 8 & 3 \end{bmatrix}$$

$$Rank(Q_O) = 2 = Rank(A)$$

So the system is output observable.

1.4 System4

Declare:

$$A = \begin{bmatrix} 2 & 1 \\ 0 & k \end{bmatrix}$$

$$B = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix}$$

$$D = 0$$

1.4.1 Question1:State Controllability

Caculate Q_C.

$$Q_C = \begin{bmatrix} B & AB \end{bmatrix} = \begin{bmatrix} 1 & 4 \\ 2 & 14 \end{bmatrix}$$

$$Rank(Q_c) = 2 = Rank(A)$$

So the system is controllable.

1.4.2 Question2:Output Controllability

Caculate Q_{CO}.

$$Q_{CO} = \begin{bmatrix} CB & CAB & D \end{bmatrix} = \begin{bmatrix} -3 & -24 & 0 \\ 4 & 22 & 0 \end{bmatrix}$$

$$Rank(Q_{CO}) = 2 = Rank(C)$$

So the system is output controllable.

1.4.3 Question3:Observability

Caculate Q_O.

$$Q_O = \begin{bmatrix} C \\ CA \end{bmatrix} = \begin{bmatrix} 1 & -2 & 2 & -13 \\ 2 & 1 & 4 & 9 \end{bmatrix}$$

$$Rank(Q_O) = 2 = Rank(A)$$

So the system is output observable.

1.5 Wolfram Codes

```
A = {{2-k,1},{0,-3}}
B = {{-1},{2+k}}
CI = {1,0}
DI = {0}
QC = Join[B,A.B,2]
QCo = Join[CI.B,CI.A.B,DI]
Qo = {CI,CI.A}
MatrixRank[QC] ==MatrixRank[A]
MatrixRank[{QCo}] == MatrixRank[{CI}]
MatrixRank[Qo] == MatrixRank[A]
TraditionalForm[QC]
TraditionalForm[QCo]
TraditionalForm[Qo]
✓ 1.5s
```

图 1: System1

```
A = {{0,1+k},{2,0}}
B = {{0},{1}}
CI = {1,-1}
DI = {0}
QC = Join[B,A.B,2]
QCo = Join[CI.B,CI.A.B,DI]
Qo = {CI,CI.A}
MatrixRank[QC] == MatrixRank[A]
MatrixRank[{QCo}] == MatrixRank[{CI}]
MatrixRank[Qo] == MatrixRank[A]
TraditionalForm[QC]
TraditionalForm[QCo]
TraditionalForm[Qo]
✓ 1.4s
```

图 2: System2

```
A = {{-2,1},{1+k,3}}
B = {{0,1},{-2,1}}
CI = {0,1}
DI = {0}
QC = Join[B,A.B,2]
QCo = Join[CI.B,CI.A.B,DI]
Qo = {CI,CI.A}
MatrixRank[QC] == MatrixRank[A]
MatrixRank[{QCo}] == MatrixRank[{CI}]
MatrixRank[Qo] == MatrixRank[A]
TraditionalForm[QC]
TraditionalForm[QCo]
TraditionalForm[Qo]
✓ 1.5s
```

图 3: System3

```
A = {{2,1},{0,k}}
B = {{1},{2}}
CI = {{1,-2},{2,1}}
DI = {{0},{0}}
QC = Join[B,A.B,2]
QCo = Join[CI.B,CI.A.B,DI,2]
Qo = Join[CI,CI.A,2]
MatrixRank[QC] == MatrixRank[A]
MatrixRank[QCo] == MatrixRank[CI]
MatrixRank[Qo] == MatrixRank[A]
TraditionalForm[QC]
TraditionalForm[QCo]
TraditionalForm[Qo]
✓ 1.5s
```

图 4: System4

2 Task2

2.1 Is reachable?

I will directly show the code:

```
A = {{0,1},{0,8}}
B = {{0},{2}}
CI = {{1,0},{0,1}}
DI = {{0},{0}}
t1 = 1
f[tao] = MatrixExp[A*(t1-tao)].B.Transpose[B].MatrixExp[Transpose[A]*(t1-tao)]
Wc = Integrate[f[tao],{tao,0,t1}]
TraditionalForm[Wc]
Det[Wc]
U[t]= Transpose[B].MatrixExp[Transpose[A]*(t1-t)].Inverse[Wc].(-MatrixExp[A*t1].{{1},{3}})
U[t] = Simplify[U[t]]
✓ 1.7s
```

图 5: Code

```
{{0,1},{0,8}}
{{0},{2}}
{{1,0},{0,1}}
{{0},{0}}
1
```

$$\left\{\left\{\frac{1}{16} e^{-16\,tao}\left(-e^8+e^{8\,tao}\right)^2, -\frac{1}{2} e^{8-16\,tao}\left(-e^8+e^{8\,tao}\right)\right\}, \left\{-\frac{1}{2} e^{8-16\,tao}\left(-e^8+e^{8\,tao}\right), 4 e^{16-16\,tao}\right\}\right\}$$

$$\left\{\left\{\frac{1}{256}\left(19-4e^8+e^{16}\right), \frac{1}{32}\left(-1+e^8\right)^2\right\}, \left\{\frac{1}{32}\left(-1+e^8\right)^2, \frac{1}{4}\left(-1+e^{16}\right)\right\}\right\}$$

$$\begin{pmatrix} \frac{1}{256}\left(19-4e^8+e^{16}\right) & \frac{1}{32}\left(e^8-1\right)^2 \\ \frac{1}{32}\left(e^8-1\right)^2 & \frac{1}{4}\left(e^{16}-1\right) \end{pmatrix}$$

$$-\frac{5}{256}+\frac{e^8}{128}+\frac{3e^{16}}{256}$$

$$\left\{\left\{-3e^8\left(\frac{e^{8-8t}\left(19-4e^8+e^{16}\right)}{128\left(-\frac{5}{256}+\frac{e^8}{128}+\frac{3e^{16}}{256}\right)}+\frac{e^{-8t}\left(-1+e^8\right)^2\left(-e^8+e^{8t}\right)}{128\left(-\frac{5}{256}+\frac{e^8}{128}+\frac{3e^{16}}{256}\right)}\right)+\left(-1-\frac{3}{8}\left(-1+e^8\right)\right)\left(-\frac{e^{8-8t}\left(-1+e^8\right)^2}{16\left(-\frac{5}{256}+\frac{e^8}{128}+\frac{3e^{16}}{256}\right)}-\frac{e^{-8t}\left(-1+e^{16}\right)\left(-e^8+e^{8t}\right)}{16\left(-\frac{5}{256}+\frac{e^8}{128}+\frac{3e^{16}}{256}\right)}\right)\right\}\right\}$$

$$\left\{\left\{\frac{2\left(5+6e^8-11e^{16}-10e^{8-8t}+58e^{16-8t}\right)}{5-2e^8-3e^{16}}\right\}\right\}$$

图 6: Answer

Note $det(W_c)$ is not 0, so it is reachable.

2.2 Model

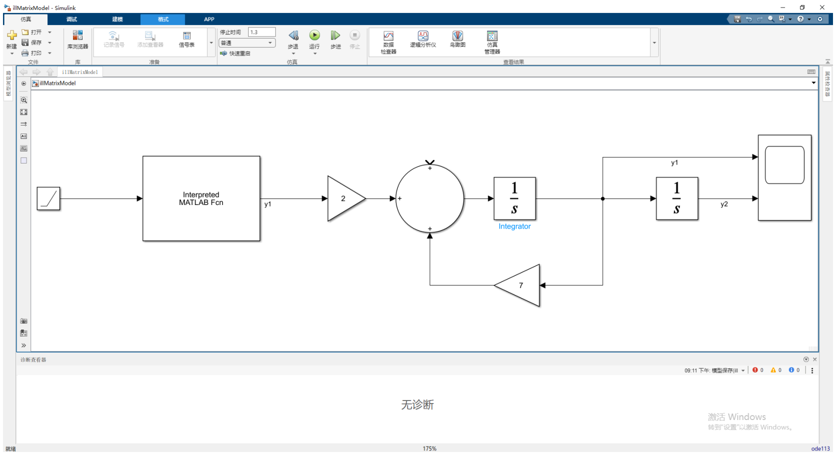

图 7: Model

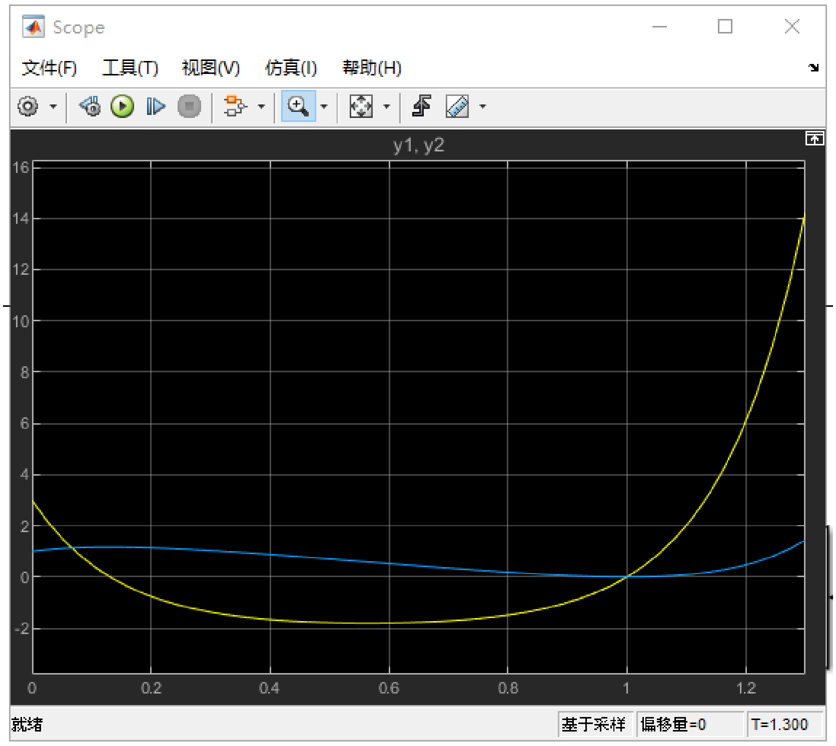

图 8: Figure of y_1, y_2