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1 Setting

1.1 vimrc

```
set nocp ai si nu et bs=2 mouse=a
set ts=4 sts=4 sw=4 hls showmatch
set ruler rulerformat=%17.(%1:%c%)
set noswapfile autoread wildmenu wildmode=list:longest
syntax on | colorscheme evening

map <F5> <ESC>:w<CR>:!g++ -g -Wall --std=c++0x -O2 %:r.cpp -o %:r && %:r < %:r.
in > %:r.out<CR>
map <F6> <ESC>:w<CR>:!g++ -g -Wall --std=c++0x -O2 %:r.cpp -o %:r && %:r < %:r.
in<CR>

map <C-t> :tabnew<CR>
```

```
command -nargs=1 PS :cd d:/ | :vi <args>.cpp | vs <args>.in | sp <args>.out
```

2 Math

2.1 Basic Arithmetic

```
// calculate a*b % m
// x86-64 only
ll large_mod_mul(ll a, ll b, ll m) {
    return ll((__int128)a*(__int128)b%m);
}

// calculate a*b % m
// |m| < 2^62, x86 available
// O(logb)
ll large_mod_mul(ll a, ll b, ll m) {
    a %= m; b %= m; ll r = 0, v = a;
    while (b) {
        if (b & 1) {
            r = r + v;
            if (r >= m) r -= m;
        }
        b >>= 1;
        v <<= 1; if (v >= m) v -= m;
    }
    return r;
}

// calculate n^k % m
ll modpow(ll n, ll k, ll m) {
    ll ret = 1;
    n %= m;
    while (k) {
        if (k & 1) ret = large_mod_mul(ret, n, m);
        n = large_mod_mul(n, n, m);
        k /= 2;
    }
    return ret;
}

// find a pair (c, d) s.t. ac + bd = gcd(a, b)
pair<ll, ll> extended_gcd(ll a, ll b) {
    if (b == 0) return { 1, 0 };
    auto t = extended_gcd(b, a % b);
    return { t.second, t.first - t.second * (a / b) };
}

// find x in [0, m) s.t. ax == gcd(a, m) (mod m)
ll modinverse(ll a, ll m) {
    return (extended_gcd(a, m).first % m + m) % m;
}
```

```
// calculate modular inverse for 1 ~ n
void calc_range_modinv(int n, int mod, int ret[]) {
    ret[1] = 1;
    for (int i = 2; i <= n; ++i)
        ret[i] = (ll)(mod - mod/i) * ret[mod%i] % mod;
}
```

2.2 Sieve Methods : Prime, Divisor, Euler phi

```
// find prime numbers in 1 ~ n
// ret[x] = false -> x is prime
// O(n*loglogn)
void sieve(int n, bool ret[]) {
    for (int i = 2; i * i <= n; ++i)
        if (!ret[i])
            for (int j = i * i; j <= n; j += i)
                ret[j] = true;
}

// calculate number of divisors for 1 ~ n
// when you need to calculate sum, change += 1 to += i
// O(n*logn)
void num_of_divisors(int n, int ret[]) {
    for (int i = 1; i <= n; ++i)
        for (int j = i; j <= n; j += i)
            ret[j] += 1;
}

// calculate euler totient function for 1 ~ n
// phi(n) = number of x s.t. 0 < x < n && gcd(n, x) = 1
// O(n*loglogn)
void euler_phi(int n, int ret[]) {
    for (int i = 1; i <= n; ++i) ret[i] = i;
    for (int i = 2; i <= n; ++i)
        if (ret[i] == i)
            for (int j = i; j <= n; j += i)
                ret[j] -= ret[j] / i;
}
```

2.3 Primality Test

```
bool test_witness(ull a, ull n, ull s) {
    if (a >= n) a %= n;
    if (a <= 1) return true;
    ull d = n >> s;
    ull x = modpow(a, d, n);
    if (x == 1 || x == n-1) return true;
    while (s-- > 1) {
        x = large_mod_mul(x, x, n);
        if (x == 1) return false;
        if (x == n-1) return true;
    }
    return false;
}
```

```

}

// test whether n is prime
// based on miller-rabin test
// O(Logn*Logn)
bool is_prime(ull n) {
    if (n == 2) return true;
    if (n < 2 || n % 2 == 0) return false;

    ull d = n >> 1, s = 1;
    for(;; (d&1) == 0; s++) d >>= 1;

#define T(a) test_witness(a##ull, n, s)
    if (n < 4759123141ull) return T(2) && T(7) && T(61);
    return T(2) && T(325) && T(9375) && T(28178)
        && T(450775) && T(9780504) && T(1795265022);
#undef T
}

```

2.4 Integer Factorization (Pollard's rho)

```

ll pollard_rho(ll n) {
    random_device rd;
    mt19937 gen(rd());
    uniform_int_distribution<ll> dis(1, n - 1);
    ll x = dis(gen);
    ll y = x;
    ll c = dis(gen);
    ll g = 1;
    while (g == 1) {
        x = (modmul(x, x, n) + c) % n;
        y = (modmul(y, y, n) + c) % n;
        y = (modmul(y, y, n) + c) % n;
        g = gcd(abs(x - y), n);
    }
    return g;
}

```

```

// integer factorization
// O(n^0.25 * Logn)
void factorize(ll n, vector<ll>& fl) {
    if (n == 1) {
        return;
    }
    if (n % 2 == 0) {
        fl.push_back(2);
        factorize(n / 2, fl);
    }
    else if (is_prime(n)) {
        fl.push_back(n);
    }
    else {
        ll f = pollard_rho(n);
        factorize(f, fl);
    }
}

```

```

        factorize(n / f, fl);
    }
}

```

2.5 Chinese Remainder Theorem

```

// find x s.t. x === a[0] (mod n[0])
//              === a[1] (mod n[1])
//              ...
// assumption: gcd(n[i], n[j]) = 1
ll chinese_remainder(ll* a, ll* n, int size) {
    if (size == 1) return *a;
    ll tmp = modinverse(n[0], n[1]);
    ll tmp2 = (tmp * (a[1] - a[0]) % n[1] + n[1]) % n[1];
    ll ora = a[1];
    ll tgcd = gcd(n[0], n[1]);
    a[1] = a[0] + n[0] / tgcd * tmp2;
    n[1] *= n[0] / tgcd;
    ll ret = chinese_remainder(a + 1, n + 1, size - 1);
    n[1] /= n[0] / tgcd;
    a[1] = ora;
    return ret;
}

```

2.6 Modular Equation

$x \equiv a \pmod{m}$, $x \equiv b \pmod{n}$ 을 만족시키는 x 를 구하는 방법.

m 과 n 을 소인수분해한 후 소수의 제곱꼴의 합동식들로 각각 쪼갬다. 이 때 특정 소수에 대하여 모순이 생기면 불가능한 경우고, 모든 소수에 대해서 모순이 생기지 않으면 전체 식을 CRT로 합치면 된다. 이제 $x \equiv x_1 \pmod{p^{k_1}}$ 과 $x \equiv x_2 \pmod{p^{k_2}}$ 가 모순이 생길 조건은 $k_1 \leq k_2$ 라고 했을 때, $x_1 \not\equiv x_2 \pmod{p^{k_1}}$ 인 경우이다. 모순이 생기지 않았을 때 답을 구하려면 CRT로 합칠 때 $x \equiv x_2 \pmod{p^{k_2}}$ 만을 남기고 합쳐주면 된다.

2.7 Catalan number

다양한 문제의 답이 되는 수열이다.

- 길이가 $2n$ 인 올바른 괄호 수식의 수
- $n + 1$ 개의 리프를 가진 풀 바이너리 트리의 수
- $n + 2$ 각형을 n 개의 삼각형으로 나누는 방법의 수

$$C_n = \frac{1}{n+1} \binom{2n}{n}$$

$$C_0 = 1 \quad \text{and} \quad C_{n+1} = \sum_{i=0}^n C_i C_{n-i}$$

$$C_0 = 1 \quad \text{and} \quad C_{n+1} = \frac{2(2n+1)}{n+2} C_n$$

2.8 Burnside's Lemma

경우의 수를 세는데, 특정 transform operation(회전, 반사, ..) 해서 같은 경우들은 하나로 친다. 전체 경우의 수는?

- 각 operation마다 이 operation을 했을 때 변하지 않는 경우의 수를 센다 (단, “아무것도 하지 않는다”라는 operation도 있어야 함!)

- 전체 경우의 수를 더한 후, operation의 수로 나눈다. (답이 맞다면 항상 나누어 떨어져야 한다)

2.9 Kirchoff's Theorem

그래프의 스패닝 트리의 개수를 구하는 정리.

무향 그래프의 Laplacian matrix L 를 만든다. 이것은 (정점의 차수 대각 행렬) - (인접행렬) 이다. L 에서 행과 열을 하나씩 제거한 것을 L' 라 하자. 어느 행/열이든 관계 없다. 그래프의 스패닝 트리의 개수는 $\det(L')$ 이다.

2.10 Lucas Theorem

```
// calculate nCm % p when p is prime
int lucas_theorem(const char *n, const char *m, int p) {
    vector<int> np, mp;
    int i;
    for (i = 0; n[i]; i++) {
        if (n[i] == '0' && np.empty()) continue;
        np.push_back(n[i] - '0');
    }
    for (i = 0; m[i]; i++) {
        if (m[i] == '0' && mp.empty()) continue;
        mp.push_back(m[i] - '0');
    }

    int ret = 1;
    int ni = 0, mi = 0;
    while (ni < np.size() || mi < mp.size()) {
        int nmod = 0, mmod = 0;
        for (i = ni; i < np.size(); i++) {
            if (i + 1 < np.size())
                np[i + 1] += (np[i] % p) * 10;
            else
                nmod = np[i] % p;
            np[i] /= p;
        }
```

```
    }
    for (i = mi; i < mp.size(); i++) {
        if (i + 1 < mp.size())
            mp[i + 1] += (mp[i] % p) * 10;
        else
            mmod = mp[i] % p;
            mp[i] /= p;
    }
    while (ni < np.size() && np[ni] == 0) ni++;
    while (mi < mp.size() && mp[mi] == 0) mi++;
    // implement binomial. binomial(m,n) = 0 if m < n
    ret = (ret * binomial(nmod, mmod)) % p;
}
return ret;
}
```

2.11 Fast Fourier Transform

```
const double PI = acos(-1);

void fft(double *r, double *im, int N, bool f) {
    for (int i = 1, j = 0; i < N; i++) {
        int k; for (k = N >> 1; j >= k; k >>= 1) j -= k;
        j += k; if (i < j) swap(r[i], r[j]), swap(im[i], im[j]);
    }
    for (int i = 1; i < N; i <<= 1) {
        double w = PI / i; if (f) w = -w;
        double c = cos(w), s = sin(w);
        for (int j = 0; j < N; j += i << 1) {
            double yr = 1, yi = 0;
            for (int k = 0; k < i; k++) {
                double zr = r[i + j + k] * yr - im[i + j + k] * yi;
                double zi = r[i + j + k] * yi + im[i + j + k] * yr;
                r[i + j + k] = r[j + k] - zr;
                im[i + j + k] = im[j + k] - zi;
                r[j + k] += zr; im[j + k] += zi;
                tie(yr, yi) = make_pair(yr * c - yi * s, yr * s + yi * c);
            }
        }
    }
}

// Compute Poly(a)*Poly(b), write to r; Indexed from 0
// O(n*logn)
int mult(int *a, int n, int *b, int m, int *r) {
    const int maxn = 1048576;
    static double ra[maxn], rb[maxn], ia[maxn], ib[maxn];
    int fn = 1;
    while (fn < n + m) fn <= 1; // n + m: interested length
    for (int i = 0; i < n; ++i) ra[i] = a[i], ia[i] = 0;
    for (int i = n; i < fn; ++i) ra[i] = ia[i] = 0;
    for (int i = 0; i < m; ++i) rb[i] = b[i], ib[i] = 0;
    for (int i = m; i < fn; ++i) rb[i] = ib[i] = 0;
    fft(ra, ia, fn, false);
```

```
fft(rb, ib, fn, false);
for (int i = 0; i < fn; ++i) {
    double real = ra[i] * rb[i] - ia[i] * ib[i];
    double imag = ra[i] * ib[i] + rb[i] * ia[i];
    ra[i] = real, ia[i] = imag;
}
fft(ra, ia, fn, true);
for (int i = 0; i < fn; ++i) r[i] = (int)floor(ra[i] / fn + 0.5);
return fn;
}
```

2.12 Number Theoretic FFT

$p = a \cdot 2^b + 1$ 꼴의 소수 p 와 p 의 원시근 x 에 대하여, $n \leq b$ 를 만족하는 모든 2^n 크기의 배열에 대해 법 p 로 FFT를 행할 수 있다. 다음은 위를 만족하는 충분히 큰 소수들 목록이다.

p	a	b	원시근	덧셈	곱셈
3221225473	3	30	5	64-bit signed	64-bit unsigned
2281701377	17	27	3	64-bit signed	64-bit signed
2013265921	15	27	31	32-bit unsigned	64-bit signed
998244353	119	23	3	32-bit signed	64-bit signed
469762049	7	26	3	32-bit signed	64-bit signed

NTT 사용 시에 자료형에 유의하여, 덧셈 혹은 곱셈에서 Integer overflow가 나지 않도록 하라.

```
const int A = 7, B = 26, P = A << B | 1, R = 3;
```

```

int Pow(int x, int y) {
    int r = 1;
    while (y) {
        if (y & 1) r = r * 111 * x % P;
        x = x * 111 * x % P;
        y >>= 1;
    }
    return r;
}

void fft(int *a, int N, bool f) {
    for (int i = 1, j = 0; i < N; i++) {
        int k; for (k = N >> 1; j >= k; k >>= 1) j -= k;
        j += k; if (i < j) swap(a[i], a[j]);
    }
    for (int i = 1; i < N; i <= 1) {
        int x = Pow(f ? Pow(R, P - 2) : R, P / i >> 1);
        for (int j = 0; j < N; j += i < 1) {
            int y = 1;
            for (int k = 0; k < i; k++) {
                int z = a[i + j + k] * 111 * y % P;
                a[i + j + k] = a[j + k] - z;
                if (a[i + j + k] < P) a[i + j + k] += P;
                a[j + k] += z;
            }
        }
    }
}

```

```

        if (a[j + k] >= P) a[j + k] -= P;
        y = y * 111 * x % P;
    }
}
}

```

2.13 FFT with AVX

실수 배열에 대하여 convolution을 FFT 한 번, Inverse FFT 한 번으로 계산할 수 있다. 새로운 배열 $a + bi$ 를 정의하자. 이 배열의 자기 자신과의 convolution의 허수부는 a 와 b 의 convolution의 2배와 같다.

```
#include <immintrin.h>
#pragma GCC target("avx2")
#pragma GCC target("fma")

const double PI = acos(-1);

__m256d mult(__m256d a, __m256d b) {
    __m256d c = _mm256_movedup_pd(a);
    __m256d d = _mm256_shuffle_pd(a, a, 15);
    __m256d cb = _mm256_mul_pd(c, b);
    __m256d db = _mm256_mul_pd(d, b);
    __m256d e = _mm256_shuffle_pd(db, db, 5);
    __m256d r = _mm256_addsub_pd(cb, e);
    return r;
}

void fft(__m128d a[], int N, bool f) {
    for (int i = 1, j = 0; i < N; i++) {
        int k; for (k = N >> 1; j >= k; k >>= 1) j -= k;
        j += k; if (i < j) swap(a[i], a[j]);
    }
    for (int i = 1; i < N; i <= 1) {
        double angle = PI / i; if (f) angle = -angle;
        __m256d w; w[0] = cos(angle), w[1] = sin(angle);
        for (int j = 0; j < N; j += i < 1) {
            __m256d y; y[0] = 1; y[1] = 0;
            for (int k = 0; k < i; k++) {
                y = _mm256_permute2f128_pd(y, y, 0);
                w = _mm256_insertf128_pd(w, a[i + j + k], 1);
                y = mult(y, w);
                __m128d vw = _mm256_extractf128_pd(y, 1);
                __m128d u = a[j + k];
                a[j + k] = _mm_add_pd(u, vw);
                a[i + j + k] = _mm_sub_pd(u, vw);
            }
        }
    }
}
```

```

int mult(int *a, int n, int *b, int m, int *r) {
    const int maxn = 1048576;
    static __m128d fv[maxn];
    int fn = 1;
    while (fn < n + m) fn <= 1;
    for (int i = 0; i < fn; i++) fv[i][0] = fv[i][1] = 0;
    for (int i = 0; i < n; i++) fv[i][0] = a[i];
    for (int i = 0; i < m; i++) fv[i][1] = b[i];
    fft(fv, fn, false);
    for (int i = 0; i < fn; i += 2) {
        __m256d a;
        a = _mm256_insertf128_pd(a, fv[i], 0);
        a = _mm256_insertf128_pd(a, fv[i + 1], 1);
        a = mult(a, a);
        fv[i] = _mm256_extractf128_pd(a, 0);
        fv[i + 1] = _mm256_extractf128_pd(a, 1);
    }
    fft(fv, fn, true);
    for (int i = 0; i < fn; i++) r[i] = round(fv[i][1] / fn * 0.5);
    return fn;
}

```

2.14 Polynomial Division

```

vll get_inv(const vll& v, int deg){
    if (deg == 1) return vll(1, fastpow(v[0], MOD - 2));

    if (deg & 1){
        vll a = get_inv(v, deg - 1);
        ll c = 0;
        for (int i = 1; i < deg - 1; i++) c = (c + a[i] * v[deg - 1 - i]) % MOD;
        ll h1 = v[deg - 1];

        ll b = MOD - (h1 * a[0] + c) % MOD * a[0] % MOD;
        if (b == MOD) b = 0; a.push_back(b);
        return a;
    }

    vll a = get_inv(v, deg >> 1);
    vll h0(v.begin(), v.begin() + (deg >> 1));
    vll h1(v.begin() + (deg >> 1), v.begin() + deg);
    vll ah0 = mult(a, h0); ah0.push_back(0);
    vll c(ah0.begin() + (deg >> 1), ah0.begin() + deg);
    vll h1a = mult(h1, a);
    vll b_ = mult(a, add(h1a, c));

    vll b(b_.begin(), b_.begin() + (deg >> 1));
    for (ll e : b) a.push_back(e ? MOD - e : 0);
    return a;
}

vll divide(const vll& F, const vll& G, bool newg = false){
    static vll G_INV;

```

```

const int N = (int)F.size() - 1, M = (int)G.size() - 1; // deg of F, G
if (N < M) return vll();
if (N == M) return vll(1, F.back()*fastpow(G.back(), MOD - 2) % MOD);

vll f = F;
if (G_INV.empty() || newg)
    vll g = G; reverse(g.begin(), g.end());
    while (g.size() < N - M + 1) g.push_back(0);
    G_INV = get_inv(g, N - M + 1);
}

reverse(f.begin(), f.end());
vll ret = mult(f, G_INV);
ret.resize(N - M + 1);
reverse(ret.begin(), ret.end());

return ret;
}

```

2.15 Gaussian Elimination

```

const double EPS = 1e-10;
typedef vector<vector<double>> VVD;

// Gauss-Jordan elimination with full pivoting.
// solving systems of linear equations (AX=B)
// INPUT:  a[][] = an n*n matrix
//         b[][] = an n*m matrix
// OUTPUT:  X      = an n*m matrix (stored in b[][])
//         A^{-1} = an n*n matrix (stored in a[][])
// O(n^3)
double gauss_jordan(VVD& a, VVD& b) {
    const int n = a.size();
    const int m = b[0].size();
    vector<int> irow(n), icol(n), ipiv(n);

    double det = 1;

    for (int i = 0; i < n; i++) {
        int pj = -1, pk = -1;
        for (int j = 0; j < n; j++) if (!ipiv[j])
            for (int k = 0; k < n; k++) if (!ipiv[k])
                if (pj == -1 || fabs(a[j][k]) > fabs(a[pj][pk])) { pj = j; pk = k; }
        if (fabs(a[pj][pk]) < EPS) return 0; // matrix is singular
        ipiv[pk]++;
        swap(a[pj], a[pk]);
        swap(b[pj], b[pk]);
        irow[i] = pj;
        icol[i] = pk;

        double c = 1.0 / a[pk][pk];
        det *= a[pk][pk];
        a[pk][pk] = 1.0;

```

```

    for (int p = 0; p < n; p++) a[pk][p] *= c;
    for (int p = 0; p < m; p++) b[pk][p] *= c;
    for (int p = 0; p < n; p++) if (p != pk) {
        c = a[p][pk];
        a[p][pk] = 0;
        for (int q = 0; q < n; q++) a[p][q] -= a[pk][q] * c;
        for (int q = 0; q < m; q++) b[p][q] -= b[pk][q] * c;
    }
}
for (int p = n - 1; p >= 0; p--) if (irow[p] != icol[p]) {
    for (int k = 0; k < n; k++) swap(a[k][irow[p]], a[k][icol[p]]);
}
return det;
}

```

2.16 Simplex Algorithm

```

// Two-phase simplex algorithm for solving linear programs of the form
//      maximize    c^T x
//      subject to  Ax <= b
//                  x >= 0
// INPUT: A -- an m x n matrix
//        b -- an m-dimensional vector
//        c -- an n-dimensional vector
//        x -- a vector where the optimal solution will be stored
// OUTPUT: value of the optimal solution (infinity if unbounded
//         above, nan if infeasible)
// To use this code, create an LPSolver object with A, b, and c as
// arguments. Then, call Solve(x).
typedef vector<double> VD;
typedef vector<VD> VVD;
typedef vector<int> VI;
const double EPS = 1e-9;

struct LPSolver {
    int m, n;
    VI B, N;
    VVD D;

    LPSolver(const VVD& A, const VD& b, const VD& c) :
        m(b.size()), n(c.size()), N(n + 1), B(m), D(m + 2, VD(n + 2)) {
        for (int i = 0; i < m; i++) for (int j = 0; j < n; j++) D[i][j] = A[i][j];
        for (int i = 0; i < m; i++) { B[i] = n + i; D[i][n] = -1; D[i][n + 1] = b[i]; }
        for (int j = 0; j < n; j++) { N[j] = j; D[m][j] = -c[j]; }
        N[n] = -1; D[m + 1][n] = 1;
    }

    void pivot(int r, int s) {
        double inv = 1.0 / D[r][s];
        for (int i = 0; i < m + 2; i++) if (i != r)
            for (int j = 0; j < n + 2; j++) if (j != s)
                D[i][j] -= D[r][j] * D[i][s] * inv;
    }
}

```

```

    for (int j = 0; j < n + 2; j++) if (j != s) D[r][j] *= inv;
    for (int i = 0; i < m + 2; i++) if (i != r) D[i][s] *= -inv;
    D[r][s] = inv;
    swap(B[r], N[s]);
}

bool simplex(int phase) {
    int x = phase == 1 ? m + 1 : m;
    while (true) {
        int s = -1;
        for (int j = 0; j <= n; j++) {
            if (phase == 2 && N[j] == -1) continue;
            if (s == -1 || D[x][j] < D[x][s] || D[x][j] == D[x][s] && N[j] <
                N[s]) s = j;
        }
        if (D[x][s] > -EPS) return true;
        int r = -1;
        for (int i = 0; i < m; i++) {
            if (D[i][s] < EPS) continue;
            if (r == -1 || D[i][n + 1] / D[i][s] < D[r][n + 1] / D[r][s] ||
                (D[i][n + 1] / D[i][s]) == (D[r][n + 1] / D[r][s]) && B[i] <
                B[r]) r = i;
        }
        if (r == -1) return false;
        pivot(r, s);
    }
}

double solve(VD& x) {
    int r = 0;
    for (int i = 1; i < m; i++) if (D[i][n + 1] < D[r][n + 1]) r = i;
    if (D[r][n + 1] < -EPS) {
        pivot(r, n);
        if (!simplex(1) || D[m + 1][n + 1] < -EPS)
            return numeric_limits<double>::infinity();
        for (int i = 0; i < m; i++) if (B[i] == -1) {
            int s = -1;
            for (int j = 0; j <= n; j++)
                if (s == -1 || D[i][j] < D[i][s] || D[i][j] == D[i][s] && N[
                    j] < N[s]) s = j;
            pivot(i, s);
        }
    }
    if (!simplex(2))
        return numeric_limits<double>::infinity();
    x = VD(n);
    for (int i = 0; i < m; i++) if (B[i] < n) x[B[i]] = D[i][n + 1];
    return D[m][n + 1];
}
};

```

2.17 Nim Game

Nim Game의 해법 : 각 더미의 돌의 개수를 모두 XOR했을 때 0이 아니면 첫번째, 0이면 두번째 플레이어가 승리.

Grundy Number : 가능한 다음 state의 Grundy Number를 모두 모은 다음, 그 set에 포함되지 않는 가장 작은 수가 현재 state의 Grundy Number가 된다. 만약 다음 state가 독립된 여러 개의 state들로 나뉠 경우, 각각의 state의 Grundy Number의 XOR 합을 생각한다.

Subtraction Game : 한 번에 k 개까지의 돌만 가져갈 수 있는 경우, 각 더미의 돌의 개수를 $k + 1$ 로 나눈 나머지를 XOR 합하여 판단한다.

Index-k Nim : 한 번에 최대 k 개의 더미를 골라 각각의 더미에서 아무렇게나 돌을 제거할 수 있을 때, 각 binary digit에 대하여 합을 $k + 1$ 로 나눈 나머지를 계산한다. 만약 이 나머지가 모든 digit에 대하여 0이라면 두번째, 하나라도 0이 아니라면 첫번째 플레이어가 승리.

3 Data Structure

3.1 Order statistic tree

```
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
#include <ext/pb_ds/detail/standard_policies.hpp>
#include <functional>
#include <iostream>
using namespace __gnu_pbds;
using namespace std;

// tree<key_type, value_type(set if null), comparator, ...>
using ordered_set = tree<int, null_type, less<int>, rb_tree_tag,
    tree_order_statistics_node_update>;

int main()
{
    ordered_set X;
    for (int i = 1; i < 10; i += 2) X.insert(i); // 1 3 5 7 9
    cout << boolalpha;
    cout << *X.find_by_order(2) << endl; // 5
    cout << *X.find_by_order(4) << endl; // 9
    cout << (X.end() == X.find_by_order(5)) << endl; // true

    cout << X.order_of_key(-1) << endl; // 0
    cout << X.order_of_key(1) << endl; // 0
    cout << X.order_of_key(4) << endl; // 2
    X.erase(3);
    cout << X.order_of_key(4) << endl; // 1
    for (int t : X) printf("%d ", t); // 1 5 7 9
}
```

3.2 Fenwick Tree

```
const int TSIZE = 100000;
int tree[TSIZE + 1];

// Returns the sum from index 1 to p, inclusive
int query(int p) {
    int ret = 0;
    for (; p > 0; p -= p & -p) ret += tree[p];
    return ret;
}

// Adds val to element with index pos
void add(int p, int val) {
    for (; p <= TSIZE; p += p & -p) tree[p] += val;
}
```

3.3 Segment Tree with Lazy Propagation

```
// example implementation of sum tree
const int TSIZE = 131072; // always 2^k form && n <= TSIZE
int segtree[TSIZE * 2], prop[TSIZE * 2];
void seg_init(int nod, int l, int r) {
    if (l == r) segtree[nod] = dat[l];
    else {
        int m = (l + r) >> 1;
        seg_init(nod << 1, l, m);
        seg_init(nod << 1 | 1, m + 1, r);
        segtree[nod] = segtree[nod << 1] + segtree[nod << 1 | 1];
    }
}

void seg_relax(int nod, int l, int r) {
    if (prop[nod] == 0) return;
    if (l < r) {
        int m = (l + r) >> 1;
        segtree[nod << 1] += (m - l + 1) * prop[nod];
        prop[nod << 1] += prop[nod];
        segtree[nod << 1 | 1] += (r - m) * prop[nod];
        prop[nod << 1 | 1] += prop[nod];
    }
    prop[nod] = 0;
}

int seg_query(int nod, int l, int r, int s, int e) {
    if (r < s || e < l) return 0;
    if (s <= l && r <= e) return segtree[nod];
    seg_relax(nod, l, r);
    int m = (l + r) >> 1;
    return seg_query(nod << 1, l, m, s, e) + seg_query(nod << 1 | 1, m + 1, r, s, e);
}

void seg_update(int nod, int l, int r, int s, int e, int val) {
    if (r < s || e < l) return;
    if (s <= l && r <= e) {
        segtree[nod] += (r - l + 1) * val;
    }
}
```



```

        prop[nod] += val;
        return;
    }
    seg_relax(nod, l, r);
    int m = (l + r) >> 1;
    seg_update(nod << 1, l, m, s, e, val);
    seg_update(nod << 1 | 1, m + 1, r, s, e, val);
    segtree[nod] = segtree[nod << 1] + segtree[nod << 1 | 1];
}
// usage:
// seg_update(1, 0, n - 1, qs, qe, val);
// seg_query(1, 0, n - 1, qs, qe);

```

3.4 Persistent Segment Tree

```

// persistent segment tree impl: sum tree
// initial tree index is 0
namespace pstree {
    typedef int val_t;
    const int DEPTH = 18;
    const int TSIZE = 1 << 18;
    const int MAX_QUERY = 262144;

    struct node {
        val_t v;
        node *l, *r;
    } npoll[TSIZE * 2 + MAX_QUERY * (DEPTH + 1)], *head[MAX_QUERY + 1];

    int pptr, last_q;

    void init() {
        // zero-initialize, can be changed freely
        memset(&npoll[TSIZE - 1], 0, sizeof(node) * TSIZE);

        for (int i = TSIZE - 2; i >= 0; i--) {
            npoll[i].v = 0;
            npoll[i].l = &npoll[i*2+1];
            npoll[i].r = &npoll[i*2+2];
        }

        head[0] = &npoll[0];
        last_q = 0;
        pptr = 2 * TSIZE - 1;
    }

    // update val to pos
    // 0 <= pos < TSIZE
    // returns updated tree index
    int update(int pos, int val, int prev) {
        head[++last_q] = &npoll[pptr++];
        node *old = head[prev], *now = head[last_q];

        int flag = 1 << DEPTH;
        for (;) {

```

```

            now->v = old->v + val;
            flag >>= 1;
            if (flag==0) {
                now->l = now->r = nullptr; break;
            }
            if (flag & pos) {
                now->l = old->l;
                now->r = &npoll[pptr++];
                now = now->r, old = old->r;
            } else {
                now->r = old->r;
                now->l = &npoll[pptr++];
                now = now->l, old = old->l;
            }
        }
        return last_q;
    }

    val_t query(int s, int e, int l, int r, node *n) {
        if (s == l && e == r) return n->v;
        int m = (l + r) / 2;
        if (m >= e) return query(s, e, l, m, n->l);
        else if (m < s) return query(s, e, m + 1, r, n->r);
        else return query(s, m, l, m, n->l) + query(m + 1, e, m + 1, r, n->r);
    }

    // query summation of [s, e] at time t
    val_t query(int s, int e, int t) {
        s = max(0, s); e = min(TSIZE - 1, e);
        if (s > e) return 0;
        return query(s, e, 0, TSIZE - 1, head[t]);
    }
}

```

3.5 Splay Tree

```

// example : https://www.acmicpc.net/problem/13159
struct node {
    node* l, * r, * p;
    int cnt, min, max, val;
    long long sum;
    bool inv;
    node(int _val) :
        cnt(1), sum(_val), min(_val), max(_val), val(_val), inv(false),
        l(nullptr), r(nullptr), p(nullptr) {}
};

node* root;

void update(node* x) {
    x->cnt = 1;
    x->sum = x->min = x->max = x->val;
    if (x->l) {
        x->cnt += x->l->cnt;

```

```

    x->sum += x->l->sum;
    x->min = min(x->min, x->l->min);
    x->max = max(x->max, x->l->max);
}
if (x->r) {
    x->cnt += x->r->cnt;
    x->sum += x->r->sum;
    x->min = min(x->min, x->r->min);
    x->max = max(x->max, x->r->max);
}
}

void rotate(node* x) {
    node* p = x->p;
    node* b = nullptr;
    if (x == p->l) {
        p->l = b = x->r;
        x->r = p;
    }
    else {
        p->r = b = x->l;
        x->l = p;
    }
    x->p = p->p;
    p->p = x;
    if (b) b->p = p;
    x->p ? (p == x->p->l ? x->p->l : x->p->r) = x : (root = x);
    update(p);
    update(x);
}

// make x into root
void splay(node* x) {
    while (x->p) {
        node* p = x->p;
        node* g = p->p;
        if (g) rotate((x == p->l) == (p == g->l) ? p : x);
        rotate(x);
    }
}

void relax_lazy(node* x) {
    if (!x->inv) return;
    swap(x->l, x->r);
    x->inv = false;
    if (x->l) x->l->inv = !x->l->inv;
    if (x->r) x->r->inv = !x->r->inv;
}

// find kth node in splay tree
void find_kth(int k) {
    node* x = root;
    relax_lazy(x);
    while (true) {
        while (x->l && x->l->cnt > k) {

```

```

            x = x->l;
            relax_lazy(x);
        }
        if (x->l) k -= x->l->cnt;
        if (!k--) break;
        x = x->r;
        relax_lazy(x);
    }
    splay(x);
}

// collect [l, r] nodes into one subtree and return its root
node* interval(int l, int r) {
    find_kth(l - 1);
    node* x = root;
    root = x->r;
    root->p = nullptr;
    find_kth(r - l + 1);
    x->r = root;
    root->p = x;
    root = x;
    return root->r->l;
}

void traverse(node* x) {
    relax_lazy(x);
    if (x->l) {
        traverse(x->l);
    }
    // do something
    if (x->r) {
        traverse(x->r);
    }
}

void uptree(node* x) {
    if (x->p) {
        uptree(x->p);
    }
    relax_lazy(x);
}

```

4 DP

4.1 Convex Hull Optimization

$O(n^2) \rightarrow O(n \log n)$

DP 점화식 풀

$$D[i] = \max_{j < i} (D[j] + b[j] * a[i]) \quad (b[k] \leq b[k+1])$$

$$D[i] = \min_{j < i} (D[j] + b[j] * a[i]) \quad (b[k] \geq b[k+1])$$

특수조건) $a[i] \leq a[i+1]$ 도 만족하는 경우, 마지막 쿼리의 위치를 저장해두면 이분검색이 필요없어지기 때문에 amortized $O(n)$ 에 해결할 수 있음

```
struct CHTLinear {
    struct Line {
        long long a, b;
        long long y(long long x) const { return a * x + b; }
    };
    vector<Line> stk;
    int qpt;
    CHTLinear() : qpt(0) { }
    // when you need maximum : (previous L).a < (now L).a
    // when you need minimum : (previous L).a > (now L).a
    void pushLine(const Line& l) {
        while (stk.size() > 1) {
            Line& l0 = stk[stk.size() - 1];
            Line& l1 = stk[stk.size() - 2];
            if ((l0.b - l1.b) * (l0.a - l1.a) > (l1.b - l0.b) * (l.a - l0.a))
                break;
            stk.pop_back();
        }
        stk.push_back(l);
    }
    // (previous x) <= (current x)
    // it calculates max/min at x
    long long query(long long x) {
        while (qpt + 1 < stk.size()) {
            Line& l0 = stk[qpt];
            Line& l1 = stk[qpt + 1];
            if (l1.a - l0.a > 0 && (l0.b - l1.b) > x * (l1.a - l0.a)) break;
            if (l1.a - l0.a < 0 && (l0.b - l1.b) < x * (l1.a - l0.a)) break;
            ++qpt;
        }
        return stk[qpt].y(x);
    }
};
```

4.2 Divide & Conquer Optimization

$O(kn^2) \rightarrow O(kn \log n)$

조건 1) DP 점화식 꼴

$$D[t][i] = \min_{j < i} (D[t-1][j] + C[j][i])$$

조건 2) $A[t][i]$ 는 $D[t][i]$ 의 답이 되는 최소의 j 라 할 때, 아래의 부등식을 만족해야 함

$$A[t][i] \leq A[t][i+1]$$

조건 2-1) 비용 C 가 다음의 사각부등식을 만족하는 경우도 조건 2)를 만족하게 됨

$$C[a][c] + C[b][d] \leq C[a][d] + C[b][c] \quad (a \leq b \leq c \leq d)$$

4.3 Knuth Optimization

$O(n^3) \rightarrow O(n^2)$

조건 1) DP 점화식 꼴

$$D[i][j] = \min_{i < k < j} (D[i][k] + D[k][j]) + C[i][j]$$

조건 2) 사각 부등식

$$C[a][c] + C[b][d] \leq C[a][d] + C[b][c] \quad (a \leq b \leq c \leq d)$$

조건 3) 단조성

$$C[b][c] \leq C[a][d] \quad (a \leq b \leq c \leq d)$$

결론) 조건 2, 3을 만족한다면 $A[i][j]$ 를 $D[i][j]$ 의 답이 되는 최소의 k 라 할 때, 아래의 부등식을 만족하게 됨

$$A[i][j-1] \leq A[i][j] \leq A[i+1][j]$$

3중 루프를 돌릴 때 위 조건을 이용하면 최종적으로 시간복잡도가 $O(n^2)$ 이 됨

5 Graph

5.1 SCC

```
const int MAXN = 100;
vector<int> graph[MAXN];
int up[MAXN], visit[MAXN], vtime;
vector<int> stk;
int scc_idx[MAXN], scc_cnt;

void dfs(int nod) {
    up[nod] = visit[nod] = ++vtime;
    stk.push_back(nod);
    for (int next : graph[nod]) {
        if (visit[next] == 0) {
            dfs(next);
            up[nod] = min(up[nod], up[next]);
        }
        else if (scc_idx[next] == 0)
            up[nod] = min(up[nod], visit[next]);
    }
    if (up[nod] == visit[nod]) {
        ++scc_cnt;
        int t;
        do {
            t = stk.back();
            stk.pop_back();
            scc_idx[t] = scc_cnt;
        } while (!stk.empty() && t != nod);
    }
}
```

```

    }
}

// find SCCs in given directed graph
// O(V+E)
// the order of scc_idx constitutes a reverse topological sort
void get_scc() {
    vtime = 0;
    memset(visit, 0, sizeof(visit));
    scc_cnt = 0;
    memset(scc_idx, 0, sizeof(scc_idx));
    for (int i = 0; i < n; ++i)
        if (visit[i] == 0) dfs(i);
}

```

5.2 2-SAT

$(b_x \vee b_y) \wedge (\neg b_x \vee b_z) \wedge (b_z \vee \neg b_x) \wedge \dots$ 같은 form을 2-CNF라고 함. 주어진 2-CNF 식을 참으로 하는 $\{b_1, b_2, \dots\}$ 가 존재하는지, 존재한다면 그 값은 무엇인지 구하는 문제를 2-SAT 이라 함.

boolean variable b_i 마다 b_i 를 나타내는 정점, $\neg b_i$ 를 나타내는 정점 2개를 만들. 각 clause $b_i \vee b_j$ 마다 $\neg b_i \rightarrow b_j$, $\neg b_j \rightarrow b_i$ 이렇게 edge를 이어줌. 그렇게 만든 그래프에서 SCC를 다 구함. 어떤 SCC 안에 b_i 와 $\neg b_i$ 가 같이 포함되어있다면 해가 존재하지 않음. 아니라면 해가 존재함.

해가 존재할 때 구체적인 해를 구하는 방법. 위에서 SCC를 구하면서 SCC DAG를 만들어 준다. 거기서 위상정렬을 한 후, 앞에서부터 SCC를 하나씩 봐준다. 현재 보고있는 SCC에 b_i 가 속해있는데 애가 $\neg b_i$ 보다 먼저 등장했다면 $b_i = \text{false}$, 반대의 경우라면 $b_i = \text{true}$, 이미 값이 assign되었다면 pass.

5.3 BCC, Cut vertex, Bridge

```

const int MAXN = 100;
vector<pair<int, int>> graph[MAXN]; // { next vertex id, edge id }
int up[MAXN], visit[MAXN], vtime;
vector<pair<int, int>> stk;

int is_cut[MAXN]; // v is cut vertex if is_cut[v] > 0
vector<int> bridge; // list of edge ids
vector<int> bcc_idx[MAXN]; // list of bccids for vertex i
int bcc_cnt;

void dfs(int nod, int par_edge) {
    up[nod] = visit[nod] = ++vtime;
    int child = 0;
    for (const auto& e : graph[nod]) {
        int next = e.first, edge_id = e.second;
        if (edge_id == par_edge) continue;
        if (visit[next] == 0) {

```

```

            stk.push_back({ nod, next });
            ++child;
            dfs(next, edge_id);
            if (up[next] == visit[next]) bridge.push_back(edge_id);
            if (up[next] >= visit[nod]) {
                ++bcc_cnt;
                do {
                    auto last = stk.back();
                    stk.pop_back();
                    bcc_idx[last.second].push_back(bcc_cnt);
                    if (last == pair<int, int>{ nod, next }) break;
                } while (!stk.empty());
                bcc_idx[nod].push_back(bcc_cnt);
                is_cut[nod]++;
            }
            up[nod] = min(up[nod], up[next]);
        }
        else
            up[nod] = min(up[nod], visit[next]);
    }
    if (par_edge == -1 && is_cut[nod] == 1)
        is_cut[nod] = 0;
}

// find BCCs & cut vertexs & bridges in undirected graph
// O(V+E)
void get_bcc() {
    vtime = 0;
    memset(visit, 0, sizeof(visit));
    memset(is_cut, 0, sizeof(is_cut));
    bridge.clear();
    for (int i = 0; i < n; ++i) bcc_idx[i].clear();
    bcc_cnt = 0;
    for (int i = 0; i < n; ++i) {
        if (visit[i] == 0)
            dfs(i, -1);
    }
}

```

5.4 Block-cut Tree

각 BCC 및 cut vertex가 block-cut tree의 vertex가 되며, BCC와 그 BCC에 속한 cut vertex 사이에 edge를 이어주면 된다.

5.5 Heavy-Light Decomposition

```

// heavy-light decomposition
//
// hld h;
// insert edges to tree[0~n-1];
// h.init(n, root);
// h.decompose(root);

```

```
// h.hldquery(u, v); // edges from u to v
struct hld {
    static const int MAXLN = 18;
    static const int MAXN = 1 << (MAXLN - 1);
    vector<int> tree[MAXN];
    int subsize[MAXN], depth[MAXN], pa[MAXLN][MAXN];

    int chead[MAXN], cidx[MAXN];
    int lchain;
    int flatpos[MAXN + 1], fptr;

    void dfs(int u, int par) {
        pa[0][u] = par;
        subsize[u] = 1;
        for (int v : tree[u]) {
            if (v == pa[0][u]) continue;
            depth[v] = depth[u] + 1;
            dfs(v, u);
            subsize[u] += subsize[v];
        }
    }

    void init(int size, int root)
    {
        lchain = fptr = 0;
        dfs(root, -1);
        memset(chead, -1, sizeof(chead));

        for (int i = 1; i < MAXLN; i++) {
            for (int j = 0; j < size; j++) {
                if (pa[i - 1][j] != -1) {
                    pa[i][j] = pa[i - 1][pa[i - 1][j]];
                }
            }
        }
    }

    void decompose(int u) {
        if (chead[lchain] == -1) chead[lchain] = u;
        cidx[u] = lchain;
        flatpos[u] = ++fptr;

        int maxchd = -1;
        for (int v : tree[u]) {
            if (v == pa[0][u]) continue;
            if (maxchd == -1 || subsize[maxchd] < subsize[v]) maxchd = v;
        }
        if (maxchd != -1) decompose(maxchd);

        for (int v : tree[u]) {
            if (v == pa[0][u] || v == maxchd) continue;
            ++lchain; decompose(v);
        }
    }
}
```

```
int lca(int u, int v) {
    if (depth[u] < depth[v]) swap(u, v);

    int logu;
    for (logu = 1; 1 << logu <= depth[u]; logu++);
    logu--;

    int diff = depth[u] - depth[v];
    for (int i = logu; i >= 0; --i) {
        if ((diff >> i) & 1) u = pa[i][u];
    }
    if (u == v) return u;

    for (int i = logu; i >= 0; --i) {
        if (pa[i][u] != pa[i][v]) {
            u = pa[i][u];
            v = pa[i][v];
        }
    }
    return pa[0][u];
}

// TODO: implement query functions
inline int query(int s, int e) {
    return 0;
}

int subquery(int u, int v) {
    int uchain, vchain = cidx[v];
    int ret = 0;
    for (;;) {
        uchain = cidx[u];
        if (uchain == vchain) {
            ret += query(flatpos[v], flatpos[u]);
            break;
        }

        ret += query(flatpos[chead[uchain]], flatpos[u]);
        u = pa[0][chead[uchain]];
    }
    return ret;
}

inline int hldquery(int u, int v) {
    int p = lca(u, v);
    return subquery(u, p) + subquery(v, p) - query(flatpos[p], flatpos[p]);
}
};
```

5.6 Bipartite Matching (Hopcroft-Karp)

```
// in: n, m, graph
// out: match, matched
// vertex cover: (reached[0][left_node] == 0) || (reached[1][right_node] == 1)
```

```
// O(E*sqrt(V))
struct BipartiteMatching {
    int n, m;
    vector<vector<int>> graph;
    vector<int> matched, match, edgeview, level;
    vector<int> reached[2];
    BipartiteMatching(int n, int m) : n(n), m(m), graph(n), matched(m, -1),
        match(n, -1) {}

    bool assignLevel() {
        bool reachable = false;
        level.assign(n, -1);
        reached[0].assign(n, 0);
        reached[1].assign(m, 0);
        queue<int> q;
        for (int i = 0; i < n; i++) {
            if (match[i] == -1) {
                level[i] = 0;
                reached[0][i] = 1;
                q.push(i);
            }
        }
        while (!q.empty()) {
            auto cur = q.front(); q.pop();
            for (auto adj : graph[cur]) {
                reached[1][adj] = 1;
                auto next = matched[adj];
                if (next == -1) {
                    reachable = true;
                }
                else if (level[next] == -1) {
                    level[next] = level[cur] + 1;
                    reached[0][next] = 1;
                    q.push(next);
                }
            }
        }
        return reachable;
    }

    int findpath(int nod) {
        for (int &i = edgeview[nod]; i < graph[nod].size(); i++) {
            int adj = graph[nod][i];
            int next = matched[adj];
            if (next >= 0 && level[next] != level[nod] + 1) continue;
            if (next == -1 || findpath(next)) {
                match[nod] = adj;
                matched[adj] = nod;
                return 1;
            }
        }
        return 0;
    }

    int solve() {
```

```
        int ans = 0;
        while (assignLevel()) {
            edgeview.assign(n, 0);
            for (int i = 0; i < n; i++)
                if (match[i] == -1)
                    ans += findpath(i);
        }
        return ans;
    }
};
```

5.7 Maximum Flow (Dinic)

```
// usage:
// MaxFlowDinic::init(n);
// MaxFlowDinic::add_edge(0, 1, 100, 100); // for bidirectional edge
// MaxFlowDinic::add_edge(1, 2, 100); // directional edge
// result = MaxFlowDinic::solve(0, 2); // source -> sink
// graph[i][edgeIndex].res -> residual
//
// in order to find out the minimum cut, use `l`.
// if l[i] == 0, i is unreachable.
//
// O(V*V*E)
// with unit capacities, O(min(V^(2/3), E^(1/2)) * E)
struct MaxFlowDinic {
    typedef int flow_t;
    struct Edge {
        int next;
        size_t inv; /* inverse edge index */
        flow_t res; /* residual */
    };
    int n;
    vector<vector<Edge>> graph;
    vector<int> q, l, start;

    void init(int _n) {
        n = _n;
        graph.resize(n);
        for (int i = 0; i < n; i++) graph[i].clear();
    }

    void add_edge(int s, int e, flow_t cap, flow_t caprev = 0) {
        Edge forward{ e, graph[e].size(), cap };
        Edge reverse{ s, graph[s].size(), caprev };
        graph[s].push_back(forward);
        graph[e].push_back(reverse);
    }

    bool assign_level(int source, int sink) {
        int t = 0;
        memset(&l[0], 0, sizeof(l[0]) * l.size());
        l[source] = 1;
        q[t++] = source;
        for (int h = 0; h < t && !l[sink]; h++) {
            int cur = q[h];
```

```

    for (const auto& e : graph[cur]) {
        if (l[e.next] || e.res == 0) continue;
        l[e.next] = l[cur] + 1;
        q[t++] = e.next;
    }
}
return l[sink] != 0;
}
flow_t block_flow(int cur, int sink, flow_t current) {
    if (cur == sink) return current;
    for (int& i = start[cur]; i < graph[cur].size(); i++) {
        auto& e = graph[cur][i];
        if (e.res == 0 || l[e.next] != l[cur] + 1) continue;
        if (flow_t res = block_flow(e.next, sink, min(e.res, current))) {
            e.res -= res;
            graph[e.next][e.inv].res += res;
            return res;
        }
    }
    return 0;
}
flow_t solve(int source, int sink) {
    q.resize(n);
    l.resize(n);
    start.resize(n);
    flow_t ans = 0;
    while (assign_level(source, sink)) {
        memset(&start[0], 0, sizeof(start[0]) * n);
        while (flow_t flow = block_flow(source, sink, numeric_limits<flow_t>::max()))
            ans += flow;
    }
    return ans;
}
};

```

5.8 Maximum Flow with Edge Demands

그래프 $G = (V, E)$ 가 있고 source s 와 sink t 가 있다. 각 간선마다 $d(e) \leq f(e) \leq c(e)$ 를 만족하도록 flow $f(e)$ 를 흘려야 한다. 이 때의 maximum flow를 구하는 문제다.

먼저 모든 demand를 합한 값 D 를 아래와 같이 정의한다.

$$D = \sum_{(u \rightarrow v) \in E} d(u \rightarrow v)$$

이제 G 에 몇개의 정점과 간선을 추가하여 새로운 그래프 $G' = (V', E')$ 을 만들 것이다. 먼저 새로운 source s' 과 새로운 sink t' 을 추가한다. 그리고 s' 에서 V 의 모든 점마다 간선을 이어주고, V 의 모든 점에서 t' 로 간선을 이어준다.

새로운 capacity function c' 을 아래와 같이 정의한다.

1. V 의 점 v 에 대해 $c'(s' \rightarrow v) = \sum_{u \in V} d(u \rightarrow v)$, $c'(v \rightarrow t') = \sum_{w \in V} d(v \rightarrow w)$
2. E 의 간선 $u \rightarrow v$ 에 대해 $c'(u \rightarrow v) = c(u \rightarrow v) - d(u \rightarrow v)$
3. $c'(t \rightarrow s) = \infty$

이렇게 만든 새로운 그래프 G' 에서 maximum flow를 구했을 때 그 값이 D 라면 원래 문제의 해가 존재하고, 그 값이 D 가 아니라면 원래 문제의 해는 존재하지 않는다.

위에서 maximum flow를 구하고 난 상태의 residual graph 에서 s' 과 t' 을 떼버리고 s 에서 t 사이의 augment path 를 계속 찾으면 원래 문제의 해를 구할 수 있다.

5.8.1 Source Code

```

struct MaxFlowEdgeDemands
{
    MaxFlowDinic mf;
    using flow_t = MaxFlowDinic::flow_t;

    vector<flow_t> ind, outd;
    flow_t D; int n;

    void init(int _n) {
        n = _n; D = 0; mf.init(n + 2);
        ind.clear(); outd.clear();
        ind.resize(n, 0); outd.resize(n, 0);
    }

    void add_edge(int s, int e, flow_t cap, flow_t demands = 0) {
        mf.add_edge(s, e, cap - demands);
        D += demands; ind[e] += demands; outd[s] += demands;
    }

    // returns { false, 0 } if infeasible
    // { true, maxflow } if feasible
    pair<bool, flow_t> solve(int source, int sink) {
        mf.add_edge(sink, source, numeric_limits<flow_t>::max());

        for (int i = 0; i < n; i++) {
            if (ind[i]) mf.add_edge(n, i, ind[i]);
            if (outd[i]) mf.add_edge(i, n + 1, outd[i]);
        }

        if (mf.solve(n, n + 1) != D) return { false, 0 };

        for (int i = 0; i < n; i++) {
            if (ind[i]) mf.graph[i].pop_back();
            if (outd[i]) mf.graph[i].pop_back();
        }

        return { true, mf.solve(source, sink) };
    }
};

```

5.9 Min-cost Maximum Flow

```
// precondition: there is no negative cycle.
// usage:
// MinCostFlow mcf(n);
// for(each edges) mcf.addEdge(from, to, cost, capacity);
// mcf.solve(source, sink); // min cost max flow
// mcf.solve(source, sink, 0); // min cost flow
// mcf.solve(source, sink, goal_flow); // min cost flow with total_flow >=
// goal_flow if possible
struct MinCostFlow {
    typedef int cap_t;
    typedef int cost_t;

    bool iszerocap(cap_t cap) { return cap == 0; }

    struct edge {
        int target;
        cost_t cost;
        cap_t residual_capacity;
        cap_t orig_capacity;
        size_t revid;
    };

    int n;
    vector<vector<edge>> graph;

    MinCostFlow(int n) : graph(n), n(n) {}

    void addEdge(int s, int e, cost_t cost, cap_t cap) {
        if (s == e) return;
        edge forward{ e, cost, cap, cap, graph[e].size() };
        edge backward{ s, -cost, 0, 0, graph[s].size() };
        graph[s].emplace_back(forward);
        graph[e].emplace_back(backward);
    }

    pair<cost_t, cap_t> augmentShortest(int s, int e, cap_t flow_limit) {
        auto infinite_cost = numeric_limits<cost_t>::max();
        auto infinite_flow = numeric_limits<cap_t>::max();
        vector<pair<cost_t, cap_t>> dist(n, make_pair(infinite_cost, 0));
        vector<int> from(n, -1), v(n);

        dist[s] = pair<cost_t, cap_t>(0, infinite_flow);
        queue<int> q;
        v[s] = 1; q.push(s);
        while(!q.empty()) {
            int cur = q.front();
            v[cur] = 0; q.pop();
            for (const auto& e : graph[cur]) {
                if (iszerocap(e.residual_capacity)) continue;
                auto next = e.target;
                auto ncost = dist[cur].first + e.cost;
                auto nflow = min(dist[cur].second, e.residual_capacity);
                if (dist[next].first > ncost) {
```

```
                    dist[next] = make_pair(ncost, nflow);
                    from[next] = e.revid;
                    if (v[next]) continue;
                    v[next] = 1; q.push(next);
                }
            }
        }

        auto p = e;
        auto pathcost = dist[p].first;
        auto flow = dist[p].second;
        if (iszerocap(flow) || (flow_limit <= 0 && pathcost >= 0)) return pair<
            cost_t, cap_t>(0, 0);
        if (flow_limit > 0) flow = min(flow, flow_limit);

        while (from[p] != -1) {
            auto nedge = from[p];
            auto np = graph[p][nedge].target;
            auto fedge = graph[p][nedge].revid;
            graph[p][nedge].residual_capacity += flow;
            graph[np][fedge].residual_capacity -= flow;
            p = np;
        }
        return make_pair(pathcost * flow, flow);
    }

    pair<cost_t, cap_t> solve(int s, int e, cap_t flow_minimum = numeric_limits<
        cap_t>::max()) {
        cost_t total_cost = 0;
        cap_t total_flow = 0;
        for(;;) {
            auto res = augmentShortest(s, e, flow_minimum - total_flow);
            if (res.second <= 0) break;
            total_cost += res.first;
            total_flow += res.second;
        }
        return make_pair(total_cost, total_flow);
    }
};
```

5.10 General Min-cut (Stoer-Wagner)

```
// implementation of Stoer-Wagner algorithm
// O(V^3)
// usage
// MinCut mc;
// mc.init(n);
// for (each edge) mc.addEdge(a,b,weight);
// mincut = mc.solve();
// mc.cut = {0,1}^n describing which side the vertex belongs to.
struct MinCutMatrix
{
    typedef int cap_t;
    int n;
```



```

vector<vector<cap_t>> graph;

void init(int _n) {
    n = _n;
    graph = vector<vector<cap_t>>(n, vector<cap_t>(n, 0));
}

void addEdge(int a, int b, cap_t w) {
    if (a == b) return;
    graph[a][b] += w;
    graph[b][a] += w;
}

pair<cap_t, pair<int, int>> stMinCut(vector<int> &active) {
    vector<cap_t> key(n);
    vector<int> v(n);
    int s = -1, t = -1;
    for (int i = 0; i < active.size(); i++) {
        cap_t maxv = -1;
        int cur = -1;
        for (auto j : active) {
            if (v[j] == 0 && maxv < key[j]) {
                maxv = key[j];
                cur = j;
            }
        }
        t = s; s = cur;
        v[cur] = 1;
        for (auto j : active) key[j] += graph[cur][j];
    }
    return make_pair(key[s], make_pair(s, t));
}

vector<int> cut;

cap_t solve() {
    cap_t res = numeric_limits<cap_t>::max();
    vector<vector<int>> grps;
    vector<int> active;
    cut.resize(n);
    for (int i = 0; i < n; i++) grps.emplace_back(1, i);
    for (int i = 0; i < n; i++) active.push_back(i);
    while (active.size() >= 2) {
        auto stcut = stMinCut(active);
        if (stcut.first < res) {
            res = stcut.first;
            fill(cut.begin(), cut.end(), 0);
            for (auto v : grps[stcut.second.first]) cut[v] = 1;
        }

        int s = stcut.second.first, t = stcut.second.second;
        if (grps[s].size() < grps[t].size()) swap(s, t);

        active.erase(find(active.begin(), active.end(), t));
        grps[s].insert(grps[s].end(), grps[t].begin(), grps[t].end());
        for (int i = 0; i < n; i++) { graph[i][s] += graph[i][t]; graph[i][t]

```

```

        ] = 0; }
        for (int i = 0; i < n; i++) { graph[s][i] += graph[t][i]; graph[t][i]
        ] = 0; }
        graph[s][s] = 0;
    }
    return res;
}

};

```

5.11 General Max Matching

```

struct DisjointSet
{
    vector<int> parent, rnk;

    DisjointSet(int n = 0) : rnk(n) {
        parent.reserve(n);
        for (int i = 0; i < n; i++) parent.push_back(i);
    }

    void reset(int n) {
        parent.clear(); rnk.assign(n, 0);
        for (int i = 0; i < n; i++) parent.push_back(i);
    }

    void increase(int n) {
        int base = parent.size();
        for (int i = base; i < base + n; i++) {
            parent.push_back(i);
            rnk.push_back(0);
        }
    }

    int find(int p) {
        return parent[p] == p ? p : parent[p] = find(parent[p]);
    }

    void merge(int a, int b) {
        a = find(a), b = find(b);
        if (a == b) return;
        if (rnk[a] < rnk[b]) swap(a, b);
        else if (rnk[a] == rnk[b]) ++rnk[a];
        parent[b] = a;
    }
};

struct MaxMatching
{
    int n;
    vector<vector<int>> gnext;
    vector<int> matched;

    int vcnt;

```

```

MaxMatching(int n) : n(n), gnext(n), matched(n, -1) {}

void AddEdge(int a, int b) {
    gnext[a].push_back(b);
    gnext[b].push_back(a);
}

int Match() {
    int ans = 0;
    while (findAugment()) ++ans;
    return ans;
}

vector<int> parent; // shrunken -> real
vector<int> forest;
vector<int> level;
vector<pair<int,int>> bridge;
queue<int> q;
DisjointSet blossomSet;
vector<int> origin; // blossomSet number to -> origin vertex
vector<int> ancestorChecker;
int ancestorCheckerValue;

vector<int> marker;
void markBlossomPath(int vv, pair<int, int> vu, int ancestor){
    int p = vv;
    marker.clear();
    while (p != ancestor) {
        int np = origin[blossomSet.find(parent[p])];
        marker.push_back(p); p = np;

        np = origin[blossomSet.find(parent[p])];
        marker.push_back(p);
        bridge[p] = vu; // need original vertex number
        q.push(p); // odd level edges were not considered
        p = np;
    }
    for (auto x : marker) blossomSet.merge(ancestor, x);
    origin[blossomSet.find(ancestor)] = ancestor;
}

void mergeBlossom(int vv, int uu, int v, int u){
    if (uu == vv) return;
    ++ancestorCheckerValue;
    int p1 = uu, p2 = vv;
    int ancestor = -1;
    for (;;) {
        if (p1 >= 0) {
            if (ancestorChecker[p1] == ancestorCheckerValue) {
                ancestor = p1;
                break;
            }
            ancestorChecker[p1] = ancestorCheckerValue;
            if (parent[p1] >= 0) p1 = origin[blossomSet.find(parent[p1])];
        }
    }
}

```

```

        else p1 = -1;
    }
    if (p2 >= 0) {
        if (ancestorChecker[p2] == ancestorCheckerValue) {
            ancestor = p2;
            break;
        }
        ancestorChecker[p2] = ancestorCheckerValue;
        if (parent[p2] >= 0) p2 = origin[blossomSet.find(parent[p2])];
        else p2 = -1;
    }
}
markBlossomPath(uu, make_pair(u, v), ancestor);
markBlossomPath(vv, make_pair(v, u), ancestor);
}

vector<int> augmentPathLink;

void getRootPath(int v, int w, bool reversed){
    if (v == w) return;
    if (level[v] & 1) {
        // odd. use bridge
        int x, y, mate = matched[v];
        tie(x,y) = tie(bridge[v].first, bridge[v].second);
        getRootPath(x, mate, !reversed);
        getRootPath(y, w, reversed);
        if (reversed) {
            augmentPathLink[y] = x;
            augmentPathLink[mate] = v;
        } else {
            augmentPathLink[v] = mate;
            augmentPathLink[x] = y;
        }
    } else {
        // even
        int mate = matched[v];
        getRootPath(parent[mate], w, reversed);
        if (reversed) {
            augmentPathLink[parent[mate]] = mate;
            augmentPathLink[mate] = v;
        } else {
            augmentPathLink[v] = mate;
            augmentPathLink[mate] = parent[mate];
        }
    }
}

void augmentPath(int v, int w) {
    augmentPathLink = vector<int>(n, -1);
    int x = forest[v];
    int y = forest[w];
    getRootPath(v, x, true);
    getRootPath(w, y, false);
    augmentPathLink[v] = w;
    int p = x;
}

```

```

for(;;) {
    int q = augmentPathLink[p];
    matched[p] = q;
    matched[q] = p;
    if (q == y) break;
    p = augmentPathLink[q];
}
}

bool findAugment() {
    parent = vector<int>(n,-1);
    forest = vector<int>(n,-1);
    level = vector<int>(n);
    bridge = vector<pair<int,int>>(n,make_pair(-1,-1));
    q = queue<int>();
    blossomSet.reset(n);
    origin = vector<int>(n);
    ancestorChecker = vector<int>(n);
    ancestorCheckerValue = 0;

    for(int i = 0; i < n; i++) {
        origin[i] = i;
        if (matched[i] == -1) {
            forest[i] = i;
            q.push(i);
        }
    }
    bool foundPath = false;
    while(!q.empty() && !foundPath) {
        int v = q.front(); q.pop();
        for(auto u : gnext[v]) {
            int vv = origin[blossomSet.find(v)];
            int uu = origin[blossomSet.find(u)];
            if (forest[uu] == -1) {
                // assert(u == uu)
                parent[uu] = v;
                forest[uu] = forest[vv];
                level[uu] = level[vv] + 1;
                parent[matched[uu]] = uu;
                forest[matched[uu]] = forest[vv];
                level[matched[uu]] = level[vv] + 2;
                q.push(matched[uu]);
            } else if (level[uu]&1) {
                // odd Level
            } else if (forest[uu] != forest[vv]){
                // found path. both are even Level
                foundPath = true;
                augmentPath(v,u);
                break;
            } else {
                // blossom formed
                mergeBlossom(vv, uu, v, u);
            }
        }
    }
}

```

```

        return foundPath;
    }
};

```

5.12 Hungarian Algorithm

```

int n, m;
int mat[MAX_N + 1][MAX_M + 1];

// hungarian method : bipartite min-weighted matching
// O(n^3) or O(m*n^2)
// http://e-maxx.ru/algo/assignment_hungary
// mat[1][1] ~ mat[n][m]
// matched[i] : matched column of row i
int hungarian(vector<int>& matched) {
    vector<int> u(n + 1), v(m + 1), p(m + 1), way(m + 1), minv(m + 1);
    vector<char> used(m + 1);
    for (int i = 1; i <= n; ++i) {
        p[0] = i;
        int j0 = 0;
        fill(minv.begin(), minv.end(), INF);
        fill(used.begin(), used.end(), false);
        do {
            used[j0] = true;
            int i0 = p[j0], delta = INF, j1;
            for (int j = 1; j <= m; ++j) {
                if (!used[j]) {
                    int cur = mat[i0][j] - u[i0] - v[j];
                    if (cur < minv[j]) minv[j] = cur, way[j] = j0;
                    if (minv[j] < delta) delta = minv[j], j1 = j;
                }
            }
            for (int j = 0; j <= m; ++j) {
                if (used[j])
                    u[p[j]] += delta, v[j] -= delta;
                else
                    minv[j] -= delta;
            }
            j0 = j1;
        } while (p[j0] != 0);
        do {
            int j1 = way[j0];
            p[j0] = p[j1];
            j0 = j1;
        } while (j0);
        for (int j = 1; j <= m; ++j) matched[p[j]] = j;
    }
    return -v[0];
}

```

6 Geometry

6.1 Basic Operations

```
const double eps = 1e-9;

inline int diff(double lhs, double rhs) {
    if (lhs - eps < rhs && rhs < lhs + eps) return 0;
    return (lhs < rhs) ? -1 : 1;
}

inline bool is_between(double check, double a, double b) {
    if (a < b)
        return (a - eps < check && check < b + eps);
    else
        return (b - eps < check && check < a + eps);
}

struct Point {
    double x, y;
    bool operator==(const Point& rhs) const {
        return diff(x, rhs.x) == 0 && diff(y, rhs.y) == 0;
    }
    Point operator+(const Point& rhs) const {
        return Point{ x + rhs.x, y + rhs.y };
    }
    Point operator-(const Point& rhs) const {
        return Point{ x - rhs.x, y - rhs.y };
    }
    Point operator*(double t) const {
        return Point{ x * t, y * t };
    }
};

struct Circle {
    Point center;
    double r;
};

struct Line {
    Point pos, dir;
};

inline double inner(const Point& a, const Point& b) {
    return a.x * b.x + a.y * b.y;
}

inline double outer(const Point& a, const Point& b) {
    return a.x * b.y - a.y * b.x;
}

inline int ccw_line(const Line& line, const Point& point) {
    return diff(outer(line.dir, point - line.pos), 0);
}
```

```
inline int ccw(const Point& a, const Point& b, const Point& c) {
    return diff(outer(b - a, c - a), 0);
}

inline double dist(const Point& a, const Point& b) {
    return sqrt(inner(a - b, a - b));
}

inline double dist2(const Point &a, const Point &b) {
    return inner(a - b, a - b);
}

inline double dist(const Line& line, const Point& point, bool segment = false) {
    double c1 = inner(point - line.pos, line.dir);
    if (segment && diff(c1, 0) <= 0) return dist(line.pos, point);
    double c2 = inner(line.dir, line.dir);
    if (segment && diff(c2, c1) <= 0) return dist(line.pos + line.dir, point);
    return dist(line.pos + line.dir * (c1 / c2), point);
}

bool get_cross(const Line& a, const Line& b, Point& ret) {
    double mdet = outer(b.dir, a.dir);
    if (diff(mdet, 0) == 0) return false;
    double t2 = outer(a.dir, b.pos - a.pos) / mdet;
    ret = b.pos + b.dir * t2;
    return true;
}

bool get_segment_cross(const Line& a, const Line& b, Point& ret) {
    double mdet = outer(b.dir, a.dir);
    if (diff(mdet, 0) == 0) return false;
    double t1 = -outer(b.pos - a.pos, b.dir) / mdet;
    double t2 = outer(a.dir, b.pos - a.pos) / mdet;
    if (!is_between(t1, 0, 1) || !is_between(t2, 0, 1)) return false;
    ret = b.pos + b.dir * t2;
    return true;
}

Point inner_center(const Point &a, const Point &b, const Point &c) {
    double wa = dist(b, c), wb = dist(c, a), wc = dist(a, b);
    double w = wa + wb + wc;
    return Point{ (wa * a.x + wb * b.x + wc * c.x) / w, (wa * a.y + wb * b.y + wc * c.y) / w };
}

Point outer_center(const Point &a, const Point &b, const Point &c) {
    Point d1 = b - a, d2 = c - a;
    double area = outer(d1, d2);
    double dx = d1.x * d1.x * d2.y - d2.x * d2.x * d1.y
        + d1.y * d2.y * (d1.y - d2.y);
    double dy = d1.y * d1.y * d2.x - d2.y * d2.y * d1.x
        + d1.x * d2.x * (d1.x - d2.x);
    return Point{ a.x + dx / area / 2.0, a.y - dy / area / 2.0 };
}
```

```

vector<Point> circle_line(const Circle& circle, const Line& line) {
    vector<Point> result;
    double a = 2 * inner(line.dir, line.dir);
    double b = 2 * (line.dir.x * (line.pos.x - circle.center.x)
        + line.dir.y * (line.pos.y - circle.center.y));
    double c = inner(line.pos - circle.center, line.pos - circle.center)
        - circle.r * circle.r;
    double det = b * b - 2 * a * c;
    int pred = diff(det, 0);
    if (pred == 0)
        result.push_back(line.pos + line.dir * (-b / a));
    else if (pred > 0) {
        det = sqrt(det);
        result.push_back(line.pos + line.dir * ((-b + det) / a));
        result.push_back(line.pos + line.dir * ((-b - det) / a));
    }
    return result;
}

vector<Point> circle_circle(const Circle& a, const Circle& b) {
    vector<Point> result;
    int pred = diff(dist(a.center, b.center), a.r + b.r);
    if (pred > 0) return result;
    if (pred == 0) {
        result.push_back((a.center * b.r + b.center * a.r) * (1 / (a.r + b.r)));
        return result;
    }
    double aa = a.center.x * a.center.x + a.center.y * a.center.y - a.r * a.r;
    double bb = b.center.x * b.center.x + b.center.y * b.center.y - b.r * b.r;
    double tmp = (bb - aa) / 2.0;
    Point cdiff = b.center - a.center;
    if (diff(cdiff.x, 0) == 0) {
        if (diff(cdiff.y, 0) == 0)
            return result; // if (diff(a.r, b.r) == 0): same circle
        return circle_line(a, Line{ Point{ 0, tmp / cdiff.y }, Point{ 1, 0 } });
    }
    return circle_line(a,
        Line{ Point{ tmp / cdiff.x, 0 }, Point{ -cdiff.y, cdiff.x } });
}

Circle circle_from_3pts(const Point& a, const Point& b, const Point& c) {
    Point ba = b - a, cb = c - b;
    Line p{ (a + b) * 0.5, Point{ ba.y, -ba.x } };
    Line q{ (b + c) * 0.5, Point{ cb.y, -cb.x } };
    Circle circle;
    if (!get_cross(p, q, circle.center))
        circle.r = -1;
    else
        circle.r = dist(circle.center, a);
    return circle;
}

Circle circle_from_2pts_rad(const Point& a, const Point& b, double r) {
    double det = r * r / dist2(a, b) - 0.25;

```

```

    Circle circle;
    if (det < 0)
        circle.r = -1;
    else {
        double h = sqrt(det);
        // center is to the left of a->b
        circle.center = (a + b) * 0.5 + Point{ a.y - b.y, b.x - a.x } * h;
        circle.r = r;
    }
    return circle;
}

```

6.2 Compare angles

```

int ccw(pair<int, int> p1, pair<int, int> p2) {
    auto ret = p1.first * 1ll * p2.second - p2.first * 1ll * p1.second;
    return ret > 0 ? 1 : (ret < 0 ? -1 : 0);
}

bool upper(pair<int, int> p) {
    return tie(p.second, p.first) > tuple<int, int>();
}

// sorting criterion: [0 ~ 2 * pi)
sort(dat.begin(), dat.end(), [](pair<int, int> a, pair<int, int> b){
    if (upper(a) != upper(b)) return upper(a) > upper(b);
    if (ccw(a, b)) return ccw(a, b) > 0;

    // optional: closest to farthest
    return hypot(a.first, a.second) < hypot(b.first, b.second);
});

```

6.3 Convex Hull

```

// find convex hull
// O(n*logn)
vector<Point> convex_hull(vector<Point>& dat) {
    if (dat.size() <= 3) return dat;
    vector<Point> upper, lower;
    sort(dat.begin(), dat.end(), [](const Point& a, const Point& b) {
        return (a.x == b.x) ? a.y < b.y : a.x < b.x;
    });
    for (const auto& p : dat) {
        while (upper.size() >= 2 && ccw(++upper.rbegin(), *upper.rbegin(), p)
            >= 0) upper.pop_back();
        while (lower.size() >= 2 && ccw(++lower.rbegin(), *lower.rbegin(), p)
            <= 0) lower.pop_back();
        upper.emplace_back(p);
        lower.emplace_back(p);
    }
    upper.insert(upper.end(), ++lower.rbegin(), --lower.rend());
    return upper;
}

```

6.4 Rotating Calipers

```
// get all antipodal pairs
// O(n)
void antipodal_pairs(vector<Point>& pt) {
    // calculate convex hull
    sort(pt.begin(), pt.end(), [](const Point& a, const Point& b) {
        return (a.x == b.x) ? a.y < b.y : a.x < b.x;
    });
    vector<Point> up, lo;
    for (const auto& p : pt) {
        while (up.size() >= 2 && ccw(*++up.rbegin(), *up.rbegin(), p) >= 0) up.
            pop_back();
        while (lo.size() >= 2 && ccw(*++lo.rbegin(), *lo.rbegin(), p) <= 0) lo.
            pop_back();
        up.emplace_back(p);
        lo.emplace_back(p);
    }

    for (int i = 0, j = (int)lo.size() - 1; i + 1 < up.size() || j > 0; ) {
        get_pair(up[i], lo[j]); // DO WHAT YOU WANT
        if (i + 1 == up.size()) --j;
        else if (j == 0) ++i;
        else if ((long long)(up[i + 1].y - up[i].y) * (lo[j].x - lo[j - 1].x)
            > (long long)(up[i + 1].x - up[i].x) * (lo[j].y - lo[j - 1].y)) ++i;
        else --j;
    }
}
```

6.5 Point in Polygon Test

```
typedef double coord_t;

inline coord_t is_left(Point p0, Point p1, Point p2) {
    return (p1.x - p0.x) * (p2.y - p0.y) - (p2.x - p0.x) * (p1.y - p0.y);
}

// point in polygon test
// http://geomalgorithms.com/a03-_inclusion.html
bool is_in_polygon(Point p, vector<Point>& poly) {
    int wn = 0;
    for (int i = 0; i < poly.size(); ++i) {
        int ni = (i + 1 == poly.size()) ? 0 : i + 1;
        if (poly[i].y <= p.y) {
            if (poly[ni].y > p.y) {
                if (is_left(poly[i], poly[ni], p) > 0) {
                    ++wn;
                }
            }
        }
        else {
            if (poly[ni].y <= p.y) {
                if (is_left(poly[i], poly[ni], p) < 0) {
                    --wn;
                }
            }
        }
    }
}
```

```
    }
}
return wn != 0;
}
```

6.6 Polygon Cut

```
// Left side of a->b
vector<Point> cut_polygon(const vector<Point>& polygon, Line line) {
    if (!polygon.size()) return polygon;
    typedef vector<Point>::const_iterator piter;
    piter la, lan, fi, fip, i, j;
    la = lan = fi = fip = polygon.end();
    i = polygon.end() - 1;
    bool lastin = diff(ccw_line(line, polygon[polygon.size() - 1]), 0) > 0;
    for (j = polygon.begin(); j != polygon.end(); j++) {
        bool thisin = diff(ccw_line(line, *j), 0) > 0;
        if (lastin && !thisin) {
            la = i;
            lan = j;
        }
        if (!lastin && thisin) {
            fi = j;
            fip = i;
        }
        i = j;
        lastin = thisin;
    }
    if (fi == polygon.end()) {
        if (!lastin) return vector<Point>();
        return polygon;
    }
    vector<Point> result;
    for (i = fi; i != lan; i++) {
        if (i == polygon.end()) {
            i = polygon.begin();
            if (i == lan) break;
        }
        result.push_back(*i);
    }
    Point lc, fc;
    get_cross(Line{ *la, *lan - *la }, line, lc);
    get_cross(Line{ *fip, *fi - *fip }, line, fc);
    result.push_back(lc);
    if (diff(dist2(lc, fc), 0) != 0) result.push_back(fc);
    return result;
}
```

6.7 Pick's theorem

격자점으로 구성된 simple polygon이 주어짐. i 는 polygon 내부의 격자점 수, b 는 polygon 선분 위 격자점 수, A 는 polygon의 넓이라고 할 때, 다음과 같은 식이 성립한다.

$$A = i + \frac{b}{2} - 1$$

7 String

7.1 KMP

```
typedef vector<int> seq_t;

void calculate_pi(vector<int>& pi, const seq_t& str) {
    pi[0] = -1;
    for (int i = 1, j = -1; i < str.size(); i++) {
        while (j >= 0 && str[i] != str[j + 1]) j = pi[j];
        if (str[i] == str[j + 1])
            pi[i] = ++j;
        else
            pi[i] = -1;
    }
}

// returns all positions matched
// O(|text|+|pattern|)
vector<int> kmp(const seq_t& text, const seq_t& pattern) {
    vector<int> pi(pattern.size()), ans;
    if (pattern.size() == 0) return ans;
    calculate_pi(pi, pattern);
    for (int i = 0, j = -1; i < text.size(); i++) {
        while (j >= 0 && text[i] != pattern[j + 1]) j = pi[j];
        if (text[i] == pattern[j + 1]) {
            j++;
            if (j + 1 == pattern.size()) {
                ans.push_back(i - j);
                j = pi[j];
            }
        }
    }
    return ans;
}
```

7.2 Z Algorithm

```
// Z[i] : maximum common prefix length of &s[0] and &s[i]
// O(|s|)
using seq_t = string;
vector<int> z_func(const seq_t &s) {
    vector<int> z(s.size());
```

```
    z[0] = s.size();
    int l = 0, r = 0;

    for (int i = 1; i < s.size(); i++) {
        if (i > r) {
            int j;
            for (j = 0; i + j < s.size() && s[i + j] == s[j]; j++) ;
            z[i] = j; l = i; r = i + j - 1;
        } else if (z[i - l] < r - i + 1) {
            z[i] = z[i - l];
        } else {
            int j;
            for (j = 1; r + j < s.size() && s[r + j] == s[r - i + j]; j++) ;
            z[i] = r - i + j; l = i; r += j - 1;
        }
    }

    return z;
}
```

7.3 Aho-Corasick

```
struct AhoCorasick
{
    const int alphabet;
    struct node {
        node() {}
        explicit node(int alphabet) : next(alphabet) {}
        vector<int> next, report;
        int back = 0, output_link = 0;
    };
    int maxid = 0;
    vector<node> dfa;
    explicit AhoCorasick(int alphabet) : alphabet(alphabet), dfa(1, node(
        alphabet)) {}
    template<typename InIt, typename Fn> void add(int id, InIt first, InIt last,
        Fn func) {
        int cur = 0;
        for (; first != last; ++first) {
            auto s = func(*first);
            if (auto next = dfa[cur].next[s]) cur = next;
            else {
                cur = dfa[cur].next[s] = (int)dfa.size();
                dfa.emplace_back(alphabet);
            }
        }
        dfa[cur].report.push_back(id);
        maxid = max(maxid, id);
    }
    void build() {
        queue<int> q;
        vector<char> visit(dfa.size());
        visit[0] = 1;
        q.push(0);
```

```

while(!q.empty()) {
    auto cur = q.front(); q.pop();
    dfa[cur].output_link = dfa[cur].back;
    if (dfa[dfa[cur].back].report.empty())
        dfa[cur].output_link = dfa[dfa[cur].back].output_link;
    for (int s = 0; s < alphabet; s++) {
        auto &next = dfa[cur].next[s];
        if (next == 0) next = dfa[dfa[cur].back].next[s];
        if (visit[next]) continue;
        if (cur) dfa[next].back = dfa[dfa[cur].back].next[s];
        visit[next] = 1;
        q.push(next);
    }
}
}
template<typename InIt, typename Fn> vector<int> countMatch(InIt first, InIt
last, Fn func) {
    int cur = 0;
    vector<int> ret(maxid+1);
    for (; first != last; ++first) {
        cur = dfa[cur].next[func(*first)];
        for (int p = cur; p; p = dfa[p].output_link)
            for (auto id : dfa[p].report) ret[id]++;
    }
    return ret;
}
};

```

7.4 Suffix Array with LCP

```

typedef char T;

// calculates suffix array.
// O(n*logn)
vector<int> suffix_array(const vector<T>& in) {
    int n = (int)in.size(), c = 0;
    vector<int> temp(n), pos2bckt(n), bckt(n), bpos(n), out(n);
    for (int i = 0; i < n; i++) out[i] = i;
    sort(out.begin(), out.end(), [&](int a, int b) { return in[a] < in[b]; });
    for (int i = 0; i < n; i++) {
        bckt[i] = c;
        if (i + 1 == n || in[out[i]] != in[out[i + 1]]) c++;
    }
    for (int h = 1; h < n && c < n; h <= 1) {
        for (int i = 0; i < n; i++) pos2bckt[out[i]] = bckt[i];
        for (int i = n - 1; i >= 0; i--) bpos[bckt[i]] = i;
        for (int i = 0; i < n; i++)
            if (out[i] >= n - h) temp[bpos[bckt[i]]++] = out[i];
        for (int i = 0; i < n; i++)
            if (out[i] >= h) temp[bpos[pos2bckt[out[i] - h]]++] = out[i] - h;
        c = 0;
        for (int i = 0; i + 1 < n; i++) {
            int a = (bckt[i] != bckt[i + 1]) || (temp[i] >= n - h)
                || (pos2bckt[temp[i + 1] + h] != pos2bckt[temp[i] + h]);

```

```

        bckt[i] = c;
        c += a;
    }
    bckt[n - 1] = c++;
    temp.swap(out);
}
return out;
}

// calculates lcp array. it needs suffix array & original sequence.
// O(n)
vector<int> lcp(const vector<T>& in, const vector<int>& sa) {
    int n = (int)in.size();
    if (n == 0) return vector<int>();
    vector<int> rank(n), height(n - 1);
    for (int i = 0; i < n; i++) rank[sa[i]] = i;
    for (int i = 0, h = 0; i < n; i++) {
        if (rank[i] == 0) continue;
        int j = sa[rank[i] - 1];
        while (i + h < n && j + h < n && in[i + h] == in[j + h]) h++;
        height[rank[i] - 1] = h;
        if (h > 0) h--;
    }
    return height;
}

```

7.5 Manacher's Algorithm

```

// find longest palindromic span for each element in str
// O(|str|)
void manacher(const string& str, int plen[]) {
    int r = -1, p = -1;
    for (int i = 0; i < str.length(); ++i) {
        if (i <= r)
            plen[i] = min((2 * p - i >= 0) ? plen[2 * p - i] : 0, r - i);
        else
            plen[i] = 0;
        while (i - plen[i] - 1 >= 0 && i + plen[i] + 1 < str.length()
            && str[i - plen[i] - 1] == str[i + plen[i] + 1]) {
            plen[i] += 1;
        }
        if (i + plen[i] > r) {
            r = i + plen[i];
            p = i;
        }
    }
}

```

8 Miscellaneous

8.1 Fast I/O


```

namespace fio {
    const int BSIZE = 524288;
    char buffer[BSIZE];
    int p = BSIZE;
    inline char readChar() {
        if(p == BSIZE) {
            fread(buffer, 1, BSIZE, stdin);
            p = 0;
        }
        return buffer[p++];
    }
    int readInt() {
        char c = readChar();
        while ((c < '0' || c > '9') && c != '-') {
            c = readChar();
        }
        int ret = 0; bool neg = c == '-';
        if (neg) c = readChar();
        while (c >= '0' && c <= '9') {
            ret = ret * 10 + c - '0';
            c = readChar();
        }
        return neg ? -ret : ret;
    }
}

```

8.2 Magic Numbers

소수 : 10 007 , 10 009 , 10 111 , 31 567 , 70 001 , 1 000 003 , 1 000 033 , 4 000 037 , 99 999 989 , 999 999 937 , 1 000 000 007 , 1 000 000 009 , 9 999 999 967 , 99 999 999 977

8.3 Java Examples

```

import java.util.Scanner;

class example
{
    final static int BSIZE = 524288;
    static byte[] buffer = new byte[BSIZE];
    static int p = BSIZE;

    static byte readByte()
    {
        if (p==BSIZE) {
            try {
                System.in.read(buffer); p=0;
            } catch (Exception e) {
                return '\0';
            }
        }
        return buffer[p++];
    }
}

```

```

public static void main(String[] args)
{
    Scanner in = new Scanner(System.in);
    int T = in.nextInt();
    while (T --> 0) {
        String str = in.next();
        if (str.matches("[A-F]?A+F+C+[A-F]?"))
            System.out.println("Infected!");
        else
            System.out.println("Good");
    }
}

```

8.4 체계적인 접근을 위한 질문들

“알고리즘 문제 해결 전략” 에서 발췌함

- 비슷한 문제를 풀어본 적이 있던가?
- 단순한 방법에서 시작할 수 있을까? (brute force)
- 내가 문제를 푸는 과정을 수식화할 수 있을까? (예제를 직접 해결해보면서)
- 문제를 단순화할 수 없을까?
- 그림으로 그려볼 수 있을까?
- 수식으로 표현할 수 있을까?
- 문제를 분해할 수 있을까?
- 뒤에서부터 생각해서 문제를 풀 수 있을까?
- 순서를 강제할 수 있을까?
- 특정 형태의 답만을 고려할 수 있을까? (정규화)