

For simplicity, let $n = q - 1$.

Observe that, when $k \neq 0$,

$$\sum_{i=0}^{n-1} \alpha^{ik} = 0, \forall \alpha \in \mathbb{Z}_q^*,$$

and when $k = 0$, obviously

$$\sum_{i=0}^{n-1} \alpha^{ik} = \sum_{i=0}^{n-1} \alpha^0 = n, \forall \alpha \in \mathbb{Z}_q^*.$$

Therefore we have

$$\begin{aligned} f(i) &= \frac{1}{n} \sum_{k,j} f(j) \alpha^{kj-ki} \\ &= \frac{1}{n} \sum_{k,j} f(j) \alpha^{kj} \alpha^{-ik} \\ &= \frac{1}{n} \sum_k F(k) \alpha^{-ik}, \end{aligned}$$

where $F(k) \stackrel{\text{def}}{=} \sum_j f(j) \alpha^{kj}$.