DDA 5002 Optimization Fall 2025
Homework # 1
Due: Thursday, September 18 at 11:59pm

Please note: Problem 2 and 4 asks you to solve a model using computer programming (Python with COPT recommended). Put the source code files you create and the solution output in a single zip file and submit it via Blackboard. Please name your file `Lastname_StudentID.zip`. Also, give meaningful names to your decision variables and constraints, and add comments to your code liberally. Both the problem set and the code are due at midnight (11:59pm) on Thursday, September 18th.

1. A company produces two types of products. A product of the first type requires 1/4 hours of assembly labor, 1/8 hours of testing, and \$1.2 worth of raw materials. A product of the second type requires 1/3 hours of assembly, 1/3 hours of testing, and \$0.9 worth of raw materials. Given the current personnel of the company, there can be at most 90 hours of assembly labor and 80 hours of testing each day. Products of the first and second type have a market value of \$9 and \$8, respectively.

(a) Formulate a linear program that can be used to maximize the daily profit of the company.

(b) Consider the following two modifications to the original problem:

(i) Suppose that up to 50 hours of overtime assembly labor can be scheduled, at a cost of \$7 per hour.

(ii) Suppose that the raw material supplier provides a 10% discount if the daily bill is above \$300.

Which of the above two elements can be easily incorporated into the linear program formulation and how? If one or both are not easy to incorporate, indicate how you might nevertheless solve the problem.

2. During a recent solar car race, you gathered the following data on daily power consumption and total mileage traveled.

Power consumed	10	8	13	15	9
Distance	60	55	75	80	64

Table 1: Solar car race data.

There is believed to be a linear relationship between power and distance, that is, of the form:

$$\text{distance} = m(\text{power}) + b,$$

for some unknown values of m and b, along with some random error term. In a statistics class, for N data points, you would generate a least-squares regression line that minimizes the sum of squared difference between the observed distances and the predicted distances, based on the linear model, that is,

$$\min \sum_{k=1}^{N} (\text{distance}_k - (m(\text{power}_k) + b))^2 .$$

Suppose instead you want to minimize the sum of the absolute deviations between the observed distances and predicted distances, that is,

$$\min \sum_{k=1}^{N} |\text{distance}_k - (m(\text{power}_k) + b)|.$$

(a) Formulate a linear program to obtain the best fit parameters m and b. Note that both m and b should be nonnegative. Clearly define the index sets, data, decision variables, and constraints.

(b) For the given data in Table 1, what values of m and b will give the best fit? Solve the optimization model using computer programming. Report your solution in a fashion that can be understood by people who do not have an optimization background (i.e., do not just point to the code output).

3. A company has two manufacturing plants (A and B) and three sales outlets (I, II, and III). Shipping costs from the plants to the outlets are in \$/unit and are as follows:

	Outlet		
Plant	I	II	III
A	4	6	8
B	7	4	3

Table 2: Shipping costs from the plants to the outlets in \$/unit

The company wants to plan production, shipping, and sales for the next two periods. The manufacturing and demand data for the two periods are shown in the following tables (Table 3 and Table 4). The plants can store products produced in one period for sale in the next. The maximum storage at each plant is 50 units and the inventory cost is \$1 per unit.

	Manufacturing Data			
	Plant A		Plant B	
Period	Unit cost (\$/unit)	Capacity	Unit cost (\$/unit)	Capacity
1	8	175	7	200
2	10	150	8	170

Table 3: Manufacturing cost and capacity for Plant A and B in two time periods

	Demand Data					
	Selling price ($/unit)			Maximum Sales		
Period	I	II	III	I	II	III
1	15	20	14	100	200	150
2	18	17	21	150	300	150

Table 4: Selling price and maximum sales at each outlet for period 1 and 2

Formulate a **data-independent** linear program to maximize the total profit of the company with all production, sales, shipment and storage constraints satisfied. State your index sets, data, decision variables and constraints clearly. You **do not** need to solve the linear program—just provide the formulations.

4. A company has one manufacturing plant and three sales outlets (A, B and C). Unit shipping costs from the plant to outlets A,B, and C are \$4, \$6, and \$8, respectively. The company wishes to develop a production, shipping, and sales plan for three periods. The corresponding data are as follows.

Manufacturing Data			Selling Price			Maximum Sales		
Unit Period	Production Cost ($)	Capacity (units)	Outlet A ($)	Outlet B ($)	Outlet C ($)	Outlet A	Outlet B	Outlet C
1	8	180	15	20	13	50	100	75
2	10	200	19	16	21	75	150	75
3	11	150	15	18	18	20	80	50

The plant has the capability of storing up to 100 units of product from one period to the next. The storage cost is \$1 per unit per period.

(a) Draw the network diagram using "soft" demands.

(b) Formulate a minimum cost network flow (MCNF) model as a **data-independent** linear program. Be sure to define your parameters and decision variables clearly.

(c) Implement and solve your model in computer programming. Report the solution "by hand" in a format that the decision maker, who knows nothing of linear program, will understand.

5. An airline company must schedule its reservation salesclerks around the clock to have at least r_t on duty during each 1-hour period starting at (24-hour) clock hour $t = 0, \ldots, 23$, where the index t denotes an hour period rather than a specific point in time. A shift beginning at hour t extends for 9 consecutive hours with 1 hour out for meal in the fourth, fifth, or sixth hours of the shift. Shifts beginning at hour t cost the company c_t per day.
Formulate an LP model to compute a minimum total cost daily shift schedule.
Hint: Use the following decision variables:

$x_t \triangleq$ number of clerks working a shift starting at period t,

$y_{t,i} \triangleq$ number of clerks working a shift starting at period t who take lunch during period i.