MATHEMATICS FORMULAE

Trigonometric functions

1. Length of an arc $= r\theta$
2. Area of sector $= \frac{1}{2}r^2\theta$

Trigonometric functions of standard angle

Function	0^0	30^0	45^0	60^0	90^0
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞
cosec	∞	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
sec	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	∞
cot	∞	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

Trigonometric functions of sum and difference

1. $\sin(A+B) = \sin A\cos B + \cos A\sin B$
2. $\sin(A-B) = \sin A\cos B - \cos A\sin B$
3. $\cos(A+B) = \cos A\cos B - \sin A\sin B$
4. $\cos(A-B) = \cos A\cos B + \sin A\sin B$
5. $\tan(A+B) = \frac{\tan A+\tan B}{1-\tan A\tan B}$
6. $\tan(A-B) = \frac{\tan A-\tan B}{1+\tan A\tan B}$

Trigonometric functions of 2θ

1. $\sin 2\theta = 2\sin\theta\cos\theta$
2. $\cos 2\theta = 2\cos^2\theta - 1$
3. $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$
4. $\cos 2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta}$
5. $\cos 2\theta = \cos^2\theta - \sin^2\theta$
6. $\cos 2\theta = 1 - 2\sin^2\theta$
7. $\sin 2\theta = \frac{2\tan\theta}{1+\tan^2\theta}$

Trigonometric functions of half angles

1. $\sin\theta = 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}$
2. $\cos\theta = 2\cos^2\frac{\theta}{2} - 1$
3. $\tan\theta = \frac{2\tan\frac{\theta}{2}}{1-\tan^2\frac{\theta}{2}}$
4. $\cos\theta = \frac{1-\tan^2\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}$
5. $\cos\theta = \cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2}$
6. $\cos\theta = 1 - 2\sin^2\frac{\theta}{2}$
7. $1+\cos\theta = 2\cos^2\frac{\theta}{2}$
8. $\sin\theta = \frac{2\tan\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}$

Trigonometric functions of 3θ

1. $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$
2. $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$
3. $\tan 3\theta = \frac{3\tan\theta - \tan^3\theta}{1-3\tan^2\theta}$

Factorization Formulae

1. $Sin C + Sin D$
$= 2\,Sin\left(\frac{C+D}{2}\right)Cos\left(\frac{C-D}{2}\right)$

2. $SinC - SinD$
$= 2\,Cos\left(\frac{C+D}{2}\right)Sin\left(\frac{C-D}{2}\right)$

3. $CosC + CosD$
$= 2\,Cos\left(\frac{C+D}{2}\right)Cos\left(\frac{C-D}{2}\right)$

4. $CosC - CosD = -2\,Sin\left(\frac{C+D}{2}\right)\,\text{Sin}\left(\frac{C-D}{2}\right)$

Defactorizaion Formulae

2 SinA CosB = Sin(A+B) + Sin(A-B)
2 CosA SinB = Sin(A+B) - Sin(A-B)
2 CosA CosB = Cos(A+B) + Cos(A-B)
2 SinA SinB = Cos(A-B) - Cos(A+B)

Trigonometric functions of Angles of Triangle

1. $Sin\,(A+B) = Sin\,C$
2. $Sin\,(\frac{A+B}{2}) = Cos(\frac{C}{2})$
3. $Cos\,(\frac{A+B}{2}) = Sin(\frac{C}{2})$

Determinants

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2c_3 - b_3c_2) - a_2(b_1c_3 - b_3c_1) + a_3(b_1c_2 - b_2c_1)$$

Minor & Cofactor of 3×3 Determinants

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

Minor Cofactor

$M_{11} = \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix}$ $A_{11} = \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix}$

$M_{12} = \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix}$ $A_{12} = (-)\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix}$

$M_{13} = \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$ $A_{13} = \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$

$M_{21} = \begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix}$ $A_{21} = (-)\begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix}$

$M_{22} = \begin{vmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{vmatrix}$ $A_{22} = \begin{vmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{vmatrix}$

$M_{23} = \begin{vmatrix} a_{11} & a_{12} \\ a_{31} & a_{32} \end{vmatrix}$ $A_{23} = (-)\begin{vmatrix} a_{11} & a_{12} \\ a_{31} & a_{32} \end{vmatrix}$

$M_{31} = \begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix}$ $A_{31} = \begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix}$

$M_{32} = \begin{vmatrix} a_{11} & a_{13} \\ a_{21} & a_{23} \end{vmatrix}$ $A_{32} = (-)\begin{vmatrix} a_{11} & a_{13} \\ a_{21} & a_{23} \end{vmatrix}$

$M_{33} = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$ $A_{33} = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$

Matrices

Multiplication of Two Matrices

A product AB exists if number of columns of A is equal to number of rows of B.

If $A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \& B = \begin{pmatrix} b_{11} \\ b_{21} \end{pmatrix}$

then product AB exists & is equal to

$$AB = \begin{pmatrix} a_{11}.b_{11} + a_{12}.b_{21} \\ a_{21}b_{11} + a_{22}b_{21} \end{pmatrix}$$

Logarithms

Law of Logarithms:

Law of product:

$log_{10}ab = log_{10}a + log_{10}b$

Law of Exponent:

$log_{10}a^m = mlog_{10}a$

Change of Base Law:

$$log_b a = \frac{log_e a}{log_e b}$$

Complex numbers

Conjugate of Complex numbers:

If z = a+ib then $\bar{z}$ = a-ib

Modulus of Complex numbers:

$|z| = \sqrt{a^2 + b^2}$

Argument of Complex numbers:

$\theta = \tan^{-1}\left(\frac{b}{a}\right)$ where

$0 \leq \theta \leq 2\pi$

Division of Complex numbers:

If $z_1 = a_1 + i\, b_1$ & $z_2 = a_2 + i\, b_2$

Then, $\frac{z_1}{z_2} = \frac{a_1 + i\, b_1}{a_2 + i\, b_2}$

$$= \frac{a_1 + i\, b_1}{a_2 + i\, b_2} \times \frac{a_2 - i\, b_2}{a_2 - i\, b_2}$$

Binomial Theorem

$$(x+a)^n = \sum_{k=0}^{n} \binom{n}{k} x^k a^{n-k}$$

Mathematical Logic

Truth Table for ‘And ‘statement

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Truth Table for ‘Or ‘statement

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Conditional statement’s truth table

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

Truth Table for DOUBLE Implication statement

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Probability

Addition Theorem

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Conditional Probability

$$P(A/B) = \frac{n(A \cap B)}{n(B)}$$

$$P(B/A) = \frac{n(A \cap B)}{n(A)}$$

Statistics

Quartile Deviation

If given data is to be divided into four parts then that number is called as quartile.

If Q_1= First Quartile

Q_2=Second Quartile

Q_3=Third Quartile

$Q_r = L + \frac{h}{f}\left(\frac{rN}{4} - C.F.\right)$

Q.D. $= \frac{Q_3 - Q_1}{2}$

Variance & standard deviation

Variance $=\sigma^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2$

Standard Deviation= $\sigma = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2}$

Applications of Definite Integral

Area bounded by the curve $y = f(x), x = a \,\&\, x = b$ is given by

$$A = \int_a^b y dx$$

Permutation

Arrangement of r objects among n objects: ${}^nP_r = \frac{n!}{(n-r)!}$

Circular Permutations: The no. of circular permutations of n objects is equal to $(n - 1)!$

Combinations: ${}^nC_r = \frac{n!}{r!(n-r)!}$

Property of combination:

${}^nC_r = {}^nC_{n-r}$

Vectors

1. Collinearity of points: Three points are said to be collinear if $\overline{AB} \times \overline{AC} = 0$ or $\overline{AB} = K.\overline{AC}$ (Where K is Scalar)

2. Section Formula

If $\bar{r}$ divides $\bar{a}\&\bar{b}$ internally in the ratio m: n then

$$\bar{r} = \frac{m\bar{b} + n\bar{a}}{m + n}$$

If $\bar{r}$ divides $\bar{a}\&\bar{b}$ externally in the ratio m:n then

$$\bar{r} = \frac{m\bar{b} - n\bar{a}}{m - n}$$

3. Mid Point formula: If $\bar{r}$ is mid-point of $\bar{a}\&\bar{b}$ then

$$\bar{r} = \frac{\bar{a} + \bar{b}}{2}$$

4. Centroid

If $\bar{g}$ is centroid of the triangle with vertices $\bar{a}, \bar{b}, \&\bar{c}$ then

$\bar{g} = \frac{\bar{a}+\bar{b}+\bar{c}}{3}$

5. Condition of coplanerity $\bar{a}, \bar{b}\&\bar{c}$

Are coplanar if $[\,\bar{a}, \,\bar{b}\,, \bar{c}] = 0$

$\bar{a}, \bar{b}, \bar{c}\&\bar{d}$ are coplanar if $\overline{AB}.\left(\overline{AC} \times \overline{AD}\right) = 0$

6. Volume

Volume of parallelepiped = $[\bar{a}\bar{b}\bar{c}\,]$

Volume of tetrahedron = $\frac{1}{6}[\bar{a}\bar{b}\bar{c}\,]$

Three Dimensional Geometry

1. Direction Cosines: If a line makes an angle a, b, & c with positive directions of x, y & z axes then direction cosines are given as

$l = cosa, m = cosb,$

$n = cosc \,\& l^2 + m^2 + n^2 = 1$

2. Direction Ratio: a, b, c are direction ratios such that

$$\frac{l}{a} = \frac{m}{b} = \frac{n}{c}$$

$$l = \pm\frac{a}{\sqrt{a^2 + b^2 + c^2}} ,$$

$m = \pm\frac{b}{\sqrt{a^2+b^2+c^2}}$, $n = \pm\frac{c}{\sqrt{a^2+b^2+c^2}}$

3. Direction Ratios of Two Lines

Let $a_1, b_1, c_1 \& a_2, b_2, c_2$ are direction ratios of two lines then

$$\cos\theta = \left|\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}\right|$$

Where θ is angle between two line.

Line

1. Vector Equation of Line:

Equation of line passing through $\overline{r} = a\hat{\imath} + b\hat{\jmath} + c\hat{k}$& parallel to vector is

$$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$$

2. Length of perpendicular from

$(\overline{a})$:on $\overline{r} = \overline{c} + \lambda\,\overline{d}$ is

$$\sqrt{|\overline{a}-\overline{c}|^2 - \left[\frac{(\overline{a}-\overline{c}).\overline{b}}{|\overline{d}|}\right]^2}$$

3. Shortest distance between the lines (d) =

$$= \frac{\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}{\sqrt{(a_1b_2-a_2b_1)^2+(a_1c_2-a_2c_1)^2+(b_1c_2-b_2c_1)^2}}$$

$d = \left|\frac{(\overline{a}_2-\overline{a}_1)\times\overline{b}}{|\overline{b}|}\right|$(For parallel lines)

Plane

1. Distance of the point$(x_1, y_1, z_1)$from the plane $ax+by+cz+d=0$

$$D = \left|\frac{ax_1+by_1+cz_1+d}{\sqrt{a^2+b^2+c^2}}\right|$$

2. Angle between plane & line:

$Plane = ax + by + cz + d = 0$

$$Line = \frac{x-x_1}{l} = \frac{y-y_1}{m} = \frac{z-z_1}{n}$$

$$\sin\theta = \frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$$

Continuity

Type of Discontinuity:

It is removable discontinuity if

$$\lim_{x\to a^+} f(x) = \lim_{x\to a^-} f(x) = f(a)$$

It is Irremovable discontinuity if

$$\lim_{x\to a^+} f(x) \neq \lim_{x\to a^-} f(x) \neq f(a)$$

Differentiation

1. Derivative of composite function: (chain rule)

$$\frac{dy}{dx} = \frac{dy}{du}\times\frac{du}{dx}$$

2. Logarithmic Differentiation

When $y = [f(x)]^{g(x)}$ then solve problem by taking log on both sides

3. Derivative of parametric function: If $x = f(t) \& y = g(t)$ then

$$\frac{dx}{dt} = f'(t) \& \frac{dy}{dt} = g'(t)$$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{g'(t)}{f'(t)}$$

4. Higher Order Derivatives:

If$y = f(x)$then

$\frac{dy}{dx} = f'(x) \& \frac{d^2y}{dx^2} = f''(x)$ Is called second order derivative.

Application of Derivatives

1. Equation of Tangent:

If $y = f(x)$ is equation of curve then slope of tangent at point (x_1, y_1) is

Slope=$\left[\frac{dy}{dx}\right]_{(x_1, y_1)} = f'(x)_{at(x_1, y_1)}$

2. Velocity & Acceleration:

If Displacement of particle = s = f (t) where t is time then

$\frac{ds}{dt}$ *is velocity* & $\frac{d^2s}{dt^2}$ *or* $\frac{dv}{dt}$ *is acceleration.*

3. Approximation:

$f(a+h) \cong h.f'(a) + f(a)$

4. Maxima & Minima:

For any function f (x)

4.1Find $f'(x)$& *put* $f'(x) = 0$

Find roots

x = a, x =b

4.2. Find $f''(x)$, *If* $f''(x) > 0$ then function is minimum at x = a otherwise if $f''(x) < 0$ then function is maximum at x= a.

Here a & b is called stationary pt. of f(x)

Integration

Integration by Parts:

1.$\int uv\, dx = u \int v\, dx - \int (\frac{du}{dx} \int v\, dx)\, dx$

2.$\int e^x [f(x) + f'(x)]\, dx = e^x . f(x) + c$

3. $\int \frac{f'(x)}{f(x)} dx = \log[f(x)] + c$

4. $\int [f(x)]^n f'(x) dx = \frac{[f(x)]^{n+1}}{(n+1)} + c$

5. $\int \sqrt{a^2 - x^2}\, dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1}\frac{x}{a} + c$

6. $\int \sqrt{x^2 - a^2}\, dx = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2} \log[x + \sqrt{x^2 - a^2}] + c$

7. $\int \sqrt{x^2 + a^2}\, dx = \frac{x}{2}\sqrt{x^2 + a^2} + \frac{a^2}{2} \log[x + \sqrt{x^2 + a^2}] + c$

8. $\int \frac{dx}{x^2 - a^2} = \frac{1}{2\mathrm{a}} \log\left|\frac{\mathrm{x} - \mathrm{a}}{\mathrm{x} + \mathrm{a}}\right| + c$

9. $\int \frac{dx}{a^2 - x^2} = \frac{1}{2\mathrm{a}} \log\left|\frac{\mathrm{a} + \mathrm{x}}{\mathrm{a} - \mathrm{x}}\right| + c$

10. $\int \frac{dx}{x^2 + a^2} = \frac{1}{\mathrm{a}} \tan^{-1}\frac{\mathrm{x}}{\mathrm{a}} + c$

11. $\int \frac{dx}{\sqrt{x^2 + a^2}} = \log\left[x + \sqrt{x^2 + a^2}\right] + c$

12. $\int \frac{dx}{\sqrt{x^2 - a^2}} = \log\left[x + \sqrt{x^2 - a^2}\right] + c$

13. $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\frac{x}{a} + c$

Definite Integral

Propertiesof Definite Integral:

1. $\int_a^b f(x)dx = -\int_b^a f(x)\,dx$

2. $\int_a^b f(x)dx = \int_a^b f(t)\,dt$

3. $\int_a^b f(x)dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx,$

$where\ a < c < b$

4. $\int_0^a f(x)dx = \int_0^a f(a-x)\,dx$

5. $\int_a^b f(x)dx = \int_a^b f(a+b-x)\,dx$

6. $\int_0^{2a} f(x)dx = \int_0^a f(x)\,dx + \int_0^a f(2a - x)dx$

7. $\int_{-a}^{a} f(x)dx =$
$2\int_0^a f(x)\,dx$,for even function
$= 0$ for odd function

Applications of Definite Integral

1. Area under the curve:

Area bounded by curves $y = f(x) or\ x = f(y)$ are given as

$$A = \int_a^b ydx\ or\ A = \int_a^b xdy$$

Distance Formula

If P(x1, x2) and Q(x2,y2) be two points, then Distance

$$PQ = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$$

Section Formula

1. Internal Division

If $A(x_1, y_1)$ and $B(x_2,y_2)$ be two points and P(x, y) divides line segment AB internally in the ratio m:n then,

$$x = \frac{mx_2 + nx_1}{m+n} \,\&\, y = \frac{my_2 + ny_1}{m+n}$$

2. External Division

If $A(x_1, y_1)$ and $B(x_2, y_2)$ be two points and P(x, y) divides line segment AB externally in the ratio m:n then,

$$x = \frac{mx_2 - nx_1}{m-n} \,\&\, y = \frac{my_2 - ny_1}{m-n}$$

3. Mid Point Formula

If P(x,y) is mid point of $A(x_1,y_1)$ & $B(x_2,y_2)$ then,

$$x = \frac{x_1 + x_2}{2} \,\&\, y = \frac{y_1 + y_2}{2}$$

4. Centroid Formula

If P(x,y) is centroid of triangle with vertices $A(x_1,y_1)$, $B(x_2,y_2)$, $C(x_3,y_3)$ then,

$$x = \frac{x_1 + x_2 + x_3}{3} \;\&$$

$$y = \frac{y_1 + y_2 + y_3}{3}$$

Slope Point form

Equation of the line having slope m and passing through (x_1,y_1) is given by,

$y - y_1 = m(x - x_1)$

Two Point Form

The Equation of a straight line passing through (x_1,y_1) and (x_2,y_2) is,

$\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$

Slope intercept form

Y= mx+c where c is y-intercept.

Double intercept form

$\frac{x}{a} + \frac{y}{b} = 1$ Where a is x- intercept and b is y- intercept.

Normal Form

$$x\cos\alpha + y\sin\alpha = OP$$

Where OP is a distance of origin from straight line AB and α is angle made by perpendicular with positive x-axis.

General equation of the line:

ax+by+c=0

From this equation slope of line= $\frac{-a}{b}$

X-intercept= $\frac{-c}{a}$, Y-intercept= $\frac{-c}{b}$

Angle between two straight lines

Where $\tan\theta = \left|\frac{m_1-m_2}{1+m_1m_2}\right|$, θ is angle between two lines and m_1& m_2 are slopes of two intersecting lines.

Distance of a point from a line

Perpendicular distance of a point $P(x_1,y_1)$ from the line ax+by+c=0 is = $\left|\frac{ax_1+by_1+c}{\sqrt{a^2+b^2}}\right|$

Distance between two parallel lines

Distance between two parallel lines $ax+by+c_1=0$ & $ax+by+c_2=0$ is given as Perpendicular Distance $= \left|\frac{c_1-c_2}{\sqrt{a^2+b^2}}\right|$.

Circle

Centre radius form

Equation of a circle with centre (h,k) and radius r is given as

$(x-h)^2 + (y-k)^2 = r^2$

Diameter form

If $A(x_1,y_1)$ & $B(x_2,y_2)$ are end point of diameter of a circle then equation of a circle is given as,

$(x-x_1)(x-x_2)+ (y-y_1)(y-y_2)=0$

General equation of a circle:

$x^2 + y^2 + 2gx + 2fy + c = 0$ *where centre is* $(-g, -f)$ *and radious* $r = \sqrt{g^2 + f^2 - c}$

Vectors:

Unit Vector along $\hat{a}=\frac{\vec{a}}{|\vec{a}|}$

Scalar product (dot product) if

$\vec{a} = a_1\vec{i} + a_2\vec{j}+a_3\vec{k}\&\vec{b}$

$= b_1\vec{i} + b_2\vec{j}+b_3\vec{k}$

$then$

$\vec{a} \bullet \vec{b} = a_1b_1 + a_2b_2 + a_3b_3$

Cross product

$\vec{a} = a_1\vec{i} + a_2\vec{j}+a_3\vec{k}\&$

$\vec{b} = b_1\vec{i} + b_2\vec{j}+b_3\vec{k}\ then$

$\vec{a} \bullet \vec{b} = a_1b_1 + a_2b_2 + a_3b_3$

$$\vec{a}\mathrm{x}\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

Differential Equations:

Order & Degree of D.E.: Order is the highest order of derivatives and power of highest order derivative which is free of radicals & fraction is degree.

Formation of Differential Equation:

Order of D.E. depends on arbitrary constants present in the equation. If arbitrary constants are n in numbers then order of D.E. is n.

Probability Distribution:

Expected Value $E(X) = \mu = \sum x_i\, p_i$

Variance = var (x) = $\sum x_i^2\, p_i - \mu^2$

Standard Deviation =

$\sigma = \sqrt{var(x)}$

Binomial Distribution

The probability of x successes is

$P(X = x) = {}^nC_x\, p^x q^{n-x}$

Geometric progression:

$t_1, t_1^2, t_1^3 \ldots\ldots t_1^n$ n^{th} term is given as $t_n = t_1 r^{n-1}$

Sum of S_n terms for a G.P.

$S_n = t_1\left(\frac{1-r^n}{1-r}\right)$ for r < 1

$= t_1\left(\frac{r^n-1}{r-1}\right)$ for r > 1

Arithmetic Mean (A.M.)

If a,b,c are in arithmetic progression then, Arithmetic Mean= b = $\frac{a+c}{2}$

Geometric mean (G.M.):-

If a,b,c are in G.P. then ,

G.M.= b $=\pm\sqrt{ac}$

Harmonic mean (H.M.):-

If a,b,c are in G.P. then ,

H.M.= b $=\frac{2ac}{a+c}$

Relation between A.M., G.M. & H. M.

$(G.M.)^2 = (A.M.)\ (H.M.)$

Exponential Series

$e^x = 1+\frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots\ldots\ldots\ldots$

$e^{-x} = 1-\frac{x}{1!} + \frac{x^2}{2!} - \frac{x^3}{3!} + \ldots\ldots\ldots\ldots$

Permutations and Combinations

It is a ratio of factorials.

${}^nP_r = \frac{\boldsymbol{n}!}{(\boldsymbol{n}-\boldsymbol{r})!}$

Circular Permutation:

The number of circular permutations around a object = (n-1)!

Combinations

It is the selection of 'r' object from 'h'given objects.

${}^{n}C_{r} = \frac{n!}{r!(n-r)!}$

Limits

$$\lim_{x\to a} f(x) = f(a)$$

$$\lim_{x\to 0}\left(\frac{a^x - 1}{x}\right) = loga$$

$$\lim_{x\to 0}\left(\frac{e^x - 1}{x}\right) = loge = 1$$

$$\lim_{x\to 0}\frac{\log(1+x)}{x} = loge = 1$$

$$\lim_{x\to 0}\left(\frac{x^n - a^n}{x-a}\right) = na^{n-1}$$

Limits of trigonometric functions

$$\lim_{x\to 0}\frac{sinx}{x} = 1 \quad \lim_{x\to 0}\frac{sinkx}{x} = k$$

$$\lim_{x\to 0}\frac{tanx}{x} = 1 \qquad \lim_{x\to 0}\frac{tankx}{x} = k$$

Limits of exponential functions

$$\lim_{x\to 0}(1+mx)^{1/mx} = e$$

$$\lim_{x\to 0}\left(\frac{e^{mx} - 1}{x}\right) = m$$

$$\lim_{x\to 0}\left(\frac{a^{mx} - 1}{x}\right) = mloga$$

Limits of logarithmic functions

$$\lim_{x\to 0}\frac{\log(1+x)}{x} = loge = 1$$

$$\lim_{x\to 0}\frac{\log(1+mx)}{x} = mloge = m$$

Integration

Integration of some standard functions:-

1. $\int x^n\,dx = \frac{x^{n+1}}{n+1}$ +c

2. $\int \frac{dx}{x} = logx + c$

3. $\int e^x\,dx = e^x$ +c

4. $\int a^x\,dx = a^x/loga$ +c

5. $\int sinx\,dx = -cosx$ +c

6. $\int cosx\,dx$

$= sinx + c$ 7. $\int sec^2 x\,dx$

$= tanx + c$

8. $\int cosec^2 x\,dx = -cotx + c$

9. $\int secxtanx\,dx = secx$ +c

10. $\int cotx\,dx = \log(sinx)$ +c

11. $\int secx\,dx = \log(secx + tanx)$ +c

12. $\int cosecx\,dx = \log(cosecx - cotx)$ +c

13. $\int cosecxcotx\,dx = -cosecx$ +c

Rules of Integration

I) If u and v are two functions of x then

$$\int (u \pm v)dx = \int u\,dx \pm \int v\,dx$$

Conics Different Types of Parabola

Term Type	$y^2=4ax$	$y^2=-4ax$	$x^2=4ay$	$x^2=-4ay$
Focus	S(a,0)	S(-a,0)	S(0,a)	S(0,-a)
Equation of Directrix	x+a=0	x-a=0	y+a=0	y-a=0
Length of latus rectum	4a	4a	4a	4a
End point of latus rectum	(a,2a) (a,-2a)	(-a,2a) (-a,-2a)	(2a,a) (-2a,a)	(2a,-a) (-2a,-a)
Axis of symmetry	x-axis	x-axis	y-axis	y-axis
Equation of axis	y=0	y=0	x=0	x=0
Tangent of vertex	y-axis	y-axis	x-axis	x-axis

Different Types of Ellipse

Term Type	$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ $(a>b)$	$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ $(b>a)$
Length of axis	Major axis=2a Minor axis=2b	Major axis=2b Minor axis=2a
Equation of axis	Major axis->y=0 Minor axis->x=0	Major axis->x=0 Minor axis->y=0
Focus	S(ae,0) S1(-ae,0)	S(0,be) S1(0,-be)
Endpoint of latus rectum	$L^1(ae,\frac{b^2}{a})$ $L^1(ae,\frac{-b^2}{a})$	$L^1(\frac{a^2}{b},be)$, $L^1(\frac{a^2}{b},-be)$
Relation between a,b,e	b2=a2(1-e2)	a2=b2(1-e2)

Eccentricity	$e = \sqrt{\dfrac{a^2 - b^2}{a^2}}$	$e = \sqrt{\dfrac{b^2 - a^2}{b^2}}$
Equation of matrix	$x = \pm\dfrac{a}{e}$	$y = \pm\dfrac{b}{e}$
Length of latus rectum	$\dfrac{2b^2}{a}$	$\dfrac{2a^2}{b}$

Types of Hyperbola

Term Type	Standard Hyperbola	Congugate Hyperbola
Equation	$\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$	$\dfrac{y^2}{b^2} - \dfrac{x^2}{a^2} = 1$
Co-ordinates of Foci	S(ae,0) S1(-ae,0)	S(0,be) S1(0,-be)
Eccentricity	$e = \dfrac{\sqrt{a^2 + b^2}}{a}$	$e = \dfrac{\sqrt{a^2 - b^2}}{b}$
Equation of directrices	$x = \pm ae$	$y = \pm be$
Distance between directrices	$\dfrac{2a}{e}$	$\dfrac{2b}{e}$
Length of latus-rectum	$\dfrac{2b^2}{a}$	$\dfrac{2a^2}{b}$
Length of transverse axis	2a	2b
Length of conjugate axis	2b	2a