Homework 1

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Problem 1.

(a)

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Problem 2.

(a)

$$\nabla f(x) = \begin{bmatrix} 4x_1^3 + 6x_1^2 - 4x_1 x_2 \\ -2x_1^2 + 8x_2 \end{bmatrix}$$

$$\text{Let } \nabla f(x) = \mathbf{0} \quad \Rightarrow \begin{cases} x_1 = 0 \\ x_2 = 0 \end{cases} \text{or} \begin{cases} x_1 = -2 \\ x_2 = 1 \end{cases}$$

$$\text{While } \nabla^2 f(x) = \begin{bmatrix} 12x_1^2 + 12x_1 - 4x_2 & -4x_1 \\ -4x_1 & 8 \end{bmatrix}$$

$$\nabla^2 f(0,0) = \begin{bmatrix} 0 & 0 \\ 0 & 8 \end{bmatrix}, \quad \nabla^2 f(-2,1) = \begin{bmatrix} 20 & 8 \\ 8 & 8 \end{bmatrix} \text{(PD)}$$

Thus, $(0,0)$ is a saddle point, while $(-2,1)$ is a local minimum.

Problem 3.

(a) Using the Leading Principle Minors method:

A_1 is indefinite;

A_2 is positive semidefinite;

A_3 is positive definite;

A_4 is indefinite.

Calculating the eigenvalues of A_3 and A_4:

$$|A_3 - \lambda I| = \begin{vmatrix} 1-\lambda & 0 & 1 \\ 0 & 1-\lambda & -1 \\ 0 & 2 & 4-\lambda \end{vmatrix} = (1-\lambda)(2-\lambda)(3-\lambda) = 0$$

$$\Rightarrow \lambda_1 = 1, \lambda_2 = 2, \lambda_3 = 3$$

$$|A_4 - \lambda I| = \begin{vmatrix} 1-\lambda & 0 & 1 \\ 0 & -1-\lambda & -1 \\ -1 & 1 & -\lambda \end{vmatrix} = -\lambda(\lambda^2+1) = 0$$

$$\Rightarrow \lambda_1 = 0, \lambda_2 = i, \lambda_3 = -i$$