## PRACTICE 1 Report From Xu Ziyang, 22320607

#### **Overview:**

- 1. Equations and graph corresponding to each system
- 2.Matlab Things
- 3. Calculate code in Wolfram Language

#### Part 1:

Equation1:

$$y'(t) + ky(t) = 0$$

Which means:

$$y'(t) + 7y(t) = 0$$

Character Equation:

$$\lambda + 7 = 0$$

Character Roots:

$$\{\lambda = -7\}$$

General Solution:

$$y=c_1e^{-7t}$$

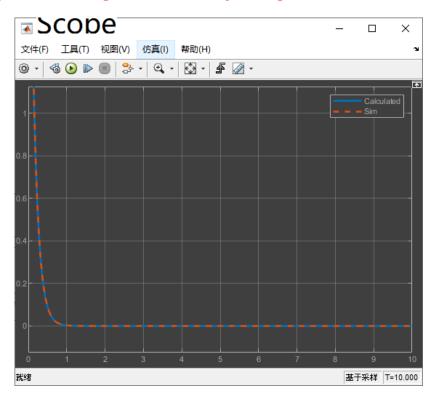
**Initial Condition:** 

$$y(0) = 3$$

Solution:

$$y = 3e^{-7t}$$

Graph: (You can see bugs with Windows, just forgive me and Microsoft...)



## Equation2:

$$y''(t) + (k+1)y'(t) + ky(t) = 0$$

Which means:

$$y''(t) + 8y'(t) + 7y(t) = 0$$

**Character Equation:** 

$$\lambda^2 + 8\lambda + 7 = 0$$

**Character Roots:** 

$$\{\lambda = -7, \lambda = -1\}$$

General Solution:

$$y = c_1 e^{-7t} + c_2 e^{-t}$$

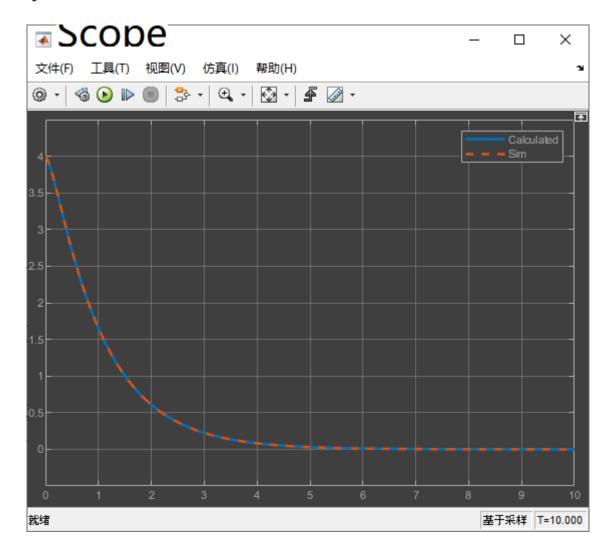
**Initial Condition:** 

$$y(0) = 4, y'(0) = -1$$

Solution:

$$y = 1/2*e^{-7t} (9e^{6t}-1)$$

## Graph:



Equation3:

$$y''(t) + k^2y(t) = 0$$

Which means:

$$y''(t) + 49 y(t) = 0$$

**Character Equation:** 

$$\lambda^2 + 49 = 0$$

**Character Roots:** 

$$\{\lambda = -7i, \lambda = 7i\}$$

General Solution:

$$y = c_1 \sin(7t) + c_2 \cos(7t)$$

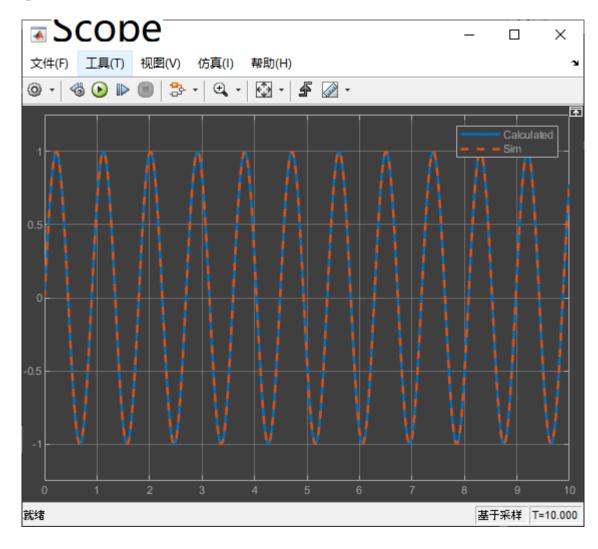
**Initial Condition:** 

$$y(0) = 0, y'(0) = -7$$

Solution:

$$y = \sin(7t)$$

Graph:



## Equation4:

$$y''(t) + 4y'(t) + 13y(t) = 0$$

Character Equation:

$$\lambda^2 + 4\lambda + 13 = 0$$

**Character Roots:** 

$$\{\lambda = -2-3i, \lambda = -2+3i\}$$

General Solution:

$$y = e^{-2t}(c_1 \sin(3t) + c_2 \cos(3t))$$

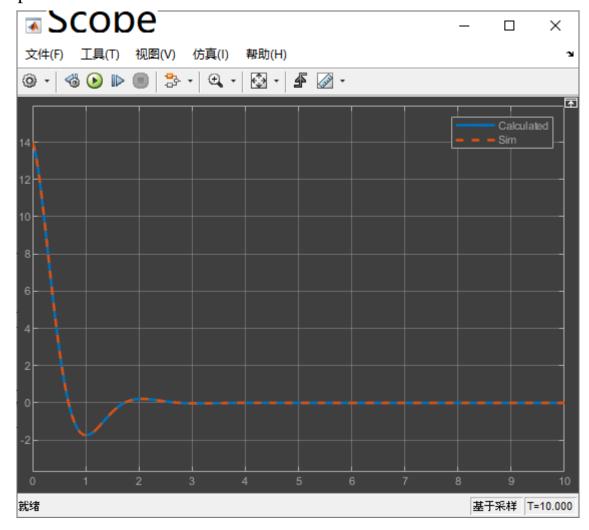
**Initial Condition:** 

$$y(0) = 14, y'(0) = -7$$

Solution:

$$y = 7e^{-2t}(\sin(3t) + 2\cos(3t))$$

## Graph:



#### Equation5:

Notice that y(t) is not 0(t) since y(0) is not 0.

$$y''(t) + ay'(t) + ky(t) = 0$$

Which means:

$$y''(t) + ay'(t) + 7y(t) = 0$$

**Character Equation:** 

$$\lambda^2 + a\lambda + 7 = 0$$

**Character Roots:** 

$$\left\{\operatorname{lambda} \to \frac{1}{2} \left(-\sqrt{a^2-28}-a\right)\right\}, \left\{\operatorname{lambda} \to \frac{1}{2} \left(\sqrt{a^2-28}-a\right)\right\}$$

General Solution:

$$\left\{ \left\{ y \rightarrow \left( \{t\} \mapsto c_1 \stackrel{1}{=} \left( -\sqrt{a^2 - 28} - a \right)^t + c_2 \stackrel{1}{=} \left( \sqrt{a^2 - 28} - a \right)^t \right) \right\} \right\}$$

Taking a = 2\*sqrt(7)(Let delta be zero):

Taking Initial Condition:

$$y(0) = 1, y'(0) = 0$$

Solution:

$$\left\{\left\{y\rightarrow\left(\left\{t\right\}\mapsto e^{-\sqrt{7}t}\left(\sqrt{7}t+1\right)\right)\right\}\right\}$$

Taking a = 2\*sqrt(5):

Taking Initial Condition:

$$y(0) = 1, y'(0) = 0$$

Solution:

$$\left\{ \left\{ y \to \left\{ \{t\} \mapsto \frac{1}{2} e^{-\sqrt{5}t} \left( \sqrt{10} \sin\left(\sqrt{2}t\right) + 2\cos\left(\sqrt{2}t\right) \right) \right\} \right\}$$

Taking a = 6:

Taking Initial Condition:

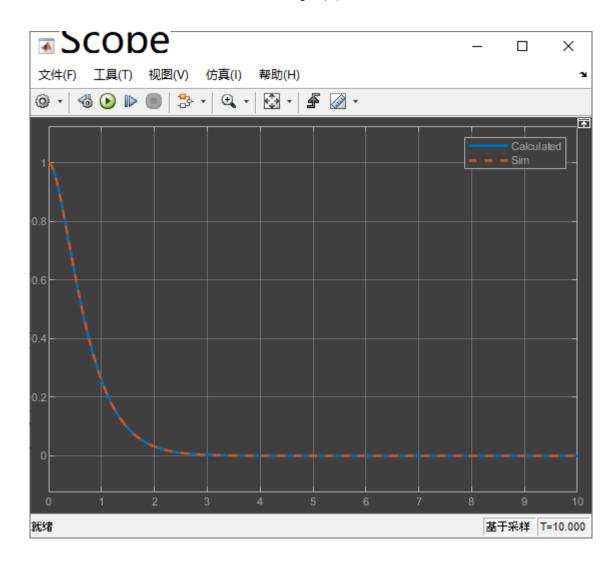
$$y(0) = 1, y'(0) = 0$$

Solution:

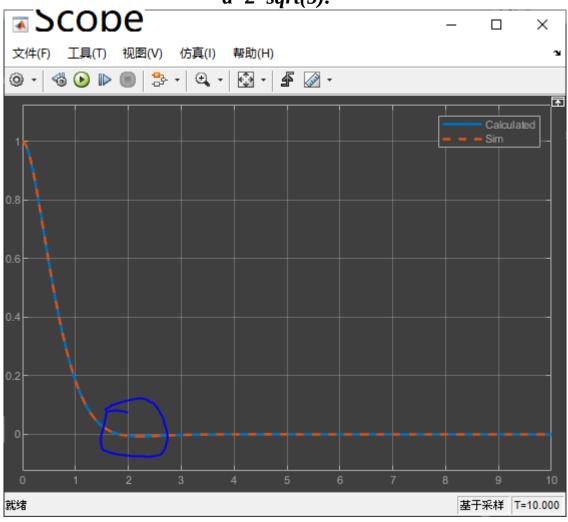
$$\left\{ \left\{ y \to \left( \{t\} \mapsto \frac{1}{4} \left( 2 e^{\left( -3 - \sqrt{2} \right) t} - 3 \sqrt{2} e^{\left( -3 - \sqrt{2} \right) t} + 2 e^{\left( \sqrt{2} - 3 \right) t} + 3 \sqrt{2} e^{\left( \sqrt{2} - 3 \right) t} \right) \right) \right\} \right\}$$

## Graphs:

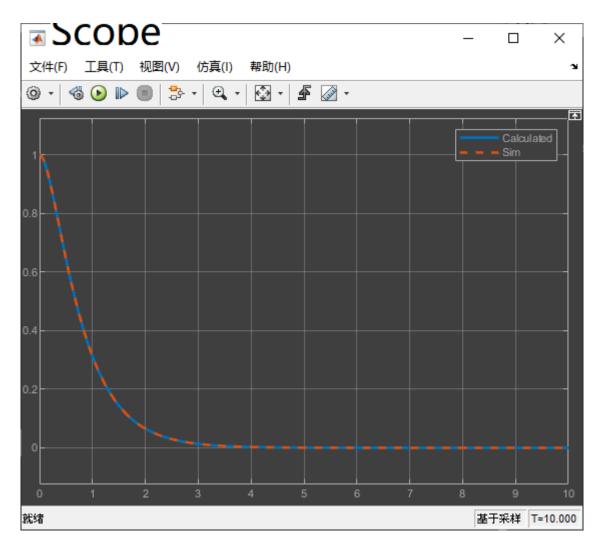
## a=2\*sqrt(7):



a=2\*sqrt(5):

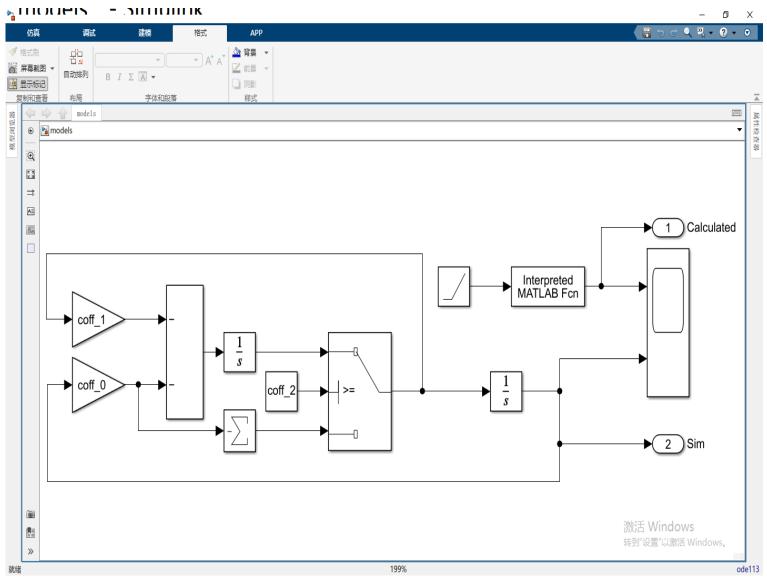


a=6:



## Part 2:

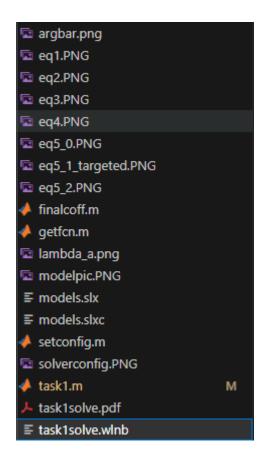
## Simulink Model:



Solver Configure:

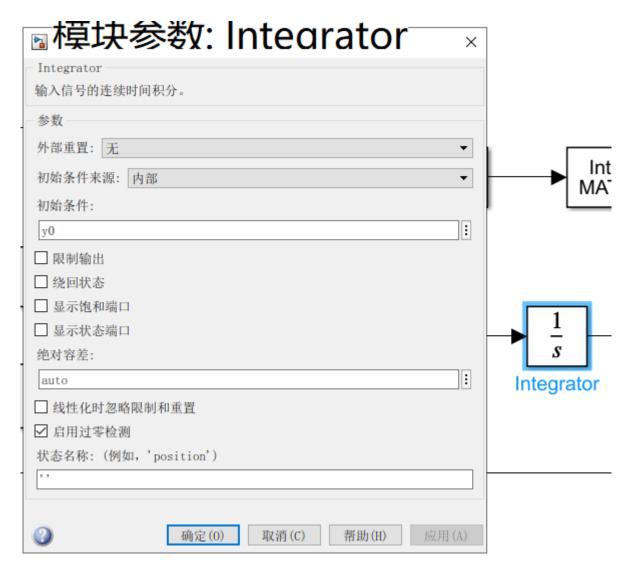


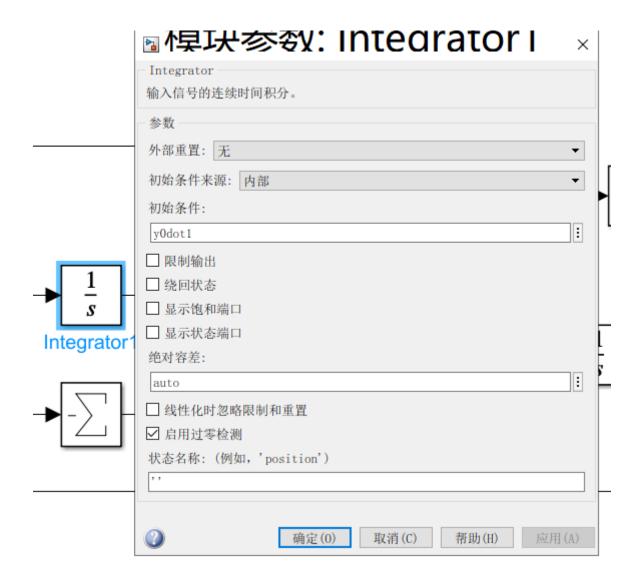
## Repository:



#### Model info:







# Codes(Why LibreOffice cannot copy format correctly?): finalcoff.m:

```
function [coff_dot2,coff_dot1,coff_nodot] = finalcoff(modeltesting,k,a)
    switch modeltesting
        case 1
            coff_dot1 = 1;
            coff dot2 = 0;
            coff_nodot = k;
        case 2
            coff_dot1 = k+1;
            coff_dot2 = 1;
            coff_nodot = k;
        case 3
            coff_dot1 = 0;
            coff_dot2 = 1;
            coff\ nodot = k^2;
        case 4
            coff_dot1 = 4;
            coff\_dot2 = 1;
            coff_nodot = 13;
        case 5+0
            coff_dot1 = a;
            coff_dot2 = 1;
            coff_nodot = 7;
        case 5+1
            coff_dot1 = a;
            coff_dot2 = 1;
            coff_nodot = 7;
        case 5+2
            coff\_dot1 = a;
            coff_dot2 = 1;
            coff_nodot = k;
    end
end
   getfcn.m:
function f = getfcn(modeltesting)
    switch modeltesting
        case 1
            f = @(t)3*exp(-7*t);
        case 2
            f = Q(t)exp(-7*t)/2*(9*exp(6*t)-1);
        case 3
            f = \mathcal{Q}(t)\sin(7^*t);
        case 4
            f = @(t)7*exp(-2*t)*(sin(3*t)+2*cos(3*t));
        case (5+0)
            f = @(t)exp(-sqrt(7)*t)*(sqrt(7)*t+1);
        case (5+1)
            f = Q(t)1/2*exp(-sqrt(5)*t)*(sqrt(10)*sin(t*sqrt(2))+2*cos(t*sqrt(2)));
        case (5+2)
            f = Q(t)1/4*(2*exp((-3-sqrt(2))*t)-3*sqrt(2)*exp((-3-sqrt(2))*t)
+2*exp((sqrt(2)-3)*t)+3*sqrt(2)*exp((sqrt(2)-3)*t));
    end
end
```

```
setconfig.m:
function [y0,y0dot1,a] = setconfig(modeltesting,k)
    switch modeltesting
        case 1
            y0 = 3;
            y0dot1= 0;
            a = 0;
        case 2
            y\theta = 4;
            y0dot1 = -1;
            a = 0;
        case 3
            y\theta = \theta;
            y0dot1 = k;
            a = 0;
        case 4
            y0 = 2*k;
            y0dot1 = -k;
             a = 0;
        case 5+0
            y0 = 1;
            y0dot1 = 0;
            a=sqrt(28);
        case 5+1
            y\theta = 1;
            y0dot1 = 0;
            a=sqrt(20);
        case 5+2
            y\theta = 1;
            y0dot1 = 0;
             a=sqrt(36);
    end
end
   task1.m(No tabs so it's working):
modeltesting = 7;\%changes for each model
[y0,y0dot1,a]=setconfig(modeltesting,k);
[coff_2,coff_1,coff_0]=finalcoff(modeltesting,k,a);
fcn = getfcn(modeltesting);
```

## Part 3:

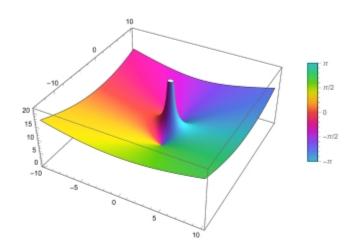
#### 接 Esc 退出全屏 in[:]:= Remove[a, x, y, lambda] in[-]:- k = 7 TraditionalForm[equation1 = $\{y'[t] + y[t] * k == 0\}$ ] TraditionalForm[charaEq1 = {x + k == 0}] TraditionalForm[Solve[charaEq1, x]] TraditionalForm[DSolve[equation1, y, t]] $TraditionalForm[init1 = {y[0] = 3}]$ TraditionalForm[DSolve[{equation1, init1}, y, t]] Out[-]-Out[-]//TraditionalForm- $\{y'(t) + 7 y(t) = 0\}$ Out[-]//TraditionalForm- $\{x + 7 = 0\}$ Out[-]//TraditionalForm= $\{\{x \to -7\}\}\$ Out[-]//TraditionalForm- $\left\{\left\{y\rightarrow\left(\left\{t\right\}\mapsto c_{1}\,e^{-7\,t}\right)\right\}\right\}$ Out[-]//TraditionalForm= $\{y(0) = 3\}$ Out[-]//TraditionalForm- $\{\{y \to (\{t\} \mapsto 3 e^{-7t})\}\}\$ ln[+]:= TraditionalForm[equation2 = $\{y''[t] + (k+1) * y'[t] + k * y[t] = \emptyset\}$ ] $TraditionalForm[charaEq2 = \{x^2 + (k+1) * x + k = 0\}]$ TraditionalForm[Solve[charaEq2]] TraditionalForm[DSolve[equation2, y, t]] TraditionalForm[init2 = $\{y[\theta] = 4, y'[\theta] = -1\}$ ] TraditionalForm[DSolve[{equation2, init2}, y, t]] Out[-]//TraditionalFor $\{y''(t)+8\ y'(t)+7\ y(t)=0\}$ Out[+]//TraditionalForm- $\{x^2 + 8x + 7 = 0\}$ Out[+]//TraditionalForm= $\{\{x \to -7\}, \{x \to -1\}\}\$ Out[+]//TraditionalForm= $\{\{y \rightarrow (\{t\} \mapsto c_1 e^{-7t} + c_2 e^{-t})\}\}$ Out[-]//TraditionalForm= $\{y(0) = 4, \ y'(0) = -1\}$ $\begin{array}{c} \text{Out[-]//TraditionalForm-} \\ \left\{ \left\{ y \rightarrow \left( \{t\} \longmapsto \frac{1}{2} \ e^{-7t} \left( 9 \ e^{6t} - 1 \right) \right) \right\} \right\} \end{array}$

```
ln[+]:= TraditionalForm[equation3 = {y''[t] + k^2 * y[t] == 0}]
          TraditionalForm[charaEq3 = \{x^2 + k^2 = 0\}]
          TraditionalForm[Solve[charaEq3, x]]
          TraditionalForm[DSolve[equation3, y, t]]
          TraditionalForm[init3 = \{y[\theta] = \theta, y'[\theta] = k\}]
          TraditionalForm[DSolve[{equation3, init3}, y, t]]
Out[-]//TraditionalFo
         \{y''(t) + 49 \ y(t) = 0\}
Out[+]//TraditionalFort
         {x^2 + 49 = 0}
Out[-]//TraditionalForm-
         \{\{x \to -7\ i\},\ \{x \to 7\ i\}\}
Out[-]//TraditionalForm=
         \{\{y \to (\{t\} \mapsto c_1 \cos(7t) + c_2 \sin(7t))\}\}
Out[-]//TraditionalForm-
         {y(0) = 0, y'(0) = 7}
Out[+]//TraditionalForm=
         \{\{y \rightarrow (\{t\} \mapsto \sin(7\ t))\}\}\
  ln[\cdot]:= TraditionalForm[equation4 = \{y''[t] + 4 * y'[t] + 13 * y[t] = 0\}]
          TraditionalForm[charaEq4 = \{x^2 + 4 * x + 13 = \theta\}]
          TraditionalForm[Solve[charaEq4, x]]
          TraditionalForm[DSolve[equation4, y, t]]
          TraditionalForm[init4 = \{y[\theta] = 2 * k, y'[\theta] = -k\}]
          TraditionalForm[DSolve[{equation4, init4}, y, t]]
Out[-]//TraditionalForm-
         \{y''(t) + 4y'(t) + 13y(t) = 0\}
Out[-]//TraditionalForm-
         \{x^2 + 4x + 13 = 0\}
Out[-]//TraditionalForm-
         \{\{x\to -2-3\,i\},\,\{x\to -2+3\,i\}\}
Out[-]//TraditionalForm-
         \{\{y \to (\{t\}) \mapsto c_2 e^{-2t} \cos(3t) + c_1 e^{-2t} \sin(3t)\}\}
Out[-]//TraditionalForm-
         \{y(0) = 14, y'(0) = -7\}
Out[-]//TraditionalForm-
         \left\{\left\{y\rightarrow\left(\left\{t\right\}\longmapsto7\;e^{-2\,t}\left(\sin(3\,t)+2\cos(3\,t)\right)\right\}\right\}
```

$$\begin{split} & \text{TraditionalForm[Solve[lambda^2 + a * lambda + k = 0, lambda]]} \\ & \text{TraditionalForm[Solve[lambda^2 + a * lambda + k = 0, a]]} \\ & \text{ComplexPlot3D[(-p^2 - 8) / p, \{p, -10 - 10 * Sqrt[2] I, 10 + 10 I\}, PlotLegends } \rightarrow \text{Automatic]} \\ & \text{TraditionalForm[Solve[- (lambda^2 + 8) = 0]]} \\ & \text{TraditionalForm[DSolve[y''[t] + a * y'[t] + k * y[t] = 0, y, t]]} \\ & \text{TraditionalForm[DSolve[y''[t] + k * y[t] = 0, y, t]]} \\ & \text{a = Sqrt[28]} \\ & \text{TraditionalForm[DSolve[\{y''[t] + a * y'[t] + k * y[t] = 0, y[0] = 1, y'[0] = 0\}, y, t]]} \\ & \text{a = Sqrt[20]} \\ & \text{TraditionalForm[DSolve[\{y''[t] + a * y'[t] + k * y[t] = 0, y[0] = 1, y'[0] = 0\}, y, t]]} \\ & \text{a = Sqrt[36]} \\ & \text{TraditionalForm[DSolve[\{y''[t] + a * y'[t] + k * y[t] = 0, y[0] = 1, y'[0] = 0\}, y, t]]} \\ & \text{Out[-]//TeaditionalForm-} \\ & \text{\{ [lambda } \rightarrow \frac{1}{2} \left( -\sqrt{a^2 - 28} - a \right) \}, \left\{ lambda \rightarrow \frac{1}{2} \left( \sqrt{a^2 - 28} - a \right) \} \right\} \\ & \text{Out[-]//TeaditionalForm-} \\ & \text{Out[-]//TeaditionalForm-} \\ & \text{(lambda } \rightarrow \frac{1}{2} \left( -\sqrt{a^2 - 28} - a \right) \}, \left\{ lambda \rightarrow \frac{1}{2} \left( \sqrt{a^2 - 28} - a \right) \} \right\} \\ & \text{Out[-]//TeaditionalForm-} \\ & \text{(lambda } \rightarrow \frac{1}{2} \left( -\sqrt{a^2 - 28} - a \right) \}, \left\{ lambda \rightarrow \frac{1}{2} \left( \sqrt{a^2 - 28} - a \right) \right\} \\ & \text{(lambda } \rightarrow \frac{1}{2} \left( -\sqrt{a^2 - 28} - a \right) \}, \left\{ lambda \rightarrow \frac{1}{2} \left( \sqrt{a^2 - 28} - a \right) \right\} \right\} \\ & \text{(lambda } \rightarrow \frac{1}{2} \left( -\sqrt{a^2 - 28} - a \right) \}, \\ & \text{(lambda } \rightarrow \frac{1}{2} \left( -\sqrt{a^2 - 28} - a \right) \}, \\ & \text{(lambda } \rightarrow \frac{1}{2} \left( -\sqrt{a^2 - 28} - a \right) \}, \\ & \text{(lambda } \rightarrow \frac{1}{2} \left( -\sqrt{a^2 - 28} - a \right) \}, \\ & \text{(lambda } \rightarrow \frac{1}{2} \left( -\sqrt{a^2 - 28} - a \right) \}, \\ & \text{(lambda } \rightarrow \frac{1}{2} \left( -\sqrt{a^2 - 28} - a \right) \}, \\ & \text{(lambda } \rightarrow \frac{1}{2} \left( -\sqrt{a^2 - 28} - a \right) \}, \\ & \text{(lambda } \rightarrow \frac{1}{2} \left( -\sqrt{a^2 - 28} - a \right) \}, \\ & \text{(lambda } \rightarrow \frac{1}{2} \left( -\sqrt{a^2 - 28} - a \right) \}, \\ & \text{(lambda } \rightarrow \frac{1}{2} \left( -\sqrt{a^2 - 28} - a \right) \}, \\ & \text{(lambda } \rightarrow \frac{1}{2} \left( -\sqrt{a^2 - 28} - a \right) \}, \\ & \text{(lambda } \rightarrow \frac{1}{2} \left( -\sqrt{a^2 - 28} - a \right) \}, \\ & \text{(lambda } \rightarrow \frac{1}{2} \left( -\sqrt{a^2 - 28} - a \right) \}, \\ & \text{(lambda } \rightarrow \frac{1}{2} \left( -\sqrt{a^2 - 28} - a \right) \}, \\ & \text{(lambda } \rightarrow \frac{1}{2$$

$$\{a \rightarrow \frac{-lambda^2 - 7}{lambda}\}$$

Out[-]-



Out[-]//TraditionalForm-

$$\{\{\text{lambda} \rightarrow -2 \ i \ \sqrt{2}\}, \{\text{lambda} \rightarrow 2 \ i \ \sqrt{2}\}\}$$

$$\{\{y \rightarrow \{t\} \mapsto c_1 e^{\frac{1}{2} \left(-\sqrt{a^2-28}-a\right)t} + c_2 e^{\frac{1}{2} \left(\sqrt{a^2-28}-a\right)t}\}\}$$

Out[-]//TraditionalForm-

$$\{\{y \rightarrow (\{t\} \mapsto c_1 \cos(\sqrt{7} t) + c_2 \sin(\sqrt{7} t))\}\}$$

Out[-]-

$$\begin{cases} \left\{ y \to \left( \{t\} \longmapsto e^{-\sqrt{7} \ t} \left( \sqrt{7} \ t+1 \right) \right) \right\} \end{cases}$$

4 |

$$\left\{ \left\{ y \to \left( \{t\} \mapsto \frac{1}{2} \ e^{-\sqrt{5} \ t} \left( \sqrt{10} \ \sin \left( \sqrt{2} \ t \right) + 2 \cos \left( \sqrt{2} \ t \right) \right) \right\} \right\}$$

Out[-]=

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$$\left\{ \left\{ y \to \left( \{t\} \longmapsto \frac{1}{4} \left( 2 \, e^{\left( -3 - \sqrt{2} \, \right) \, t} - 3 \, \sqrt{2} \, e^{\left( -3 - \sqrt{2} \, \right) \, t} + 2 \, e^{\left( \sqrt{2} - 3 \right) \, t} + 3 \, \sqrt{2} \, e^{\left( \sqrt{2} - 3 \right) \, t} \right) \right\} \right\}$$