RL PROJECT

CALL OPTIONS PRICING

USING REINFORCEMENT LEARNING ALGORITHMS FOR CALL OPTION PRICING

INTRODUCTION

- *A call option is a financial contract that gives its holder the right, but not the obligation, to buy specified securities at a predetermined price known as the strike price until a specific fixed date known as the expiry date.*

- In our project, an investor has a call option to purchase one share of a stock for a fixed price p and has T days to exercise it. For simplicity, we assume that the investor takes a decision at the start of each day.

- The investor may decide not to exercise the option but if he does exercise the option when the stock price is s, he effectively gets (s - p).

- The price of the stock is assumed to be varied with independent increments, *i.e.*, the price on day t + 1 is $S_{t+1} = S_t + W_t$

INTRODUCTION

- We assume $p \in N$ and W_t to be (discrete) uniformly distributed with endpoints $-\varepsilon$ and $+\varepsilon$ for some $\varepsilon \in N$. For example, for $\varepsilon=5$, W_t $U\{-5,5\}$.

- We developed an agent that determines whether to exercise (or not exercise) the call option over a certain time period of T days.

- Specifically, our agent provides an optimal price for each of the T days, and if the stock price on that day exceeds the optimal price, the investor should sell his call option on that day.

ENVIRONMENT

- Action Space is defined as { 0,1 }.
    - a_t = 0 indicates that we hold
    - a_t = 1 indicates that we sell (i.e. exercise the call option)

- A state is defined as s_t = { X_t , t } where X_t is the current price and t is the days spent.

ENVIRONMENT

- The Reward Function is defined as :
  - $R_t(s_t, a_t) = 0$ if $a_t = 0$
  - $R_t(s_t, a_t) = max(0, X_t - P_s)$ if $a_t = 1$ where P_s is the strike price.
  - The reward function is basically defined as the profit we gain by selling our option.

- The transition from one state to another is defined as $S_{t+1} = S_t + W_t$ where ,W_t Uniform(-val, val) {val is the max deviation possible}.

RL ALGORITHMS USED

We implemented a finite horizon version of the following algorithms:

1. Value Iteration
2. Policy Iteration
3. Monte Carlo control
4. Q-Learning
5. TD control using SARSA(λ)

VALUE ITERATION

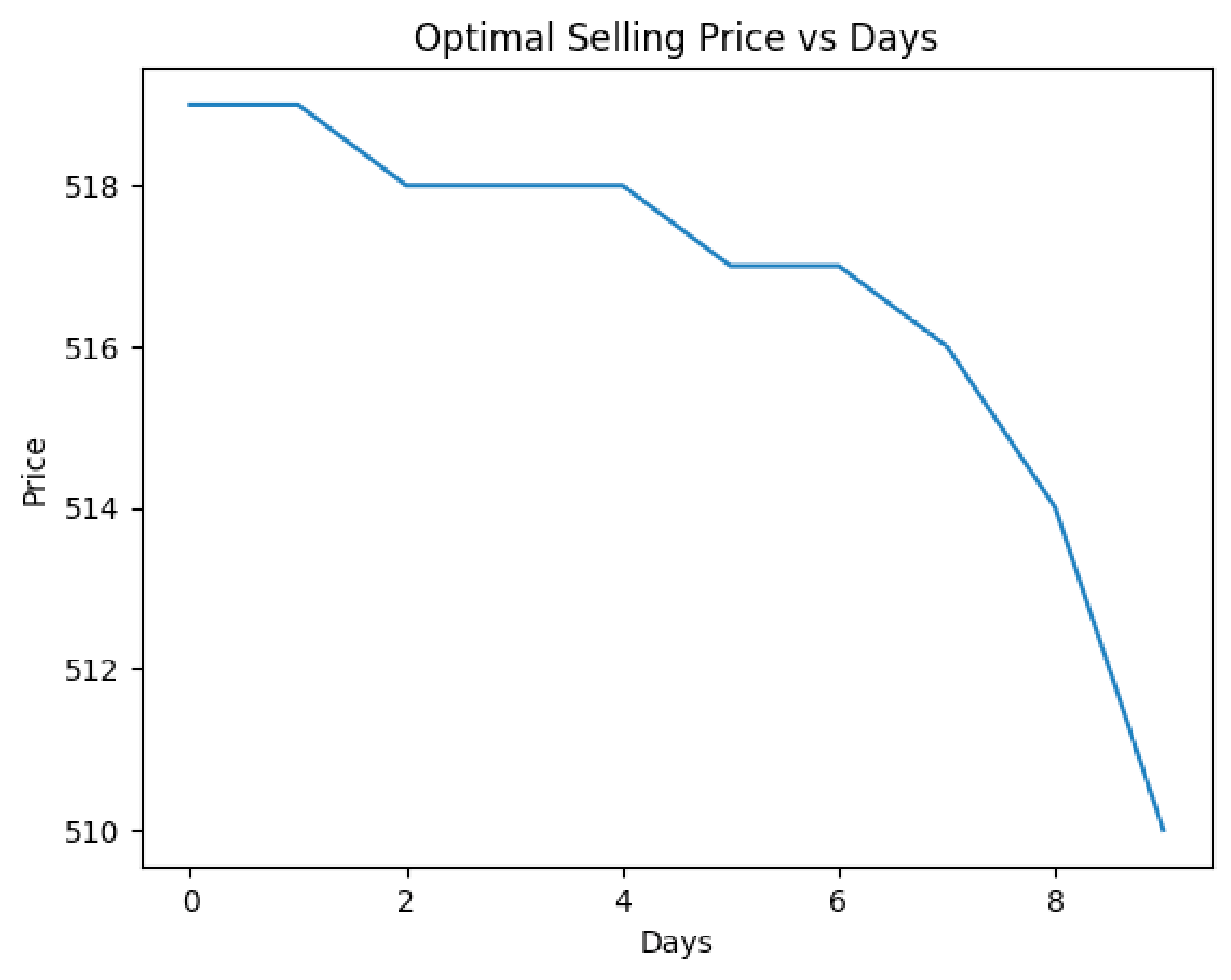
Pseudo Code:

Algorithm 1 Finite Horizon Value Iteration

for $t = T-1, T-2, \ldots, 0$ **do**
  for $s \in \mathcal{S}$ **do**
    $\pi_t(s), V_t(s) = \text{maximize}_a \mathbb{E}\left[r_t + V_{t+1}(s_{t+1})\right]$
  end for
end for

Result Obtained:

(strike price = 510, start price = 500, drift = 5 and T = 10)



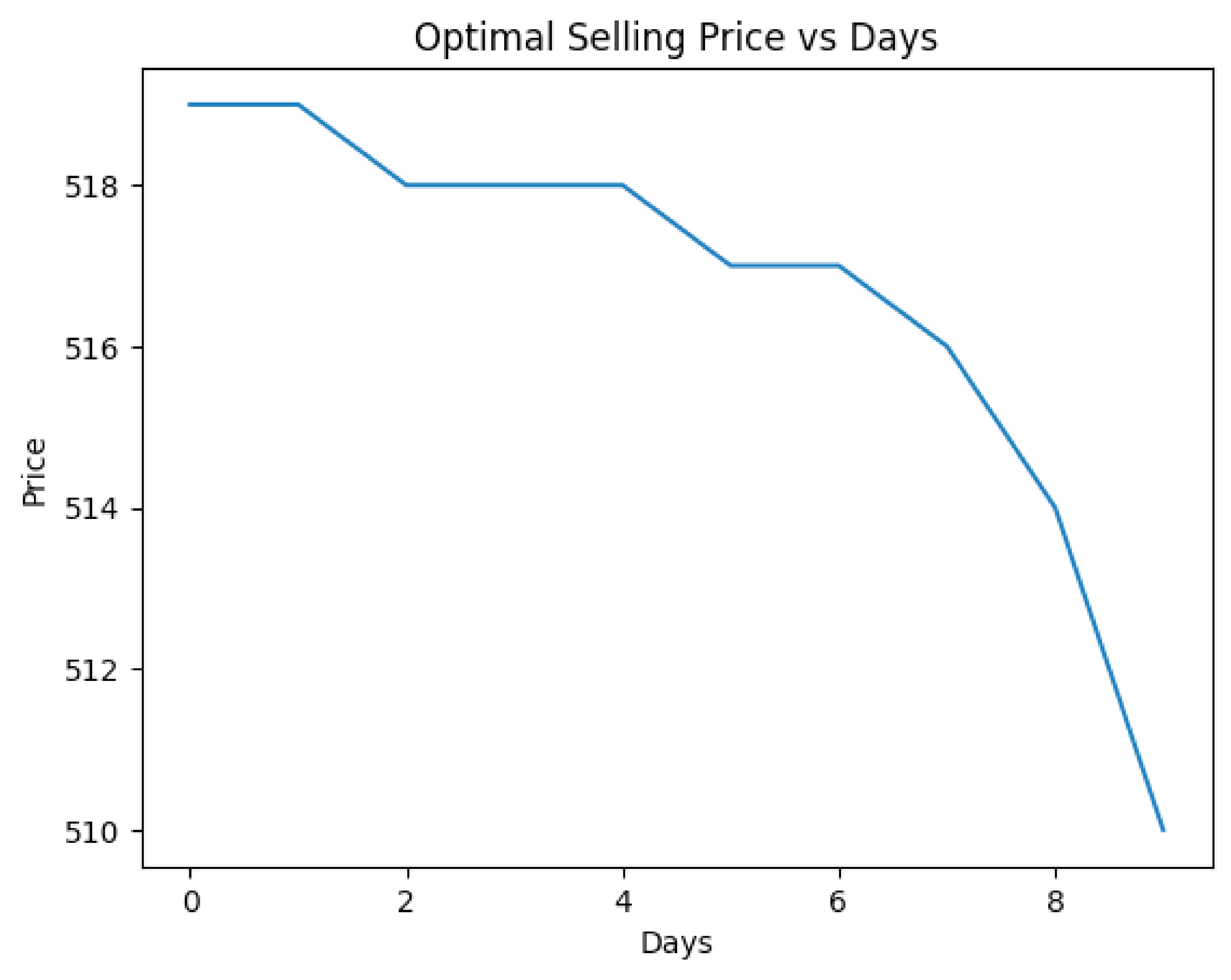
POLICY ITERATION

Pseudo Code

Algorithm 4 Policy Iteration

Initialize $\pi^{(0)}$.
for $n = 1, 2, \ldots$ **do**
  $V^{(n-1)} = \mathrm{Solve}[V = \mathcal{T}^{\pi^{(n-1)}} V]$
  $\pi^{(n)} = \mathcal{G} V^{\pi^{(n-1)}}$
end for

Result Obtained:
(strike price = 510, start price = 500, drift = 5 and T = 10)



MONTE CARLO

We have implemented On-policy every-visit Monte Carlo.

Pseudo Code Followed

On-policy first-visit MC control (for ε-soft policies), estimates $\pi \approx \pi_*$

Algorithm parameter: small $\varepsilon > 0$
Initialize:
  $\pi \leftarrow$ an arbitrary ε-soft policy
  $Q(s, a) \in \mathbb{R}$ (arbitrarily), for all $s \in \mathcal{S}$, $a \in \mathcal{A}(s)$
  $Returns(s, a) \leftarrow$ empty list, for all $s \in \mathcal{S}$, $a \in \mathcal{A}(s)$

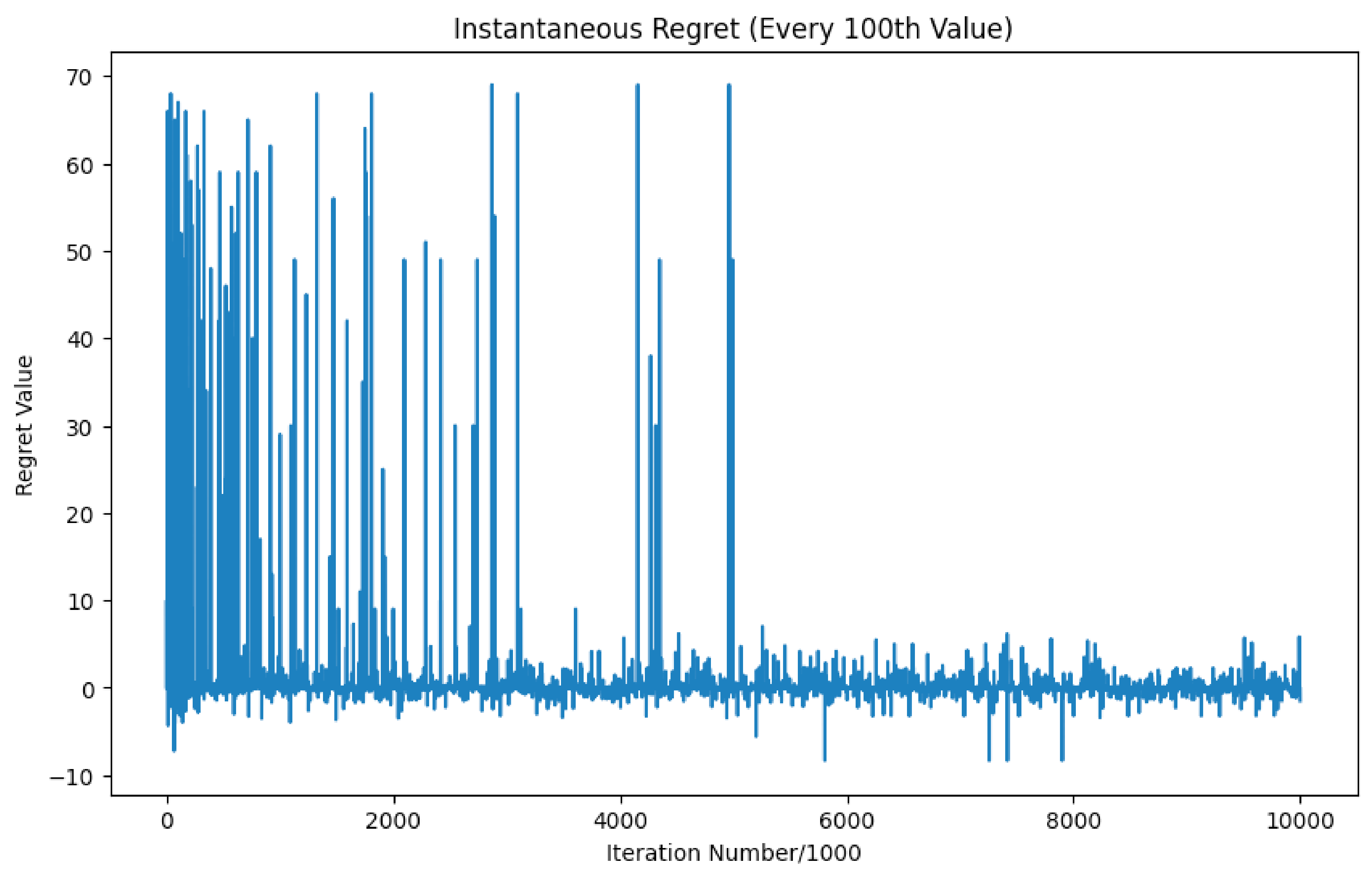
Repeat forever (for each episode):
  Generate an episode following π: $S_0, A_0, R_1, \ldots, S_{T-1}, A_{T-1}, R_T$
  $G \leftarrow 0$
  Loop for each step of episode, $t = T-1, T-2, \ldots, 0$:
    $G \leftarrow \gamma G + R_{t+1}$
    Unless the pair S_t, A_t appears in $S_0, A_0, S_1, A_1 \ldots, S_{t-1}, A_{t-1}$:
      Append G to $Returns(S_t, A_t)$
      $Q(S_t, A_t) \leftarrow \text{average}(Returns(S_t, A_t))$
      $A^* \leftarrow \arg\max_a Q(S_t, a)$ (with ties broken arbitrarily)
      For all $a \in \mathcal{A}(S_t)$:

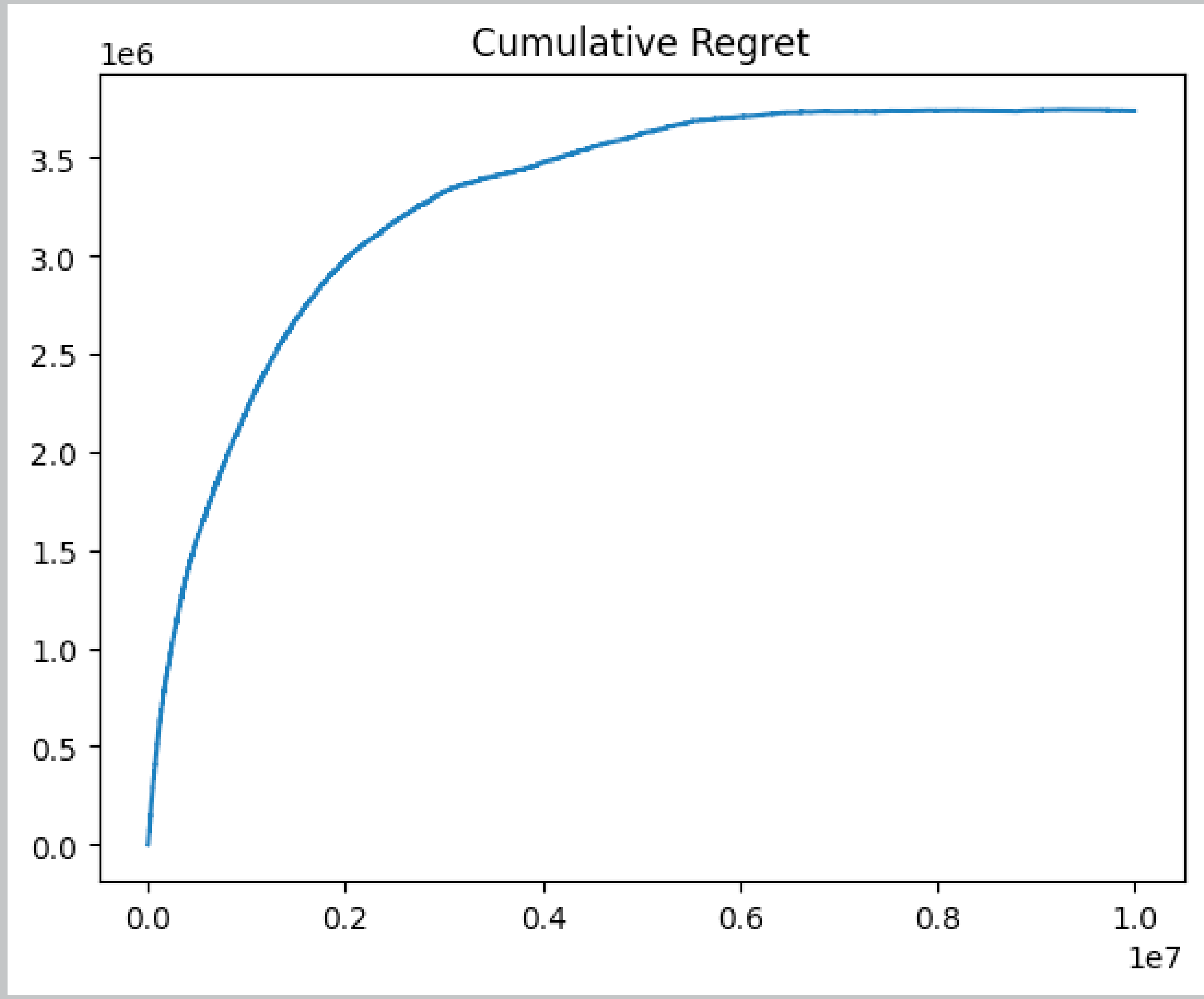
$$\pi(a|S_t) \leftarrow \begin{cases} 1 - \varepsilon + \varepsilon/|\mathcal{A}(S_t)| & \text{if } a = A^* \\ \varepsilon/|\mathcal{A}(S_t)| & \text{if } a \neq A^* \end{cases}$$

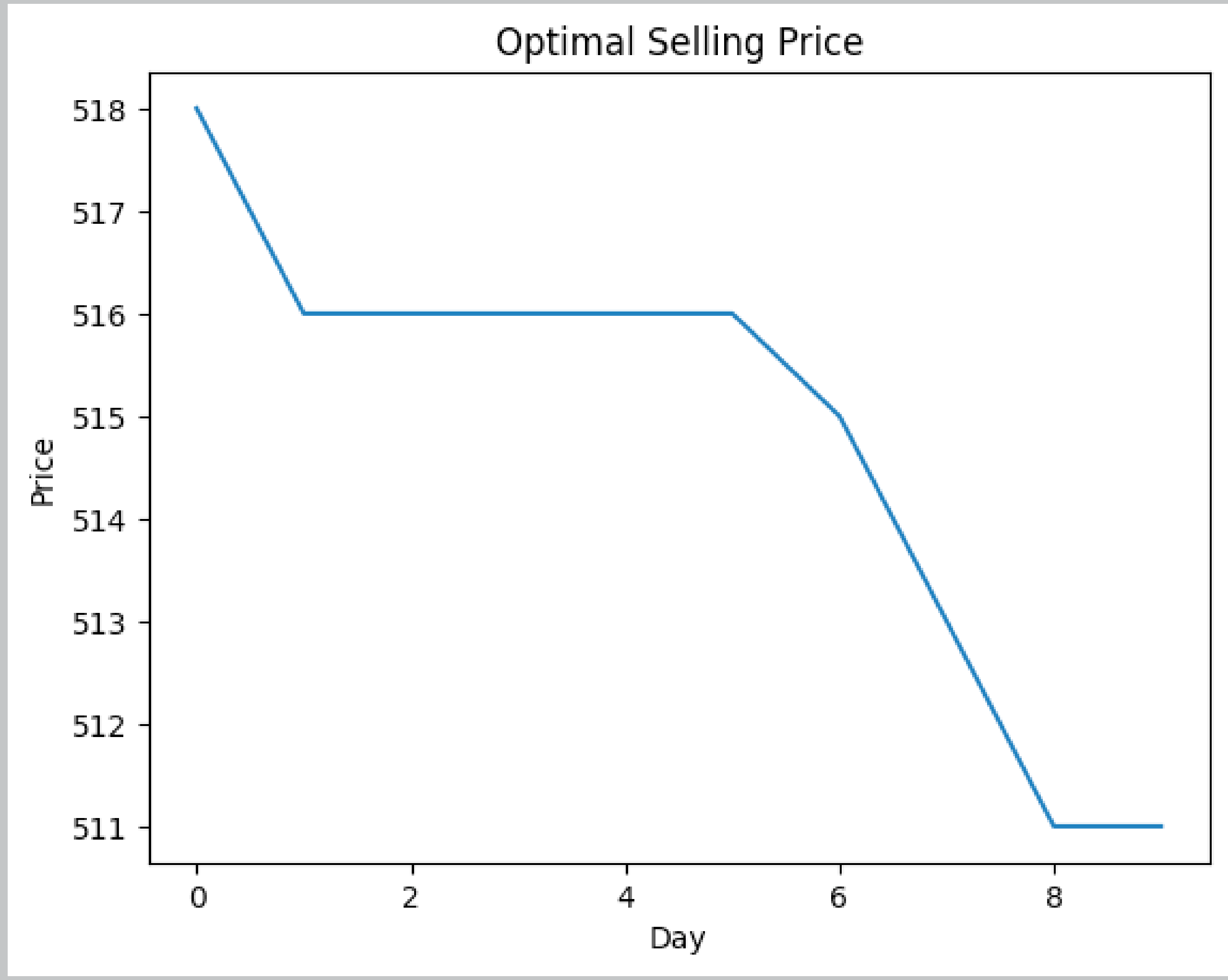
MONTE CARLO

Results Obtained:

(strike price = 510, start price = 500, drift = 8 and T = 10)





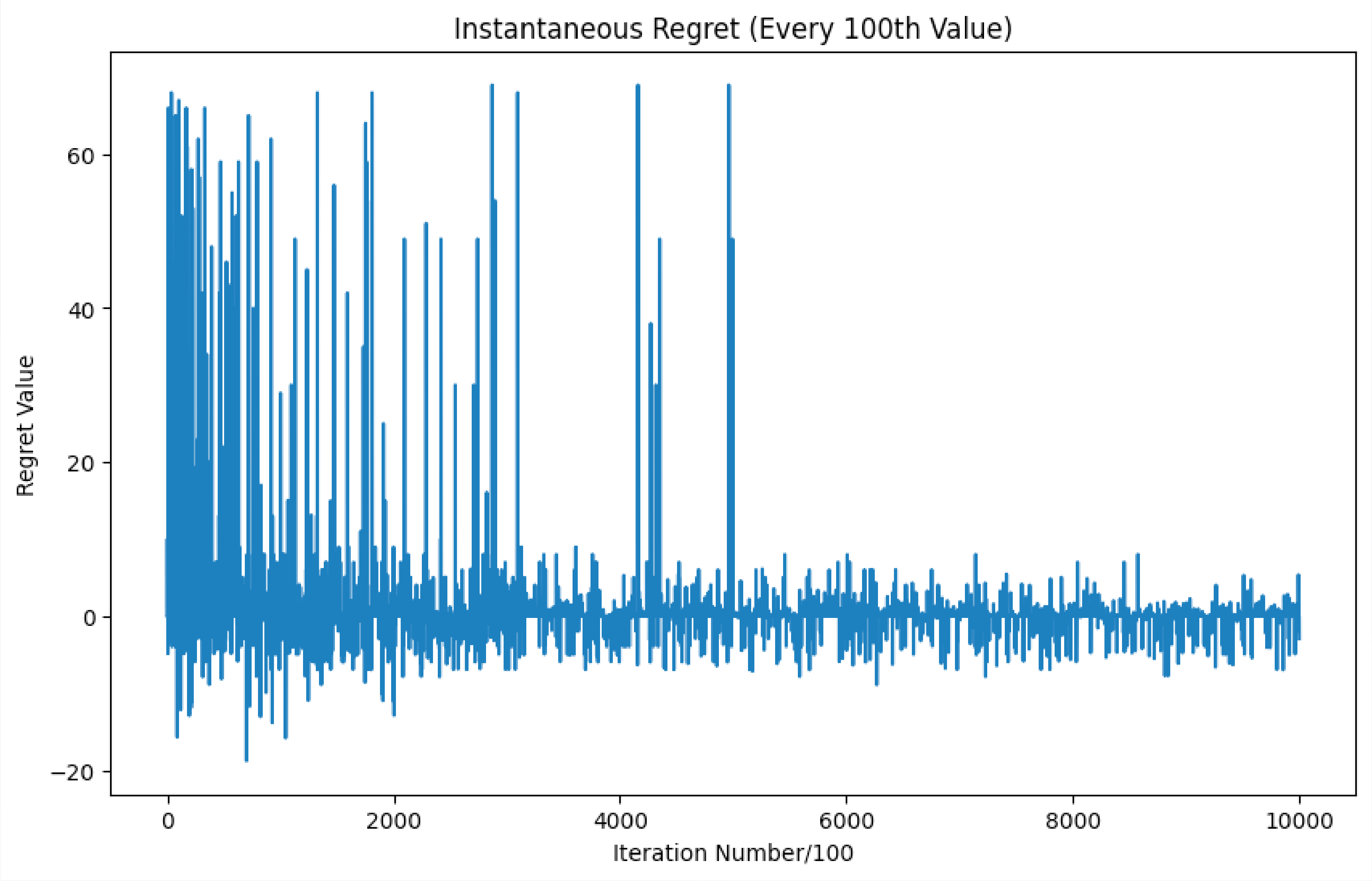


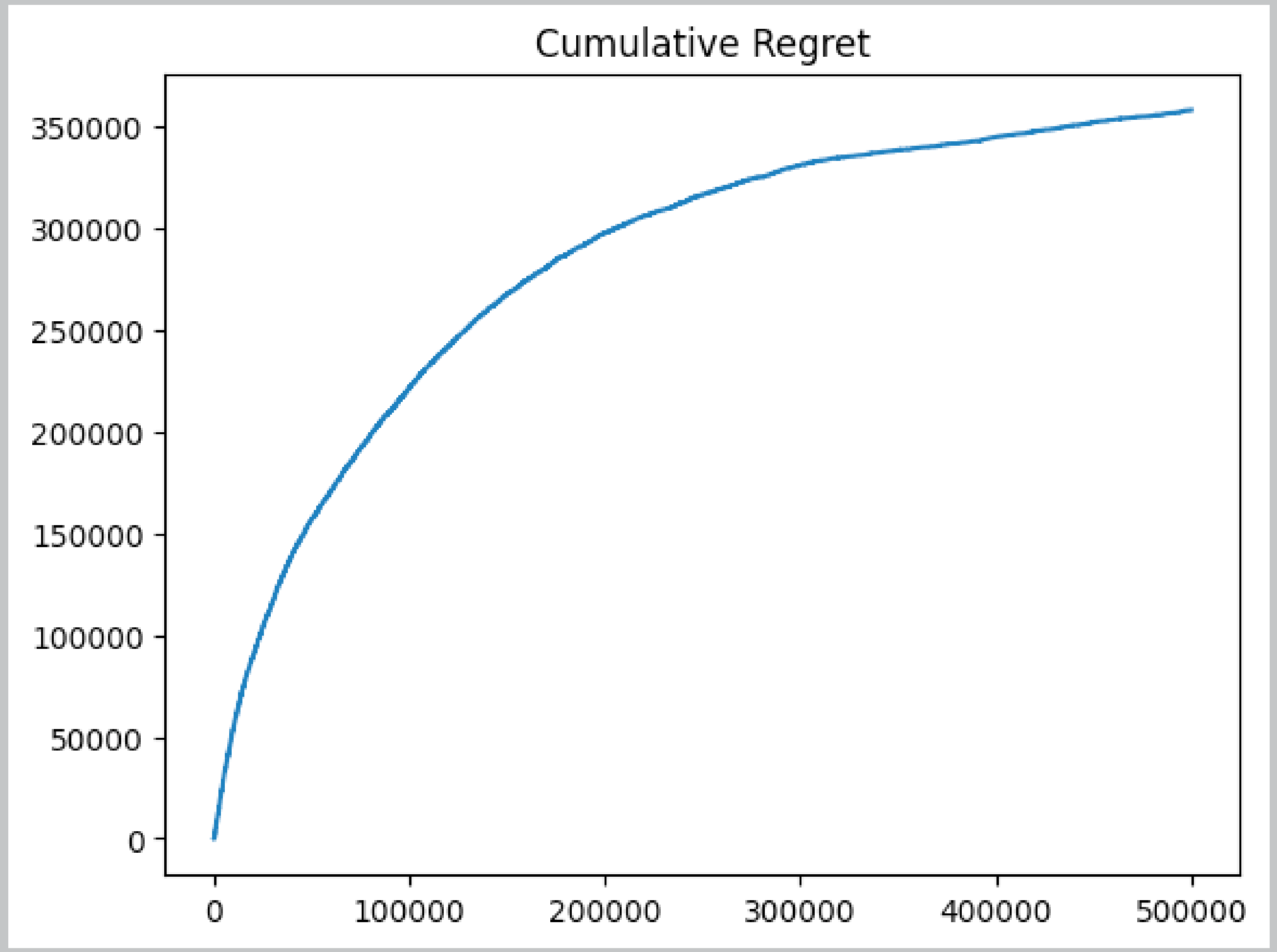
MONTE CARLO

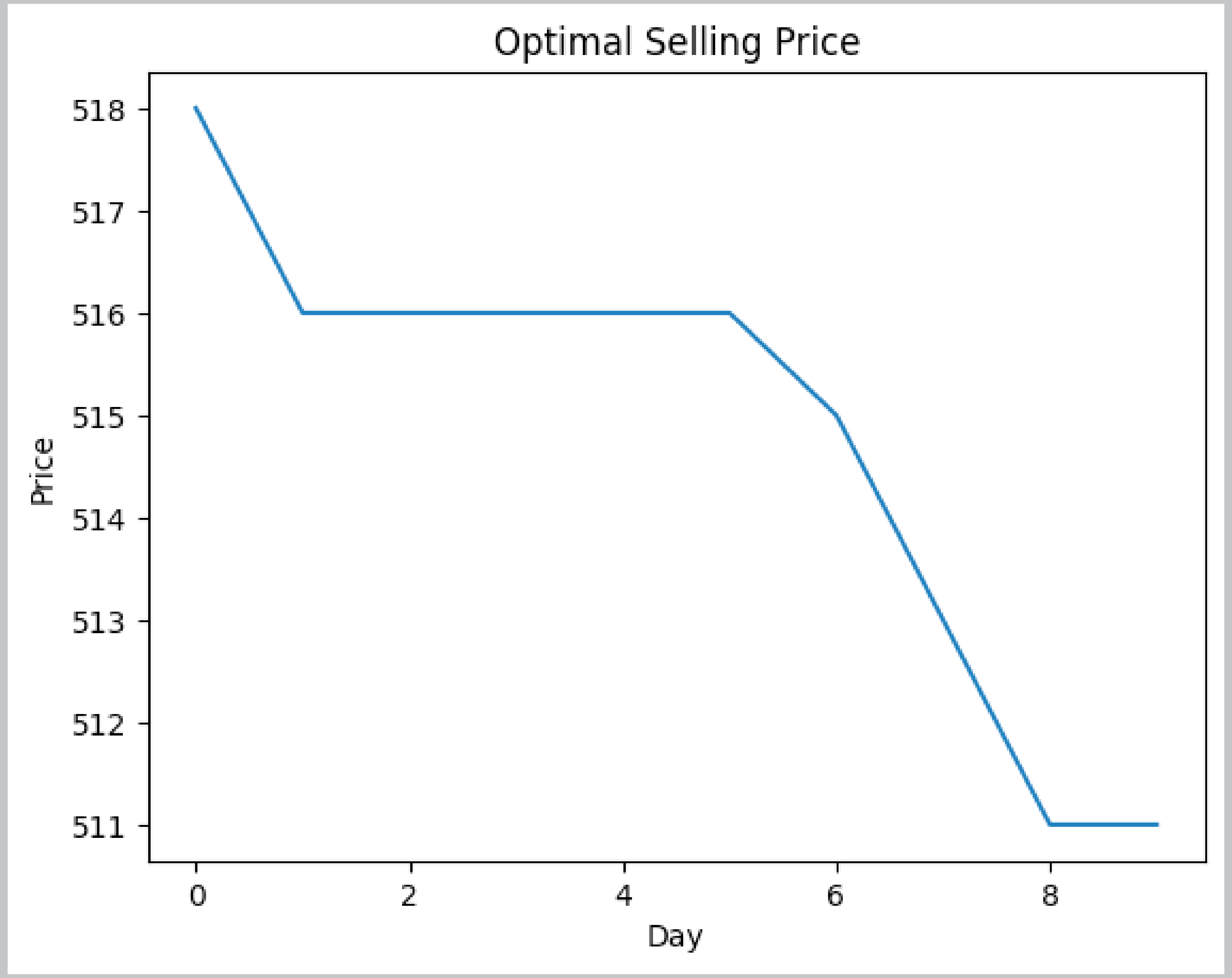
Updated Values of Regret:

Results Obtained:

(strike price = 510, start price = 500, drift = 8 and T = 10)







Q-LEARNING

Pseudo Code followed:

Algorithm 1 Finite Horizon Q-Learning

Notation:

$Q_n^m(i,a)$: Q-value for state i, action a, stage n, recursion m.

$a(m)$: step-size at recursion index m

$Q_N(i,a)$: Q-value for state i and action a at terminal stage (N).

$g_n(i,a,j)$: Single stage cost for stage n where current state is i, action is a and next state is j.

$g_N(i)$: Terminal reward at the N^{th} stage when terminal state is i.

$A(j)$: Set of feasible actions in state j.

$\eta(i,a)$: Sampling function taking input (i,a) as state-action pair that returns the next state.

Input: Samples of the form $\big(i$ (current state), a (action), g (cost), j (next state)$\big)$.

Output: Updated Q-value $Q_n^{m+1}(i,a)$ estimated after m iterations of the algorithm.

Initialization: $Q_n^0(i,a) = 0,\ \forall(i,a), n = 0,\dots,N-1,$ and $Q_N^0(i,a) = g_N(i), \forall(i,a)$

procedure Finite Horizon Q-Learning:

$a(m) = \left\lceil \frac{1}{(m+1)/10} \right\rceil$

$j = \eta(i,a)$ (from samples)

$Q_n^{m+1}(i,a) = \big(1 - a(m)\big)\Big(Q_n^m(i,a)\Big) + a(m)$

$\times\Big(g_n(i,a) + \min_{b \in A(j)} Q_{n+1}^m(j,b)\Big), n = 0, 1, \dots, N-1,$

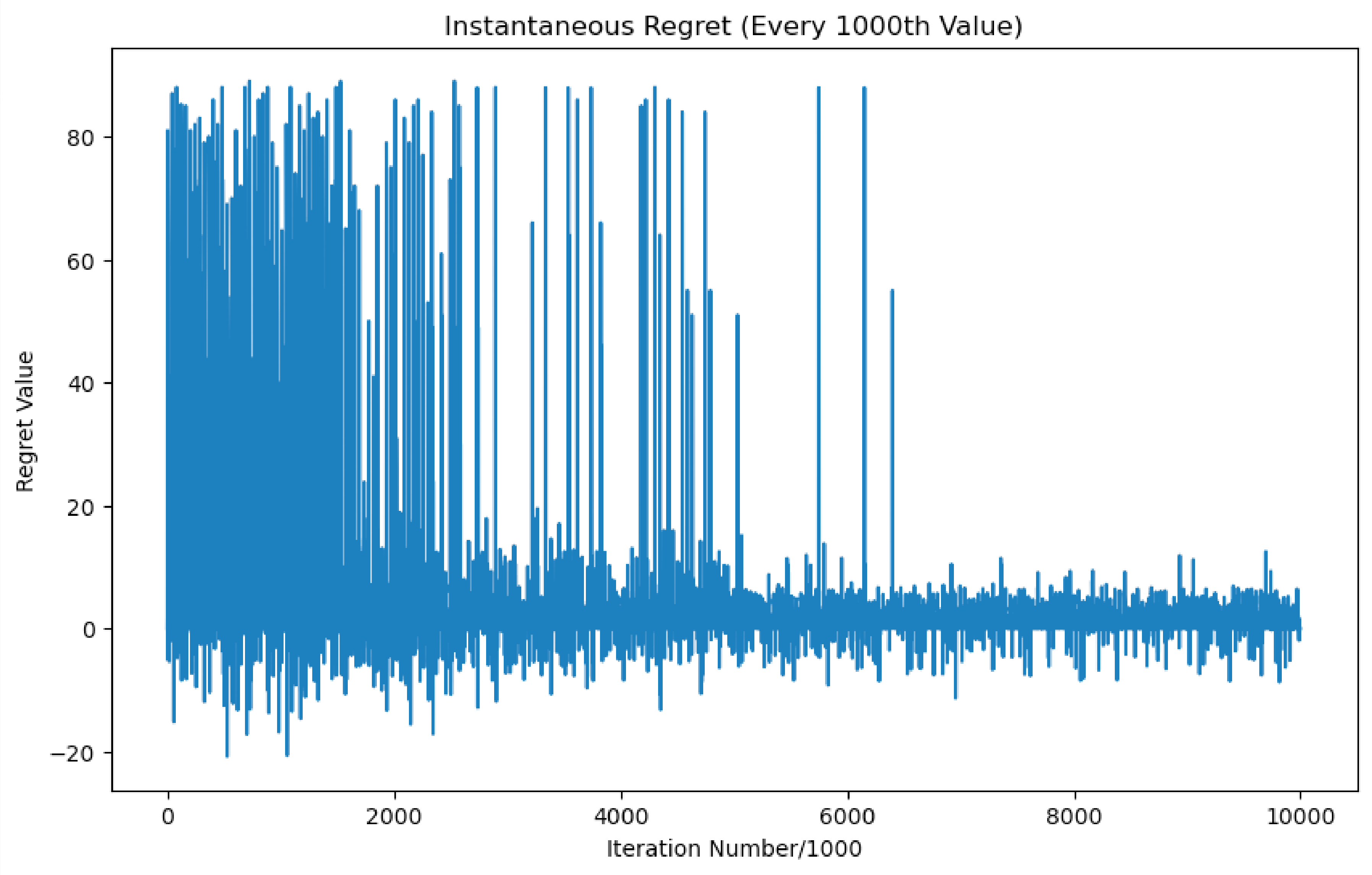
$Q_N^{m+1}(i,a) = g_N(i),\ \forall(i,a)$ tuples.

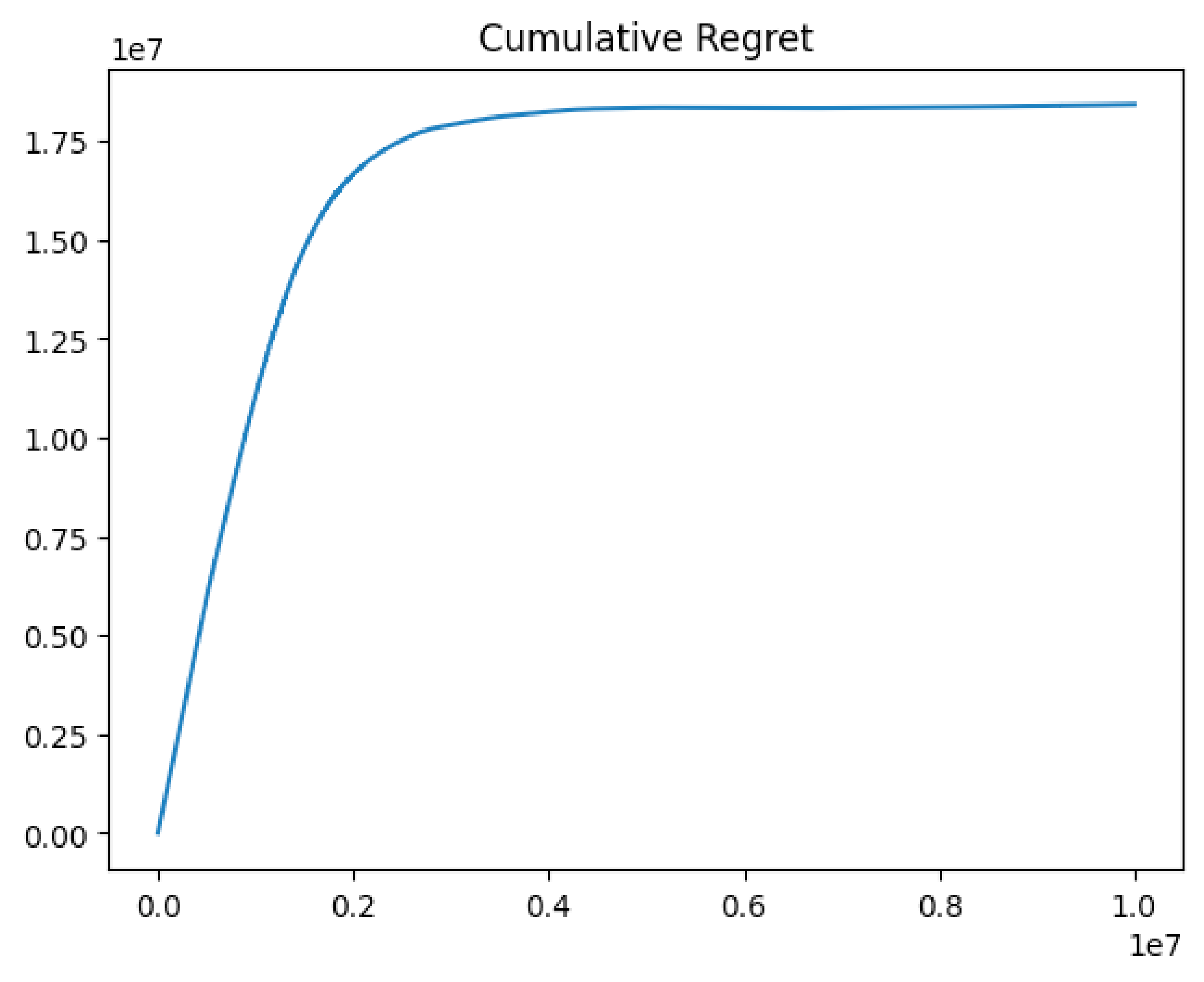
return $Q_n^{m+1}(i,a)$

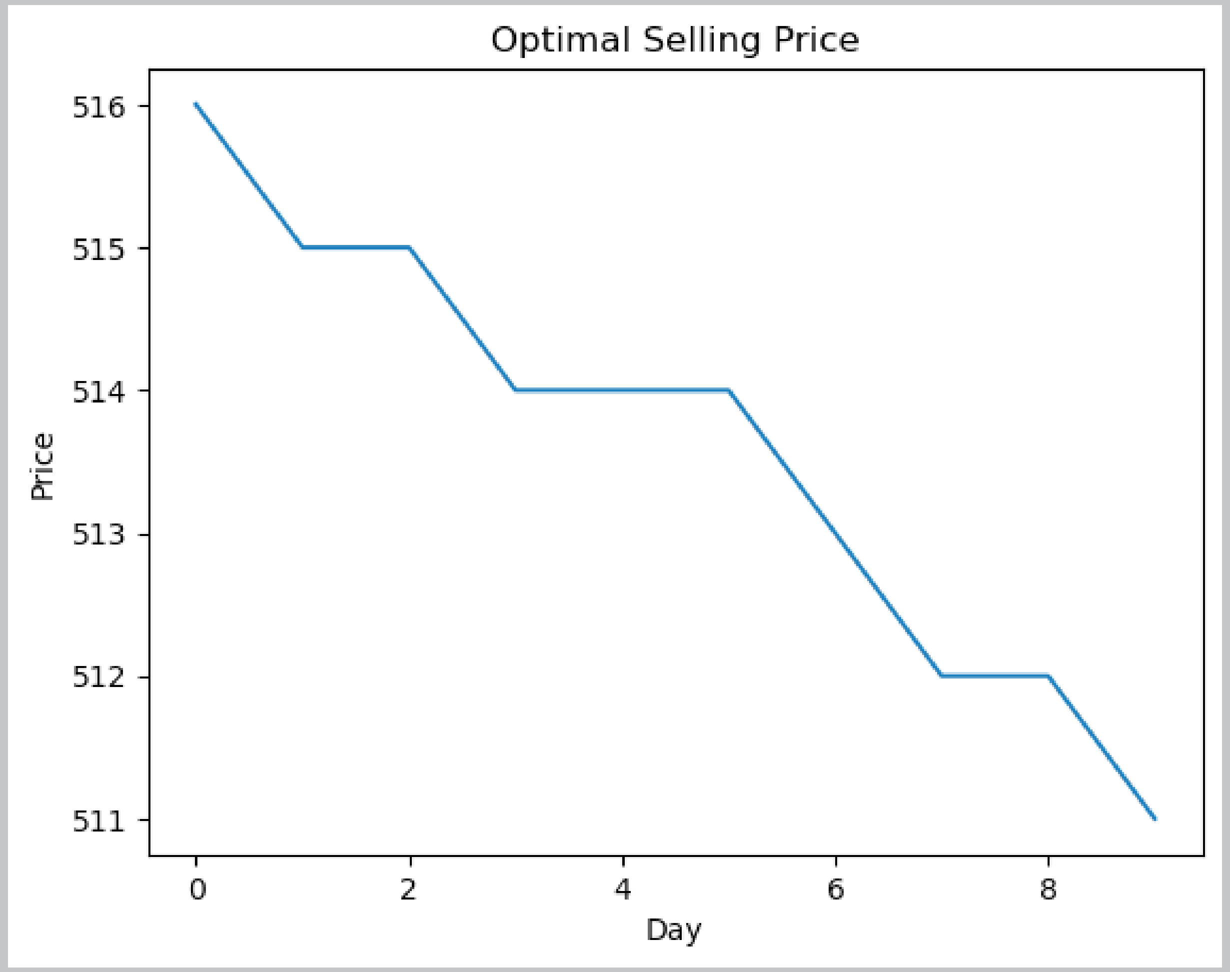
Q-LEARNING

Results Obtained:

(strike price = 510, start price = 500, drift = 5 and T = 10)





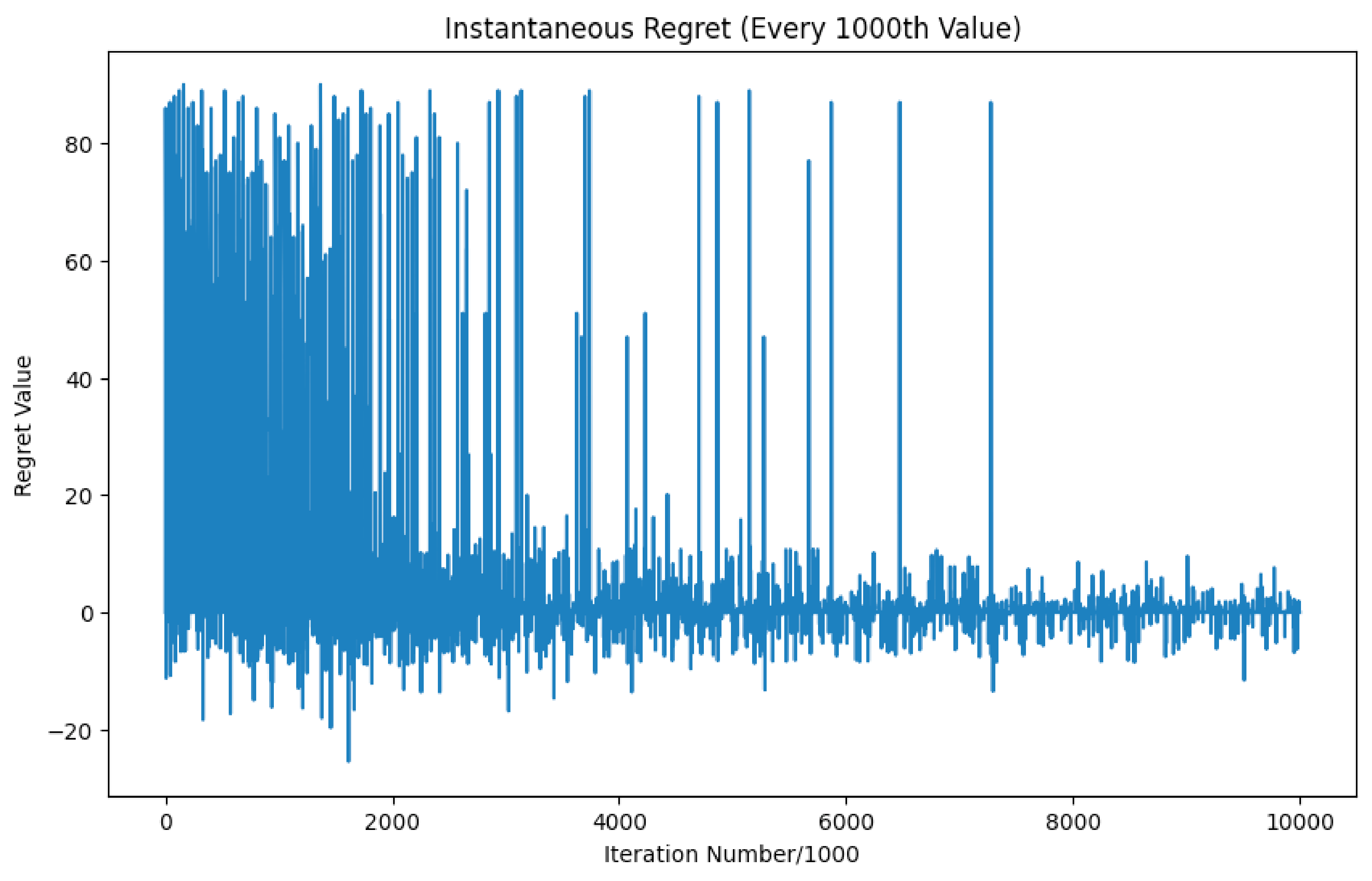


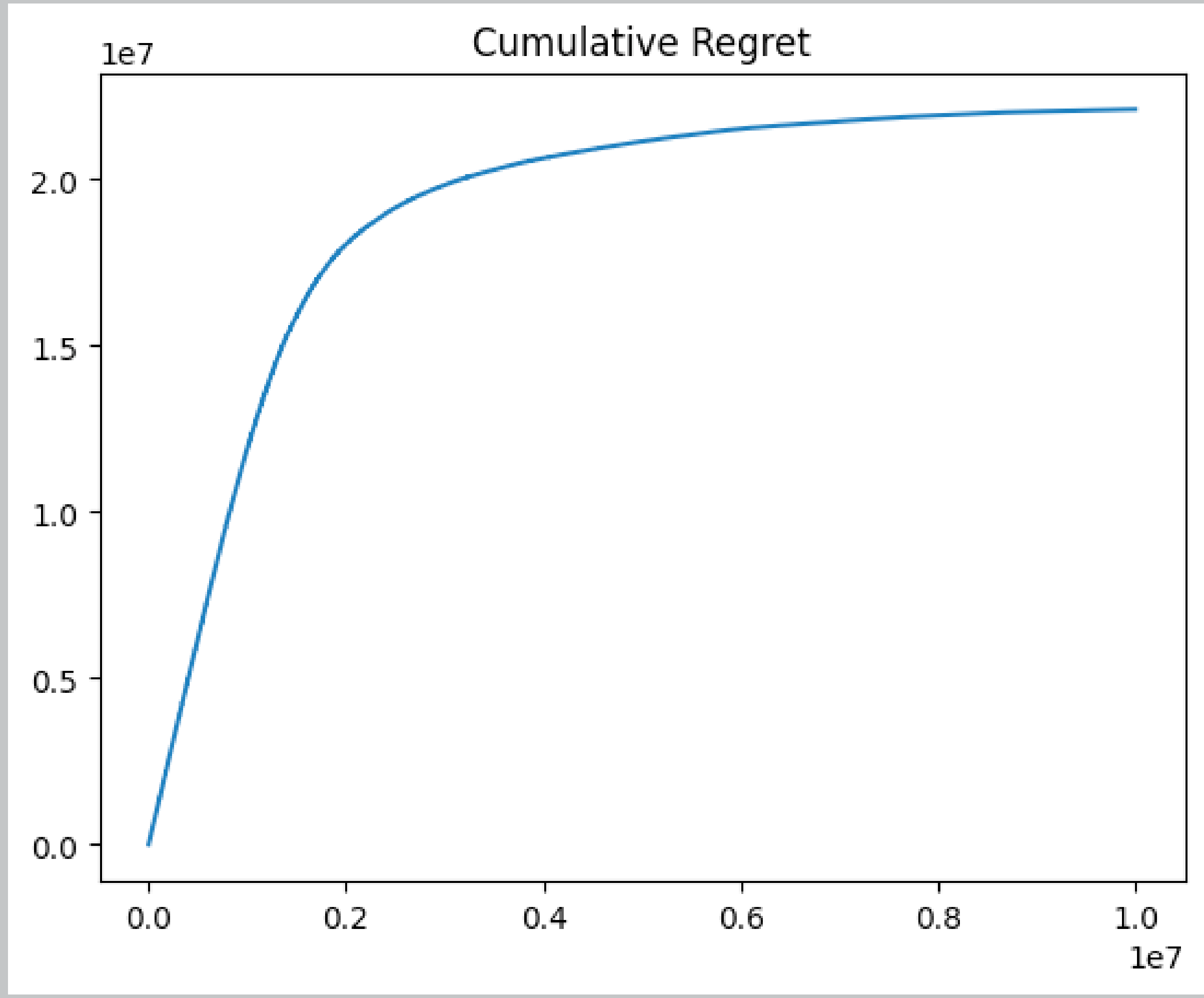
Q-LEARNING

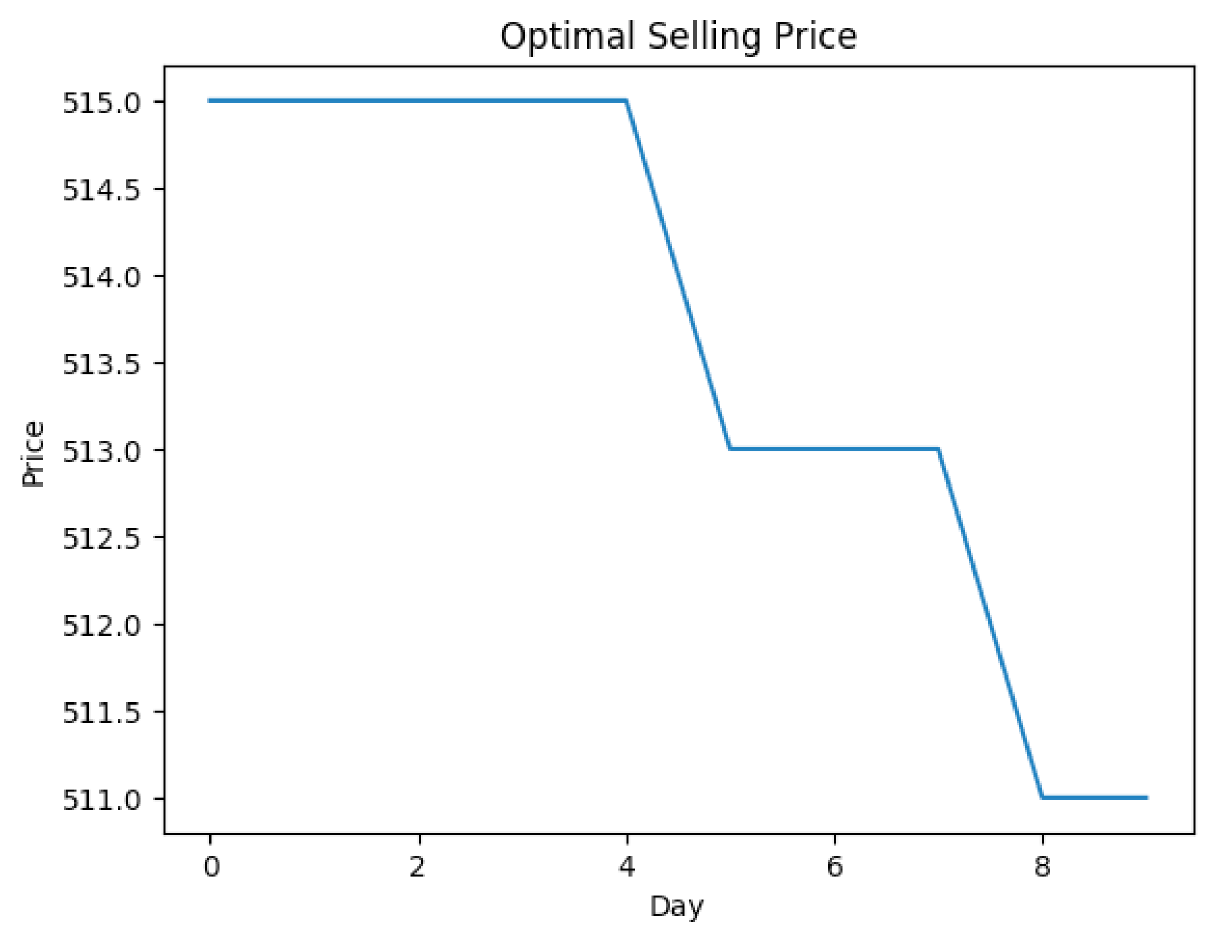
Updated Values of Regret:

Results Obtained:

(strike price = 510, start price = 500, drift = 10 and T = 10)







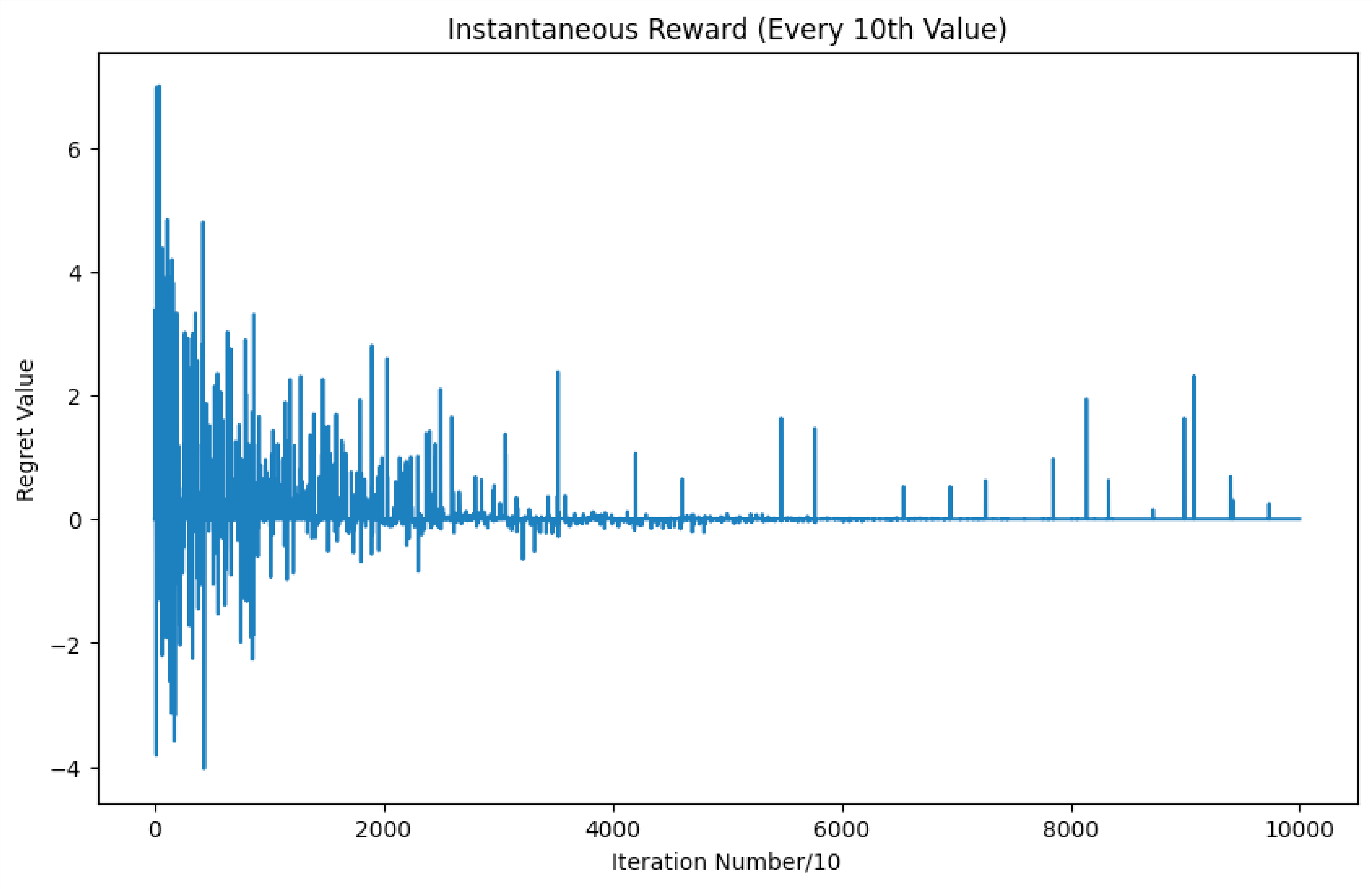
TD CONTROL
USING SARSA(λ)

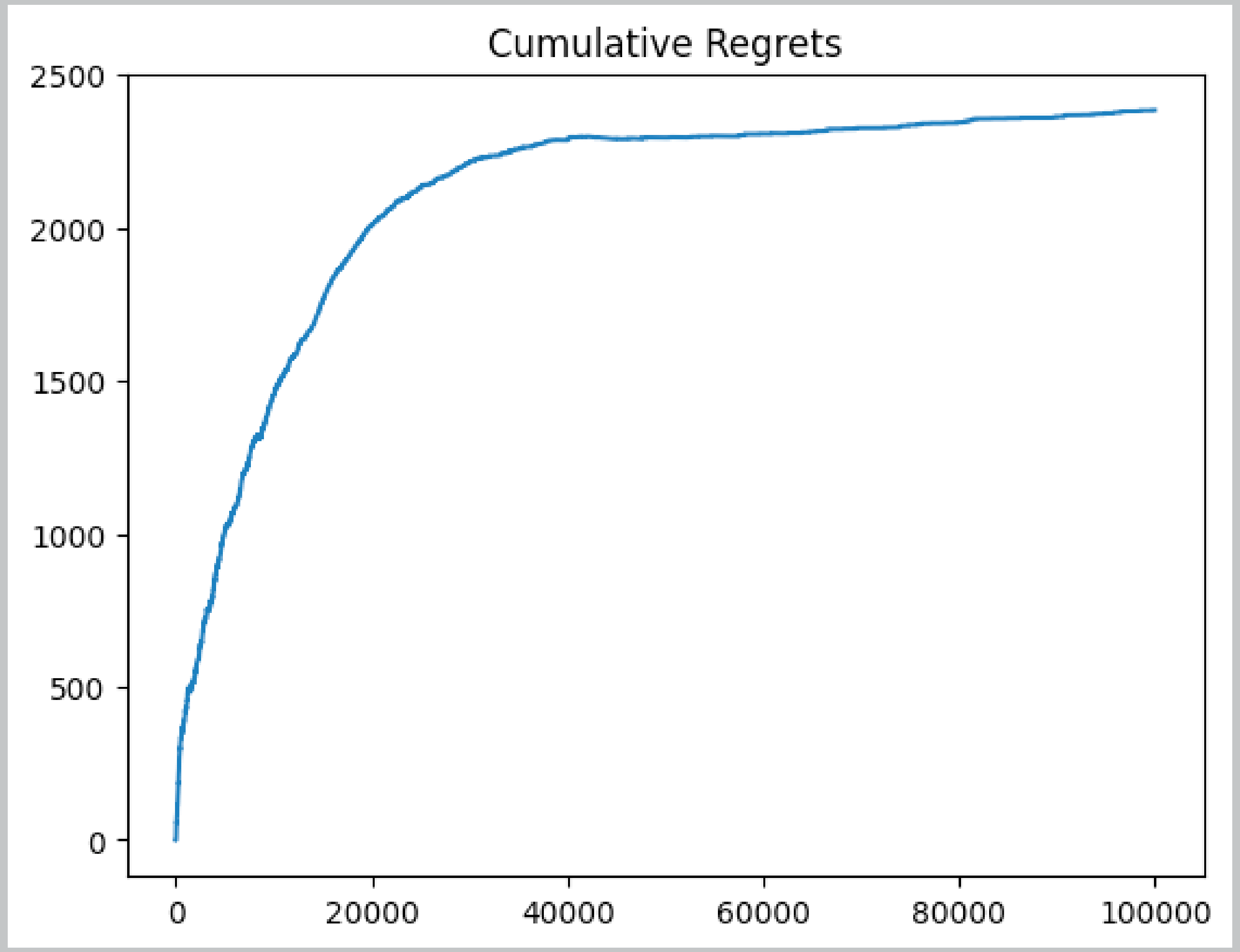
Pseudo Code followed:

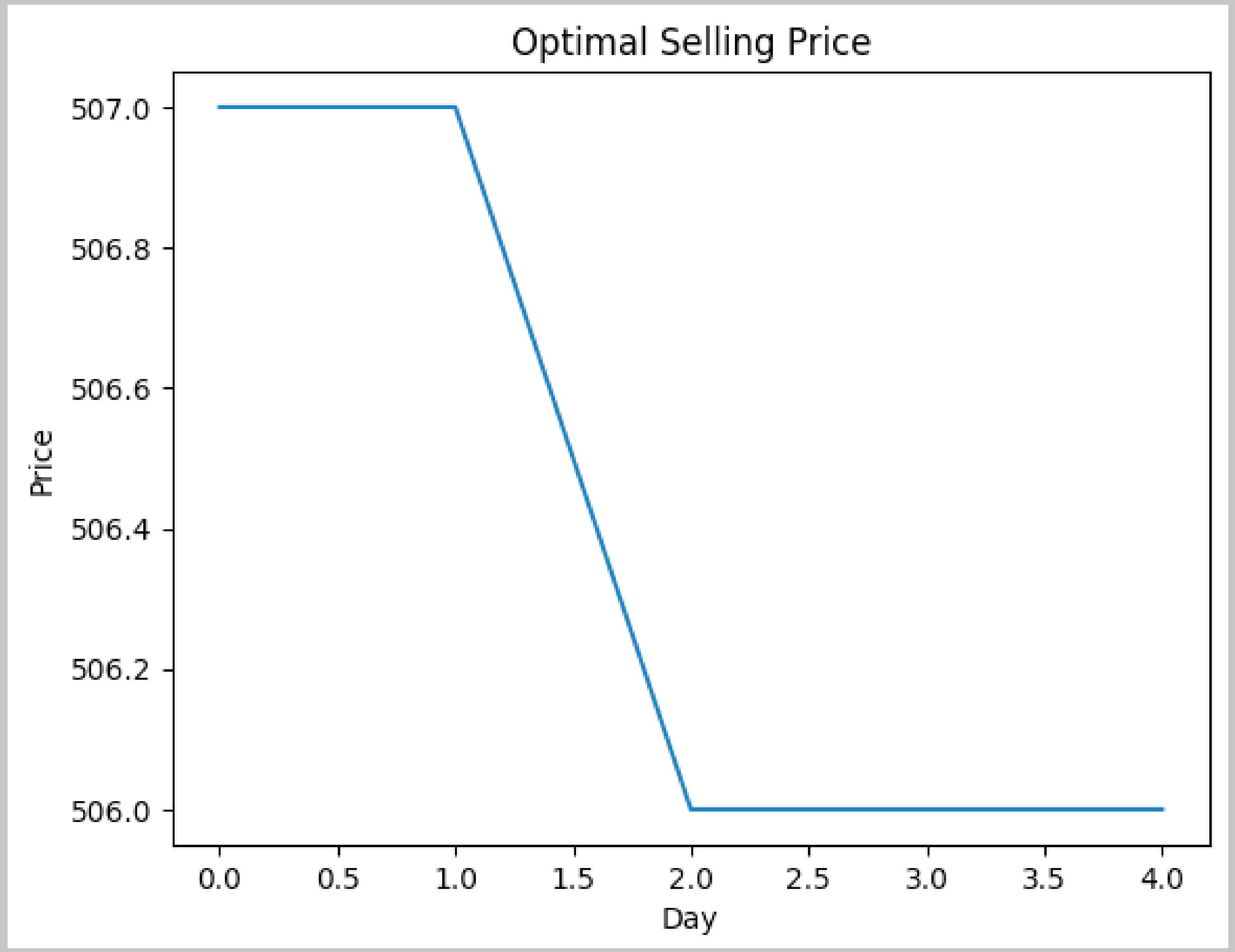
Initialize $Q(s,a)$ arbitrarily, for all $s \in \mathcal{S}, a \in \mathcal{A}(s)$
Repeat (for each episode):
    $E(s,a) = 0$, for all $s \in \mathcal{S}, a \in \mathcal{A}(s)$
    Initialize S, A
    Repeat (for each step of episode):
        Take action A, observe R, S'
        Choose A' from S' using policy derived from Q (e.g., ε-greedy)
        $\delta \leftarrow R + \gamma Q(S', A') - Q(S, A)$
        $E(S,A) \leftarrow E(S,A) + 1$
        For all $s \in \mathcal{S}, a \in \mathcal{A}(s)$:
            $Q(s,a) \leftarrow Q(s,a) + \alpha \delta E(s,a)$
            $E(s,a) \leftarrow \gamma \lambda E(s,a)$
        $S \leftarrow S'$; $A \leftarrow A'$
    until S is terminal

TD CONTROL
USING SARSA(λ)

Results Obtained:
(strike price = 505, start price = 500, drift = 5 and T = 5)





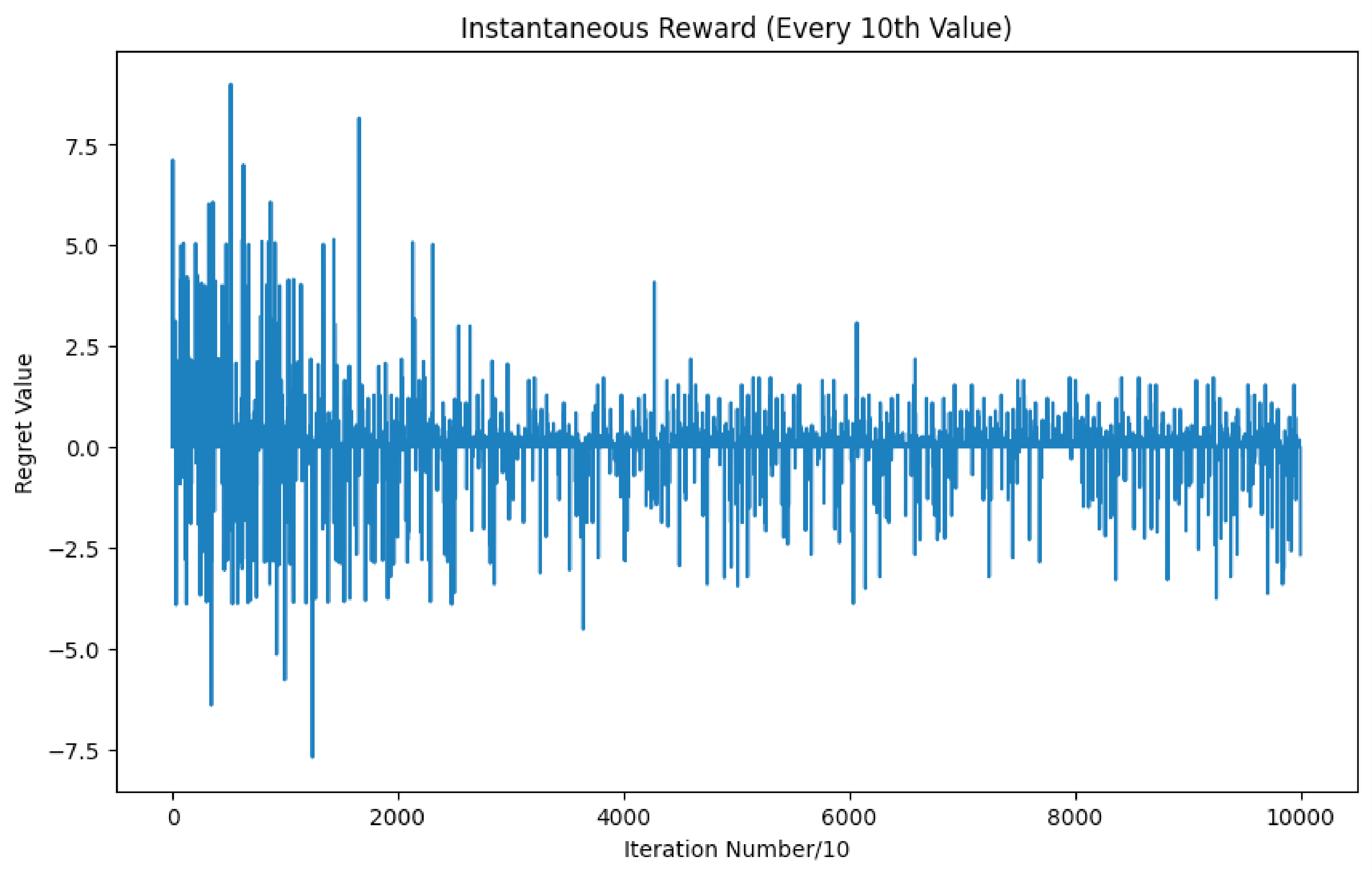


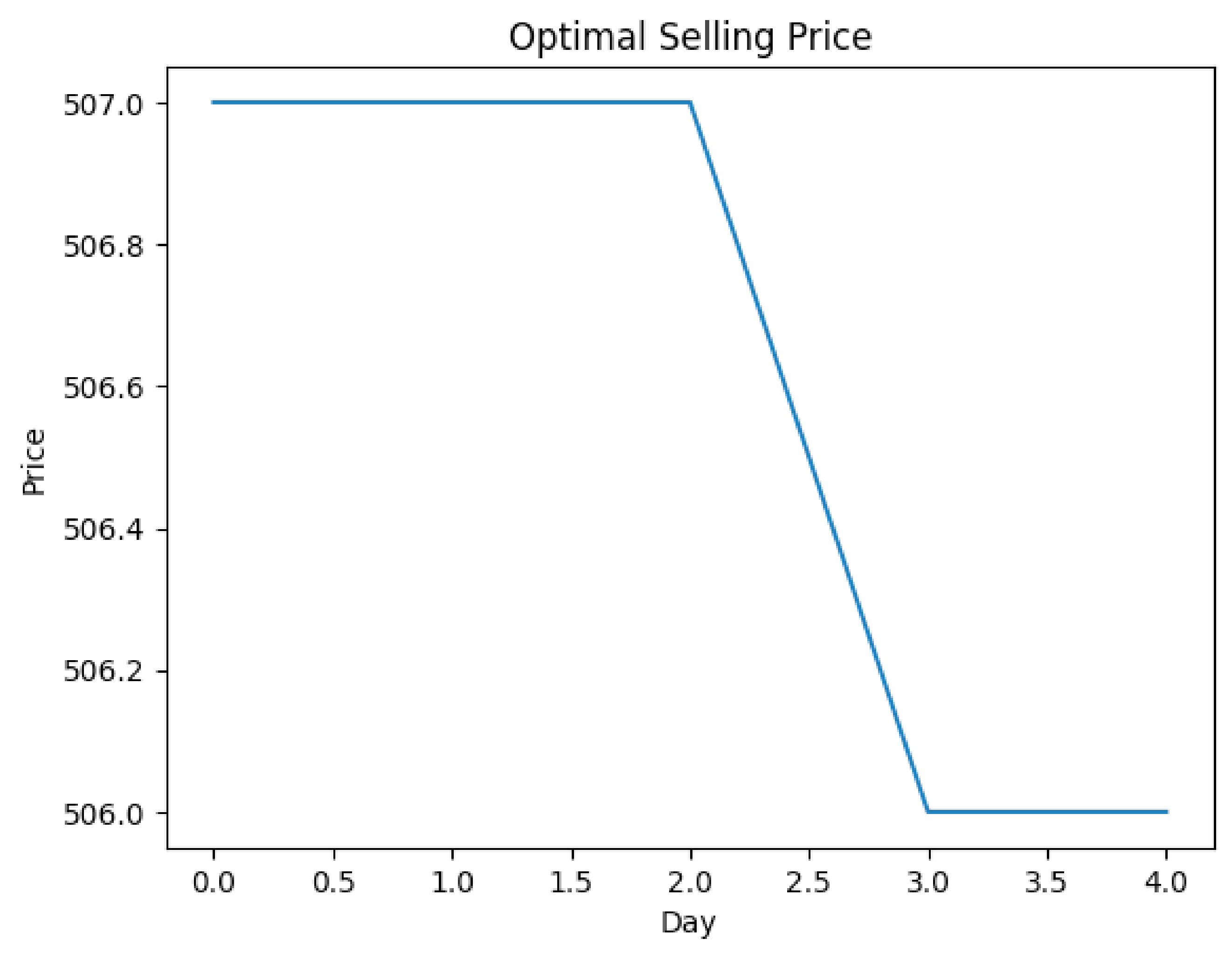
TD CONTROL
USING SARSA(λ)

Updated Values of Regret:

Results Obtained:

(strike price = 505, start price = 500, drift = 5 and T = 5)





Conclusion

An optimal policy will follow the below given theorem :

There exist numbers $s_1 \geqslant s_2 \geqslant \cdots \geqslant s_T$ such that it is optimal to exercise an option at time t iff $x_t \geqslant s_t$. Hence, the optimal strategy is of threshold type.

We see an identical trend with the RL algorithms we implemented, confirming the results' correctness.

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Thank You!