Homework 1

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Problem 1.

(a) The solving process is as follows:

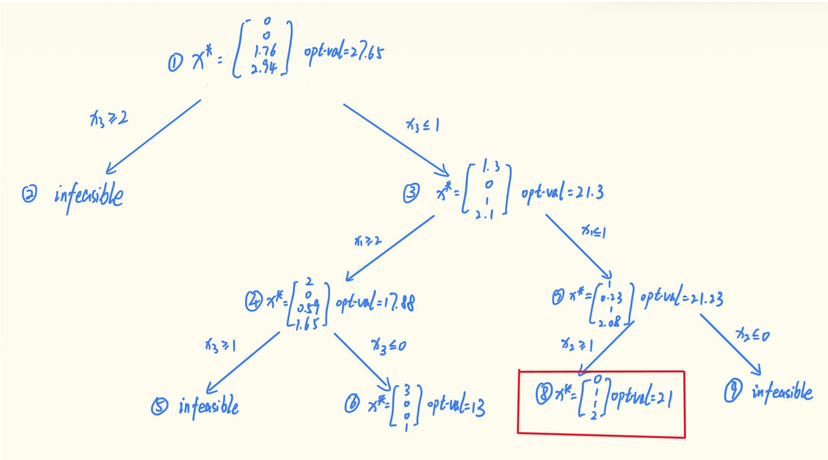

Figure 1: Solving process for (a)

(b) Yes, but one condition needs to be met: the objective function coefficients are all integers. Because if the coefficients of the objective function are all integers, then the objective value of any integer solution must also be an integer, so the optimal integer solution z must also be an integer.

In this question, the objective function coefficients are 2, 3, 4, and 7, all of which are integers, so this condition is satisfied.

Problem 2.

(a)

$$\nabla f(x) = \begin{bmatrix} 4x_1^3 + 6x_1^2 - 4x_1x_2 \\ -2x_1^2 + 8x_2 \end{bmatrix}$$

$$\text{Let } \nabla f(x) = \mathbf{0} \quad \Rightarrow \begin{cases} x_1 = 0 \\ x_2 = 0 \end{cases} \text{or} \begin{cases} x_1 = -2 \\ x_2 = 1 \end{cases}$$

$$\text{While } \nabla^2 f(x) = \begin{bmatrix} 12x_1^2 + 12x_1 - 4x_2 & -4x_1 \\ -4x_1 & 8 \end{bmatrix}$$

$$\nabla^2 f(0,0) = \begin{bmatrix} 0 & 0 \\ 0 & 8 \end{bmatrix}, \quad \nabla^2 f(-2,1) = \begin{bmatrix} 20 & 8 \\ 8 & 8 \end{bmatrix} \text{(PD)}$$

Thus, $(0,0)$ is a saddle point, while $(-2,1)$ is a local minimum.

(b) The contour plot is as follows:

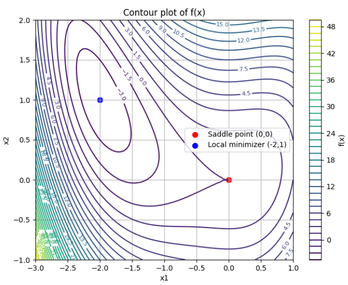

Figure 2: Contour plot of $f(x)$

From the contour map, it can be seen that the function presents a concave shape near the point $(-2,1)$ and is the only local minimum point.

When $x_1 \to \infty or x_2 \to \infty$, $f(x) \to +\infty$, so there is a global minimum at $(-2,1)$.

Problem 3.

(a) Using the Leading Principle Minors method:

A_1 is indefinite;

A_2 is positive semidefinite;

A_3 is positive definite;

A_4 is indefinite.

Calculating the eigenvalues of A_3 and A_4:

$$|A_3 - \lambda I| = \begin{vmatrix} 1-\lambda & 0 & 1 \\ 0 & 1-\lambda & -1 \\ 0 & 2 & 4-\lambda \end{vmatrix} = (1-\lambda)(2-\lambda)(3-\lambda) = 0$$

$$\Rightarrow \lambda_1 = 1, \lambda_2 = 2, \lambda_3 = 3$$

$$|A_4 - \lambda I| = \begin{vmatrix} 1-\lambda & 0 & 1 \\ 0 & -1-\lambda & -1 \\ -1 & 1 & -\lambda \end{vmatrix} = -\lambda(\lambda^2+1) = 0$$

$$\Rightarrow \lambda_1 = 0, \lambda_2 = i, \lambda_3 = -i$$