PROBABILISTIC ROBOTICS: SIMULTANEOUS LOCALIZATION AND MAPPING

Pierre-Paul TACHER

1

Let d the dimension of state space in EKF SLAM, and N the number of landmarks in the map. We have $d = 3N + 3$. The complexity of motion update in EKF SLAM is asymptotically dominated by order d matrix multiplications, so the complexity is $O(d^\omega)$ or equivalently $O(N^\omega)$, where $\omega < 2.3728639$(cf. [1]). On the other hand, the complexity of EKF algorithm for localization is dominated by the loop over all sensings, which is executed $O(N)$ times and contains a constant number of constant size matrix multiplications / inversions; hence an asymptotic complexity of $O(N)$.

2

A solution to initialize landmarks during bearing-only SLAM can be found in paper [2]; It uses the framework described in [3], [4] to manage relationships between spatial uncertainties. I will give only a summary of the main ideas here and refer the reader to the papers for the details.

In [4], the concept of *approximate transformation* (AT) is defined: it expresses the uncertain relative location of one frame $\mathbf{B} = (X_2, Y_2, \theta_2)$ with respect to another frame $\mathbf{A} = (X_1, Y_1, \theta_1)$. The transformation has a mean and a covariance matrix. Think of it as the pose of the robot or the location of a landmark.

(1) The ATs can be *compounded*: knowing the AT $\mathbf{A}$ with respect to $\mathbf{W}$ and the AT $\mathbf{B}$ with respect to $\mathbf{A}$, compute the AT $\mathbf{B}$ with respect to $\mathbf{W}$, in which case the uncertainty is the sum of the original ones (think of 2 successive motion update in Kalman filter without sensing).
(2) The ATs can be *merged*; knowing a first AT $\mathbf{A_1}$ with respect to $\mathbf{W}$ and a second AT of the same frame $\mathbf{A_2}$ with respect to $\mathbf{W}$, compute the AT $\mathbf{A}$ with respect to $\mathbf{W}$ by combining the 2 spatial relationships, in which case the resulting uncertainty is less than both of original uncertainties (think of motion update followed by sensing a known location landmark).

An interesting analogy can be made with series / parallel electric resistances.

Back to the landmark initialization problem: Obviously, a single bearing measurement of a landmark does not allow to determine a location for it (cf. 1). The idea is to accumulate over time evidences of the presence of a new landmark before integrating it in the state variable. We see in figure 2 that potential landmarks location arise at the intersections of measurements from different positions. For each of the intersection, we *compound* the covariance Σ_t of robot uncertain pose at time t with covariance of the measurement Q to get an approximate landmark location $\hookrightarrow \mathcal{N}(\mu, C_1)$. We do it again from location at time $t + 1$ to get another estimate $\hookrightarrow \mathcal{N}(\mu, C_2)$ and then *merge* both estimates (cf. figure 3) to get a clearer position $\mathcal{N}(\mu, C_3)$.

Note that every measurements intersection is not necessarily a landmark, that is why we need to introduce the notion of *persistance*: as the persistence count increases, the certainty that the gaussian corresponds to an actual landmark increases. In figure 4 again estimate is build from measurement intersection $\mathcal{N}(\mu_4, C_4)$. This estimate is close enough (we need to define a distance between Gaussian distributions) from the previous one, so the estimates are merged and persistance count is incremented. When the count reaches a certain threshold, the landmark is appended to state variable and initialized with current estimate's mean and covariance.

We need to modify the loop on measurements in EKF algorithm Table 10.2 of the book: we have seen previously that for a given known landmark k, the r.v. $\pi_k = (z - h(y_t, k))\Psi_k^{-1}(z - h(y_t, k))$ is

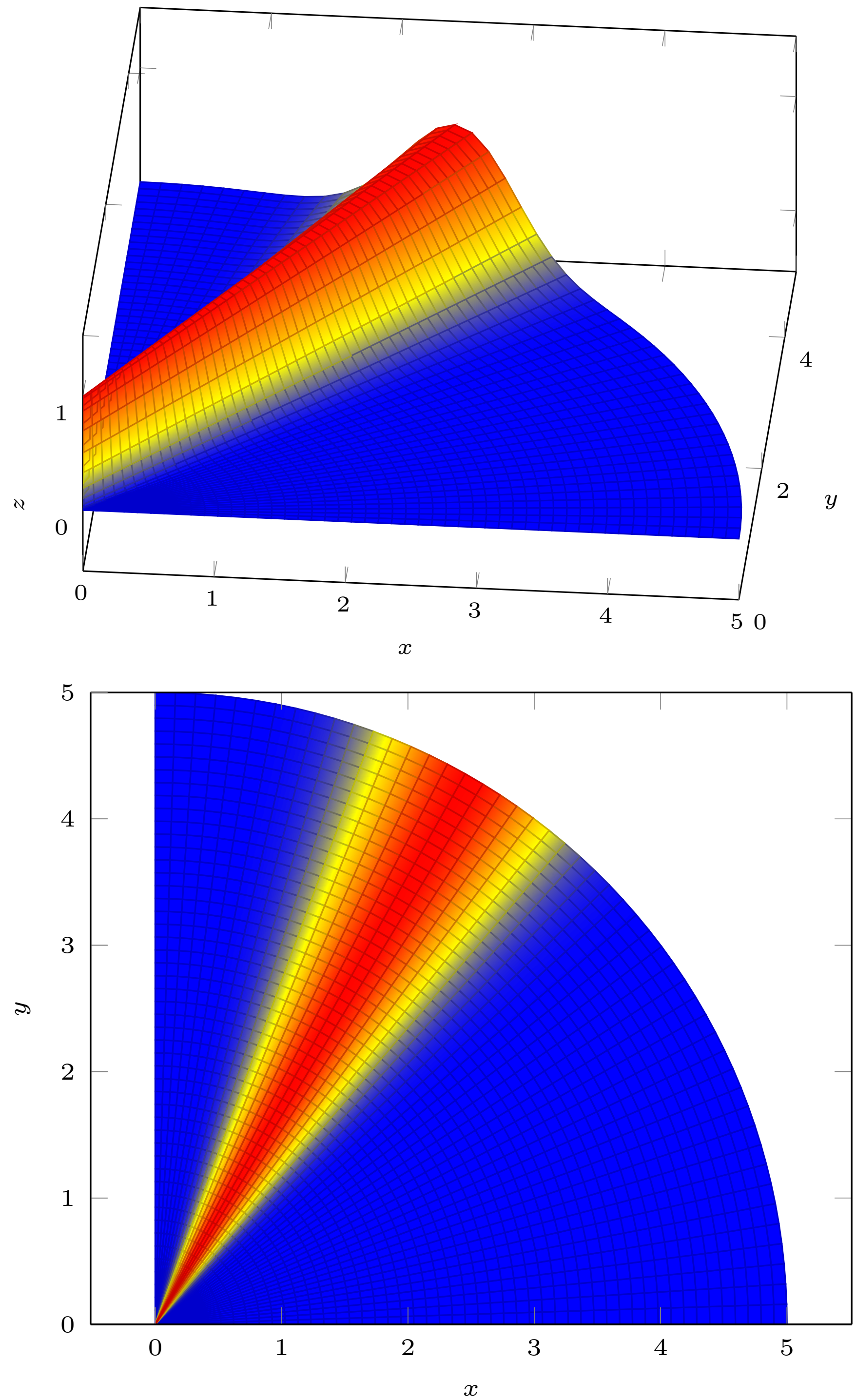

FIGURE 1. scaled probability density of location in robot local frame of a landmark, detected at $\phi = \frac{\pi}{3}$ with noise variance $\sigma^2 = 0.01$

a chi-squared r.v. if landmark k is the actual target of measurement z;so instead of maximizing π_k over k, we can do a Chi-squared test and decide a threshold above which the landmark is associated to the measurement (we need to choose a policy to handle incompatibilities); if the measurement cannot be associated to any already known landmark, it is used in above initialization algorithm.

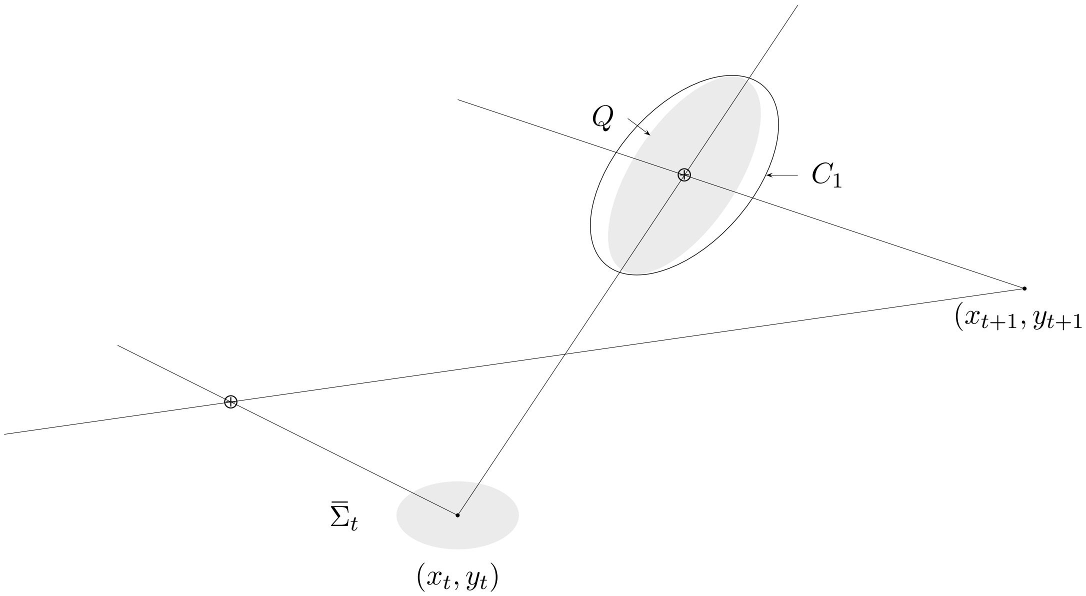

FIGURE 2. Uncertainty of the robot pose is compounded with noise uncertainty to estimate landmark position

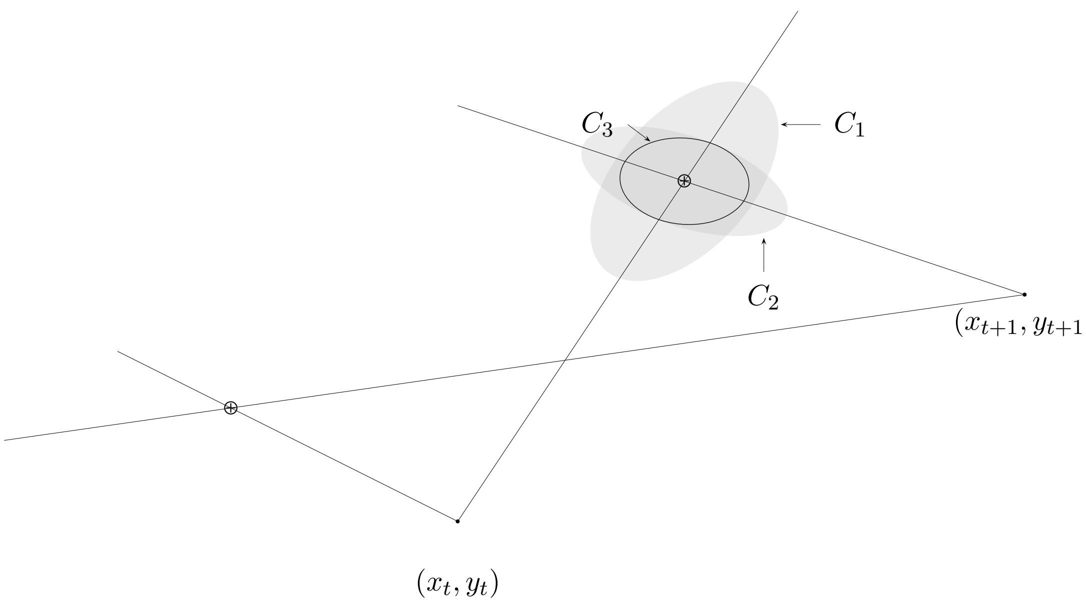

FIGURE 3. Merging of estimates from 2 different positions

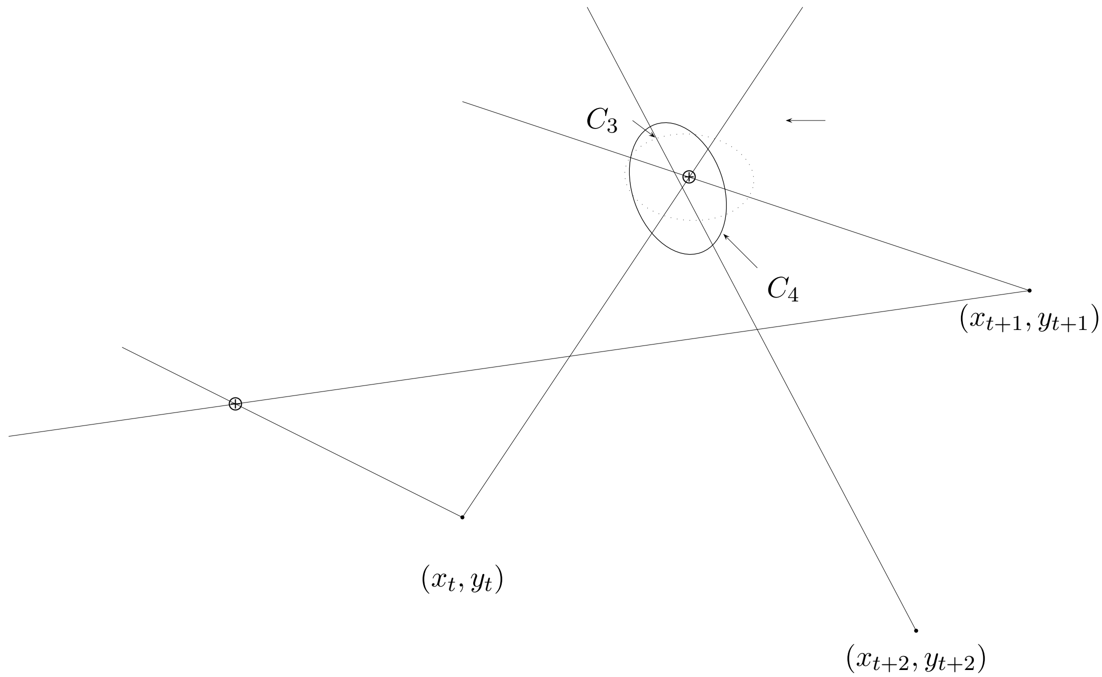

FIGURE 4. Persistance count of landmark is increased when seen again at a later stage

References

[1] Le Gall, François: *Powers of tensors and fast matrix multiplication*, Proceedings of the 39th International Symposium on Symbolic and Algebraic Computation (2014)

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[3] Smith, Randall; Self, Matthew; Cheeseman, Peter: *Estimating uncertain spatial relationships in robotics*, Machine Intelligence and Pattern Recognition (1986)

[4] Smith, Randall; Cheeseman, Peter: *On the representation and estimation of spatial uncertainty*, The International Journal of Robotics Research Vol. 5, No. 4 (1986)