Homework 1

PENG Qiheng

Student ID 225040065

November 11, 2025

Problem 1.

1.

$$\begin{aligned}
\text{state space}\quad & \mathcal{S} = \{0, 1, \ldots, n\} \\
\text{action space}\quad & \mathcal{A} = \{'c',' r'\} \\
\text{transition function}\quad & P(s'|s,a) = \begin{cases} \frac{n-i}{n}, & a =' c', s' = s+1 \\ \frac{i}{n}, & a =' c', s' = s-1 \\ \frac{1}{n}, & a =' r', \forall s' \in \mathcal{S} \\ 0, & \text{otherwise} \end{cases} \\
\text{reward function}\quad & R(s,a,s') = \begin{cases} 1, & s' = n \\ 0, & \text{otherwise} \end{cases} \\
\text{optimal value function}\quad & V^*(s) = \\
& \max_{a \in \mathcal{A}} \sum_{s' \in \mathcal{S}} P(s'|s,a) \left(R(s,a,s') + \gamma V^*(s') \right)
\end{aligned}$$

2. ***

Problem 2.

$$
\begin{aligned}
V^* &= R + \gamma P V^* \\
(I - \gamma P) V^* &= R \\
V^* = (I - \gamma P)^{-1} R &= \hat{V} = \Phi w \\
R &= (I - \gamma P) \Phi w
\end{aligned}
$$

So we need Φ to satisfy the following equation:

$$R = (I - \gamma P) \Phi w$$

Thus we can guarantees that $V^* = \Phi w$.

Problem 3.

1. ***

2. ***

3. ***

4. ***

Problem 4. ***