Report of Assignment 4 Problem 1

Zhihan Li

1600010653

April 28, 2018

Question 1

Proof The setting is shown in Figure 1. From reflection law, $\mathbf{r}$, $\mathbf{n}$ and $\mathbf{l}$ lie in the same plane and $\angle(\mathbf{r}, \mathbf{n}) = \angle(\mathbf{n}, \mathbf{l})$. From the definition of $\mathbf{h}$, we know that $\mathbf{v}$, $\mathbf{h}$ and $\mathbf{l}$ lie in the same plane and $\angle(\mathbf{v}, \mathbf{h}) = \angle(\mathbf{h}, \mathbf{l})$.

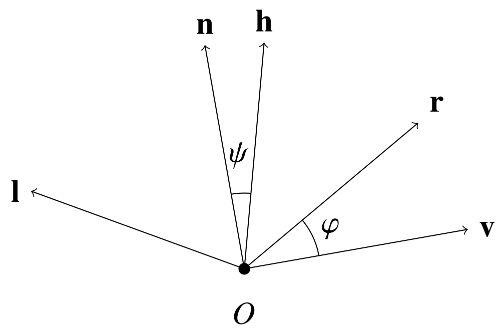

Figure 1 The angles ϕ and φ

If $\mathbf{v}$ lies in the plane of $\mathbf{l}$, $\mathbf{n}$ and $\mathbf{r}$, then all the five vectors lie in the same plane. Therefore, we have

$$\begin{aligned} \psi &= \angle(\mathbf{h}, \mathbf{n}) = \angle(\mathbf{h}, \mathbf{l}) - \angle(\mathbf{n}, \mathbf{l}) \\ &= \frac{1}{2}\left(\angle(\mathbf{v}, \mathbf{l}) - \angle(\mathbf{r}, \mathbf{l})\right) \\ &= \frac{1}{2}\angle(\mathbf{v}, \mathbf{r}) = \frac{1}{2}\varphi \end{aligned} \tag{1}$$

and equivalently $2\psi = \varphi$ as desired.

$\square$