Equation for solution of Assignment 2.2. By Orlando Pereira.

$$Q_{out} = A\,.[G.\eta_0 - a_1(Tm - Ta) - a_2(Tm - Ta)^2]$$

$$\dot{m}_f\,.C_f.(T_{fout} - T_{fin}) = A\,.[G.\eta_0 - a_1(Tm - Ta) - a_2(Tm - Ta)^2]$$

$$A\,.m\,.C_f.(T_{fout} - T_{fin}) = A\,.[G.\eta_0 - a_1(Tm - Ta) - a_2(Tm - Ta)^2]$$

$$m\,.C_f.(T_{fout} - T_{fin}) = G.\eta_0 - a_1(Tm - Ta) - a_2(Tm - Ta)^2$$

$$G.\eta_0 - a_1(Tm - Ta) - a_2(Tm - Ta)^2 - m\,.C_f.\left(T_{fout} - T_{fin}\right) = 0$$

- Since: $Tm = \left(\frac{T_{fout} + T_{fin}}{2}\right) \rightarrow \quad T_{fout} - T_{fin} = 2\,(Tm - T_{fin})$

$$G.\eta_0 - a_1(Tm - Ta) - a_2(Tm - Ta)^2 - m\,.C_f.2\left(Tm - T_{fin}\right) = 0$$

-Then, rearranging the equation:

$$a_2\,.Tm^2 + \left(a_1 - 2.a_2.Ta + 2.m.C_f\right)Tm - \left(G.\eta_0 + a_1.Ta - a_2.Ta^2 + 2.m.C_f.T_{fin}\right) = 0$$

- It's a 2nd degree equation, being Tm the variable. It can be solved using the following formula:

Given a general quadratic equation of the form

$$ax^2 + bx + c = 0$$

with x representing an unknown, a, b and c representing constants with $a \neq 0$, the quadratic formula is:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

where the plus–minus symbol "±" indicates that the quadratic equation has two solutions.[1] Written separately, they become:

$$x_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad \text{and} \quad x_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

$a = a_2$
$b = a_1 - 2.a_2.Ta + 2.m.C_f$
$c = -\left(G.\eta_0 + a_1.Ta - a_2.Ta^2 + 2.m.C_f.T_{fin}\right)$

-Then, having calculated Tm: $T_{fout} = 2.Tm - T_{fin}$