

MATHEMATICS FORMULAE

Trigonometric functions

1. Length of an arc = $r\theta$

2. Area of sector = $\frac{1}{2}r^2\theta$

Trigonometric functions of standard angle

| Function | 0° | 30° | 45° | 60° | 90° |
|----------|-----------|----------------------|----------------------|----------------------|------------|
| sin | 0 | $\frac{1}{2}$ | $\frac{1}{\sqrt{2}}$ | $\frac{\sqrt{3}}{2}$ | 1 |
| cos | 1 | $\frac{\sqrt{3}}{2}$ | $\frac{1}{\sqrt{2}}$ | $\frac{1}{2}$ | 0 |
| tan | 0 | $\frac{1}{\sqrt{3}}$ | 1 | $\sqrt{3}$ | ∞ |
| cosec | ∞ | 2 | $\sqrt{2}$ | $\frac{2}{\sqrt{3}}$ | 1 |
| sec | 1 | $\frac{2}{\sqrt{3}}$ | $\sqrt{2}$ | 2 | ∞ |
| cot | ∞ | $\sqrt{3}$ | 1 | $\frac{1}{\sqrt{3}}$ | 0 |

Trigonometric functions of sum and difference

1. $\sin(A + B) = \sin A \cos B + \cos A \sin B$

2. $\sin(A - B) = \sin A \cos B - \cos A \sin B$

3. $\cos(A + B) = \cos A \cos B - \sin A \sin B$

4. $\cos(A - B) = \cos A \cos B + \sin A \sin B$

5. $\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

6. $\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$

Trigonometric functions of 2θ

1. $\sin 2\theta = 2 \sin \theta \cos \theta$

2. $\cos 2\theta = 2\cos^2 \theta - 1$

3. $\tan 2\theta = \frac{2\tan \theta}{1 - \tan^2 \theta}$

4. $\cos 2\theta = \frac{1 - \tan^2 \theta}{1 + \tan^2 \theta}$

5. $\cos 2\theta = \cos^2 \theta - \sin^2 \theta$

6. $\cos 2\theta = 1 - 2\sin^2 \theta$

7. $\sin 2\theta = \frac{2\tan \theta}{1 + \tan^2 \theta}$

Trigonometric functions of half angles

1. $\sin \theta = 2 \sin \frac{\theta}{2} \cos \frac{\theta}{2}$

2. $\cos \theta = 2\cos^2 \frac{\theta}{2} - 1$

3. $\tan \theta = \frac{2\tan \frac{\theta}{2}}{1 - \tan^2 \frac{\theta}{2}}$

4. $\cos \theta = \frac{1 - \tan^2 \frac{\theta}{2}}{1 + \tan^2 \frac{\theta}{2}}$

5. $\cos \theta = \cos^2 \frac{\theta}{2} - \sin^2 \frac{\theta}{2}$

6. $\cos \theta = 1 - 2\sin^2 \frac{\theta}{2}$

7. $1 + \cos \theta = 2\cos^2 \frac{\theta}{2}$

8. $\sin \theta = \frac{2\tan \frac{\theta}{2}}{1 + \tan^2 \frac{\theta}{2}}$

Trigonometric functions of 3θ

1. $\sin 3\theta = 3\sin \theta - 4\sin^3 \theta$

2. $\cos 3\theta = 4\cos^3 \theta - 3\cos \theta$

3. $\tan 3\theta = \frac{3\tan \theta - \tan^3 \theta}{1 - 3\tan^2 \theta}$

Factorization Formulae

1. $\sin C + \sin D$
 $= 2 \sin \left(\frac{C + D}{2} \right) \cos \left(\frac{C - D}{2} \right)$

2. $\sin C - \sin D$
 $= 2 \cos \left(\frac{C + D}{2} \right) \sin \left(\frac{C - D}{2} \right)$

$$3. \cos C + \cos D$$

$$= 2 \cos\left(\frac{C+D}{2}\right) \cos\left(\frac{C-D}{2}\right)$$

$$4. \cos C - \cos D = -2 \sin\left(\frac{C+D}{2}\right) \sin\left(\frac{C-D}{2}\right)$$

Defactorization Formulae

$$2 \sin A \cos B = \sin(A+B) + \sin(A-B)$$

$$2 \cos A \sin B = \sin(A+B) - \sin(A-B)$$

$$2 \cos A \cos B = \cos(A+B) + \cos(A-B)$$

$$2 \sin A \sin B = \cos(A-B) - \cos(A+B)$$

Trigonometric functions of Angles of Triangle

$$1. \sin(A+B) = \sin C$$

$$2. \sin\left(\frac{A+B}{2}\right) = \cos\left(\frac{C}{2}\right)$$

$$3. \cos\left(\frac{A+B}{2}\right) = \sin\left(\frac{C}{2}\right)$$

Determinants

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2c_3 - b_3c_2) - a_2(b_1c_3 - b_3c_1) + a_3(b_1c_2 - b_2c_1)$$

Minor & Cofactor of 3×3 Determinants

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

Minor Cofactor

$$M_{11} = \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} \quad A_{11} = \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix}$$

$$M_{12} = \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} \quad A_{12} = (-) \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix}$$

$$M_{13} = \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix} \quad A_{13} = \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

$$M_{21} = \begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix} \quad A_{21} = (-) \begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix}$$

$$M_{22} = \begin{vmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{vmatrix} \quad A_{22} = \begin{vmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{vmatrix}$$

$$M_{23} = \begin{vmatrix} a_{11} & a_{12} \\ a_{31} & a_{32} \end{vmatrix} \quad A_{23} = (-) \begin{vmatrix} a_{11} & a_{12} \\ a_{31} & a_{32} \end{vmatrix}$$

$$M_{31} = \begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix} \quad A_{31} = \begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix}$$

$$M_{32} = \begin{vmatrix} a_{11} & a_{13} \\ a_{21} & a_{23} \end{vmatrix} \quad A_{32} = (-) \begin{vmatrix} a_{11} & a_{13} \\ a_{21} & a_{23} \end{vmatrix}$$

$$M_{33} = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} \quad A_{33} = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$$

Matrices

Multiplication of Two Matrices

A product AB exists if number of columns of A is equal to number of rows of B.

$$\text{If } A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \& B = \begin{pmatrix} b_{11} \\ b_{21} \end{pmatrix}$$

then product AB exists & is equal to

$$AB = \begin{pmatrix} a_{11} \cdot b_{11} + a_{12} \cdot b_{21} \\ a_{21} \cdot b_{11} + a_{22} \cdot b_{21} \end{pmatrix}$$

Logarithms

Law of Logarithms:

Law of product:

$$\log_{10} ab = \log_{10} a + \log_{10} b$$

Law of Exponent:

$$\log_{10} a^m = m \log_{10} a$$

Change of Base Law:

$$\log_b a = \frac{\log_e a}{\log_e b}$$

Complex numbers

Conjugate of Complex numbers:

If $z = a + ib$ then $\bar{z} = a - ib$

Modulus of Complex numbers:

$$|z| = \sqrt{a^2 + b^2}$$

Argument of Complex numbers:

$$\theta = \tan^{-1} \left(\frac{b}{a} \right) \quad \text{where}$$

$$0 \leq \theta \leq 2\pi$$

Division of Complex numbers:

$$\text{If } z_1 = a_1 + i b_1 \text{ \& } z_2 = a_2 + i b_2$$

$$\begin{aligned} \text{Then, } \frac{z_1}{z_2} &= \frac{a_1 + i b_1}{a_2 + i b_2} \\ &= \frac{a_1 + i b_1}{a_2 + i b_2} \times \frac{a_2 - i b_2}{a_2 - i b_2} \end{aligned}$$

Binomial Theorem

$$(x + a)^n = \sum_{k=0}^n \binom{n}{k} x^k a^{n-k}$$

Mathematical Logic

Truth Table for 'And' statement

| p | q | $p \wedge q$ |
|-----|-----|--------------|
| T | T | T |
| T | F | F |
| F | T | F |
| F | F | F |

Truth Table for 'Or' statement

| p | q | $p \vee q$ |
|-----|-----|------------|
| T | T | T |
| T | F | T |
| F | T | T |
| F | F | F |

Conditional statement's truth table

| p | q | $p \rightarrow q$ |
|-----|-----|-------------------|
| T | T | T |
| T | F | F |
| F | T | T |
| F | F | T |

Truth Table for DOUBLE Implication statement

| p | q | $p \leftrightarrow q$ |
|-----|-----|-----------------------|
| T | T | T |
| T | F | F |
| F | T | F |
| F | F | T |

Probability

Addition Theorem

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Conditional Probability

$$P(A/B) = \frac{n(A \cap B)}{n(B)}$$

$$P(B/A) = \frac{n(A \cap B)}{n(A)}$$

Statistics

Quartile Deviation

If given data is to be divided into four parts then that number is called as quartile.

If Q_1 = First Quartile

Q_2 = Second Quartile

Q_3 = Third Quartile

$$Q_r = L + \frac{h}{f} \left(\frac{rN}{4} - C.F. \right)$$

$$Q.D. = \frac{Q_3 - Q_1}{2}$$

Variance & standard deviation

$$\text{Variance} = \sigma^2 = \frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^2$$

Standard Deviation = $\sigma =$

$$\sqrt{\frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^2}$$

Applications of Definite Integral

Area bounded by the curve

$y = f(x)$, $x = a$ & $x = b$ is given by

$$A = \int_a^b y dx$$

Permutation

Arrangement of r objects among

n objects: ${}^n P_r = \frac{n!}{(n-r)!}$

Circular Permutations: The no. of circular permutations of n objects is equal to $(n-1)!$

Combinations: ${}^n C_r = \frac{n!}{r!(n-r)!}$

Property of combination:

$${}^n C_r = {}^n C_{n-r}$$

Vectors

1. Co linearity of points: Three points are said to be collinear if $\overrightarrow{AB} \times \overrightarrow{AC} = 0$ or $\overrightarrow{AB} = K \cdot \overrightarrow{AC}$ (Where K is Scalar)

2. Section Formula

If \bar{r} divides \bar{a} & \bar{b} internally in the ratio $m:n$ then

$$\bar{r} = \frac{m\bar{b} + n\bar{a}}{m+n}$$

If \bar{r} divides \bar{a} & \bar{b} externally in the ratio $m:n$ then

$$\bar{r} = \frac{m\bar{b} - n\bar{a}}{m-n}$$

3. Mid Point formula: If \bar{r} is mid-point of \bar{a} & \bar{b} then

$$\bar{r} = \frac{\bar{a} + \bar{b}}{2}$$

4. Centroid

If \bar{g} is centroid of the triangle with vertices $\bar{a}, \bar{b}, \bar{c}$ then

$$\bar{g} = \frac{\bar{a} + \bar{b} + \bar{c}}{3}$$

5. Condition of coplanarity $\bar{a}, \bar{b}, \bar{c}$

Are coplanar if $[\bar{a}, \bar{b}, \bar{c}] = 0$

$\bar{a}, \bar{b}, \bar{c}$ & \bar{d} are coplanar if $\overrightarrow{AB} \cdot (\overrightarrow{AC} \times \overrightarrow{AD}) = 0$

6. Volume

Volume of parallelepiped =

$$[\bar{a}\bar{b}\bar{c}]$$

Volume of tetrahedron = $\frac{1}{6} [\bar{a}\bar{b}\bar{c}]$

Three Dimensional Geometry

1. Direction Cosines: If a line makes an angle $a, b,$ & c with positive directions of x, y & z axes then direction cosines are given as

$$l = \cos a, m = \cos b,$$

$$n = \cos c \text{ \& } l^2 + m^2 + n^2 = 1$$

2. Direction Ratio: a, b, c are direction ratios such that

$$\frac{l}{a} = \frac{m}{b} = \frac{n}{c}$$

$$l = \pm \frac{a}{\sqrt{a^2 + b^2 + c^2}},$$

$$m = \pm \frac{b}{\sqrt{a^2 + b^2 + c^2}}, n = \pm \frac{c}{\sqrt{a^2 + b^2 + c^2}}$$

3. Direction Ratios of Two Lines

Let a_1, b_1, c_1 & a_2, b_2, c_2 are direction ratios of two lines then

$$\cos \theta = \frac{a_1 a_2 + b_1 b_2 + c_1 c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2} \sqrt{a_2^2 + b_2^2 + c_2^2}}$$

Where θ is angle between two line.

Line

1. Vector Equation of Line:

Equation of line passing through

$\vec{r} = a\hat{i} + b\hat{j} + c\hat{k}$ & parallel to vector is

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$$

2. Length of perpendicular form

(\bar{a}) : on $\vec{r} = \vec{c} + \lambda \vec{d}$ is

$$\sqrt{|\bar{a} - \vec{c}|^2 - \left[\frac{(\bar{a} - \vec{c}) \cdot \vec{d}}{|\vec{d}|} \right]^2}$$

3. Shortest distance between the lines (d) =

$$= \frac{\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}{\sqrt{(a_1 b_2 - a_2 b_1)^2 + (a_1 c_2 - a_2 c_1)^2 + (b_1 c_2 - b_2 c_1)^2}}$$

$$d = \frac{|(\bar{a}_2 - \bar{a}_1) \times \vec{b}|}{|\vec{b}|} \text{ (For parallel lines)}$$

Plane

1. Distance of the point (x_1, y_1, z_1) from the plane

$$ax + by + cz + d = 0$$

$$D = \frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2 + b^2 + c^2}}$$

2. Angle between plane & line:

$$\text{Plane} = ax + by + cz + d = 0$$

$$\begin{aligned} \text{Line} &= \frac{x - x_1}{l} = \frac{y - y_1}{m} = \frac{z - z_1}{n} \\ \sin \theta &= \frac{a_1 a_2 + b_1 b_2 + c_1 c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2} \sqrt{a_2^2 + b_2^2 + c_2^2}} \end{aligned}$$

Continuity

Type of Discontinuity:

It is removable discontinuity if

$$\lim_{x \rightarrow a^+} f(x) = \lim_{x \rightarrow a^-} f(x) = f(a)$$

It is Irremovable discontinuity if

$$\lim_{x \rightarrow a^+} f(x) \neq \lim_{x \rightarrow a^-} f(x) \neq f(a)$$

Differentiation

1. Derivative of composite function: (chain rule)

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

2. Logarithmic Differentiation

When $y = [f(x)]^{g(x)}$ then solve problem by taking log on both sides

3. Derivative of parametric function: If $x = f(t)$ & $y = g(t)$ then

$$\frac{dx}{dt} = f'(t) \text{ \& } \frac{dy}{dt} = g'(t)$$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{g'(t)}{f'(t)}$$

4. Higher Order Derivatives:

If $y = f(x)$ then

$\frac{dy}{dx} = f'(x)$ & $\frac{d^2 y}{dx^2} = f''(x)$ Is called second order derivative.

Application of Derivatives

1. Equation of Tangent:

If $y = f(x)$ is equation of curve
then slope of tangent at point

(x_1, y_1) is

$$\text{Slope} = \left[\frac{dy}{dx} \right]_{(x_1, y_1)} = f'(x)_{at(x_1, y_1)}$$

2. Velocity & Acceleration:

If Displacement of particle = $s = f(t)$ where t is time then

$$\frac{ds}{dt} \text{ is velocity } \& \frac{d^2s}{dt^2} \text{ or } \frac{dv}{dt}$$

is acceleration.

3. Approximation:

$$f(a+h) \cong h \cdot f'(a) + f(a)$$

4. Maxima & Minima:

For any function $f(x)$

4.1 Find $f'(x)$ & put $f'(x) = 0$

Find roots

$$x = a, x = b$$

4.2. Find $f''(x)$, If $f''(x) > 0$ then

function is minimum at $x = a$

otherwise if $f''(x) < 0$ then

function is maximum at $x = a$.

Here a & b is called stationary pt.
of $f(x)$

Integration

Integration by Parts:

$$1. \int uv \, dx = u \int v \, dx - \int (u \, dv) \, dx$$

$$2. \int e^x [f(x) + f'(x)] \, dx = e^x \cdot f(x) + c$$

$$3. \int \frac{f'(x)}{f(x)} \, dx = \log[f(x)] + c$$

$$4. \int [f(x)]^n f'(x) \, dx$$

$$= \frac{[f(x)]^{n+1}}{(n+1)} + c$$

$$5. \int \sqrt{a^2 - x^2} \, dx = \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} + c$$

$$6. \int \sqrt{x^2 - a^2} \, dx = \frac{x}{2} \sqrt{x^2 - a^2} - \frac{a^2}{2} \log [x + \sqrt{x^2 - a^2}] + c$$

$$7. \int \sqrt{x^2 + a^2} \, dx = \frac{x}{2} \sqrt{x^2 + a^2} + \frac{a^2}{2} \log [x + \sqrt{x^2 + a^2}] + c$$

$$8. \int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \log \left| \frac{x-a}{x+a} \right| + c$$

$$9. \int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \log \left| \frac{a+x}{a-x} \right| + c$$

$$10. \int \frac{dx}{x^2 + a^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + c$$

$$11. \int \frac{dx}{\sqrt{x^2 + a^2}} = \log [x + \sqrt{x^2 + a^2}] + c$$

$$12. \int \frac{dx}{\sqrt{x^2 - a^2}} = \log [x + \sqrt{x^2 - a^2}] + c$$

$$13. \int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1} \frac{x}{a} + c$$

Definite Integral

Properties of Definite Integral:

$$1. \int_a^b f(x) \, dx = - \int_b^a f(x) \, dx$$

$$2. \int_a^b f(x) dx = \int_a^b f(t) dt$$

$$3. \int_a^b f(x) dx = \int_a^c f(x) dx + \int_c^b f(x) dx,$$

where $a < c < b$

$$4. \int_0^a f(x) dx = \int_0^a f(a-x) dx$$

$$5. \int_a^b f(x) dx = \int_a^b f(a+b-x) dx$$

$$6. \int_0^a f(x) dx = \int_0^a f(x) dx + \int_0^a f(2a-x) dx$$

$$7. \int_{-a}^a f(x) dx = \begin{cases} 2 \int_0^a f(x) dx, & \text{for even function} \\ 0 & \text{for odd function} \end{cases}$$

Applications of Definite Integral

1. Area under the curve:

Area bounded by curves

$y = f(x)$ or $x = f(y)$ are given as

$$A = \int_a^b y dx \text{ or } A = \int_a^b x dy$$

Distance Formula

If $P(x_1, x_2)$ and $Q(x_2, y_2)$ be two points, then Distance

$$PQ = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$$

Section Formula

1. Internal Division

If $A(x_1, y_1)$ and $B(x_2, y_2)$ be two points and $P(x, y)$ divides line segment AB internally in the ratio $m:n$ then,

$$x = \frac{mx_2 + nx_1}{m+n} \text{ \& } y = \frac{my_2 + ny_1}{m+n}$$

2. External Division

If $A(x_1, y_1)$ and $B(x_2, y_2)$ be two points and $P(x, y)$ divides line segment AB externally in the ratio $m:n$ then,

$$x = \frac{mx_2 - nx_1}{m-n} \text{ \& } y = \frac{my_2 - ny_1}{m-n}$$

3. Mid Point Formula

If $P(x, y)$ is mid point of $A(x_1, y_1)$ & $B(x_2, y_2)$ then,

$$x = \frac{x_1 + x_2}{2} \text{ \& } y = \frac{y_1 + y_2}{2}$$

4. Centroid Formula

If $P(x, y)$ is centroid of triangle with vertices $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$ then,

$$x = \frac{x_1 + x_2 + x_3}{3} \text{ \& } y = \frac{y_1 + y_2 + y_3}{3}$$

Slope Point form

Equation of the line having slope m and passing through (x_1, y_1) is given by,

$$y - y_1 = m(x - x_1)$$

Two Point Form

The Equation of a straight line passing through (x_1, y_1) and (x_2, y_2) is,

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

Slope intercept form

$Y = mx + c$ where c is y -intercept.

Double intercept form

$\frac{x}{a} + \frac{y}{b} = 1$ Where a is x - intercept and b is y - intercept.

Normal Form

$$x \cos \alpha + y \sin \alpha = OP$$

Where OP is a distance of origin from straight line AB and α is angle made by perpendicular with positive x -axis.

General equation of the line:

$$ax + by + c = 0$$

From this equation slope of line = $-\frac{a}{b}$

$$X\text{-intercept} = -\frac{c}{a}, Y\text{-intercept} = -\frac{c}{b}$$

Angle between two straight lines

Where $\tan \theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$, θ is angle between two lines and m_1 & m_2 are slopes of two intersecting lines.

Distance of a point from a line

Perpendicular distance of a point $P(x_1, y_1)$ from the line

$$ax + by + c = 0 \text{ is } = \left| \frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}} \right|$$

Distance between two parallel lines

Distance between two parallel

lines $ax + by + c_1 = 0$ &

$ax + by + c_2 = 0$ is given as

Perpendicular Distance

$$= \left| \frac{c_1 - c_2}{\sqrt{a^2 + b^2}} \right|$$

Circle

Centre radius form

Equation of a circle with centre (h, k) and radius r is given as

$$(x - h)^2 + (y - k)^2 = r^2$$

Diameter form

If $A(x_1, y_1)$ & $B(x_2, y_2)$ are end point of diameter of a circle then

equation of a circle is given as,

$$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$$

General equation of a circle:

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

where centre is $(-g, -f)$

and radius $r = \sqrt{g^2 + f^2 - c}$

Vectors:

Unit Vector along $\hat{a} = \frac{\vec{a}}{|\vec{a}|}$

Scalar product (dot product) if

$$\vec{a} = a_1 \vec{i} + a_2 \vec{j} + a_3 \vec{k} \text{ and } \vec{b} = b_1 \vec{i} + b_2 \vec{j} + b_3 \vec{k}$$

$$= b_1 \vec{i} + b_2 \vec{j} + b_3 \vec{k}$$

then

$$\vec{a} \cdot \vec{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$$

Cross product

$$\vec{a} = a_1\vec{i} + a_2\vec{j} + a_3\vec{k}$$

$$\vec{b} = b_1\vec{i} + b_2\vec{j} + b_3\vec{k} \text{ then}$$

$$\vec{a} \cdot \vec{b} = a_1b_1 + a_2b_2 + a_3b_3$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

Differential Equations:

Order & Degree of D.E.: Order is the highest order of derivatives and power of highest order derivative which is free of radicals & fraction is degree.

Formation of Differential

Equation:

Order of D.E. depends on arbitrary constants present in the equation. If arbitrary constants are n in numbers then order of D.E. is n.

Probability Distribution:

$$\text{Expected Value } E(X) = \mu =$$

$$\sum x_i p_i$$

$$\text{Variance} = \text{var}(x) = \sum x_i^2 p_i - \mu^2$$

$$\text{Standard Deviation} =$$

$$\sigma = \sqrt{\text{var}(x)}$$

Binomial Distribution

The probability of x successes is

$$P(X = x) = {}^nC_x p^x q^{n-x}$$

Geometric progression:

$$t_1, t_1^2, t_1^3, \dots, t_1^n \quad n^{\text{th}} \text{ term is}$$

$$\text{given as } t_n = t_1 r^{n-1}$$

Sum of S_n terms for a G.P.

$$S_n = t_1 \left(\frac{1-r^n}{1-r} \right) \text{ for } r < 1$$

$$= t_1 \left(\frac{r^n - 1}{r - 1} \right) \text{ for } r > 1$$

Arithmetic Mean (A.M.)

If a, b, c are in arithmetic progression then, Arithmetic

$$\text{Mean} = b = \frac{a+c}{2}$$

Geometric mean (G.M.):

If a, b, c are in G.P. then ,

$$\text{G.M.} = b = \pm \sqrt{ac}$$

Harmonic mean (H.M.):

If a, b, c are in G.P. then ,

$$\text{H.M.} = b = \frac{2ac}{a+c}$$

Relation between A.M., G.M. & H. M.

$$(\text{G.M.})^2 = (\text{A.M.})(\text{H.M.})$$

Exponential Series

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

$$e^{-x} = 1 - \frac{x}{1!} + \frac{x^2}{2!} - \frac{x^3}{3!} + \dots$$

Permutations and Combinations

It is a ratio of factorials.

$${}_nP_r = \frac{n!}{(n-r)!}$$

Circular Permutation:

The number of circular permutations around a object = $(n-1)!$

Combinations

It is the selection of 'r' object from 'h' given objects.

$${}_nC_r = \frac{n!}{r!(n-r)!}$$

Limits

$$\lim_{x \rightarrow a} f(x) = f(a)$$

$$\lim_{x \rightarrow 0} \left(\frac{a^x - 1}{x} \right) = \log a$$

$$\lim_{x \rightarrow 0} \left(\frac{e^x - 1}{x} \right) = \log e = 1$$

$$\lim_{x \rightarrow 0} \frac{\log(1+x)}{x} = \log e = 1$$

$$\lim_{x \rightarrow 0} \left(\frac{x^n - a^n}{x - a} \right) = na^{n-1}$$

Limits of trigonometric functions

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \quad \lim_{x \rightarrow 0} \frac{\sin kx}{x} = k$$

$$\lim_{x \rightarrow 0} \frac{\tan x}{x} = 1 \quad \lim_{x \rightarrow 0} \frac{\tan kx}{x} = k$$

Limits of exponential functions

$$\lim_{x \rightarrow 0} (1 + mx)^{1/mx} = e$$

$$\lim_{x \rightarrow 0} \left(\frac{e^{mx} - 1}{x} \right) = m$$

$$\lim_{x \rightarrow 0} \left(\frac{a^{mx} - 1}{x} \right) = m \log a$$

Limits of logarithmic functions

$$\lim_{x \rightarrow 0} \frac{\log(1+x)}{x} = \log e = 1$$

$$\lim_{x \rightarrow 0} \frac{\log(1+mx)}{x} = m \log e = m$$

Integration

Integration of some standard functions:-

$$1. \int x^n dx = \frac{x^{n+1}}{n+1} + c$$

$$2. \int \frac{dx}{x} = \log x + c$$

$$3. \int e^x dx = e^x + c$$

$$4. \int a^x dx = a^x / \log a + c$$

$$5. \int \sin x dx = -\cos x + c$$

$$6. \int \cos x dx$$

$$= \sin x + c$$

$$7. \int \sec^2 x dx$$

$$= \tan x + c$$

$$8. \int \operatorname{cosec}^2 x dx = -\cot x + c$$

$$9. \int \sec x \tan x dx = \sec x + c$$

$$10. \int \cot x dx = \log(\sin x) + c$$

$$11. \int \sec x dx = \log(\sec x + \tan x) + c$$

$$12. \int \operatorname{cosec} x dx = \log(\operatorname{cosec} x - \cot x) + c$$

$$13. \int \operatorname{cosec} x \cot x dx = -\operatorname{cosec} x + c$$

Rules of Integration

I) If u and v are two functions of x then

$$\int (u \pm v) dx = \int u dx \pm \int v dx$$

Conics Different Types of Parabola

| Term | Type | $y^2=4ax$ | $y^2=-4ax$ | $x^2=4ay$ | $x^2=-4ay$ |
|---------------------------|------|-----------------------|-------------------------|-----------------------|-------------------------|
| Focus | | $S(a,0)$ | $S(-a,0)$ | $S(0,a)$ | $S(0,-a)$ |
| Equation of Directrix | | $x+a=0$ | $x-a=0$ | $y+a=0$ | $y-a=0$ |
| Length of latus rectum | | $4a$ | $4a$ | $4a$ | $4a$ |
| End point of latus rectum | | $(a,2a)$ $(a,-2a)$ | $(-a,2a)$ $(-a,-2a)$ | $(2a,a)$ $(-2a,a)$ | $(2a,-a)$ $(-2a,-a)$ |
| Axis of symmetry | | x-axis | x-axis | y-axis | y-axis |
| Equation of axis | | $y=0$ | $y=0$ | $x=0$ | $x=0$ |
| Tangent of vertex | | y-axis | y-axis | x-axis | x-axis |

Different Types of Ellipse

| Term | Type | $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ($a > b$) | $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ($b > a$) |
|--------------------------|------|--|--|
| Length of axis | | Major axis= $2a$ Minor axis= $2b$ | Major axis= $2b$ Minor axis= $2a$ |
| Equation of axis | | Major axis- $\rightarrow y=0$ Minor axis- $\rightarrow x=0$ | Major axis- $\rightarrow x=0$ Minor axis- $\rightarrow y=0$ |
| Focus | | $S(ae,0)$ $S1(-ae,0)$ | $S(0,be)$ $S1(0,-be)$ |
| Endpoint of latus rectum | | $L^1(ae, \frac{b^2}{a})$ $L^1(ae, -\frac{b^2}{a})$ | $L^1(\frac{a^2}{b}, be)$, $L^1(\frac{a^2}{b}, -be)$ |
| Relation between | | $b^2=a^2(1-e^2)$ | $a^2=b^2(1-e^2)$ |

| | | |
|------------------------|------------------------------------|------------------------------------|
| a,b,e | | |
| Eccentricity | $e = \sqrt{\frac{a^2 - b^2}{a^2}}$ | $e = \sqrt{\frac{b^2 - a^2}{b^2}}$ |
| Equation of matrix | $x = \pm \frac{a}{e}$ | $y = \pm \frac{b}{e}$ |
| Length of latus rectum | $\frac{2b^2}{a}$ | $\frac{2a^2}{b}$ |

Types of Hyperbola

| Term Type | Standard Hyperbola | Conjugate Hyperbola |
|------------------------------|---|---|
| Equation | $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ | $\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$ |
| Co-ordinates of Foci | S(ae,0) S1(-ae,0) | S(0,be) S1(0,-be) |
| Eccentricity | $e = \frac{\sqrt{a^2 + b^2}}{a}$ | $e = \frac{\sqrt{a^2 + b^2}}{b}$ |
| Equation of directrices | $x = \pm ae$ | $y = \pm be$ |
| Distance between directrices | $\frac{2a}{e}$ | $\frac{2b}{e}$ |
| Length of latus-rectum | $\frac{2b^2}{a}$ | $\frac{2a^2}{b}$ |
| Length of transverse axis | 2a | 2b |
| Length of conjugate axis | 2b | 2a |