

Boolean Minimization

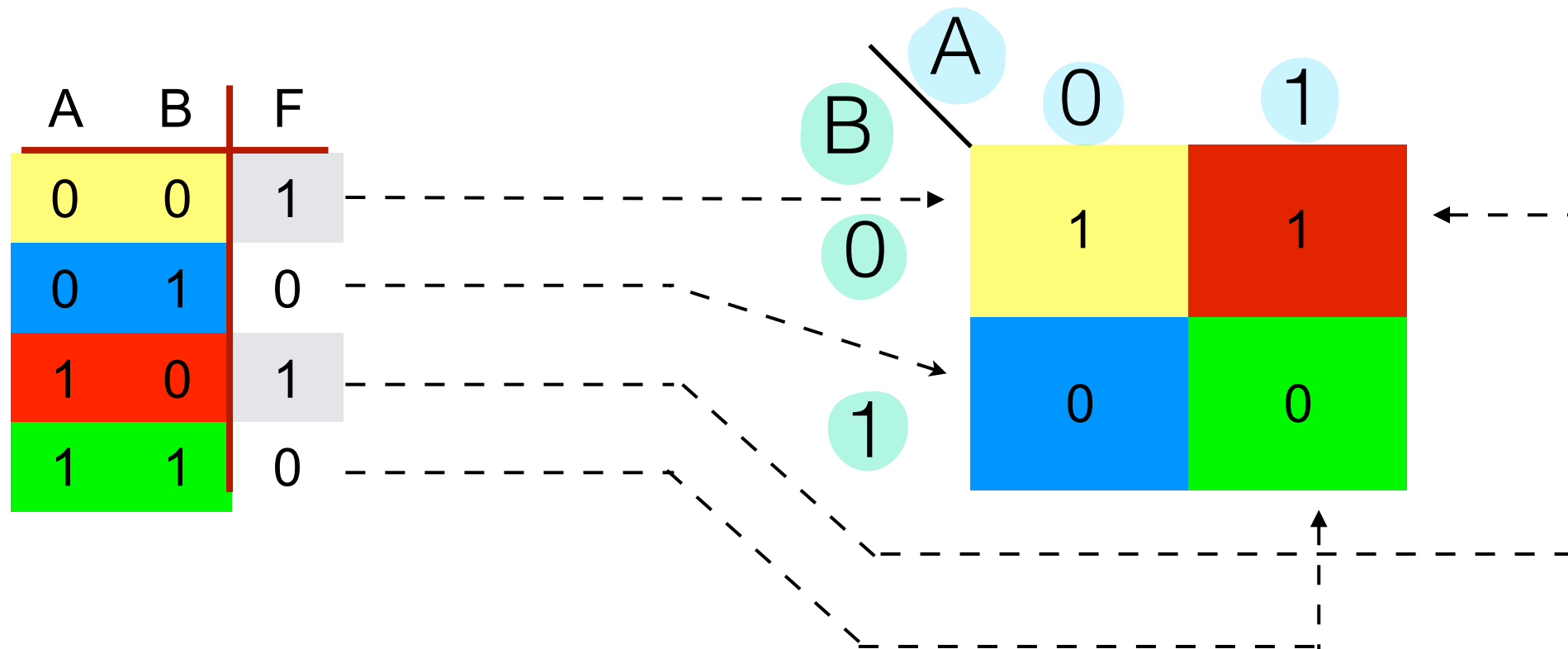
Lecture 4

Outline

- Karnaugh-map
- Circuit design

Karnaugh Map

- Karnaugh Map (K-map) อาศัยการจัดเรียงตารางค่าความจริง ในรูปแบบใหม่ ซึ่งทำให้สามารถลดรูปโดยอาศัยคุณสมบัติการคอมพลิเมนต์ได้ง่ายขึ้น
- ตัวอย่างของ K-map แบบ 2 ตัวแปร



Examples of K-Map

		AB			
		00	01	11	10
C	0				
	1				

3-variable K-map

		AB			
		00	01	11	10
CD	00				
	01				
	11				
	10				

4-variable K-map

Using K-Map

- หาช่องใน K-map ที่มีลอจิก 1 ติดกัน
- จำนวนช่องที่ใช้ลดรูปต้องมีขนาด 2, 4, 8, ... เท่านั้น
- ตัวอย่าง

A	B	F
0	0	1
0	1	0
1	0	1
1	1	0

		A	
		0	1
B	0	1	1
	1	0	0

$$F = \overline{B}$$

Using K-Map (2)

- ตัวอย่าง

A	B	C	F
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

		AB			
C		00	01	11	10
	0	0	0	1	1
	1	0	1	1	1

$$F = A + BC$$

Example

A	B	C	F
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

		AB			
		00	01	11	10
C	0	0	0	1	0
	1	0	1	1	1

$$F = BC + AB + AC$$

Example (2)

A	B	C	F
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	1

		AB			
		00	01	11	10
C	0	1	0	0	1
	1	0	0	1	1

$$F = AC + \bar{B}\bar{C}$$

Example (3)

A	B	C	D	F
0	0	0	0	1
0	0	0	1	1
0	0	1	0	0
0	0	1	1	1
0	1	0	0	0
0	1	0	1	1
0	1	1	0	0
0	1	1	1	0
1	0	0	0	1
1	0	0	1	1
1	0	1	0	0
1	0	1	1	0
1	1	0	0	0
1	1	0	1	1
1	1	1	0	0
1	1	1	1	0

		AB			
	CD	00	01	11	10
00		1	0	0	1
01		1	1	1	1
11		1	0	0	0
10		0	0	0	0

$$F = \bar{C}D + \bar{B}\bar{C} + \bar{A}BD$$

Example (4)

A	B	C	D	F
0	0	0	0	1
0	0	0	1	0
0	0	1	0	1
0	0	1	1	1
0	1	0	0	0
0	1	0	1	1
0	1	1	0	1
0	1	1	1	1
1	0	0	0	1
1	0	0	1	0
1	0	1	0	1
1	0	1	1	1
1	1	0	0	0
1	1	0	1	0
1	1	1	0	1
1	1	1	1	1

		AB			
CD		00	01	11	10
		00	01	11	10
00		1	0	0	1
01		0	1	0	0
11		1	1	1	1
10		1	1	1	1

$$F = C + \overline{B}\overline{D} + \overline{A}BD$$

Don't Cares

- การใช้ประโยชน์จาก don't cares
- ตัวอย่าง

A	B	C	F
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	x
1	1	1	x

don't cares

C \ AB				
	00	01	11	10
0	0	0	x	1
1	0	1	x	0

$$F = BC + A\overline{C}$$

Example (5)

A	B	C	F
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	x
1	0	0	x
1	0	1	1
1	1	0	0
1	1	1	1

		AB			
		00	01	11	10
C	0	1	0	0	x
	1	0	x	1	1

$$F = AC + \overline{B}\overline{C}$$

Example (6)

A	B	C	D	F
0	0	0	0	1
0	0	0	1	1
0	0	1	0	1
0	0	1	1	0
0	1	0	0	x
0	1	0	1	1
0	1	1	0	1
0	1	1	1	1
1	0	0	0	1
1	0	0	1	0
1	0	1	0	0
1	0	1	1	1
1	1	0	0	x
1	1	0	1	x
1	1	1	0	x
1	1	1	1	x

		AB			
		00	01	11	10
CD	00	1	x	x	1
	01	1	1	x	0
	11	0	1	x	1
	10	1	1	x	0

$$F = B + \overline{C}\overline{D} + ACD + \overline{A}\overline{C} + \overline{A}\overline{D}$$

Circuit Design

- ตัวอย่าง: ออกแบบ Two-bit Comparator

- Input Spec:

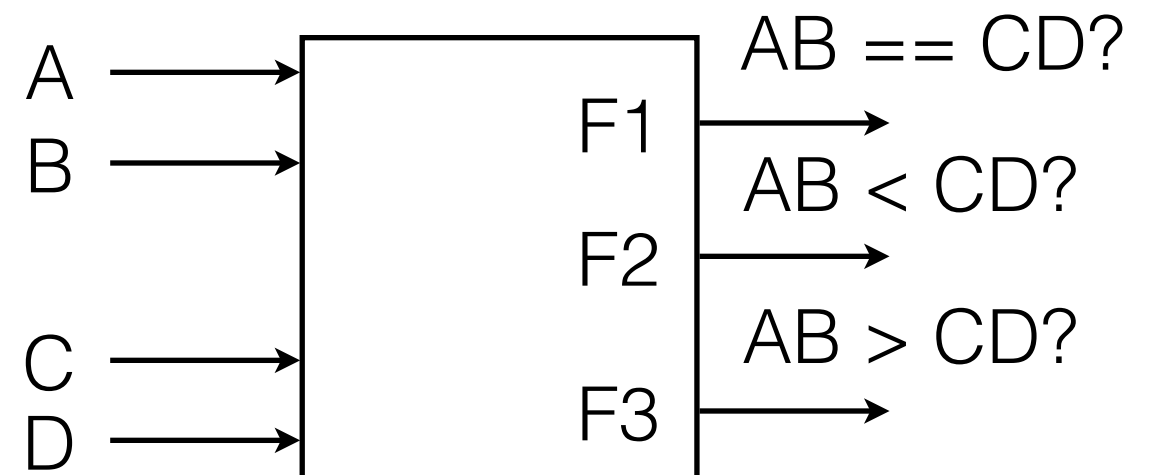
- 2-bit input จำนวน 2 ตัว (AB, CD)

- Output Spec:

- F1 เช็คว่า $AB == CD$

- F2 เช็คว่า $AB < CD$

- F3 เช็คว่า $AB > CD$



Two-bit Comparator

A	B	C	D	F1	F2	F3
0	0	0	0	1		
0	0	0	1	0		
0	0	1	0	0		
0	0	1	1	0		
0	1	0	0	0		
0	1	0	1	1		
0	1	1	0	0		
0	1	1	1	0		
1	0	0	0	0		
1	0	0	1	0		
1	0	1	0	1		
1	0	1	1	0		
1	1	0	0	0		
1	1	0	1	0		
1	1	1	0	0		
1	1	1	1	1		

		AB			
CD		00	01	11	10
00		1	0	0	0
01		0	1	0	0
11		0	0	1	0
10		0	0	0	1

$$F_1 = \bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}\bar{B}\bar{C}D + ABCD + \bar{A}BC\bar{D}$$

Two-bit Comparator (2)

A	B	C	D	F1	F2	F3
0	0	0	0		0	
0	0	0	1		1	
0	0	1	0		1	
0	0	1	1		1	
0	1	0	0		0	
0	1	0	1		0	
0	1	1	0		1	
0	1	1	1		1	
1	0	0	0		0	
1	0	0	1		0	
1	0	1	0		0	
1	0	1	1		1	
1	1	0	0		0	
1	1	0	1		0	
1	1	1	0		0	
1	1	1	1		0	

CD \ AB				
	00	01	11	10
00	0	0	0	0
01	1	0	0	0
11	1	1	0	1
10	1	1	0	0

$AB < CD?$

$$F_2 =$$

Two-bit Comparator (3)

A	B	C	D	F1	F2	F3
0	0	0	0			
0	0	0	1			
0	0	1	0			
0	0	1	1			
0	1	0	0			
0	1	0	1			
0	1	1	0			
0	1	1	1			
1	0	0	0			
1	0	0	1			
1	0	1	0			
1	0	1	1			
1	1	0	0			
1	1	0	1			
1	1	1	0			
1	1	1	1			

		AB			
CD		00	01	11	10
	00				
	01				
	11				
	10				

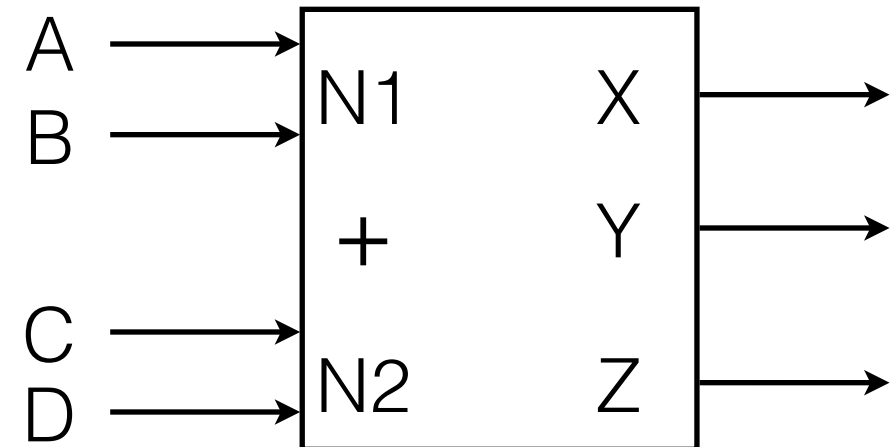
$$F_3 =$$

Two-Bit Comparator (4)

- Schematic Diagram

Two-Bit Binary Addder

- ตัวอย่าง ออกแบบ Two-bit Binary Addder
- Input Spec
 - 2-bit input จำนวน 2 ตัว (AB, CD)
- Output Spec
 - 3-bit binary (XYZ)



Two-bit Binary Adder (2)

A	B	C	D	X	Y	Z
0	0	0	0	0	0	0
0	0	0	1	0	0	1
0	0	1	0	0	1	0
0	0	1	1	0	1	1
0	1	0	0	0	0	1
0	1	0	1	0	1	0
0	1	1	0	0	1	1
0	1	1	1	1	0	0
1	0	0	0	0	1	0
1	0	0	1	0	1	1
1	0	1	0	1	0	0
1	0	1	1	1	0	1
1	1	0	0	0	1	1
1	1	0	1	1	0	0
1	1	1	0	1	0	1
1	1	1	1	1	1	0

AB \ CD		00	01	11	10
CD	00	0	0	0	0
	01	0	0	1	0
	11	0	0	1	1
	10	0	1	1	1

$$X = AC + ABD + B\bar{C}\bar{D}$$

Two-bit Binary Adder (3)

A	B	C	D	X	Y	Z
0	0	0	0			
0	0	0	1			
0	0	1	0			
0	0	1	1			
0	1	0	0			
0	1	0	1			
0	1	1	0			
0	1	1	1			
1	0	0	0			
1	0	0	1			
1	0	1	0			
1	0	1	1			
1	1	0	0			
1	1	0	1			
1	1	1	0			
1	1	1	1			

CD \ AB	00	01	11	10
00				
01				
11				
10				

$Y =$

Two-bit Binary Adder (4)

A	B	C	D	X	Y	Z
0	0	0	0			
0	0	0	1			
0	0	1	0			
0	0	1	1			
0	1	0	0			
0	1	0	1			
0	1	1	0			
0	1	1	1			
1	0	0	0			
1	0	0	1			
1	0	1	0			
1	0	1	1			
1	1	0	0			
1	1	0	1			
1	1	1	0			
1	1	1	1			

		AB			
CD		00	01	11	10
	00				
	01				
	11				
	10				

$Z =$

5-Variable K-map

A	B	C	D	E	F	G
0	0	0	0	0	0	1
0	0	0	0	1	1	0
0	0	0	1	0	1	0
0	0	0	1	1	0	0
0	0	1	0	0	1	x
0	0	1	0	1	1	x
0	0	1	1	0	1	1
0	0	1	1	1	0	1
0	1	0	0	0	0	1
0	1	0	0	1	1	0
0	1	0	1	0	1	1
0	1	0	1	1	1	1
0	1	1	0	0	0	x
0	1	1	0	1	1	1
0	1	1	1	0	1	x
0	1	1	1	1	1	x

A	B	C	D	E	F	G
1	0	0	0	0	0	1
1	0	0	0	1	1	0
1	0	0	1	0	0	0
1	0	0	1	1	0	1
1	0	1	0	0	0	1
1	0	1	0	1	1	1
1	0	1	1	0	0	1
1	0	1	1	1	0	1
1	1	0	0	0	0	1
1	1	0	0	1	1	1
1	1	0	1	0	1	1
1	1	0	1	1	1	1
1	1	1	0	0	0	x
1	1	1	0	1	1	1
1	1	1	1	0	0	1
1	1	1	1	1	1	1

$A = 0$

DE \ BC	00	01	11	10
00	0	1	0	0
01	1	1	1	1
11	0	0	1	1
10	1	1	1	1

$A = 1$

DE \ BC	00	01	11	10
00	0	0	0	0
01	1	1	1	1
11	0	0	1	1
10	0	0	0	1

$$F = \overline{D}E + BE + B\overline{C}D + \overline{A}D\overline{E} + \overline{A}\overline{B}C\overline{D}$$

5-Variable K-map (2)

A	B	C	D	E	F	G
0	0	0	0	0	0	1
0	0	0	0	1	1	0
0	0	0	1	0	1	0
0	0	0	1	1	0	0
0	0	1	0	0	1	x
0	0	1	0	1	1	x
0	0	1	1	0	1	1
0	0	1	1	1	0	1
0	1	0	0	0	0	1
0	1	0	0	1	1	0
0	1	0	1	0	1	1
0	1	0	1	1	1	1
0	1	1	0	0	0	x
0	1	1	0	1	1	1
0	1	1	1	0	1	x
0	1	1	1	1	1	x

A	B	C	D	E	F	G
1	0	0	0	0	0	1
1	0	0	0	1	1	0
1	0	0	1	0	0	0
1	0	0	1	1	0	1
1	0	1	0	0	0	1
1	0	1	0	1	1	1
1	0	1	1	0	0	1
1	0	1	1	1	0	1
1	1	0	0	0	0	1
1	1	0	0	1	1	1
1	1	0	1	0	1	1
1	1	0	1	1	1	1
1	1	1	0	0	0	x
1	1	1	0	1	1	1
1	1	1	1	0	0	1
1	1	1	1	1	1	1

$A = 0$

		BC			
DE		00	01	11	10
		1	x	x	1
01	00	0	x	1	0
11	00	0	1	x	1
10	00	0	1	x	1

$A = 1$

		BC			
DE		00	01	11	10
		1	1	x	1
01	00	0	1	1	1
11	00	0	1	1	1
10	00	1	1	1	1

$$F = C + AB + \overline{D}\overline{E} + A\overline{E} + BD$$