

# Practice4 Report

Name: Xu Ziyang 22320607

2023/11/3 YY/MM/DD

## 摘要

This Practice work is mainly about how to transform a linear system between state-space form and differential equation form. I used subsystem to show all models. The transfer function is used to check if I was right. Here is the overview of systems:

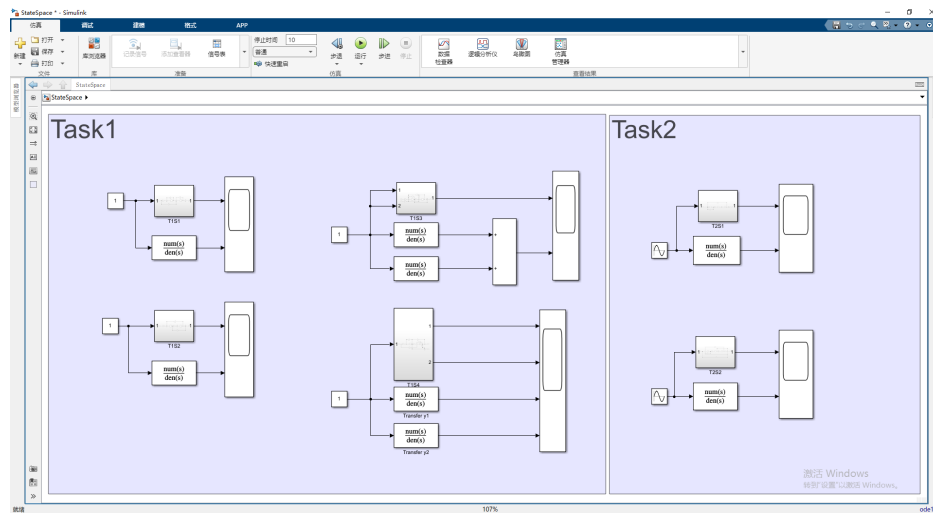


图 1: Overview of systems

# 1 Constructing Systems

Here shows the Detail of subsystem:

## 1.1 Task1

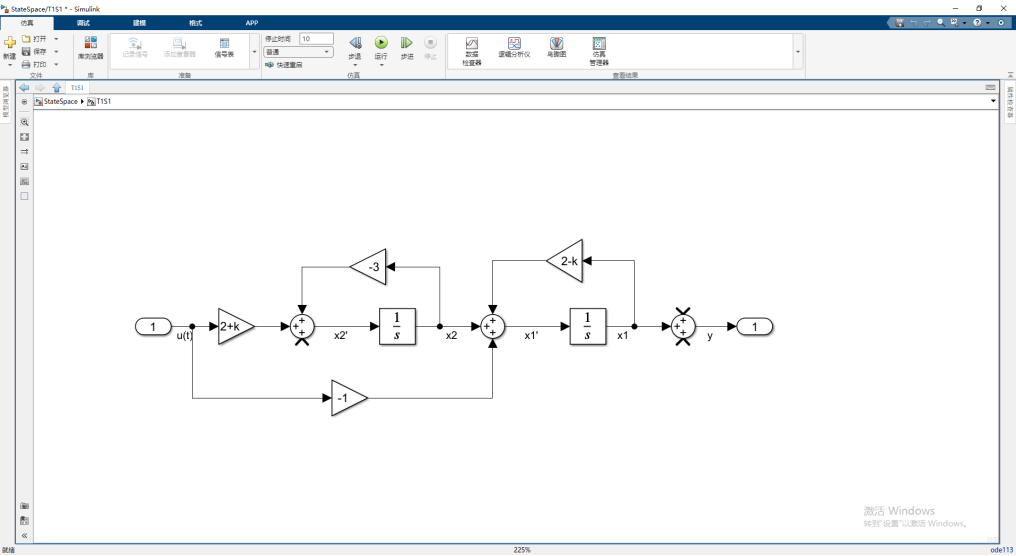


图 2: System1

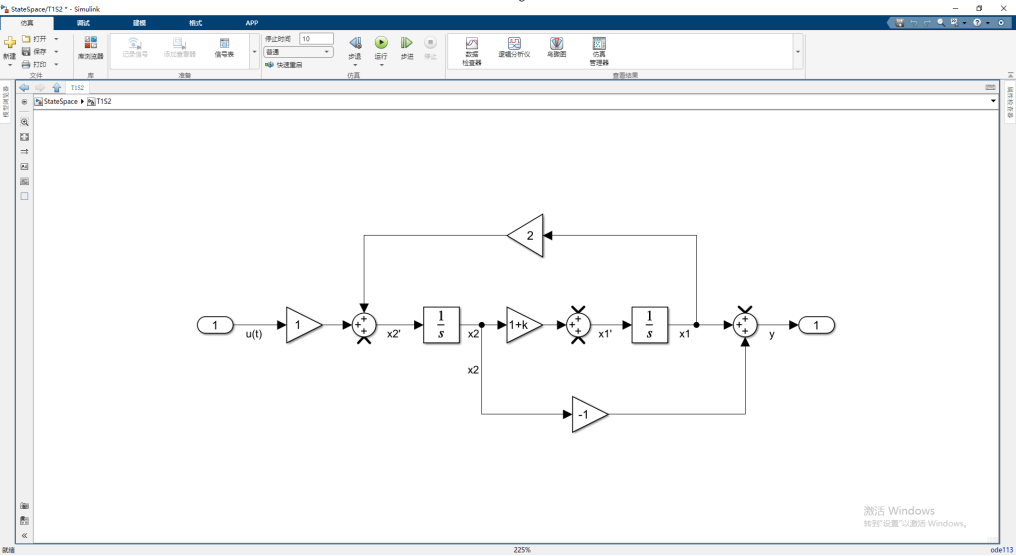


图 3: System2

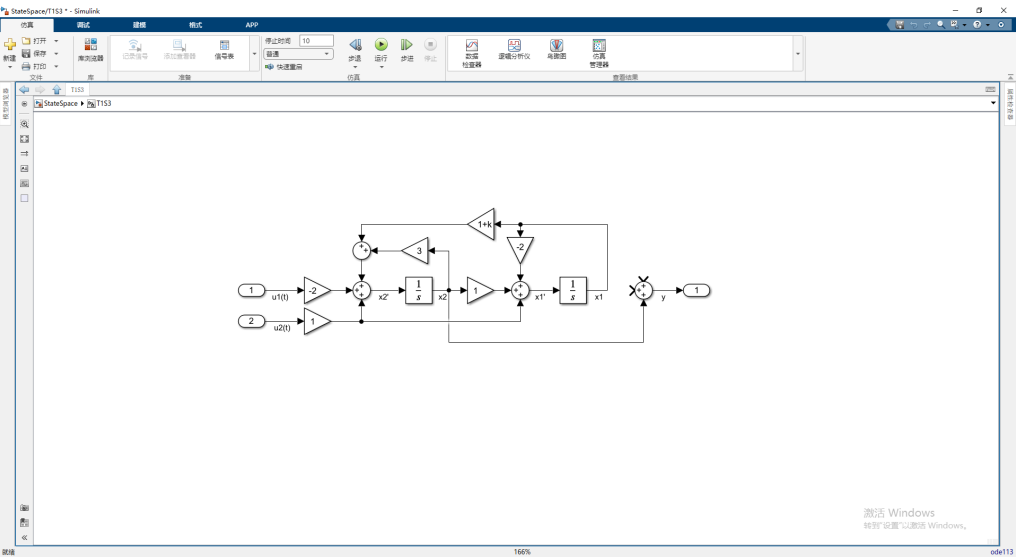


图 4: System3

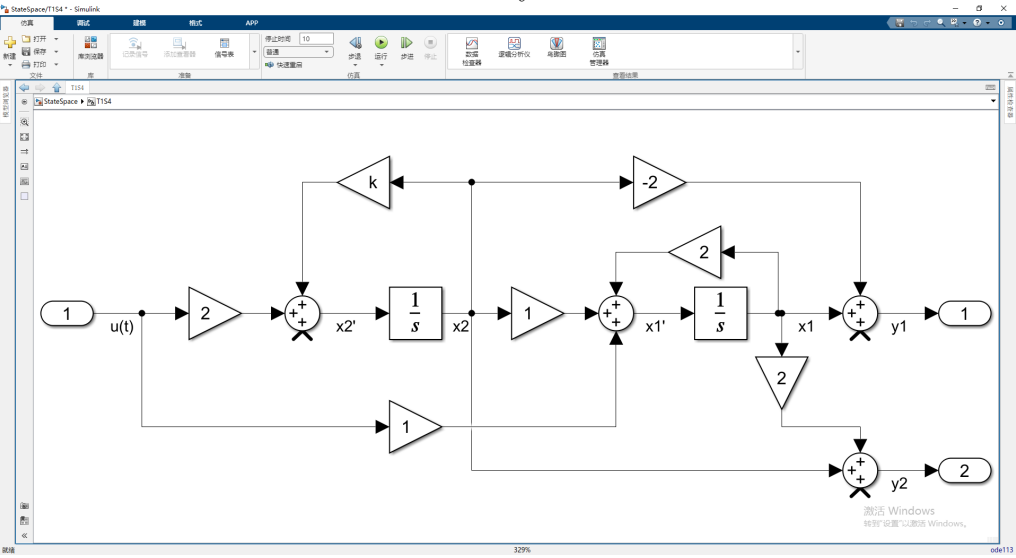


图 5: System4

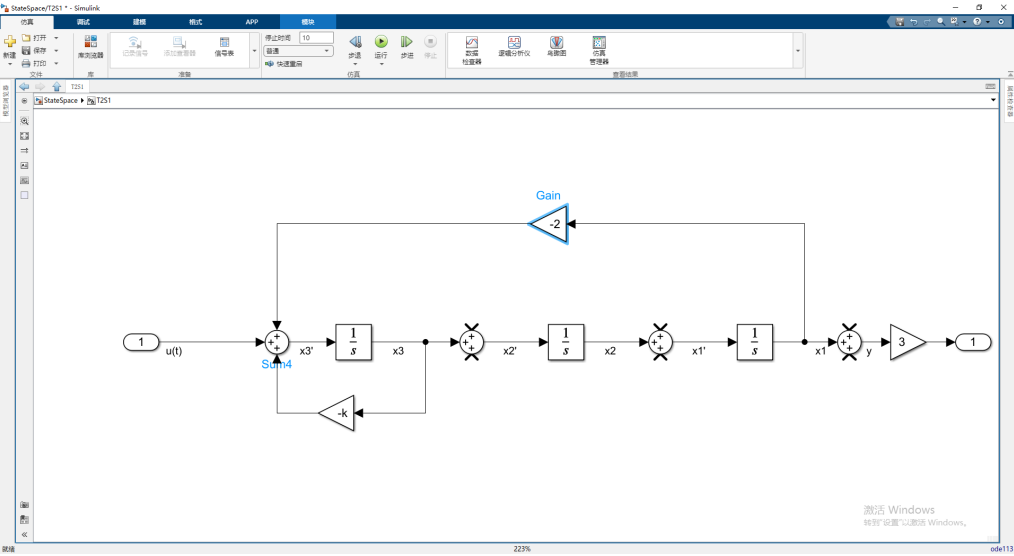


图 6: System1

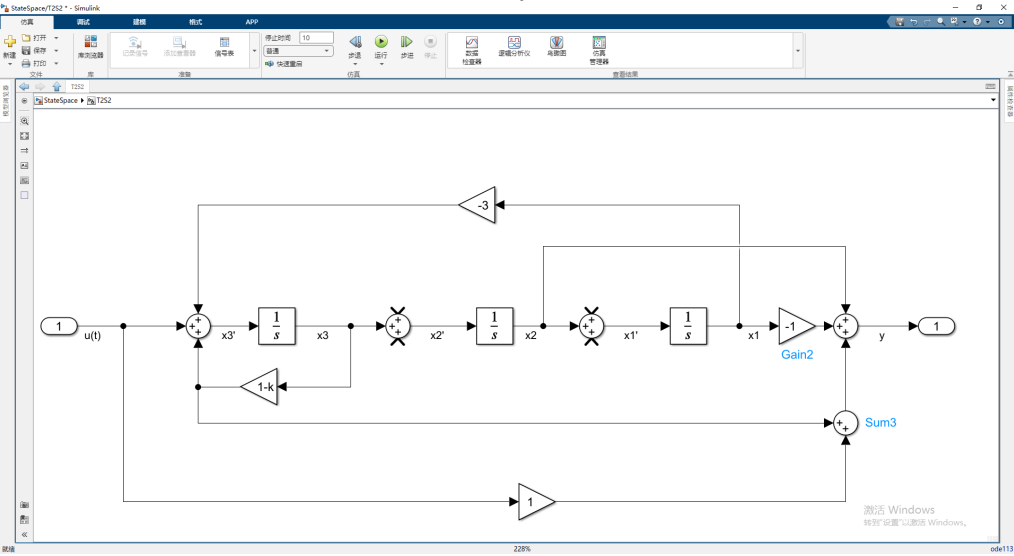


图 7: System2

## **1.2 Task2**

# **2 Task1**

## **2.1 Construct the system in MATLAB/Simulink**

See above section.

## **2.2 Represent the system in the Input-Output form**

Transfer function model Details:

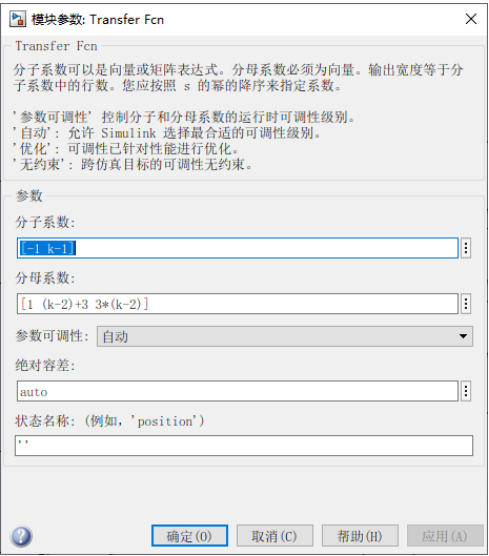


图 8: System1

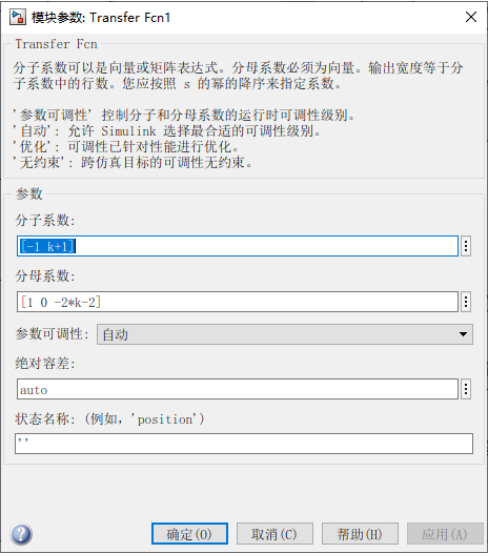


图 9: System2

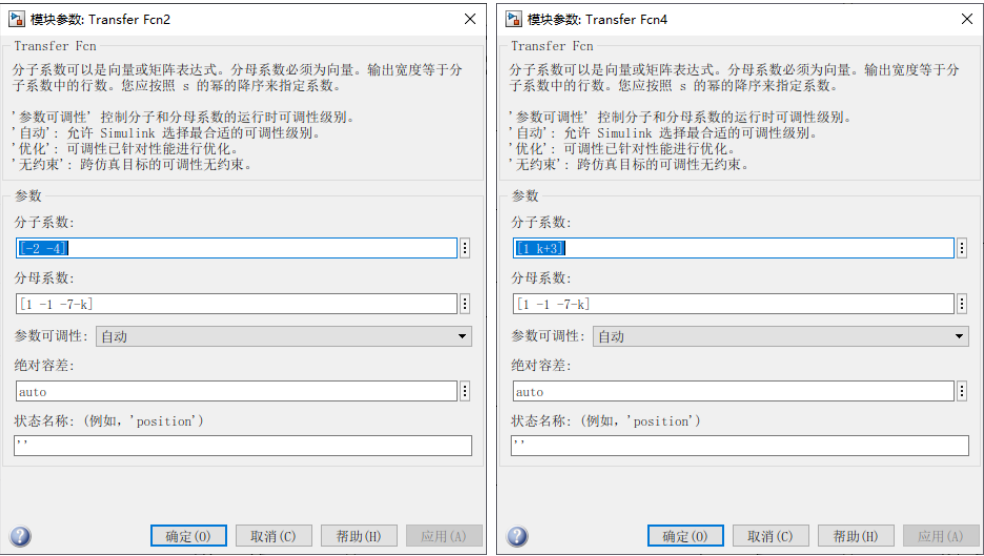


图 10: System3

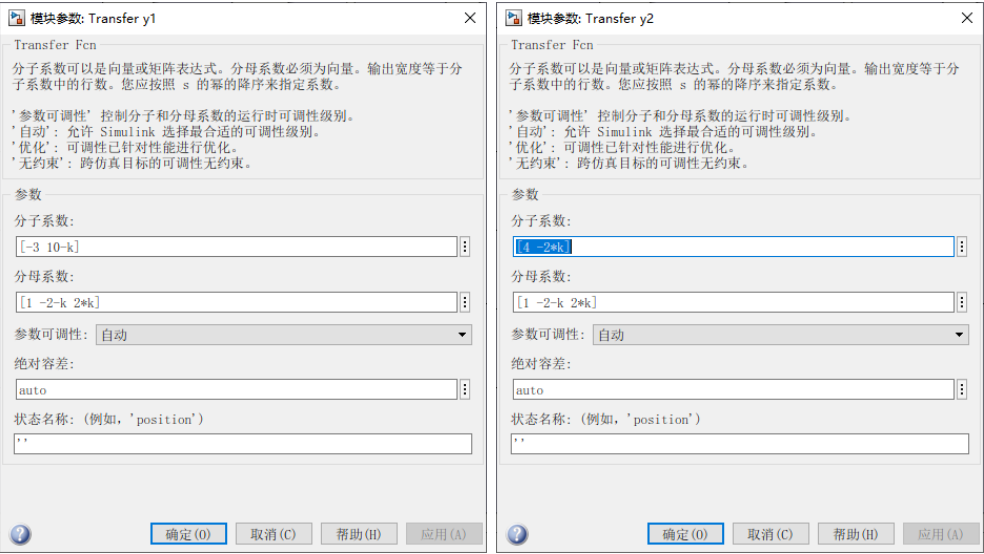


图 11: System4

Calculation by Wolfram Engine:



```

A = {{2-k,1},{0,-3}}
B = {{-1},{2+k}}
CI = {1,0}
DI = 0
TraditionalForm[Together[W = CI.Inverse[({{s,0},{0,s}}-A)].B+DI]]

```

$\begin{pmatrix} 2-k & 1 \\ 0 & -3 \end{pmatrix}$   
 $\begin{pmatrix} -1 \\ 2+k \end{pmatrix}$   
 $\begin{pmatrix} 1 & 0 \end{pmatrix}$   
 $0$   
 $\left\{ \frac{k-s-1}{(s+3)(k+s-2)} \right\}$

图 12: System1

```

A = {{0,1+k},{2,0}}
B = {{0},{1}}
CI = {1,-1}
DI = 0
TraditionalForm[Together[W = CI.Inverse[({{s,0},{0,s}}-A)].B+DI]]

```

$\begin{pmatrix} 0 & 1+k \\ 2 & 0 \end{pmatrix}$   
 $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$   
 $\begin{pmatrix} 1 & -1 \end{pmatrix}$   
 $0$   
 $\left\{ \frac{-k+s-1}{2k-s^2+2} \right\}$

图 13: System2

```

A = {{-2,1},{1+k,3}}
B = {{0,1},{-2,1}}
CI = {0,1}
DI = 0
TraditionalForm[Together[W = CI.Inverse[({{s,0},{0,s}}-A)].B+DI]]

```

$\begin{pmatrix} -2 & 1 \\ 1+k & 3 \end{pmatrix}$   
 $\begin{pmatrix} 0 & 1 \\ -2 & 1 \end{pmatrix}$   
 $\begin{pmatrix} 0 & 1 \end{pmatrix}$   
 $0$   
 $\left\{ \frac{2(s+2)}{-k+s^2-s-7}, \frac{-k-s-3}{k-s^2+s+7} \right\}$

图 14: System3

```

A = {{2,1},{0,k}}
B = {{1},{2}}
CI = {{1,-2},{2,1}}
DI = 0
TraditionalForm[Together[W = CI.Inverse[({{s,0},{0,s}}-A)].B+DI]]

```

$\begin{pmatrix} 2 & 1 \\ 0 & k \end{pmatrix}$   
 $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$   
 $\begin{pmatrix} 1 & -2 \\ 2 & 1 \end{pmatrix}$   
 $0$   
 $\left( \begin{array}{c} \frac{k+3s-10}{(s-2)(k-s)} \\ \frac{2(k-2s)}{(s-2)(k-s)} \end{array} \right)$

图 15: System4

## 2.3 Stability

$$k = 7$$

For System1:

$$\det(A - \lambda * I) = \begin{vmatrix} 2 - k - \lambda & 1 \\ 0 & -3 - \lambda \end{vmatrix} = 0$$

$$\rightarrow \lambda_1 = -3, \lambda_2 = 2 - k$$

Getting  $\lambda_1 = -3, \lambda_2 = -5$  For me. So System1 is asymptotically stable in my case, performing **Node Behavior**.

For System2:

$$\det(A - \lambda * I) = \begin{vmatrix} 0 - \lambda & 1 + k \\ 2 & 0 - \lambda \end{vmatrix} = 0$$

Getting  $\lambda_1 = 4, \lambda_2 = -4$  For me. So System2 is not stable in my case, performing **Saddle Behavior**.

For System3:

$$\det(A - \lambda * I) = \begin{vmatrix} -2 - \lambda & 1 \\ 1 + k & 3 - \lambda \end{vmatrix} = 0$$

Getting  $\lambda_1 = \frac{1-\sqrt{57}}{2}, \lambda_2 = \frac{1+\sqrt{57}}{2}$  For me. So System3 is not stable in my case, performing **Saddle Behavior**.

For System4:

$$\det(A - \lambda * I) = \begin{vmatrix} 2 - \lambda & 1 \\ 0 & k - \lambda \end{vmatrix} = 0$$

Getting  $\lambda_1 = 2, \lambda_2 = 7$  For me. So System4 is not stable in my case, performing **Node Behavior**.

### 3 Task2

#### 3.1 Represent the system in the canonical State-Space forms

System1:

**Controlable**

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -3 & 0 & k+1 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 2-3 & 1-0 & -k+1 \end{bmatrix}$$

$$D = \textcolor{red}{b}_n = 1(\text{not } b_0)$$

**Observable**

$$A = \begin{bmatrix} -3 & 1 & 0 \\ 0 & 0 & 1 \\ k+1 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} -k+1 \\ 1-0 \\ 2-3 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$$

$$D = \textcolor{red}{b}_n = 1(\text{not } b_0)$$

#### 3.2 Is system stable for $u(t) = 0$ ?

**Both not.**

### **3.3 Construct the system in MATLAB/Simulink**

See section **Constructing Systems**.

# 4 Figures

## 4.1 Task1

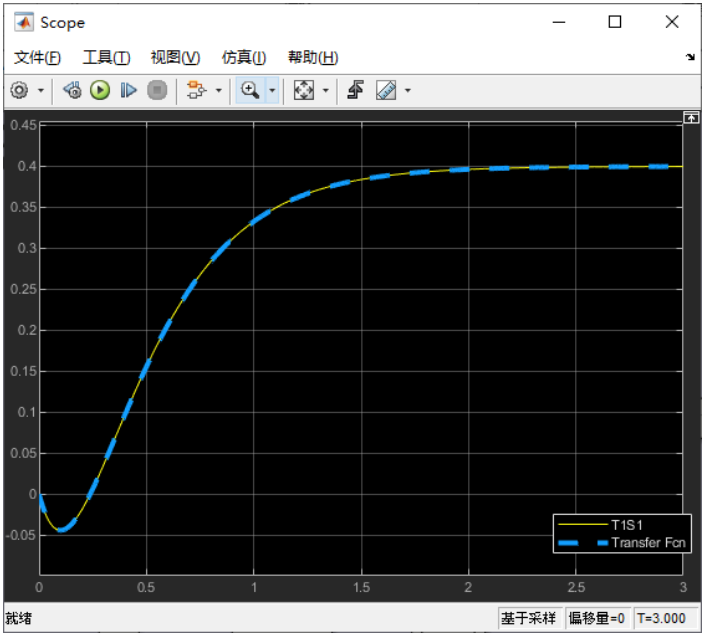


图 16: System1

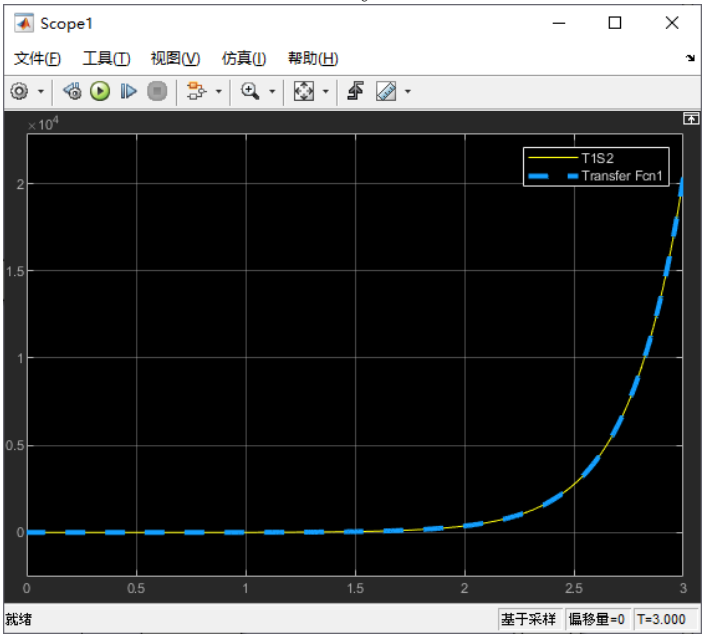


图 17: System2

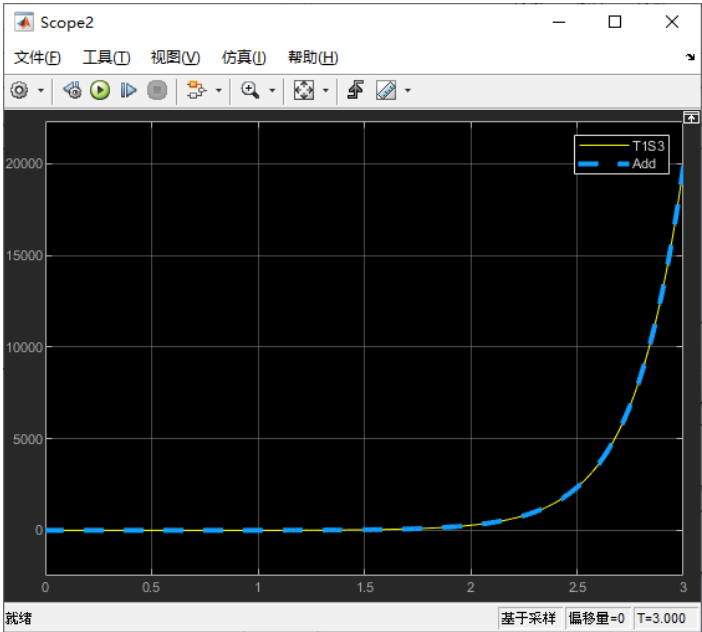


图 18: System3

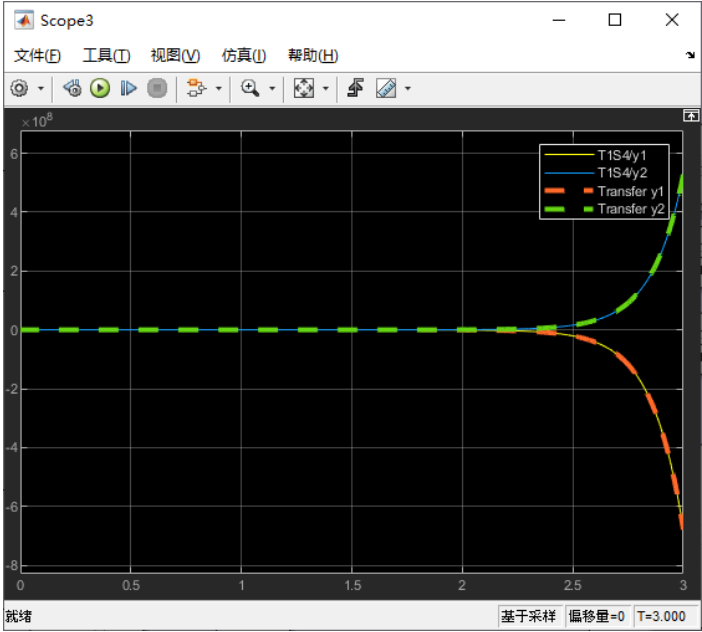


图 19: System4

4.2 Task2

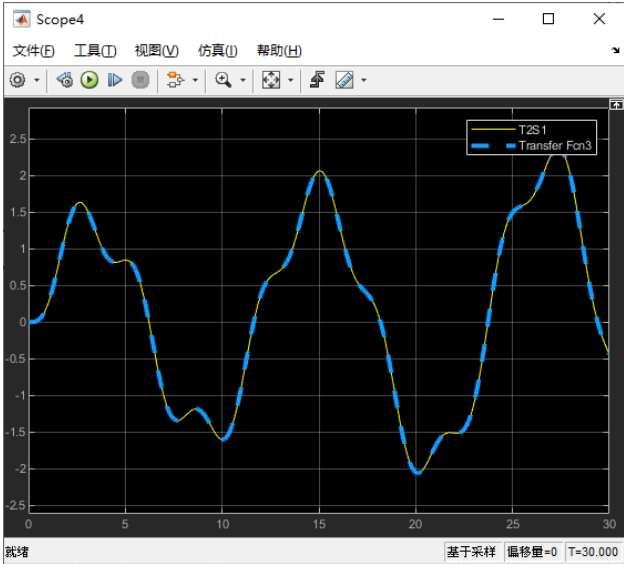


图 20: System1

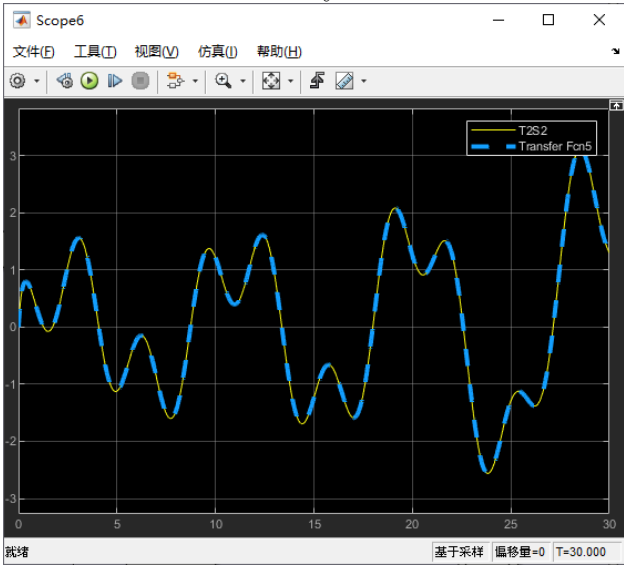


图 21: System2