Homework 3

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Problem 1.

(a1)

$$X = \begin{bmatrix} 1 & x_1 & x_1^2 & \cdots & x_1^8 \\ 1 & x_2 & x_2^2 & \cdots & x_2^8 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & x_n & x_n^2 & \cdots & x_n^8 \end{bmatrix}, \quad y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}$$

(a2)

$$\hat{\theta} = \begin{bmatrix} 0.60775979 \\ -7.25862115 \\ 15.3450059 \\ 17.26526602 \\ -46.39728808 \\ -10.63103168 \\ 41.47282583 \\ 1.75929095 \\ -11.22913232 \end{bmatrix}$$

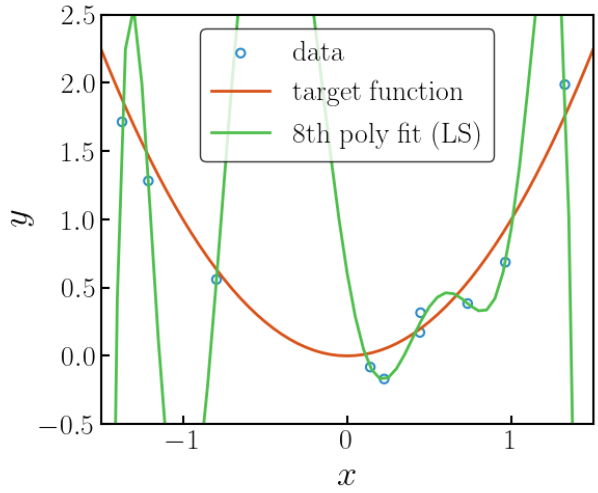


Figure 1: Ridge regression with different λ

(a3)

$$||\mathbf{X_{test}}\hat{\theta} - \mathbf{y_{test}}||_2 = 4.5607$$

(b1) The figure is as follows:

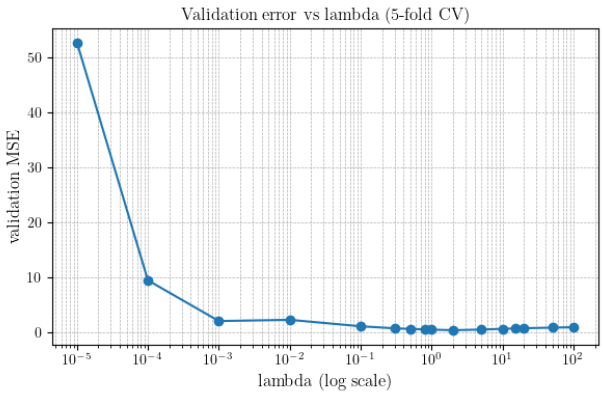
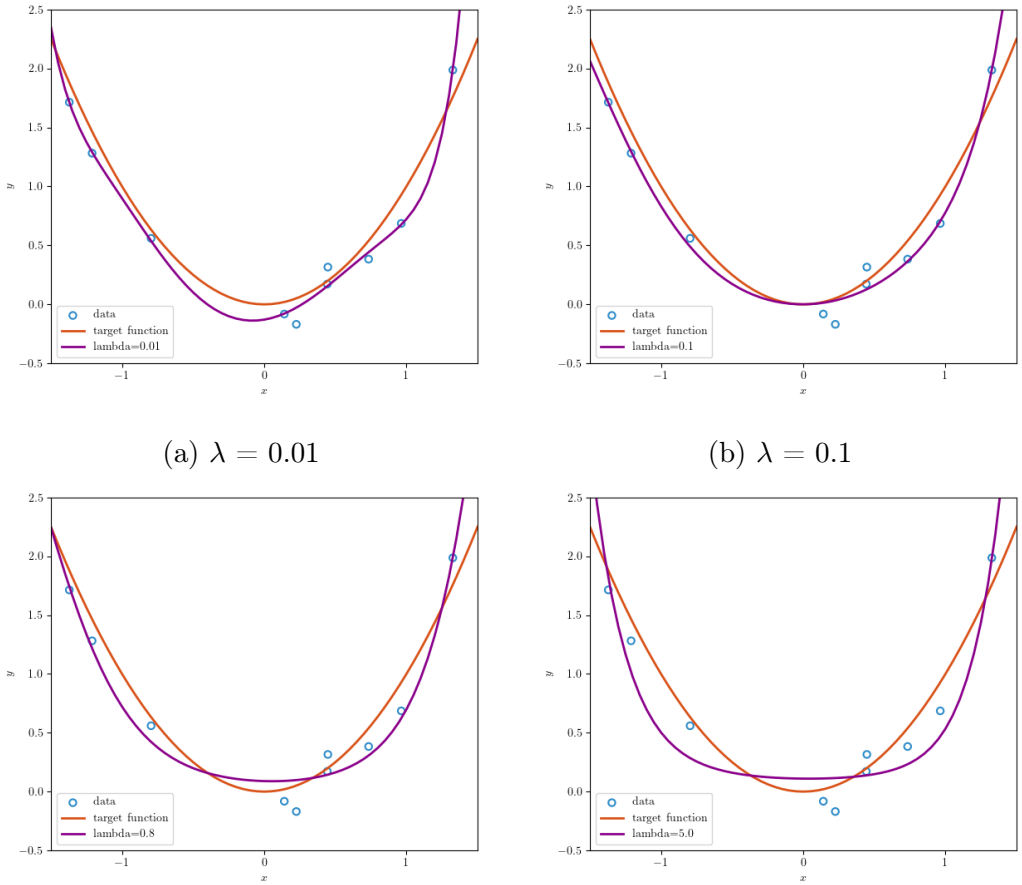


Figure 2: Lasso regression with different λ

(b2) The figures are as follows:



(a) $\lambda = 0.01$ (b) $\lambda = 0.1$

(c) $\lambda = 0.8$ (d) $\lambda = 5$

Figure 3: Lasso regression coefficients with different λ

(b3)

$$\lambda = 0.01, \quad \text{test error} = 0.6634$$

$$\lambda = 0.1, \quad \text{test error} = 0.6341$$

$$\lambda = 0.8, \quad \text{test error} = 0.6493$$

$$\lambda = 5, \quad \text{test error} = 0.8207$$

Problem 2.

(1) The top 40 eigenfaces are as follows:

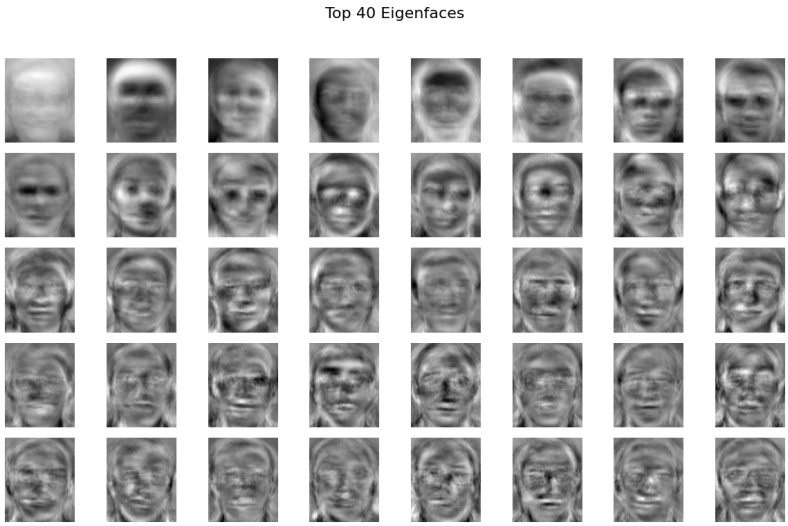


Figure 4: Top 40 eigenfaces

(2) The figure is as follows:

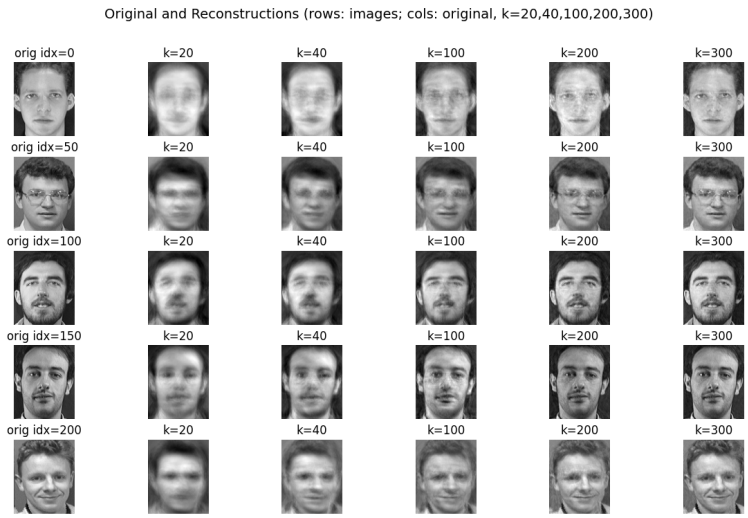


Figure 5: Corresponding reconstructed images with k

As k increases, the reconstruction details significantly increase (facial contours and expression details gradually recover), and if k is too small, important structures will be lost and blurred; Excessive k can restore noise and individual differences, tending towards overfitting the training set.

(3) The figure is as follows:

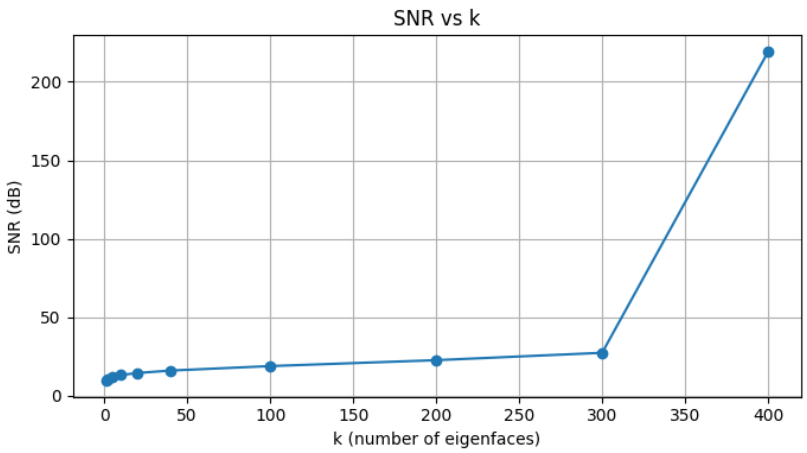
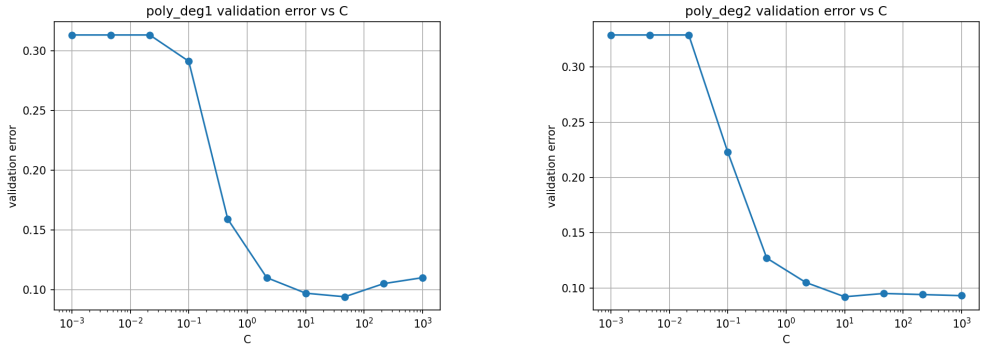


Figure 6: SNR under different choices of k

Problem 3.

(a) The figures are as follows:



(a) Polynomial degree = 1 (b) Polynomial degree = 2

Figure 7: Validation error vs C for different polynomial degrees

poly_deg1 best C = 46.4159, val_err = 0.094

poly_deg1 test error (retrained on whole training set) = 0.0441

poly_deg2 best C = 10, val_err = 0.092

poly_deg2 test error (retrained on whole training set) = 0.0355

(b) The figure is as follows:

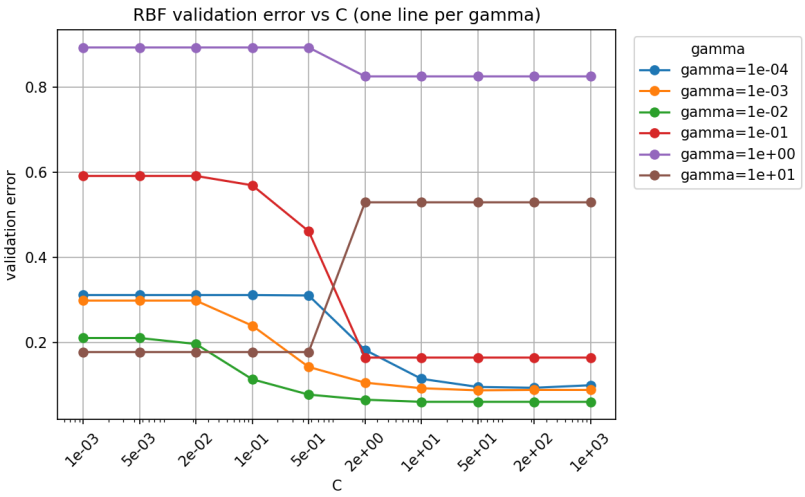


Figure 8: Validation error vs gamma and C for RBF kernel

RBF best gamma=0.01, best C=10

RBF test error (retrained on whole training set) = 0.0005