1 Problem Statement

1.1 Problem description

The design problem involves a brake system consisting of a brake disc and brake pads. Our goal is to select optimal component dimensions that fulfill structural, modal and thermal performance specifications.

This is a design optimization problem where FEA is performed iteratively to search the feasible solution space for the optimal solution. Most commonly, commercial FEA packages do not provide gradient information for performing optimization using gradient-based algorithms. Therefore, gradient-free method such as response surface optimization techniques are used. In this approach, the solution space is first sampled and FEA is performed for these data points. This data is used to train a response surface which is then incrementally refined to produce a statistical model for predicting performance characteristics for any design points within the solution space.

The exercise is performed using ANSYS response surface and optimization modules.

1.2 Objective

In this brake system design, the geometry of the brake pads is invariant and the optimization is applied strictly to the brake disc.

Our goal is to:

1. Minimize the volume of the brake disc for optimal material usage and weight
2. Minimize the peak stress on the disc for safe operational design
3. Maximize the modal response to avoid resonance with the other components in a vehicle
4. Minimize the temperature to reduce thermal stresses and expansion

2 Finite Element Analysis

The region of interest is located on the flange of the brake disc since the various loads act directly on the flange surfaces and its geometry primarily affects the output parameters. Therefore, the design variables for this exercise are chosen to be the flange thickness (P1), flange outer diameter (P2) and flange inner diameter (P3) of the brake disc.

Design geometry and material specifications are uniform across the three analyses.

2.1 Structural analysis

The brake disc experiences stresses due to the applied pressure and frictional contact from the brake pads. Furthermore, it experiences dynamic forces due to the body forces and rotational effects during operation. The structural analysis is setup to represent this scenario. An optimal design should withstand the stresses produced in this mode of operation.

The preprocessing steps begin by assigning material to brake disc and pads. Gray cast iron is chosen for the disc and Structural steel is set for the pads. Boundary conditions for the analysis are:

1. Rotational speed of the disc is 250 rad/s about the rotational axis of the disc

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2. Actuation pressure on the brake pads 10.495 MPa
3. Frictional contact between the disc and the pads, coefficient of friction is 0.22
4. Confine the lateral movement of the pads along direction normal to the flange
5. Fix the disc with respect to the Ground via rotational joint

Desired output variables:

1. Maximum stress (P4): The stress parameter is chosen as Maximum Principal stress over Von Mises stress. This is because the brake disc is made of Gray cast iron which is a brittle material. Von Mises criteria applies to ductile failure while brittle failure is modelled using Max. Principal stress theory.

2. Volume (P7): Volume probe option is used to track the volume during optimization

Global mesh element type is chosen as tetrahedrons and the size is set to 6mm. At the contact interface between to two bodies, the element size is set to 3mm for better FEA results.

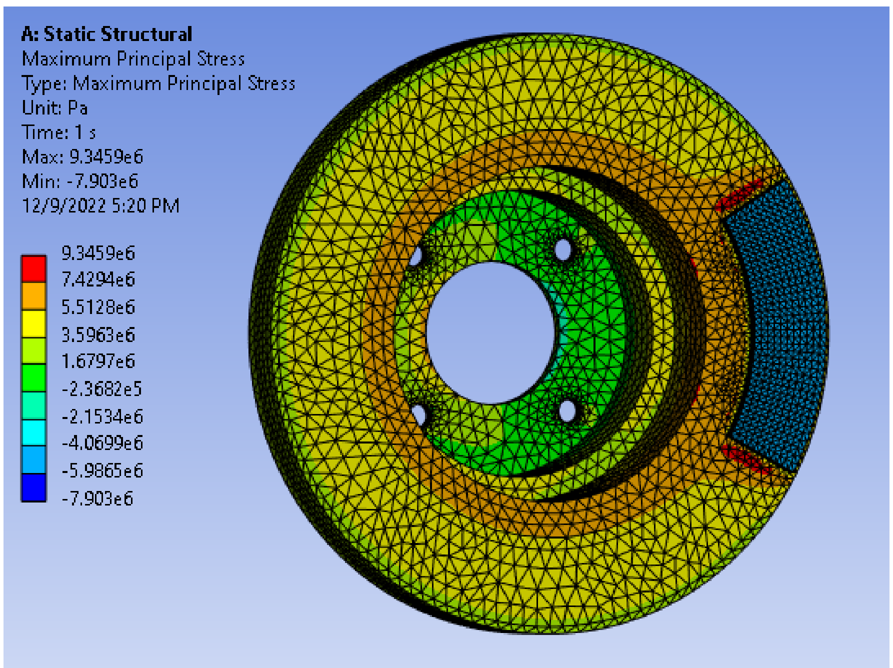

Figure 1 Static structural analysis

2.2 Modal analysis

This analysis focuses on tracking the frequency response of the brake disc to mitigate failure due to resonance with the rest of the system.

Here, we are interested in the behavior on the disc alone, so the brake pads are suppressed. The analysis is setup to find the first natural frequency of the disc (7th mode).

Desired output variables:

Frequency (P8): This the frequency due to deformation of the disc.

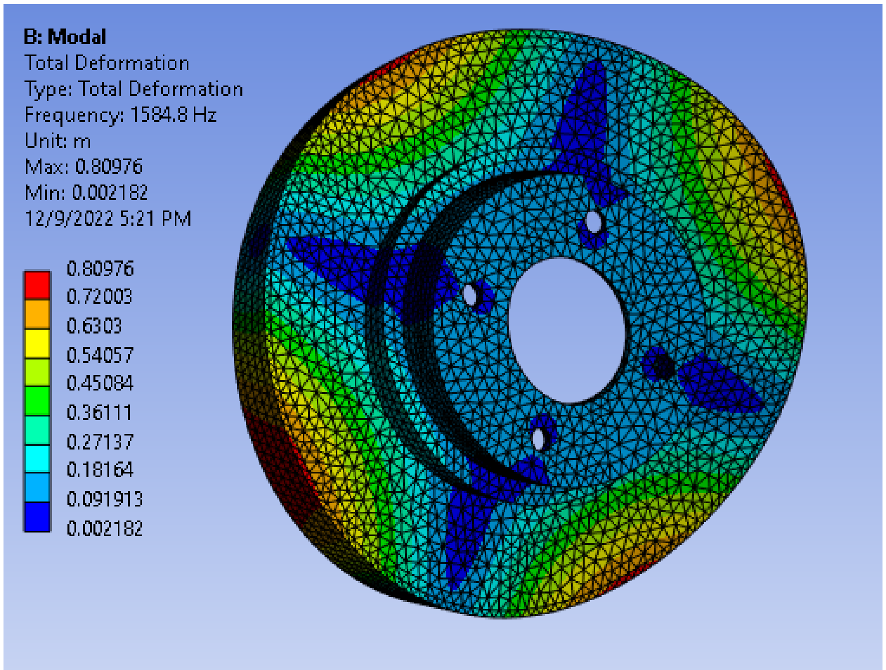

Figure 2 Modal analysis

2.3 Thermal analysis

During braking operation, a great deal of heat is generated due to friction. The disc should operate safety at elevated temperatures without yielding or cracking due to thermal expansion or stresses.

Once again, we are interested in the behavior on the disc and the brake pads are suppressed.

Boundary conditions for the analysis are:

1. Convective heat transfer is established over all the disc surfaces, where ambient temperate is 35 °C and convective coefficient is 5 $W/m^2°C$
2. Heat flux of 1.5395e+6 W/m^2 is input into the disc over the annular contact area with the brake pads

Desired output variables:

Temperature (P9): Maximum temperature on the disc.

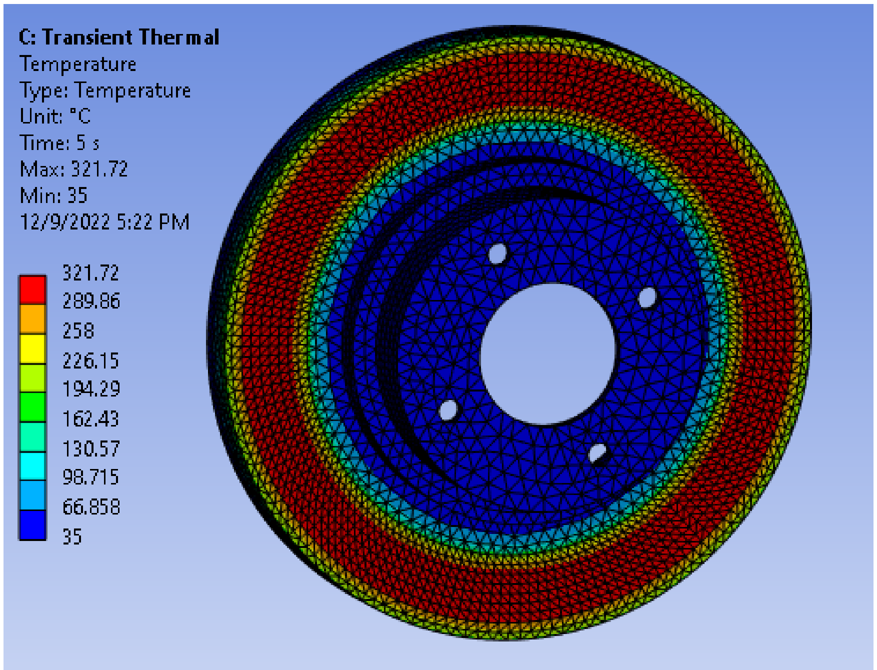

Figure 3 Thermal analysis

An initial FEA is performed for all three models at flange thickness (P1) = 25mm, flange outer diameter (P2) = 125mm and flange inner diameter (P3) = 75mm.

3 Design Optimization

To optimize the design variables P1, P2, P3 first we sample the solution space using a suitable DOE method. Then a response surface is created which upon refinement can statistically predict an optimal response for a given set of objectives and constraints.

3.1 Design of experiments

To model a response surface adequately, a large number of datapoints is necessary. However, this also increases the computational effort since FEA is performed for sample point. In ANSYS, Optimal Space-Filling and Latin Hypercube Sampling allow the user to choose the number of sample points.

In this instance, Optimal Space-Filling Sampling is performed for 50 datapoints.

Design of Experiments	
Design of Experiments Type	Optimal Space-Filling Design
Design Type	Maximum Entropy
Maximum Number Of Cycles	10
Samples Type	User-Defined Samples
Random Generator Seed	9
Number of Samples	50

Figure 4 DOE parameters

The design variables are continuous and bounded as follows:

Table of Schematic D4: Optimization				
	A	B	C	D
1	Input Parameters			
2	Name	Lower Bound	Upper Bound	
3	P1 - rotor_thickness (mm)	20	30	
4	P2 - rotor_OD (mm)	124	145	
5	P3 - rotor_ID (mm)	70	88	

Figure 5 Constraints on design variables

3.2 Response surface

The simulation results from the DOE are used to build a response surface. ANSYS provides a wide range of response surface options. Options like Kriging, Non-parametric Regression, Neural Network produce a very good fit to the data, however they produce highly non-linear surfaces with multiple local minima.

Other two methods, Genetic Aggregation and Standard Response Surface (Second order polynomial) produce smoother response surfaces. Among the two, Genetic aggregation resulted in better fit to the training data (DOE points), hence this method is selected. Here, 10 verification points (testing data) are generated to test goodness of fit.

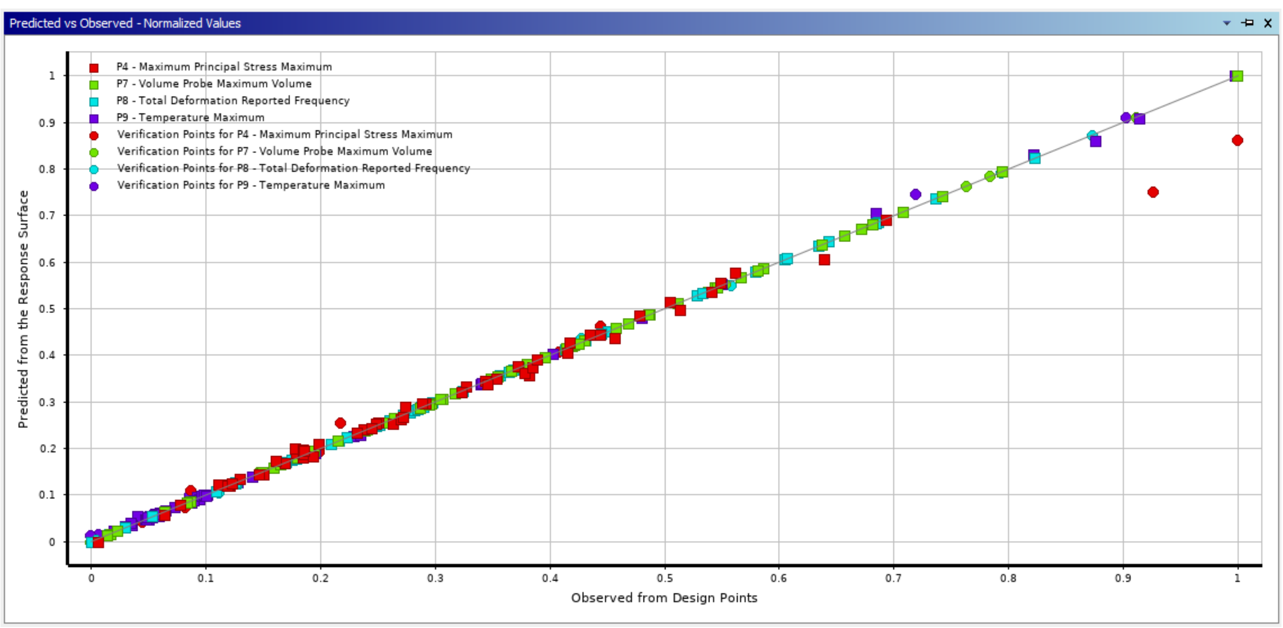

Figure 6 Goodness of fit

Table of Schematic D3: Response Surface

	A	B	C	D	E
1		P4 - Maximum Principal Stress Maximum	P7 - Volume Probe Maximum Volume	P8 - Total Deformation Reported Frequency	P9 - Temperature Maximum
2	Coefficient of Determination (Best Value = 1)				
3	Learning Points	0.99556	1	1	0.99958
4	Cross-Validation on Learning Points	0.92405	1	0.99991	0.9993
5	Root Mean Square Error (Best Value = 0)				
6	Learning Points	1.8952E+05	1.7787E-09	0.025583	0.022742
7	Verification Points	1.3371E+06	2.3261E-08	3.0364	0.05288
8	Cross-Validation on Learning Points	7.8348E+05	9.759E-09	1.3659	0.029572
9	Relative Maximum Absolute Error (Best Value = 0%)				
10	Learning Points	21.528	0	0.055953	8.6431
11	Verification Points	90.899	0.020361	4.108	10.937
12	Cross-Validation on Learning Points	98.161	0.023584	3.2792	11.28
13	Relative Average Absolute Error (Best Value = 0%)				
14	Learning Points	5.0862	0	0.012775	1.2704
15	Verification Points	21.419	0	1.4089	3.0825
16	Cross-Validation on Learning Points	21.554	0.0032625	0.6483	1.5551

Figure 7 Response surface metrics

As can be seen, the original datapoints as well as the verification points lie on the fit trendline except for a few outliers. Some of the response surfaces are shown below:

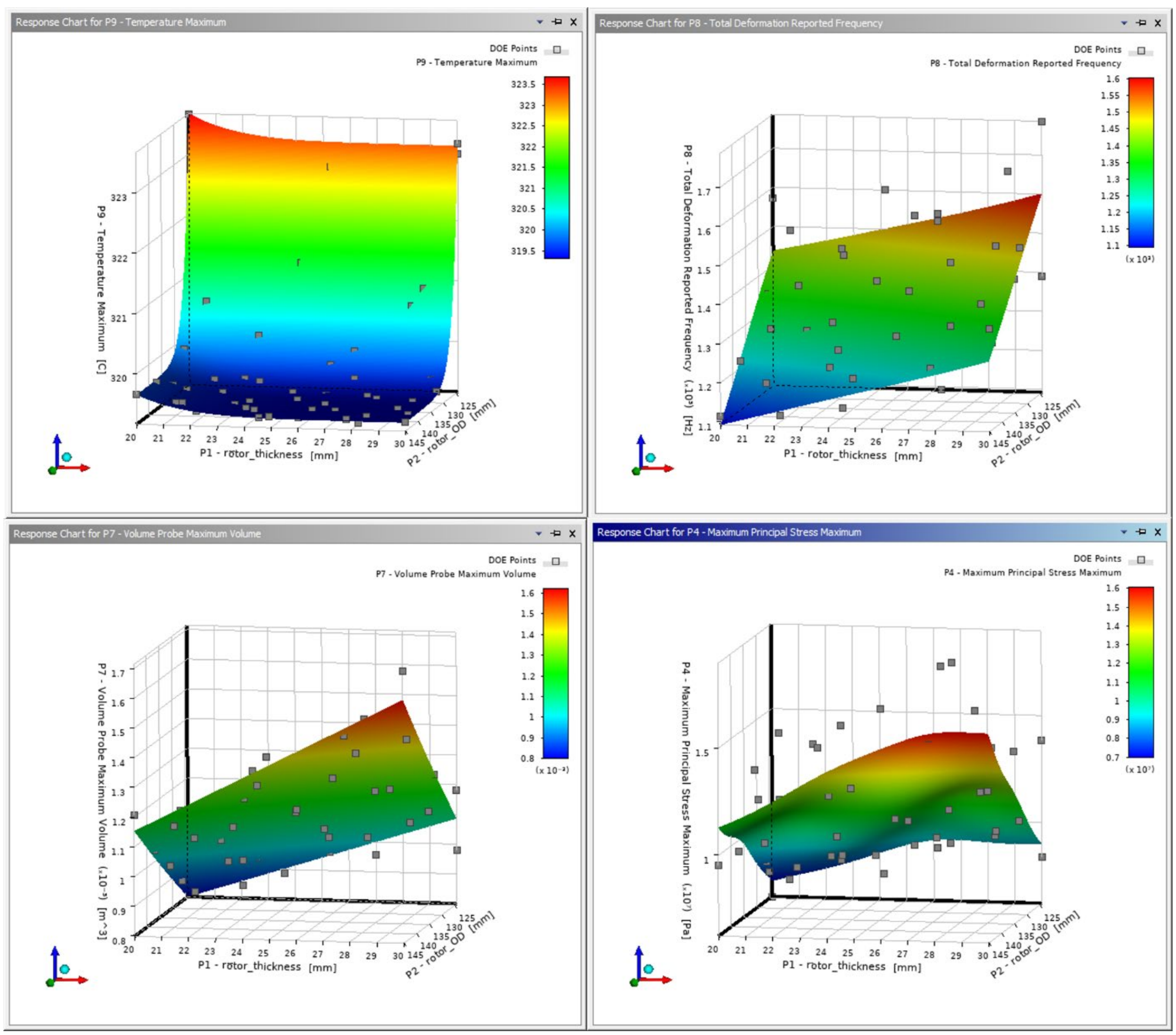

Figure 8 Response surfaces (Clockwise from top right: Temperature, Frequency, Volume, Stress)

There is some error in the stress value prediction, however this surface is good enough to move onto to the optimization phase.

3.3 Optimization

To setup the optimization solver, the objective function and constraints are formulated as below:

Table of Schematic D4: Optimization

	A	B	C	D	E	F	G	H	I
1	Name	Parameter	Objective			Constraint			
2			Type	Target	Tolerance	Type	Lower Bound	Upper Bound	Tolerance
3	Minimize P4; P4 <= 1.4E+07 Pa	P4 - Maximum Principal Stress Maximum	Minimize	0		Values <= Upper Bound		1.4E+07	1
4	Minimize P7	P7 - Volume Probe Maximum Volume	Minimize	0		No Constraint			
5	Maximize P8; P8 >= 1200 Hz	P8 - Total Deformation Reported Frequency	Maximize	5000		Values >= Lower Bound	1200		0.1
6	Minimize P9; P9 <= 400 C	P9 - Temperature Maximum	Minimize	0		Values <= Upper Bound		400	0.01
*		Select a Parameter							

Figure 9 Optimization objectives

Optimization solver is chosen as Multi-Objective Genetic Algorithm (MOGA). ANSYS defaults to two choices, MOGA and screening for this problem. Since Screening method is random in nature, MOGA is preferred.

Optimization	
Method Selection	Manual
Method Name	MOGA
Estimated Number of Evaluations	14400
Tolerance Settings	☑
Verify Candidate Points	☑
Number of Initial Samples	3000
Number of Samples Per Iteration	600
Maximum Allowable Pareto Percentage	70
Convergence Stability Percentage	2
Maximum Number of Iterations	20
Maximum Number of Candidates	3

Figure 10 Optimizer parameters

Three candidate points are verified.

Candidate Points

	Candidate Point 1	Candidate Point 1 (verified)	Candidate Point 2	Candidate Point 2 (verified)	Candidate Point 3	Candidate Point 3 (verified)
P1 - rotor_thickness (mm)	20		20.002		20.005	
P2 - rotor_OD (mm)	124		124		124	
P3 - rotor_ID (mm)	71.734		71.311		71.111	
P4 - Maximum Principal Stress Maximum (Pa)	★★ 6.1011E+06	★★ 6.212E+06	★★ 6.0924E+06	★★ 5.9708E+06	★★ 6.0913E+06	★★ 6.2788E+06
P7 - Volume Probe Maximum Volume (m^3)	0.00084298	0.00084295	0.00084544	0.00084542	0.00084666	0.00084664
P8 - Total Deformation Reported Frequency (Hz)	1557.7	1563	1561.5	1566.2	1563.3	1567.1
P9 - Temperature Maximum (C)	★ 323.68	★ 323.68	★ 323.67	★ 323.66	★ 323.67	★ 323.66

Figure 11 Candidate optimal points

The three candidates selected by the optimization routine are almost identical. The deviation in the output parameter values between the three points is also low. To further improve the response surface, these points are added to the refinement points set of the response surface. Both the response surface and the optimization routine are executed once again.

Candidate Points	Candidate Point 1	Candidate Point 1 (verified)	Candidate Point 2	Candidate Point 2 (verified)	Candidate Point 3	Candidate Point 3 (verified)
P1 - rotor_thickness (mm)	20.001		20.003		20.002	
P2 - rotor_OD (mm)	124		124		124.03	
P3 - rotor_ID (mm)	72.73		72.433		72.247	
P4 - Maximum Principal Stress Maximum (Pa)	★★ 6.1463E+06	★★ 6.1887E+06	★★ 6.145E+06	★★ 6.2884E+06	★★ 6.1503E+06	★★ 6.1474E+06
P7 - Volume Probe Maximum Volume (m^3)	0.00083733	0.00083732	0.00083908	0.00083907	0.00084057	0.00084056
P8 - Total Deformation Reported Frequency (Hz)	1550.1	1552.4	1554.3	1556.1	1556	1558
P9 - Temperature Maximum (C)	★ 323.66	★ 323.66	★ 323.66	★ 323.66	★ 323.61	★ 323.6

Figure 12 Candidate points after refinement

After refinement there is an improvement in predictions with respect to the FEA values. Once again, candidate points and their outputs are very similar. Candidate point 1 is selected as the optimal solution, since it has the lowest volume which is the primary objective. Though any of these points would be a decent solution.

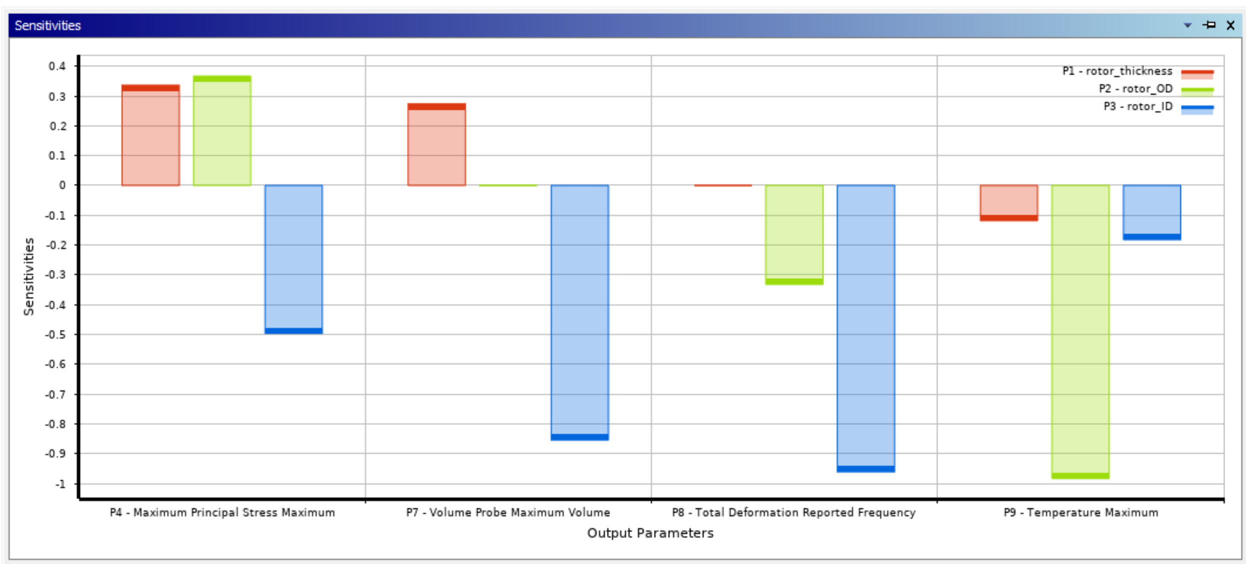

Figure 13 Sensitivity analysis

In addition to the candidate points, optimization method provides trade-off and sensitivity analysis for all combinations of the parameters. As can be seen here, the inner diameter of the disc has more affect overall on the output parameters. Insights such as this can help supplement design process.

4 Discussion

The table below compares the design parameters before and after optimization. The optimal solution manages to bring down the volume of the disc by nearly 16% and Max. stress value is reduced by 33.8%. However, no improvements in temperature and frequency are seen, in fact frequency and temperature reduce by 2% and 0.6% respectively. The primary objective of minimizing the volume is achieved and the negative impact on temperature and frequency is lower, therefore this trade-off can be accepted from this optimal solution.

Parameters	Initial	Optimal	Percentage change (Absolute)
Thickness P1, (mm)	25	20.001	19.996
Outer diameter P2, (mm)	125	124	0.8
Inner diameter P3, (mm)	75	72.73	3.026666667
Maximum Principal Stress P4, (MPa)	9.3459	6.1887	33.78165827
Volume P7, (m^3)	9.97E-04	8.37E-04	15.98824084
Frequency P8, (Hz)	1584.8	1552.4	2.044422009
Temperature P9, (°C)	321.72	323.66	0.603008828

Table 1 Optimization results

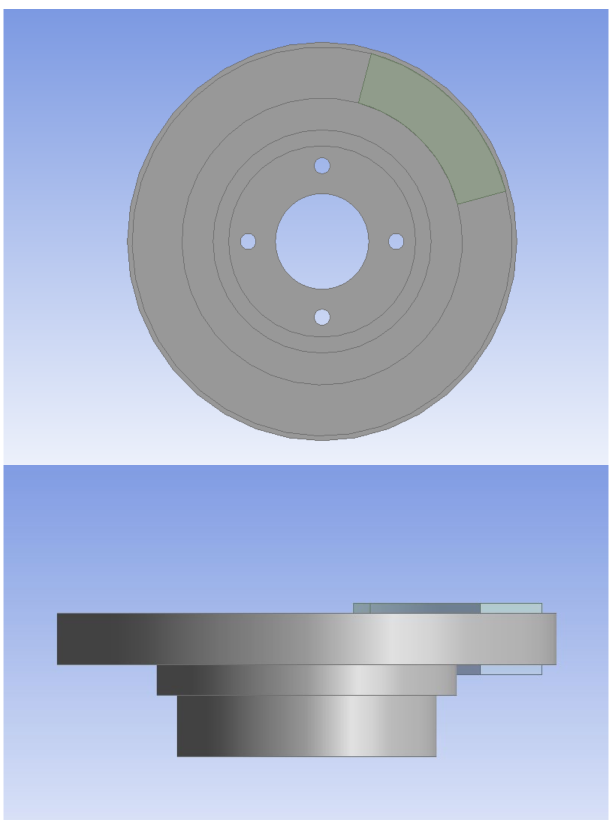

Figure 15 Initial Geometry

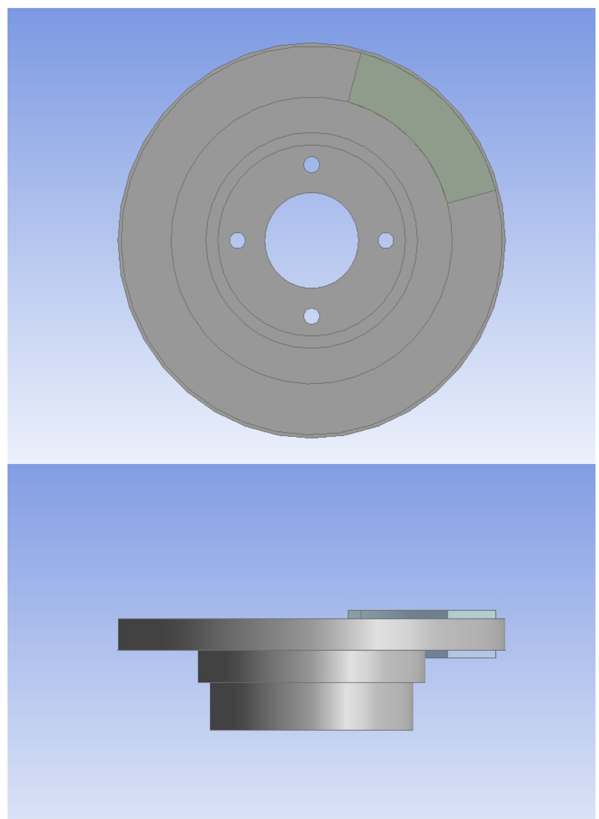

Figure 15 Final geometry

The optimization exercise yields an optimal solution and a response surface which be used to statistically predict output parameters in unexplored design domain. This together with the sensitivity and tradeoff data can help inform future design modifications.

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