Learning to bid in revenue-maximizing auctions

Link

[Paper]
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[Code]

Notes

We consider the problem of the optimization of bidding strategies in prior-dependent revenue maximizing auctions, when the seller fixes the reserve prices based on the bid distributions. Our study is done in the setting where one bidder is strategic. Using a variational approach, we study the complexity of the original objective and we introduce a relaxation of the objective functional in order to use gradient descent methods. Our approach is simple, general and can be applied to various value distributions and revenue maximizing mechanisms. The new strategies we derive yield massive uplifts compared to the traditional truthfully bidding strategy.

- Introduce the optimization problem that strategic bidders are facing when the seller is optimizing personalized reserve prices based on their bid distributions. A straightforward optimization can fail because the objective is discontinuous as a function of the bidding strategy.
- Introduce a new relaxation of the problem which is stable to local perturbations of the objective function and computationally tractable and efficient.
- Optimize this new objective through a simple neural network and get very significant improvements in bidder utility compared to truthful bidding.
- Theoretical analysis of thresholded strategies

price. It is known that the optimal reserve price of bidder i is her monopoly price equal to $\arg\max_r r(1-F_{B_i}(r))$, or equivalently[1] to $\psi_{B_i}^{-1}(0)$, where ψ_{B_i} is the usual virtual value function defined as

$$\psi_{B_i}(b) = b - \frac{1-F_{B_i}(b)}{f_{B_i}(b)}.$$

An example on Criteo Data

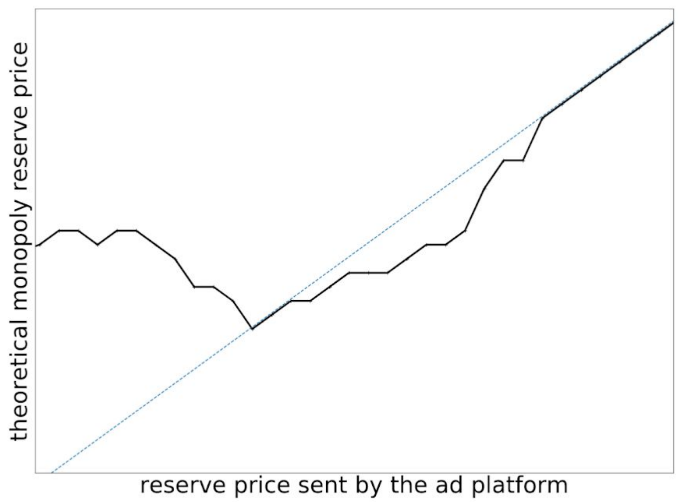

Figure: This plot was done on Criteo data. We bucketize all the requests we receive by the reserve price that was sent by a large ad platform. We then look on each bucket what would have been the optimal reserve price for Criteo. The plot is in log scale.

tion is G_i, the cdf of the maximum bid of players other than i; obviously, if the other bidders are truthful, G_i is the distribution of the maximum value of the other bidders.

Lemma

The utility of the strategic bidder using the strategy β increasing (ψ_B denotes the virtual value associated to the new distribution of bid) is given by:

$$\text{Bidder Utility}(r) = \mathbb{E}_{X_i \sim F_i}\Big((X_i - h_\beta(X_i)))G(\beta(X_i))\mathbf{1}_{[X_i \geq x_\beta]}\Big) .$$

with $h_\beta(X) = \psi_B(\beta(X)) = \beta(X) - \beta'(X)\frac{1-F(X)}{f(X)}$ *and* x_β *the reserve value.*

A. Results for the uniform distribution

Auction Type		K=2	K=3	K=4
Baselines	Utility of truthful strategy (in revenue maximizing)	0.083	0.057	0.040
	Utility of truthful strategy (in welfare maximizing)	0.166	0.083	0.050
Lazy second price auction	Utility of strategic bidder	0.141 ± 0.001	0.077 ± 0.001	0.048 ± 0.001
	Uplift vs truthful bidding	+72%	+36%	+20%
Eager second price auction	Utility of strategic bidder	0.126 ± 0.01	0.083 ± 0.01	0.050 ± 0.002
	Uplift vs revenue-maximizing	+51%	+46%	+25%
Myerson auction	Utility of strategic bidder	0.246 ± 0.001	0.131 ± 0.01	0.079 ± 0.001
	Uplift vs revenue-maximizing	+195%	+130%	+97.5%
Boosted second price	Utility of strategic bidder	0.24 ± 0.01	0.08 ± 0.01	0.055 ± 0.002
	Uplift vs revenue maximizing	+200%	+40%	+37.5%

Table 2. **All bidders have a uniform value distribution. The strategic bidder has $K-1$ opponents, all bidding truthfully. The reserve price of all other bidders is equal to 0.5. The reserve price of the strategic bidder is computed on her bid distribution. For each run, the evaluation is based on 10^6 samples. We average on 10 learnings the performance of the strategies.** The utility of the strategic bidder can be higher that in the welfare-maximizing auction because revenue maximizing auctions are removing the competition below the reserve price. We provide some examples of strategies in Appendix E.

Reference

1. First Price Auction vs. Second Price Auction