For simplicity, let $n = q - 1$.
Observe that, when $k \neq 0$,

$$\sum_{i=0}^{n-1} \alpha^{ik} = 0, \forall \alpha \in \mathbb{Z}_q^*,$$

and when $k = 0$, obviously

$$\sum_{i=0}^{n-1} \alpha^{ik} = \sum_{i=0}^{n-1} \alpha^0 = n, \forall \alpha \in \mathbb{Z}_q^*.$$

$$
\begin{aligned}
f(i) &= \frac{1}{n} \sum_{k,j} f(j) \alpha^{kj-ki} \\
&= \frac{1}{n} \sum_{k,j} f(j) \alpha^{kj} \alpha^{-ik} \\
&= \frac{1}{n} \sum_{k} F(k) \alpha^{-ik}
\end{aligned}
$$