DDA 5002 Homework # 4 Fall 2025

Due: Sunday, November 16 at 11:59pm

Please note: Problem 1 asks you to solve a series of models in Python with COPT and Problem 2 asks you to plot a nonlinear function. Put your .py files (or Jupyter notebook files) and the output screenshot files you create in a single zip file and submit it via Blackboard. Please name your file `Lastname_StudentID.zip`. Also, give meaningful names to your decision variables and constraints, and add comments to your code liberally. Both the problem set and the code are due at the end of day on Sunday, November 16th.

1.

(a) Solve the following integer program via the branch-and-bound algorithm. You should use the termination condition as the lower bound equals to the upper bound (i.e., all nodes are fathomed.) Form the branch-and-bound tree and indicate the solution associated with each node, and why each node is fathomed. You can solve each linear programming relaxation in Python, adding branching inequalities as necessary. For the linear programming relaxation nodes, you need to report the solution and what branching constraint you have added. You should include the Python codes to solve each of the LP relaxation problems.

$$\begin{aligned}
\max_{x} \quad & 2x_1 + 3x_2 + 4x_3 + 7x_4 \\
\text{s.t.} \quad & 4x_1 + 6x_2 - 2x_3 + 8x_4 = 20 \\
& x_1 + 2x_2 - 6x_3 + 7x_4 = 10 \\
& x_1, \ldots, x_4 \geq 0 \\
& x_1, \ldots, x_4 \in \mathbb{Z}.
\end{aligned}$$

(b) Suppose that at some step in using the branch-and-bound algorithm, the gap between upper and lower bounds at every node is strictly less than 1. That is, suppose $\bar{z} - \underline{z} < 1$ assuming $\bar{z}$ denotes the upper bound achieved at the certain node, and $\underline{z}$ is the current lower bound. Are we assured that the current incumbent, with objective function value $\underline{z}$, is optimal? What characteristics of the integer program are needed, i.e., sufficient, for such a property to hold?

2. Consider the unconstrained optimization problem

$$\min_{x \in \mathbb{R}^2} \; f(x) = x_1^4 + 2(x_1 - x_2)x_1^2 + 4x_2^2.$$

(a) Calculate all stationary points of the mapping f and investigate whether the stationary points are local maximizer, local minimizer, or saddle points. The stationary points are the points where each element of the gradient equals to zero.

(b) Create a 3D or contour plot of the function using Python and decide whether the problem possesses a global solution or not. In Python you can use the package `matplotlib` and `numpy` to complete this task. Please plot the function with proper legends.

3. Classify the following matrices and verify whether they are positive definite, positive semidefinite or indefinite.

$$A_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 1 & 2 & 5 \end{bmatrix}, \quad A_3 = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 2 & 4 \end{bmatrix}, \quad A_4 = \begin{bmatrix} 1 & 0 & 1 \\ 0 & -1 & -1 \\ -1 & 1 & 0 \end{bmatrix}.$$

What are the eigenvalues of the matrices A_3 and A_4?