Simulation and modelling in RE Systems
- Intro and continuous dynamic models-

Overview of today's seminar

Modelling fundamentals

Steady-state models
- Dimensional analysis
- One-dimensional models

Dynamic models
- One-dimensional models
(- Multi-dimensional models)

Systems of differential equations
- Euler-Cauhy
- Runge-Kutta, fourth order

Modeling fundamentals

What is a model?

- A **simplified representation** of a given system

Be aware: it is NOT
a copy of that system!

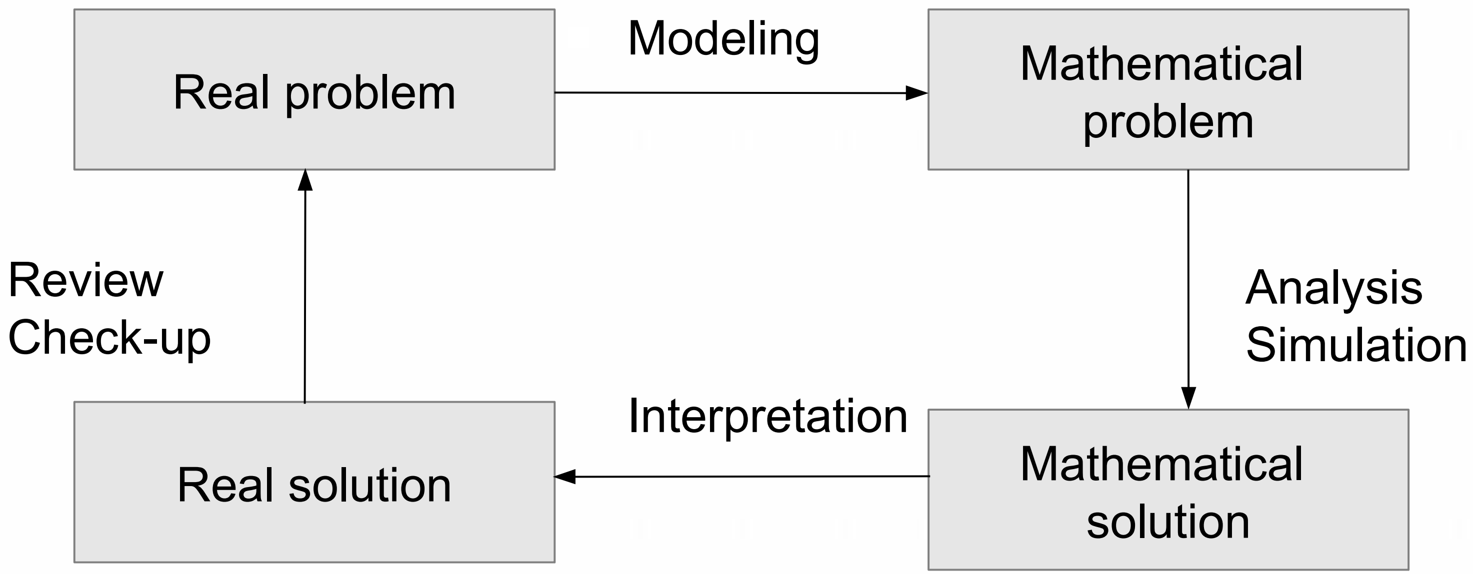
→ **Ergo:**
all models are WRONG
...
but they are useful!

Modeling fundamentals

Which is „better“?

Modeling fundamentals

The modeling process



Source: Ortlieb, 2013

Modelling fundamentals

Steps in the modeling process

- **Definition of research/modeling question:** goals and aims!

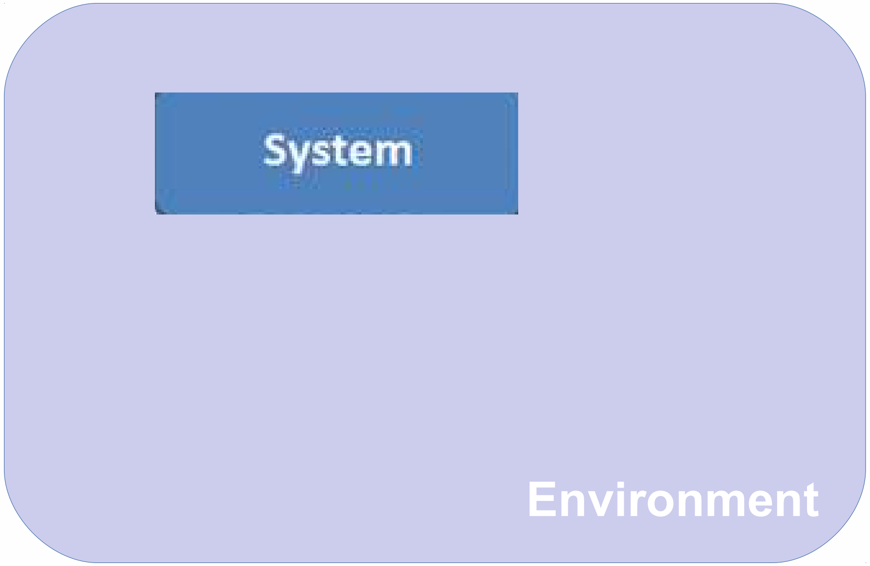
- **Underlying priciples:**
  Available theoretical approaches, mathematical laws,...

- **Relevant information:**
  relevant vs. irrelevant information, suitable simplifications,...

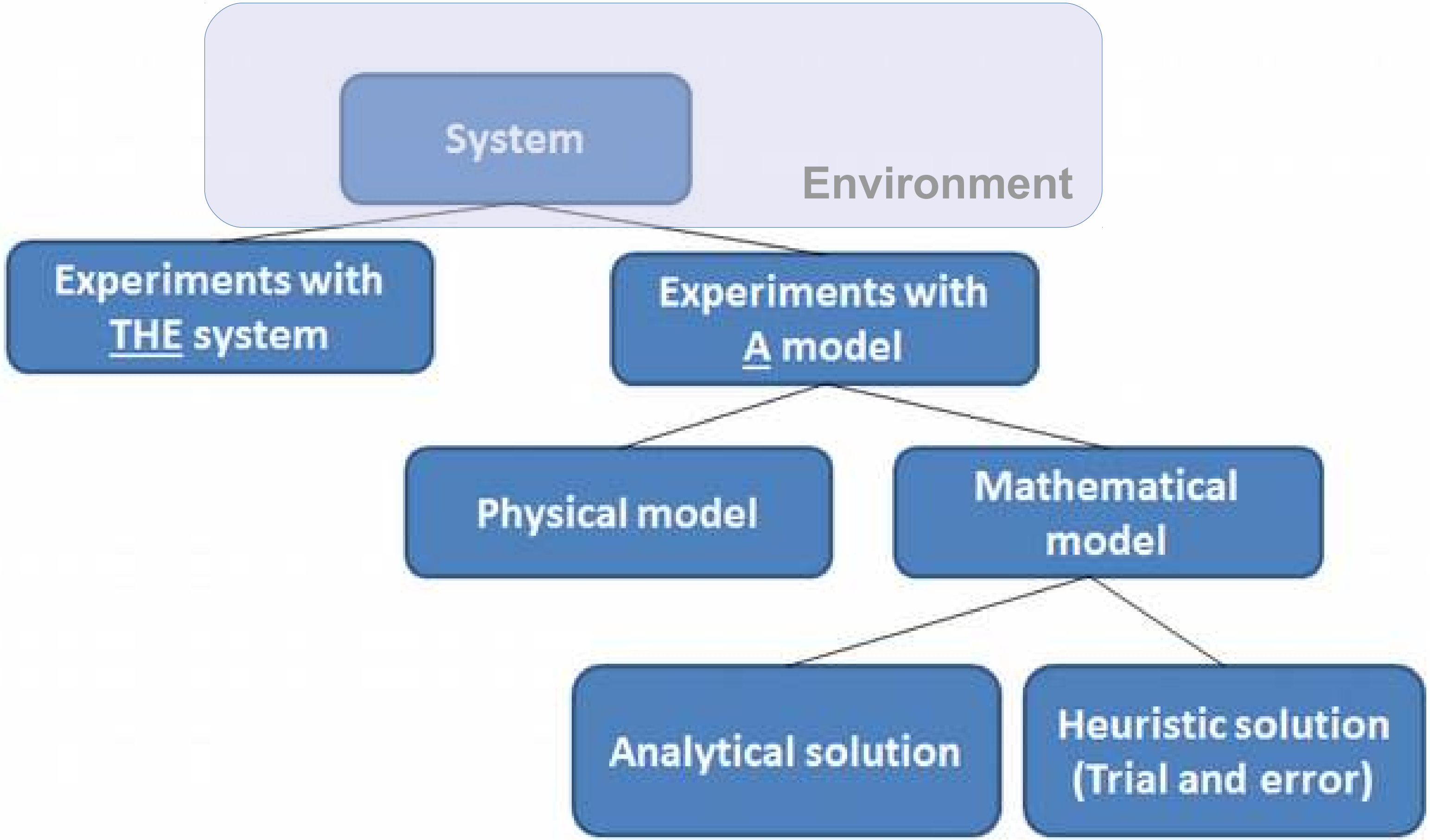
- **Variables and parameters:**
  Input ranges for relevant parameters and variables, estimation errors, initial values

Source: Ortlieb, 2013

Modeling fundamentals



Modeling fundamentals



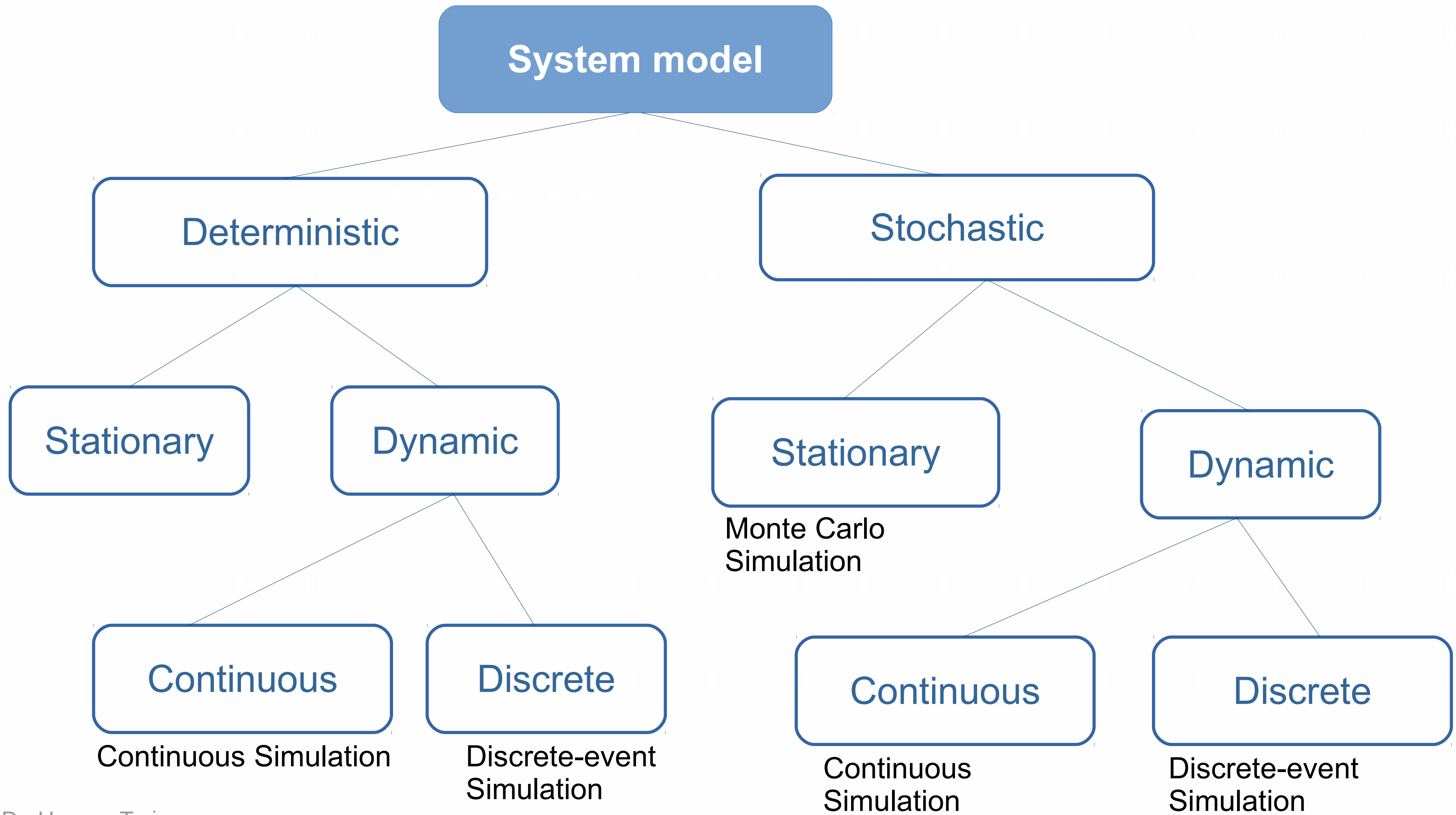
Main types of models

- **Stationary (static) / dynamic**

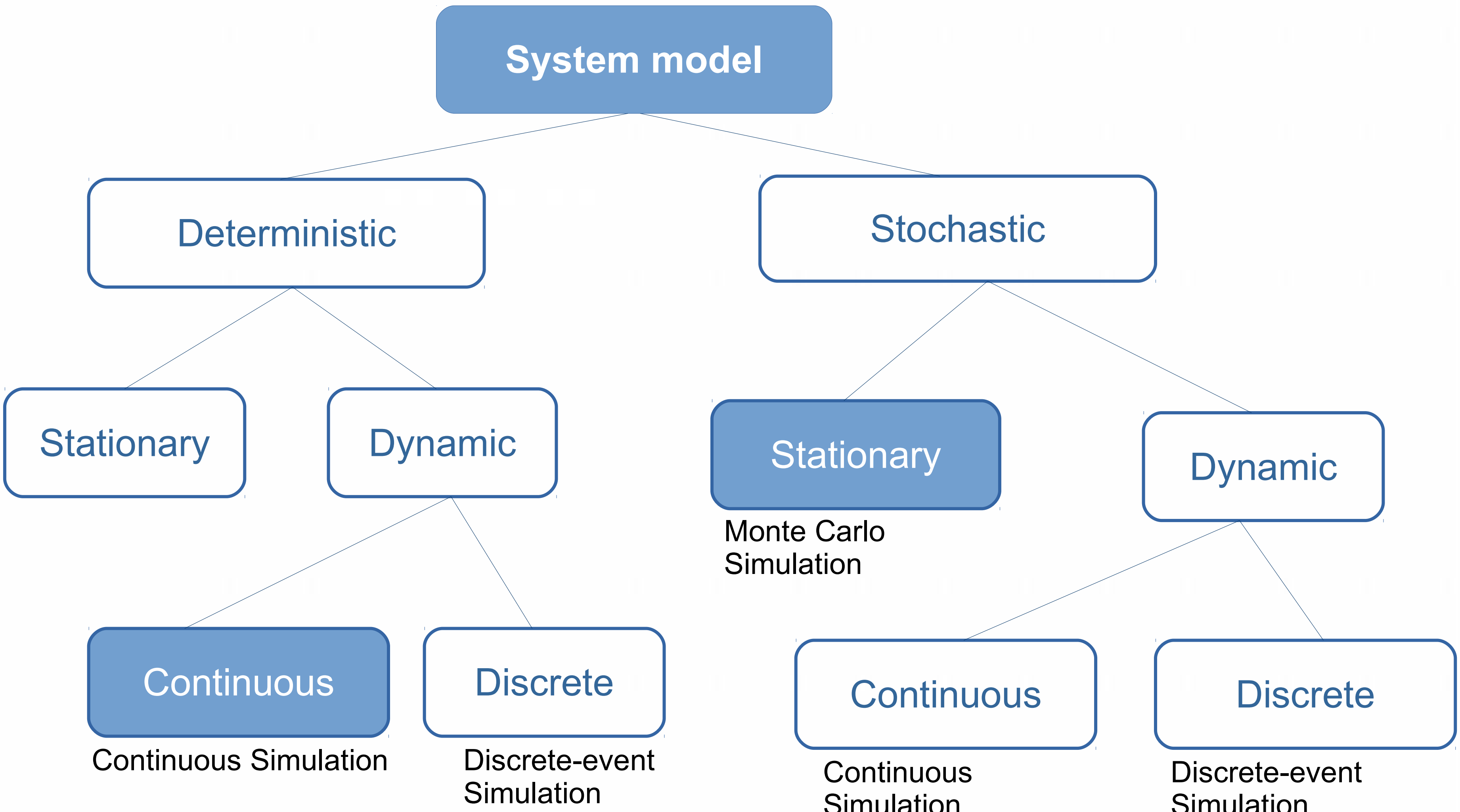
- **Discrete / continuous**

- **Deterministic / stochastic (statistic)**

Main types of models



Main types of models



Modeling fundamentals

Assessment criteria for a model

Validation: is the model doing *the right thing*?
Is the model chosen suitable/accurate for the desired purpose?

a. Plausibility check: (also called functional-validation)
- Test the model under extreme conditions
- Test the model under simplified conditions

b. Calibration: comparison with measurements, refining input parameters

Verfication: is the model doing *the right thing **right***?
Is the model is **correctly implemented** with respect to the conceptual model? Is its syntax logically acceptable?
e.g. extrapolating outside validity range...

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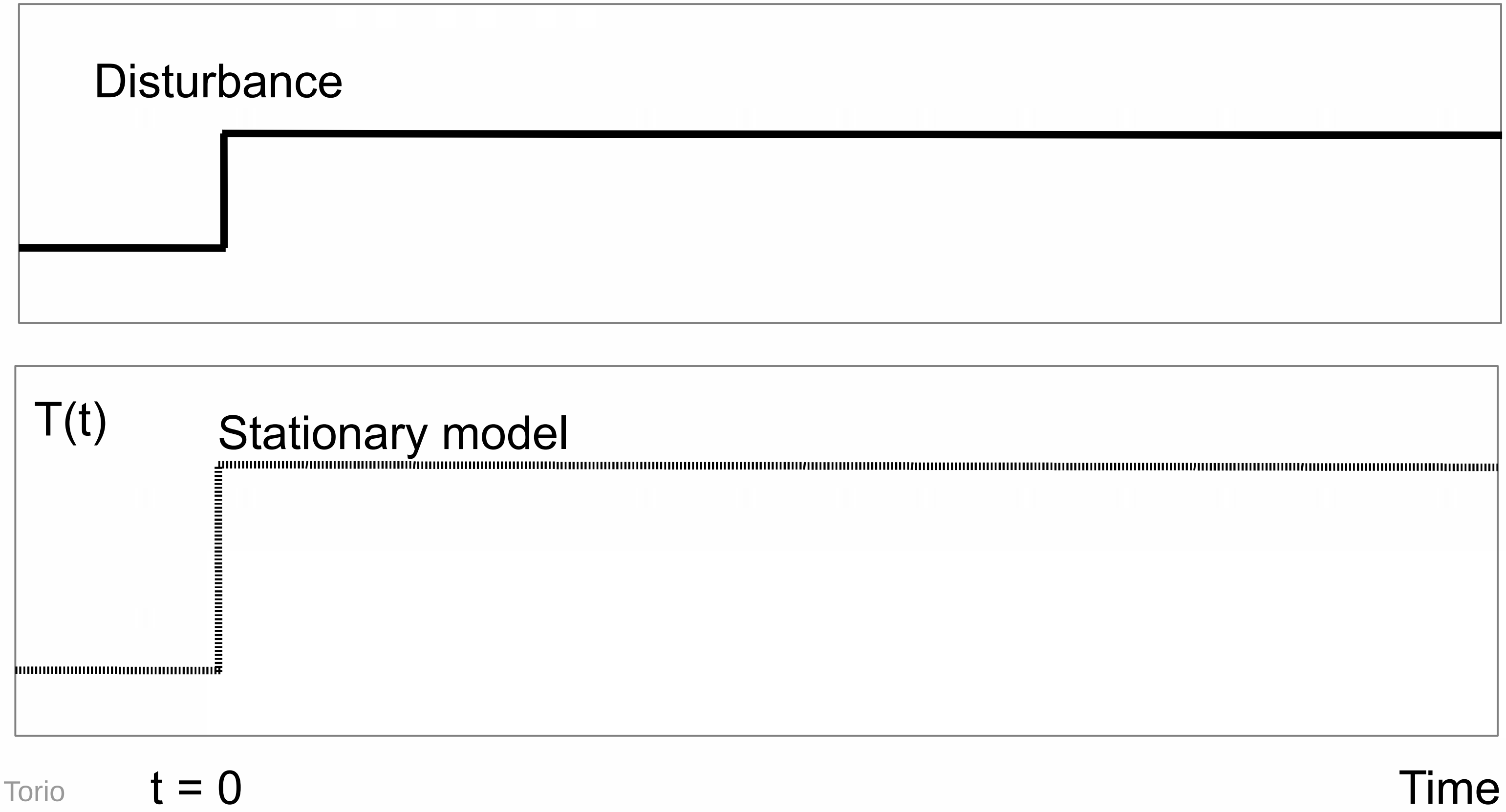
Stationary models

- no time dependency: „instantaneous adaptation“
- only equilibrium states



Stationary models

Examples of stationary models:

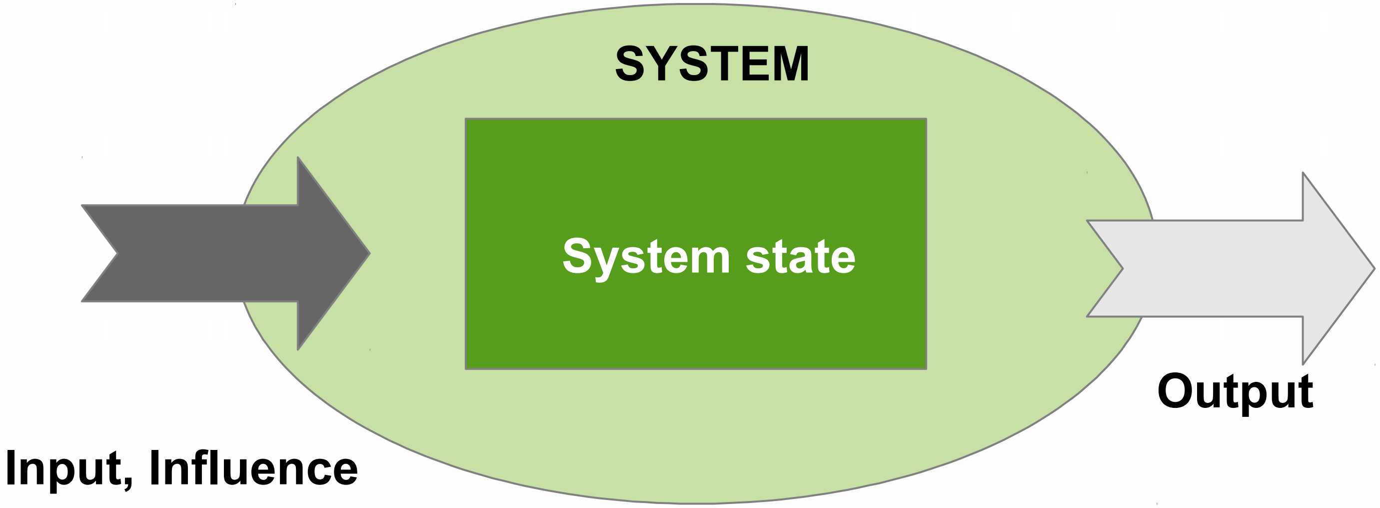
- Ohm's law $$U = I \cdot R$$

- Ideal gas law $$p \cdot V = n \cdot r \cdot T$$

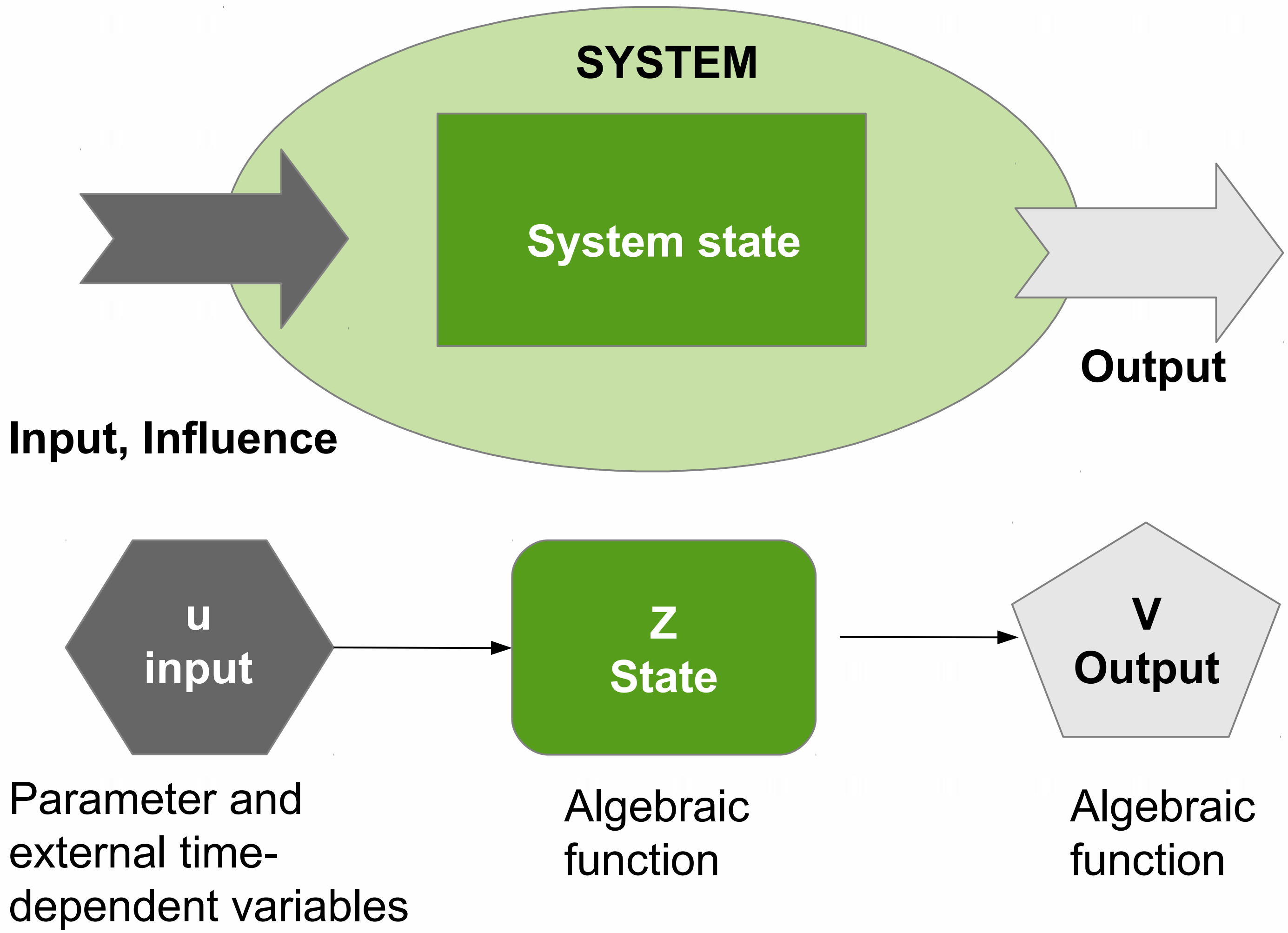
- „The“ collector equation

$$Q = A \cdot \left[G \cdot \eta_0 - a_1 \cdot (\theta_m - \theta_a) - a_2 \cdot (\theta_m - \theta_a)^2 \right]$$

Stationary models



Stationary models



Stationary models

„The“ collector equation:

$$\eta = \frac{1}{G}\cdot\left[G\cdot\eta_0 - a_1\cdot(T_m - T_a) - a_2\cdot(T_m - T_a)^2\right]$$

$$Q_{out} = A\cdot\left[G\cdot\eta_0 - a_1\cdot(T_m - T_a) - a_2\cdot(T_m - T_a)^2\right]$$

T_a [K]
ambient temperature

T_m [K]
mean fluid temperature

A [m^2]
Aperture area

η_0 [K] = $F'(\alpha\tau)_{en}$
zero-loss efficiency

a_1 [$Wm^{-2}K^{-1}$]
linear coefficient for heat losses

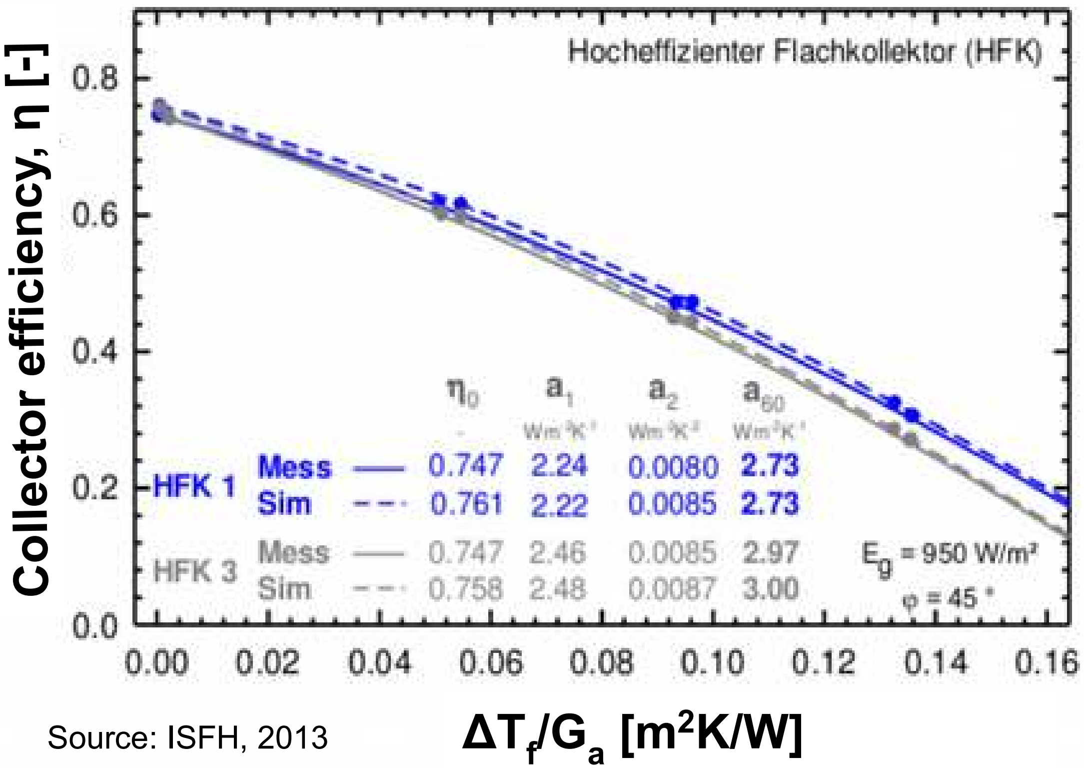
a_2 [$Wm^{-2}K^{-2}$]
cuadratic coefficient for heat losses

Stationary models

„The“ collector equation:

$$\eta = \frac{1}{G} \cdot \left[G \cdot \eta_0 - a_1 \cdot (T_m - T_a) - a_2 \cdot (T_m - T_a)^2 \right]$$

$$\dot{Q}_{out} = A \cdot \left[G \cdot \eta_0 - a_1 \cdot (T_m - T_a) - a_2 \cdot (T_m - T_a)^2 \right]$$



Source: ISFH, 2013

a_1 $[Wm^{-2}K^{-1}]$
linear coefficient for heat losses

a_2 $[Wm^{-2}K^{-2}]$
cuadratic coefficient for heat losses

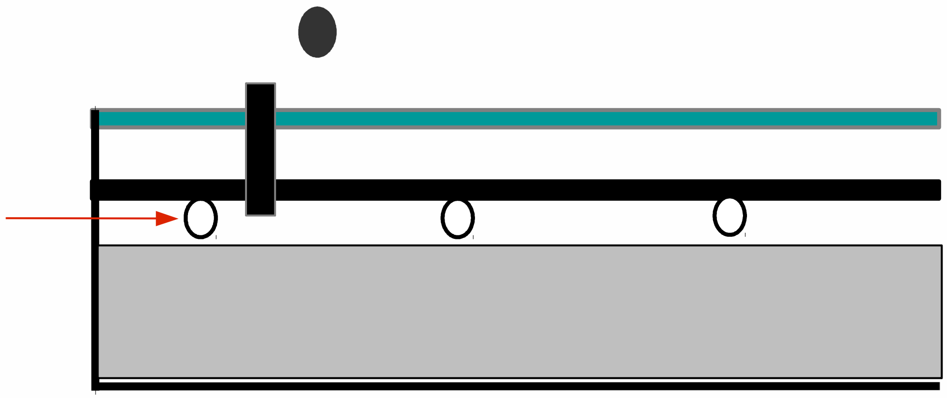
a_{60} $[Wm^{-2}K^{-1}]$
effective coefficient for heat losses at 60°C temperature difference

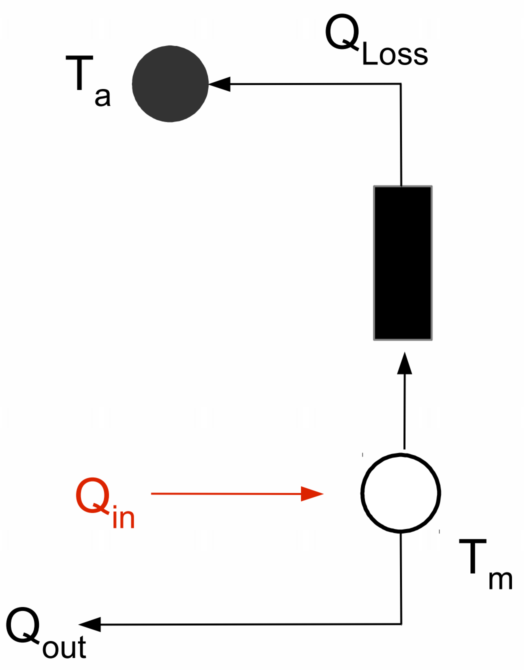
Stationary models

„The“ collector equation:

$$\eta = \frac{1}{G} \cdot \left[G \cdot \eta_0 - a_1 \cdot (T_m - T_a) - a_2 \cdot (T_m - T_a)^2 \right]$$

$$\dot{Q}_{out} = A \cdot \left[G \cdot \eta_0 - a_1 \cdot (T_m - T_a) - a_2 \cdot (T_m - T_a)^2 \right]$$



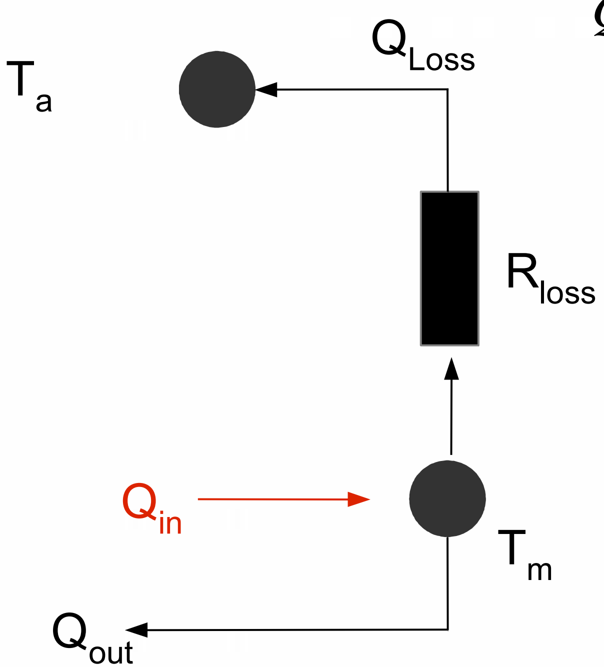


Stationary models

$$\eta = \frac{1}{G} \cdot \left[G \cdot \eta_0 - a_1 \cdot (T_m - T_a) - a_2 \cdot (T_m - T_a)^2 \right]$$

„The“ collector equation:

$$\dot{Q}_{out} = A \cdot \left[G \cdot \eta_0 - a_1 \cdot (T_m - T_a) - a_2 \cdot (T_m - T_a)^2 \right]$$



What type of model is it?
Lets give the child a name:

One dimensional *(only one system variable)*
stationary *(no dynamic processes)*
boxmodel *(not differentiated in space)*

also called

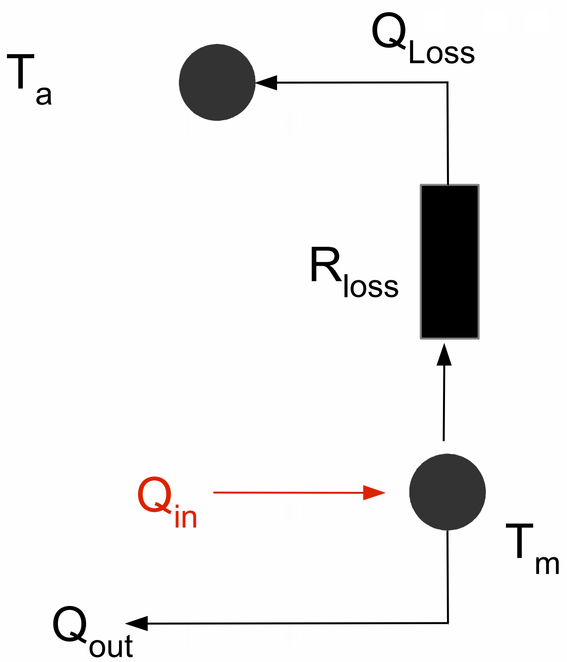
One-node (stationary) collector model

Stationary models

$$\eta = \frac{1}{G} \cdot \left[G \cdot \eta_0 - a_1 \cdot (T_m - T_a) - a_2 \cdot (T_m - T_a)^2 \right]$$

„The“ collector equation:

$$\dot{Q}_{out} = A \cdot \left[G \cdot \eta_0 - a_1 \cdot (T_m - T_a) - a_2 \cdot (T_m - T_a)^2 \right]$$



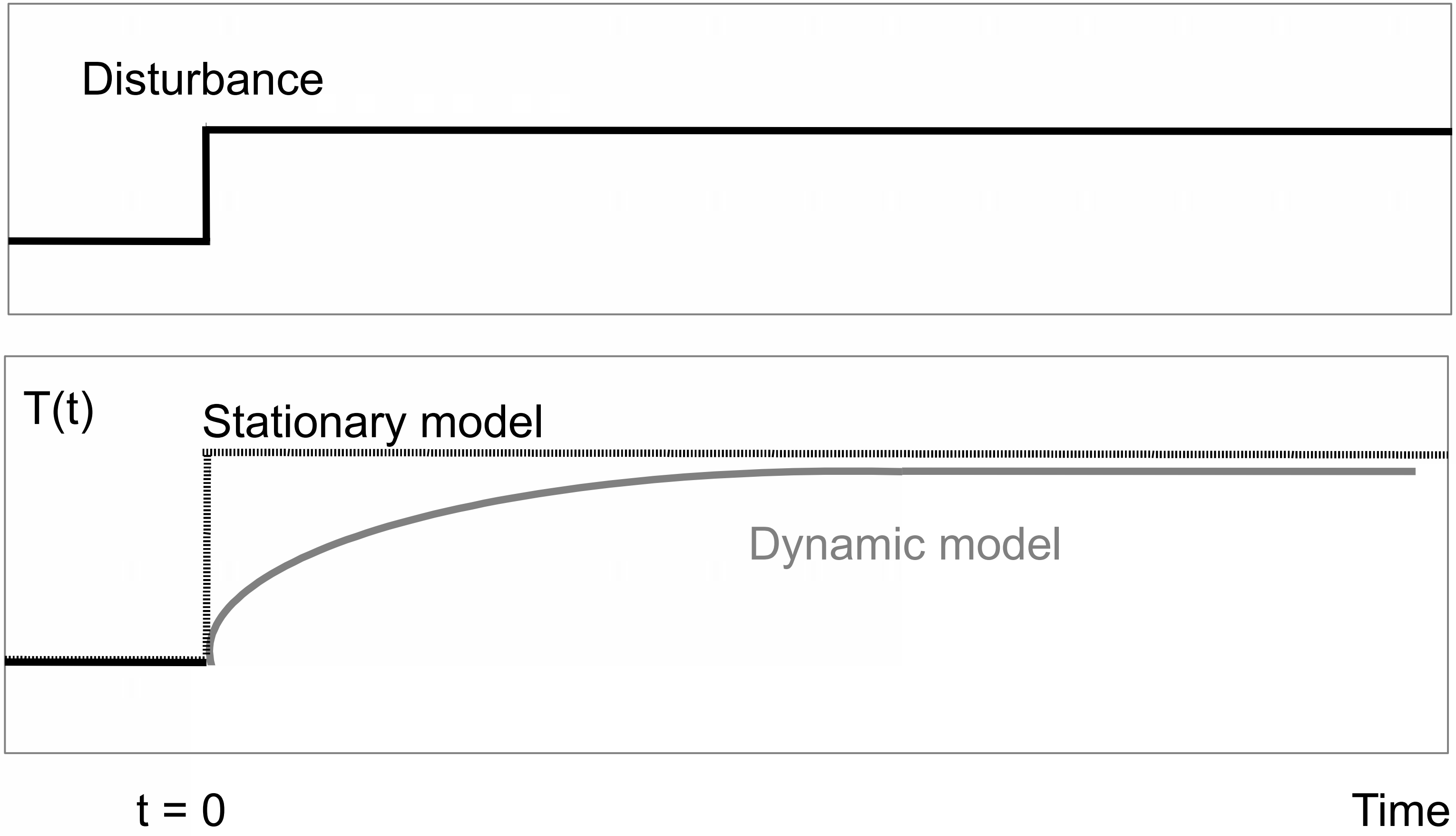
General equation form
for one dimensional box stationary models:

$$S_i = p_i \cdot \varsigma_i$$

S_i , system variable: $T_{m,}$
ς_i, outside relation/influence: G, T_a
p_i, system parameters: η_0, a_1, a_2

Main types of models

- Stationary / dynamic



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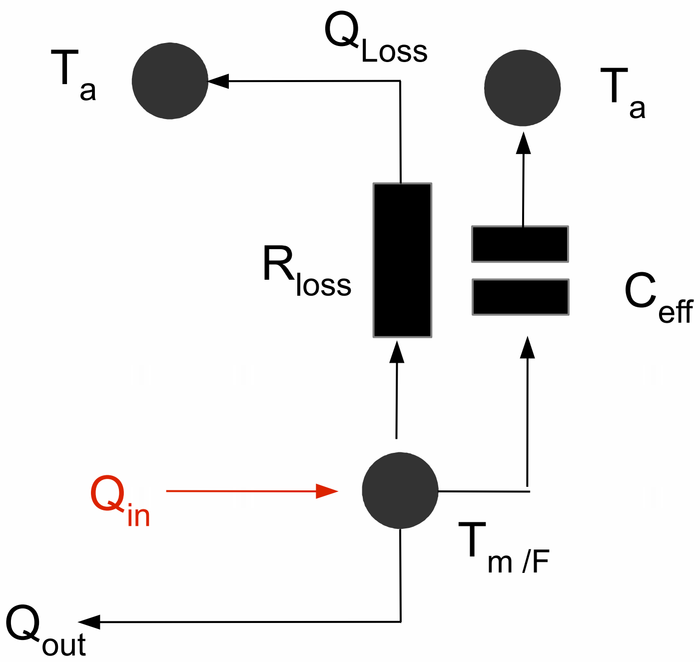
Dynamic models – One node

Stationary $\dot{Q}_{\text{out}} = A \cdot \left[G \cdot \eta_0 - a_1 \cdot (T_m - T_a) - a_2 \cdot (T_m - T_a)^2 \right]$

Dynamic

$$C_{eff} \cdot \frac{dT_m}{dt} = A \cdot \left[G \cdot \eta_0 - a_1 \cdot (T_m - T_a) - a_2 \cdot (T_m - T_a)^2 \right] - \dot{m}_F \cdot c_F \cdot (T_{Fout} - T_{Fin})$$



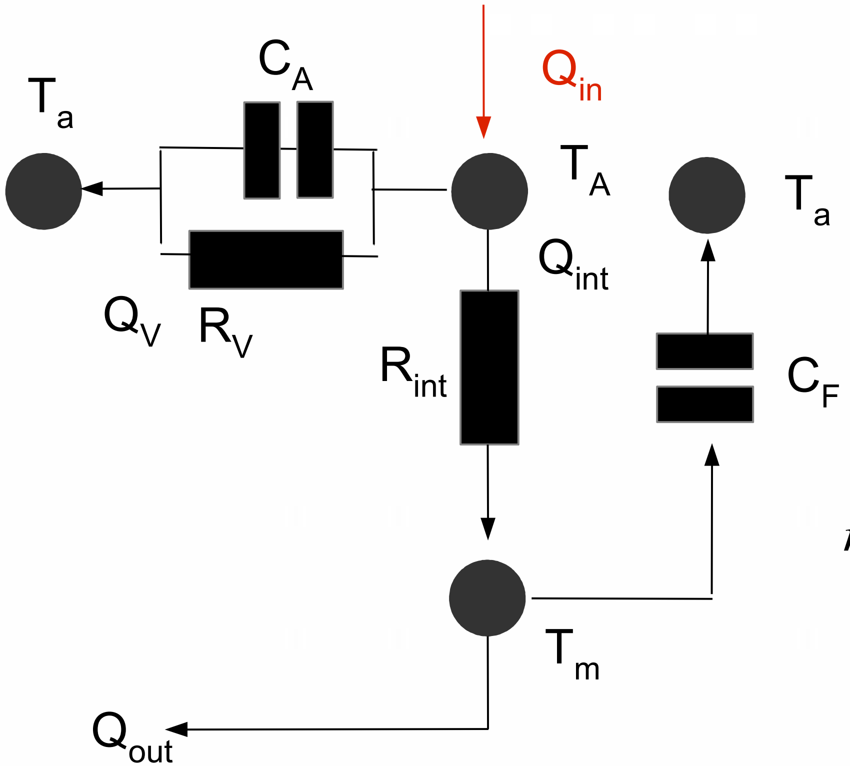
$$\dot{Q}_{\text{in}} = \dot{Q}_{loss} + \dot{Q}_{out} + \dot{Q}_{eff}$$

$$\dot{Q}_{\text{eff}} = \dot{Q}_{in} - \dot{Q}_{loss} - \dot{Q}_{out}$$

Dynamic models – Two nodes

Dynamic + multi-dimensional

Two node collector model



Temperature node: Absorber

$$\dot{Q}_{\text{in}} = \dot{Q}_{int} + \dot{Q}_V + \dot{Q}_A$$

$$\dot{Q}_{\text{in}} - \dot{Q}_{\text{int}} - \dot{Q}_V = m_A \cdot c_A \cdot \left(\frac{dT_A}{dt}\right)$$

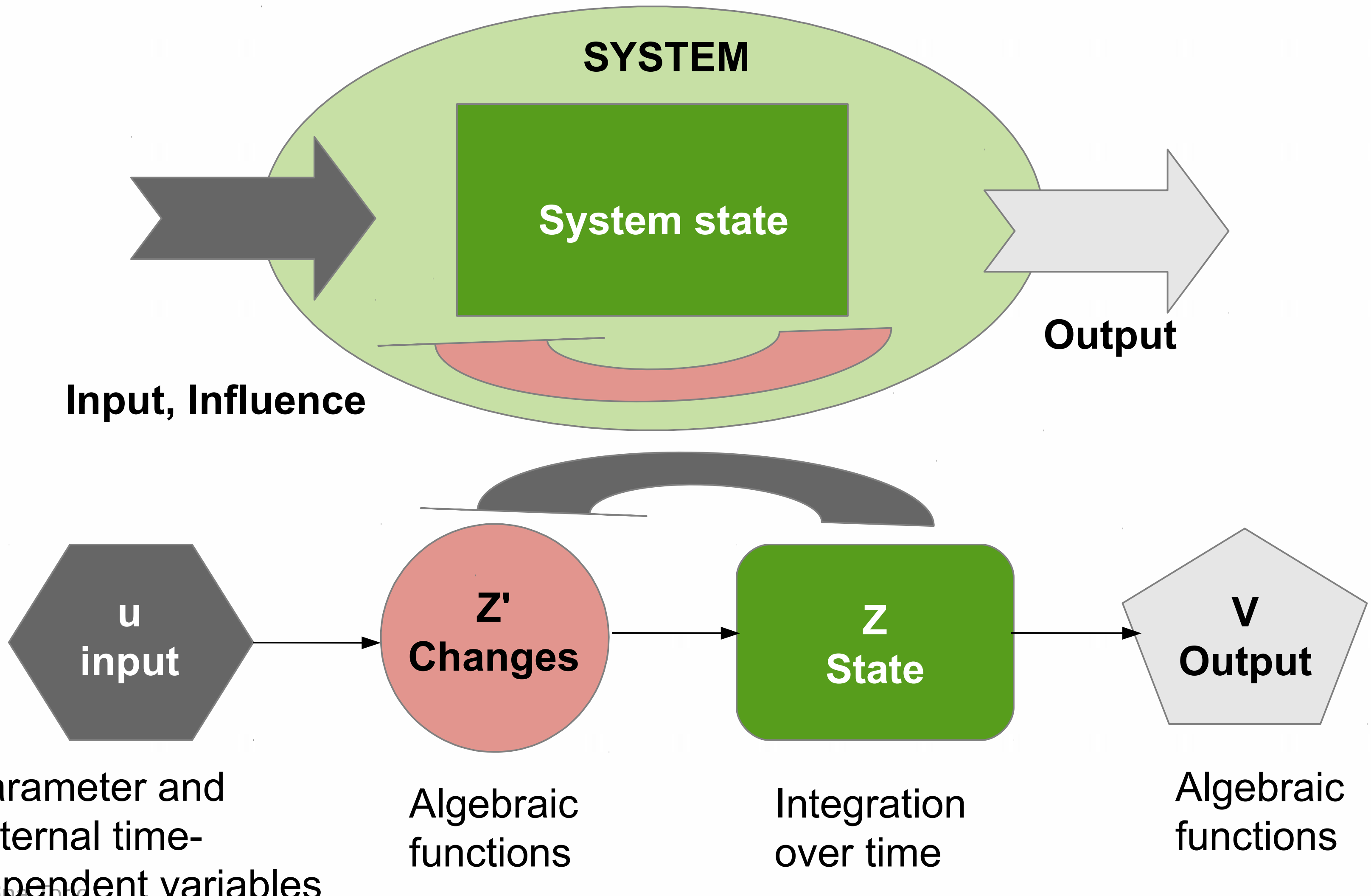
$$\dot{Q}_{in} = A \cdot G \cdot \eta_{opt}$$

Temperature node: Fluid

$$m_F \cdot c_F \cdot \left(\frac{dT_F}{dt}\right) = \dot{Q}_{int} - \dot{m}_F \cdot c_F \cdot (T_{Fout} - T_{Fin})$$

$$\dot{Q}_{\text{int}} = \frac{1}{R_{\text{int}}} \cdot (T_A - T_m) = U_{int} \cdot A_A \cdot (T_A - T_m)$$

Dynamic models



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Dynamic models – Two nodes

Dynamic

General: System of differential equations

Analytic solutions: only for linear systems and for *some* non-linear systems!!

Otherwise: **Numeric integration methods!**

- **Euler-Cauchy**
- **Runge-Kutta**

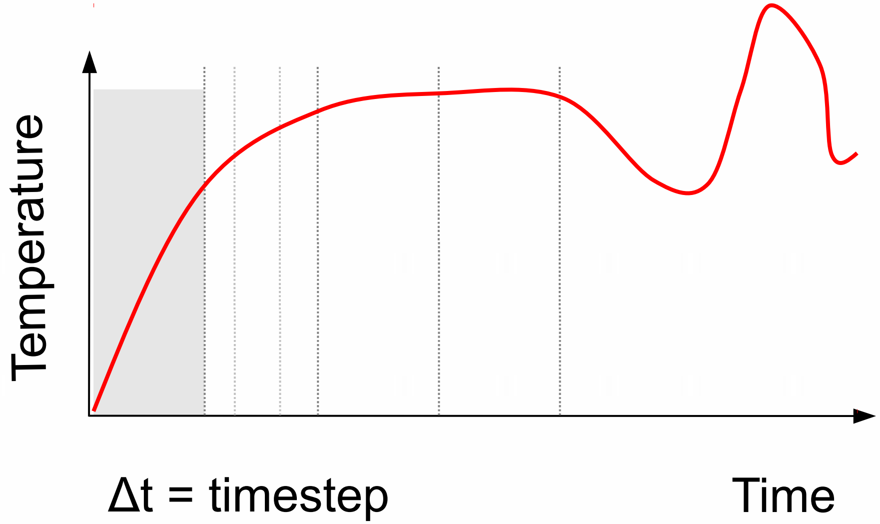
Differential equations

Dynamic – one node

$$C_{eff} \cdot \frac{dT_m}{dt} = A \cdot \left[G \cdot \eta_0 - a_1 \cdot (T_m - T_a) - a_2 \cdot (T_m - T_a)^2 \right] - \dot{m}_F \cdot c_F \cdot (T_{Fout} - T_{Fin})$$

Instead of derivative (analytical) solutions...

...”**approximative methods”: Discretization**



Dynamic models

Numeric integration methods

Discretization

$$\frac{dz}{dt} = \frac{z_{k+1} - z_k}{\Delta t}$$

Euler Cauchy

$$\frac{dz}{dt} = f\left(z,t\right)$$

$$z\left(t+\Delta t\right) = z\left(t\right) + f\left(z,t\right)\cdot \Delta t$$

Runge-Kutta

$$k_1 = f\left(z,t\right)\cdot\Delta t$$

$$k_2 = f\left(z + k_1\cdot\frac{1}{2}, t + \frac{\Delta t}{2}\right)\Delta t$$

$$k_3 = f\left(z + k_2\cdot\frac{1}{2}, t + \frac{\Delta t}{2}\right)\Delta t$$

$$k_4 = f\left(z + k_3\cdot 1, t + \Delta t\right)\cdot\Delta t$$

$$z\left(t+\Delta t\right) = z\left(t\right) + \frac{k_1 + 2\cdot k_2 + 2\cdot k_3 + k_4}{6}$$

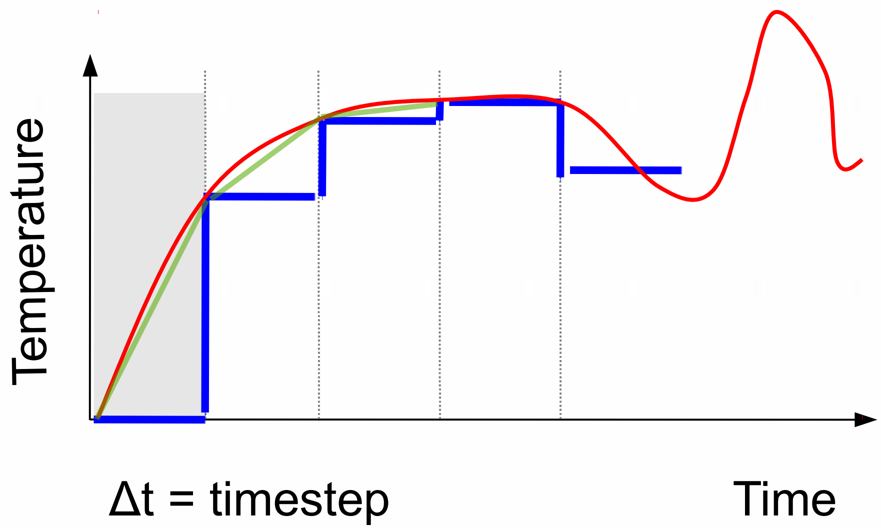
Dynamic models

Numeric integration methods

Euler Cauchy

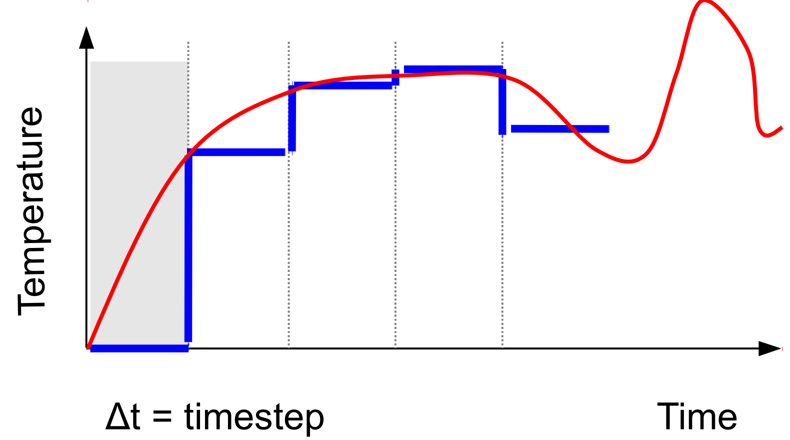
$$\frac{dz}{dt} = f\left(z,t\right)$$

$$z\left(t+\Delta t\right) = z\left(t\right) + f\left(z,t\right)\cdot \Delta t$$



Dynamic models

Numeric integration methods



Runge Kutta

$$z(t+\Delta t) = z(t) + \frac{k_1 + 2\cdot k_2 + 2\cdot k_3 + k_4}{6}$$

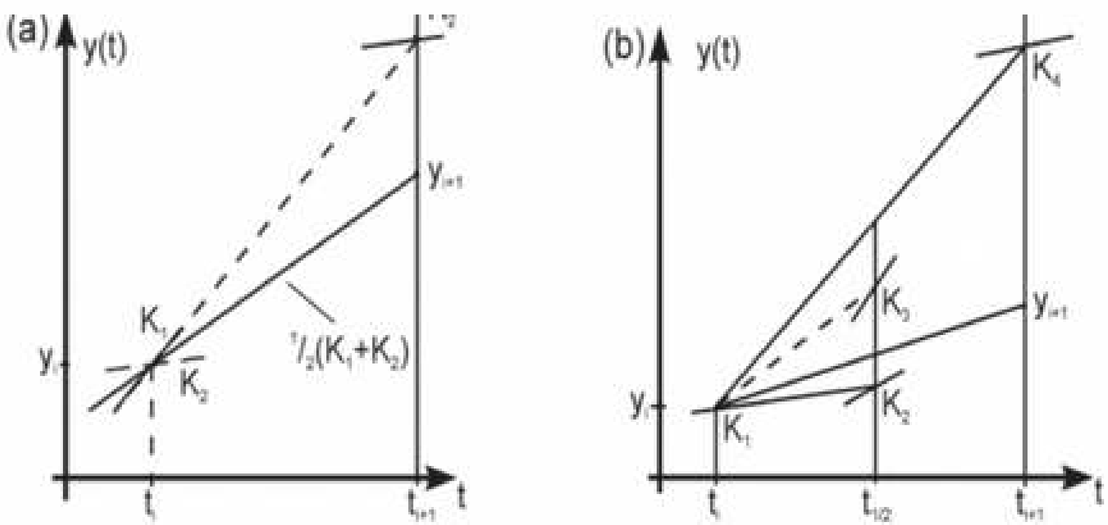


Abb. 19.4. a) Prädiktor-Korrektor-Verfahren b) Runge-Kutta-Verfahren

$$k_1 = f(z,t)\cdot\Delta t$$

$$k_2 = f\left(z + k_1\cdot\frac{1}{2}, t + \frac{\Delta t}{2}\right)\Delta t$$

$$k_3 = f\left(z + k_2\cdot\frac{1}{2}, t + \frac{\Delta t}{2}\right)\Delta t$$

$$k_4 = f(z + k_3\cdot 1, t + \Delta t)\cdot\Delta t$$

Source: Westermann

Dynamic models

Numeric integration methods

$$C_{eff} = A \cdot c_{eff}$$

$$\dot{m}_F = A \cdot m$$

Trial: Euler Cauchy and Runge-Kutta go Octave/Matlab/Excel!

$$C_{eff} \cdot \frac{dT_m}{dt} = A \cdot \left[G \cdot \eta_0 - a_1 \cdot (T_m - T_a) - a_2 \cdot (T_m - T_a)^2 \right] - \dot{m}_F \cdot c_F \cdot (T_{F,out} - T_{F,in})$$

Parameter	Value	Unit
η_0	0,651	-
a_1	1,631	$Wm^{-2}K^{-1}$
a_2	0,0096	$Wm^{-2}K^{-1}$
c_{eff}	44030	$Jm^{-2}K^{-1}$
A	1,33	m^2
m	10	$Kgh^{-1}m^{-2}$
T_a	5	°C

Parameter	Value	Unit
G	700	Wm^{-2}
$T_{F,in} = T_{F,out,0}$	10	°C
c_F	4180	$Jkg^{-1}K^{-1}$
Timestep	1	second
	1	minute
	10	minutes
	1	hour

Dynamic models

Assignment I: Numeric integration methods

a) For the parameters and (initial) conditions given on the previous slide calculate the outlet temperature of the collector using the Euler-Cauchy AND Runge-Kutta numerical methods for the following times and timesteps:

Timestep:	**Time:**
1 second	1 minute
1 minute	1 minute
	10 minutes
10 minutes	10 minutes
	60 minutes
30 minutes	30 minutes
	60 minutes

b) Calculate for the same parameters (taking into account only those applicable in the steady-state equation) the steady state collector outlet temperature

c) Draw main conclussions from your results: e.g. for which purpose(s) would a steady-state approach be justified? which numeric integration method could be used under which conditions?

References

- Stafell I. et al., 2012. A review of domestic heat pumps. Energy and Environmental Science, 2012,5, 9291-9306, DOI: 10.1039/C2EE22653G

- ISFH, 2013. Abschlussbericht zum Vorhaben Hocheffiziente Flachkollektoren mit selektiv beschichteten Zweischeibenverglasungen. 2013.

- Fischer, Müller-Steinberger. Einführung eines 2-Knotenmodells zur besseren Beschreibung der thermischen Leistungsfähigkeit von Sonnenkollektoren. OTTI - 16. Symposium Thermische Solarenergie, 2006 – Themenschwerpunkt Simulation und Planungswerkzeuge

- Westermann. Mathematik für Ingenieure – Ein anwendungsorientiertes Lehrbuch. Springer Verlag, 2011.