Some Results About Graph Contraction Problems.

Prafullkumar Tale

25th May, 2020

Graph Modification Problems

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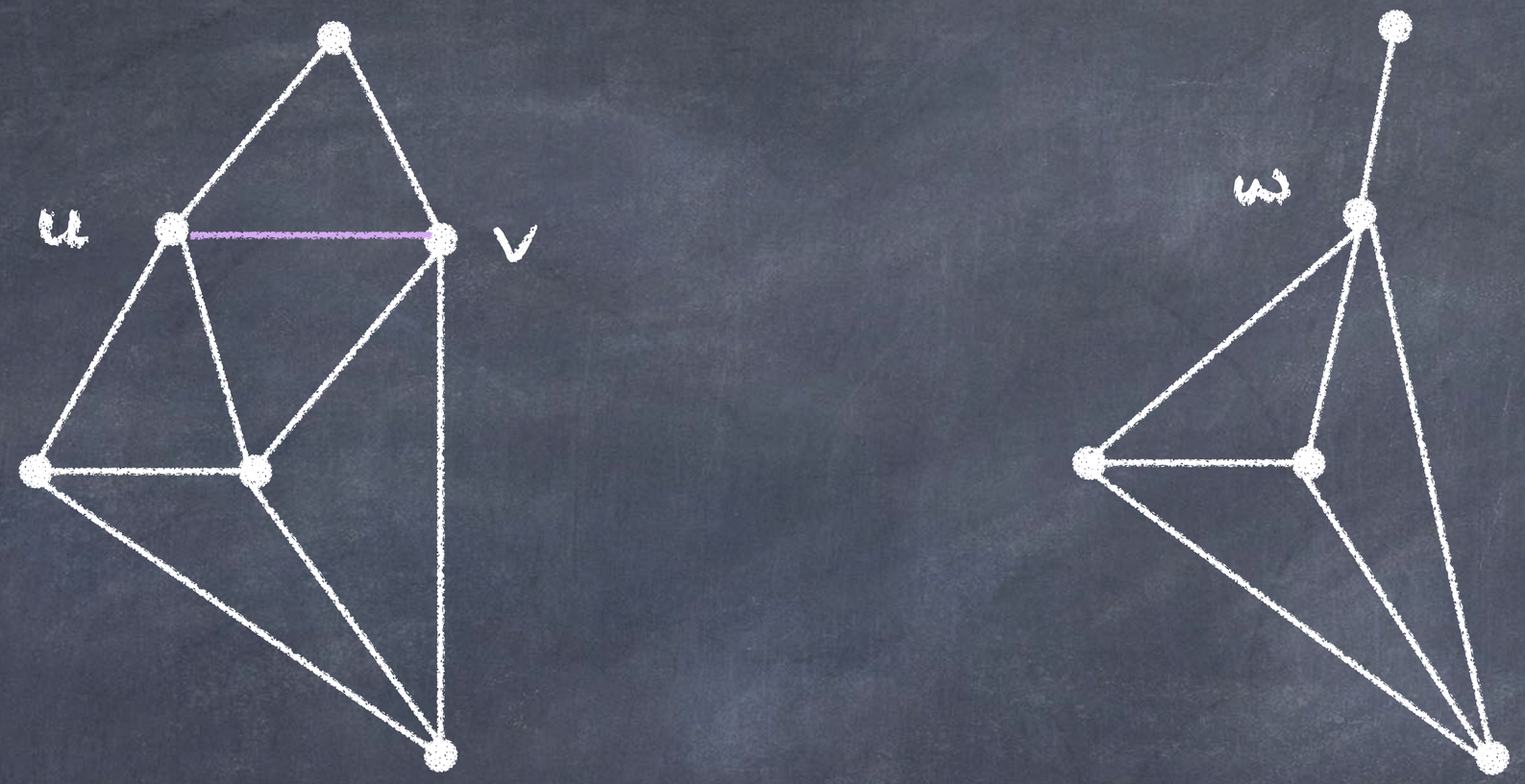
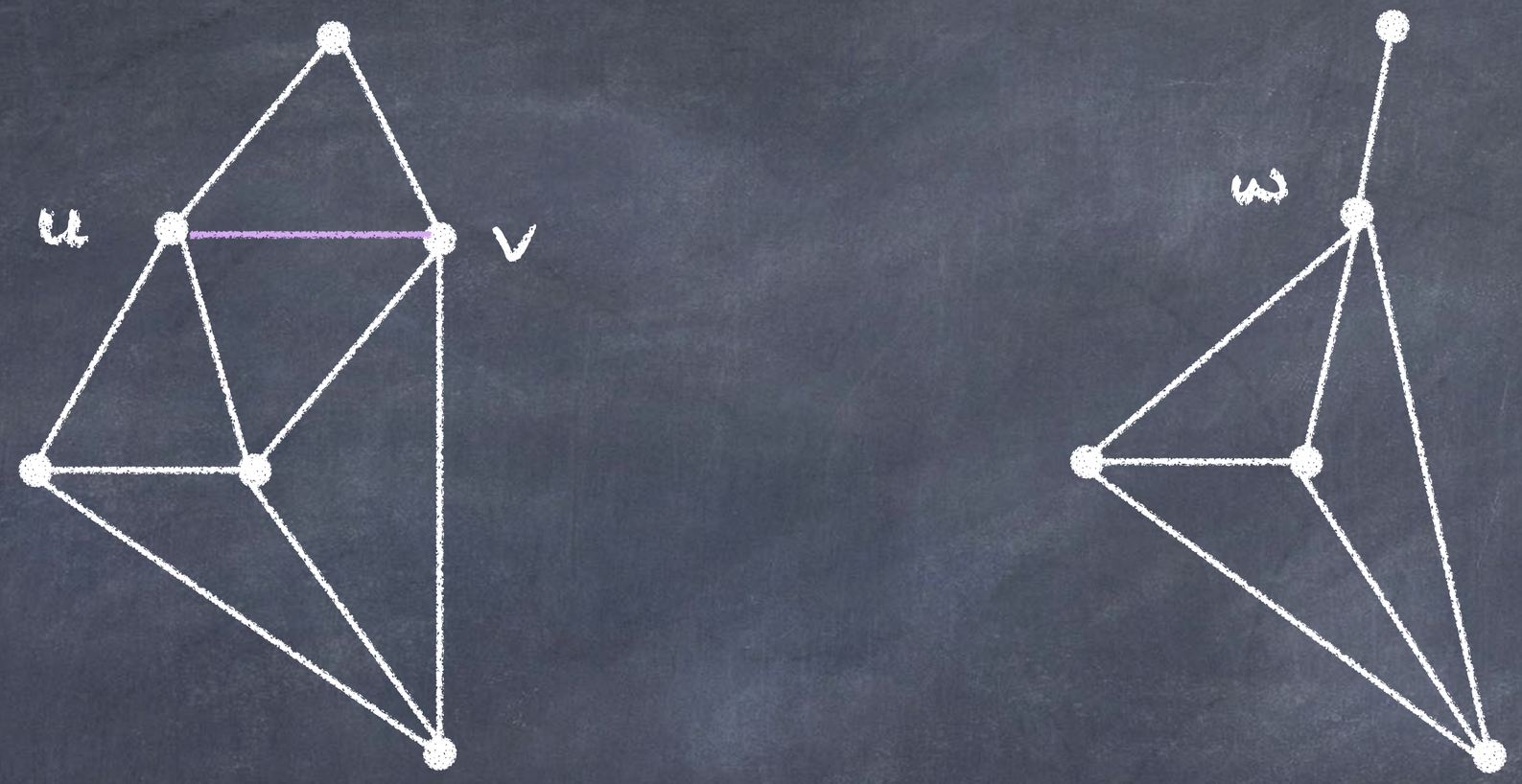
Modification : (1) vertex deletion (2) edge deletion (3) edge addition (4) edge contraction



Edge contraction



Edge contraction

No self-loop
No parallel edges

$\mathcal{G}$ – Contraction

Input: Graph G, int k

Question: Can we contract at most k edges in G s.t. resulting graph is in $\mathcal{G}$?

Parameterized Complexity and Known Results

Overview Of Our Contributions

Brief description: Lossy Kernel for Clique Cont.

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- Objective : Find an algorithm f(k) poly(n)

  f(k) – function depending only on k
  k – collection of parameters

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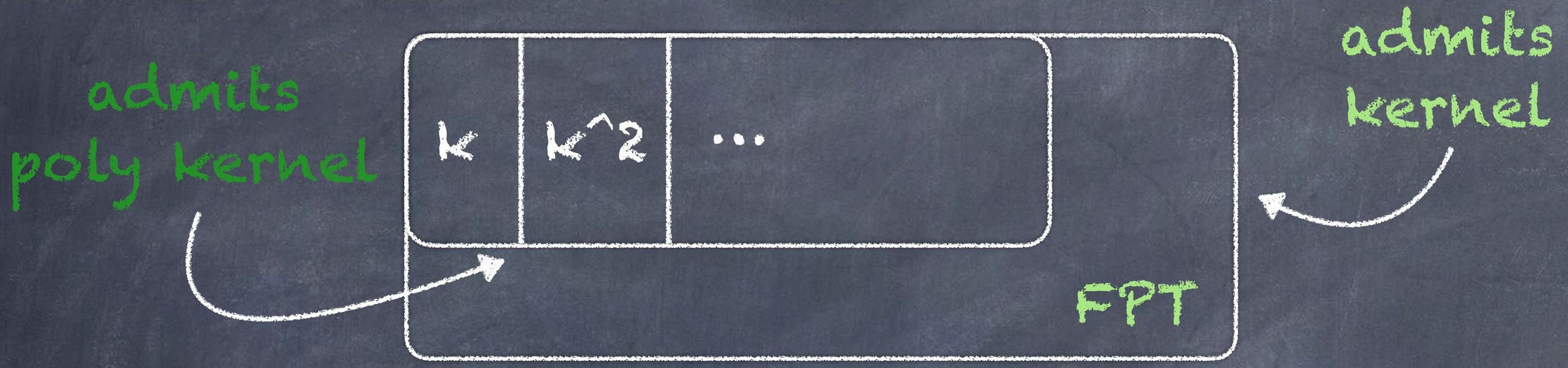

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Guo & Cai (2015):	Clique	exp(k log k)

$\mathcal{G}$ – Contraction: Kernel

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Q2. Can we interpolate between these two extremes?

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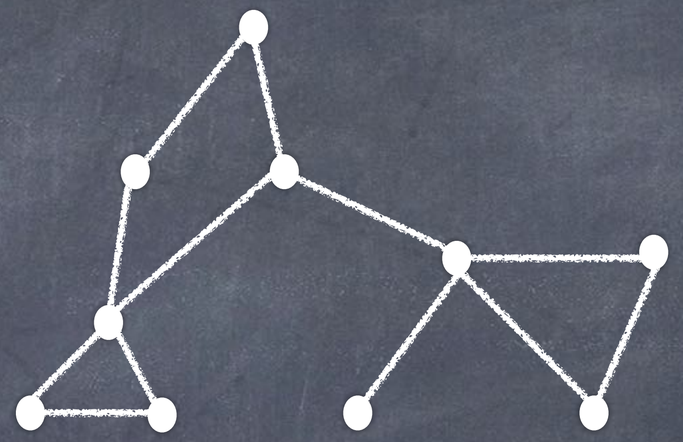

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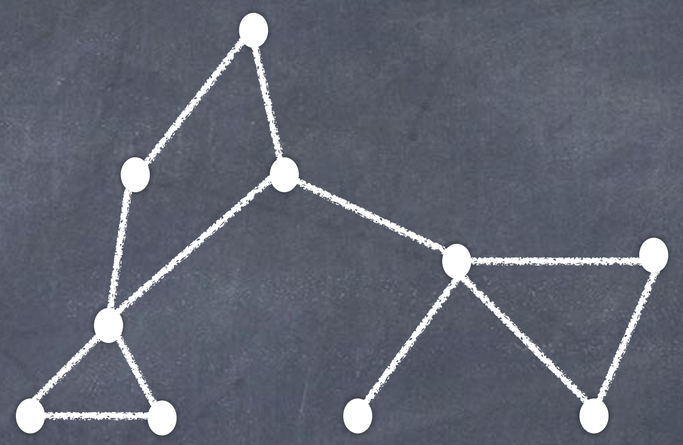

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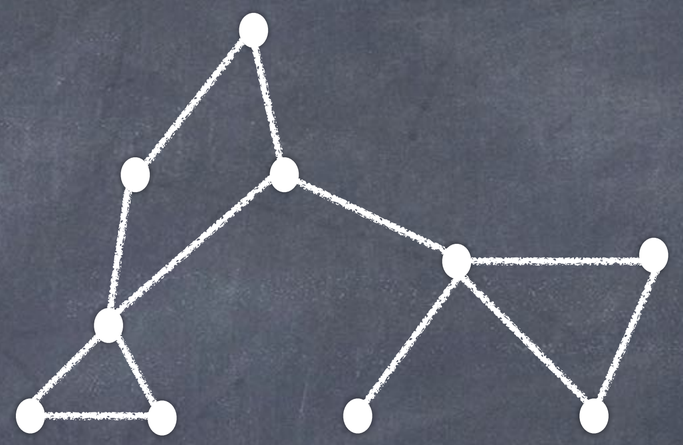

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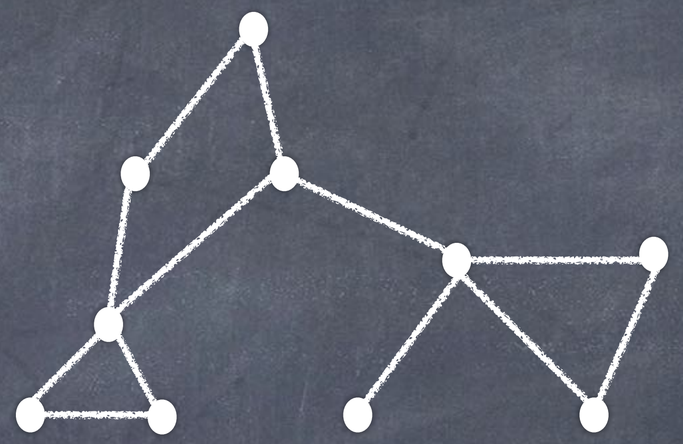

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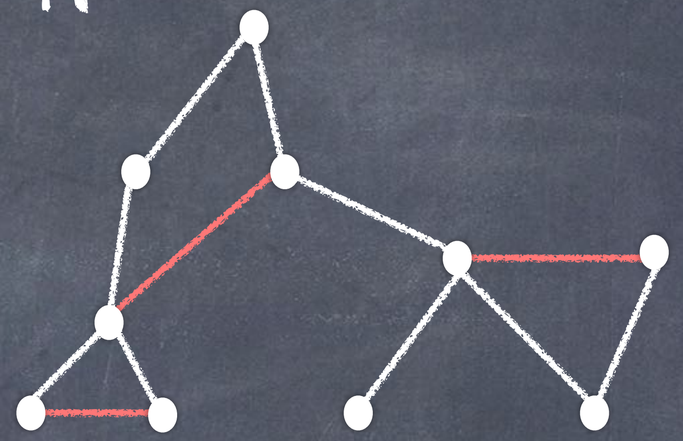

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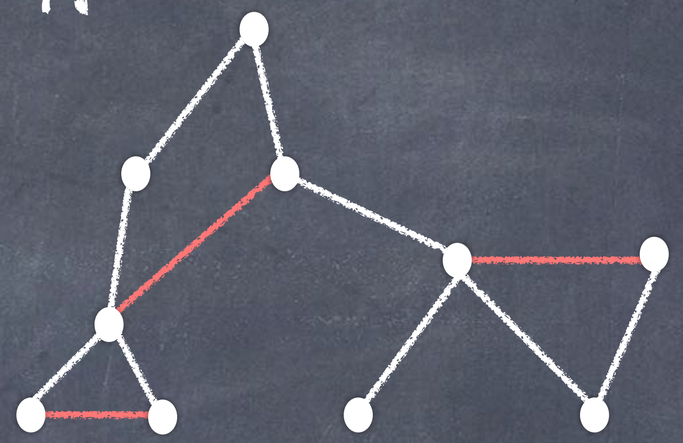

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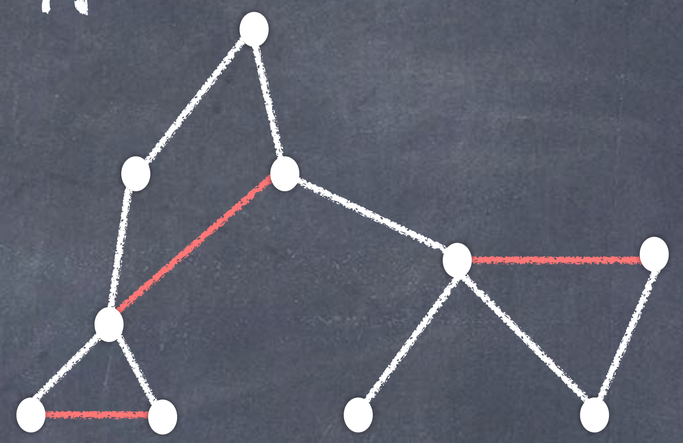

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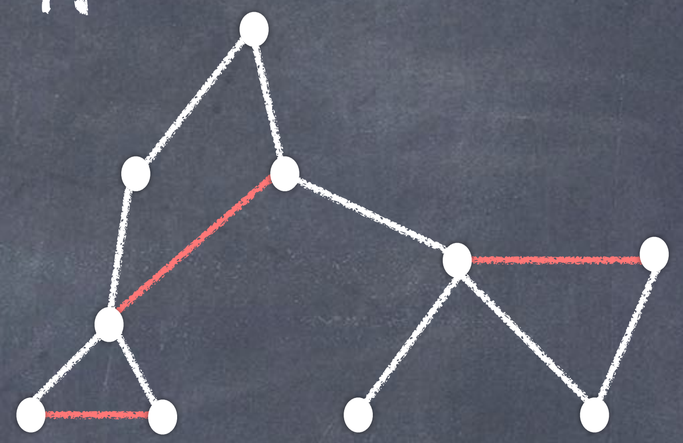

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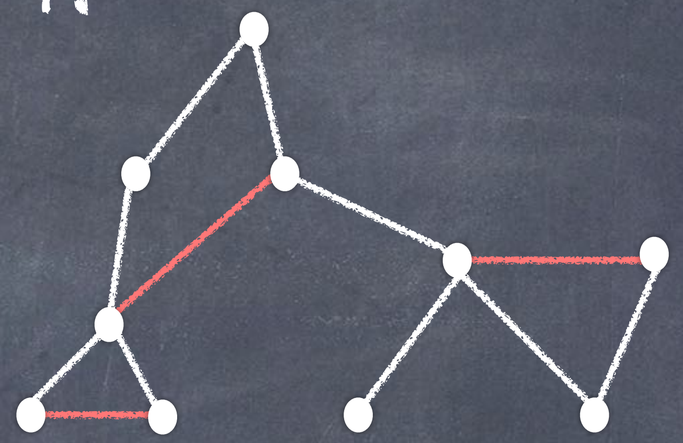

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– Agrawal, Kanesh, Saurabh, and T. (CIAC 2017)

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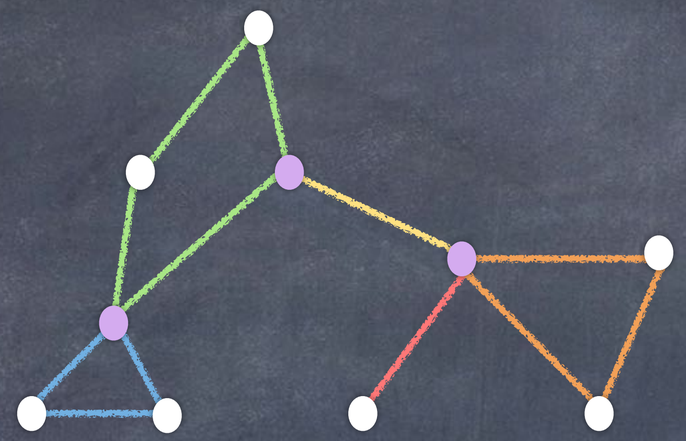
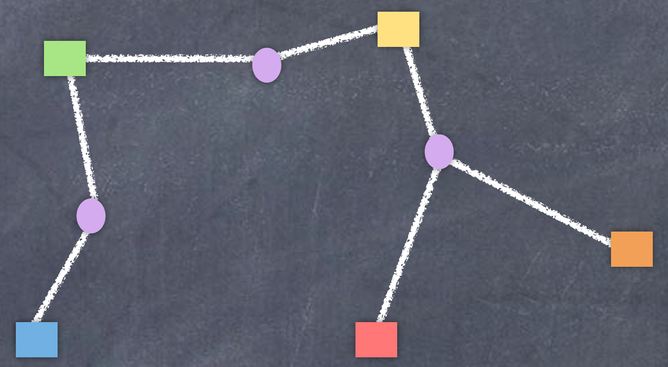
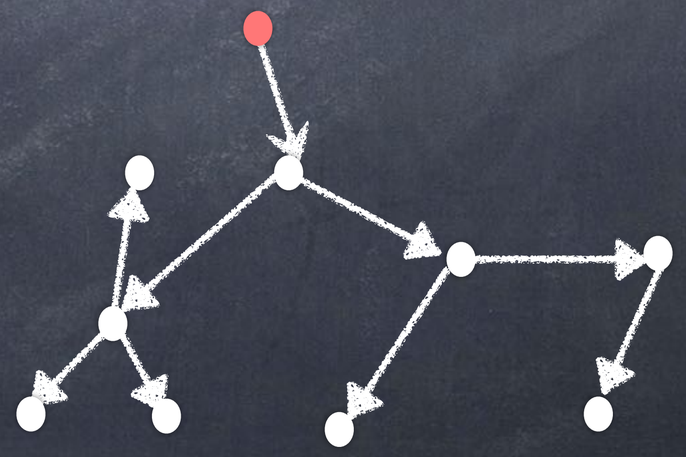




– Bounded Out-tree Contraction

rooted-tree, directed, arc
away from root



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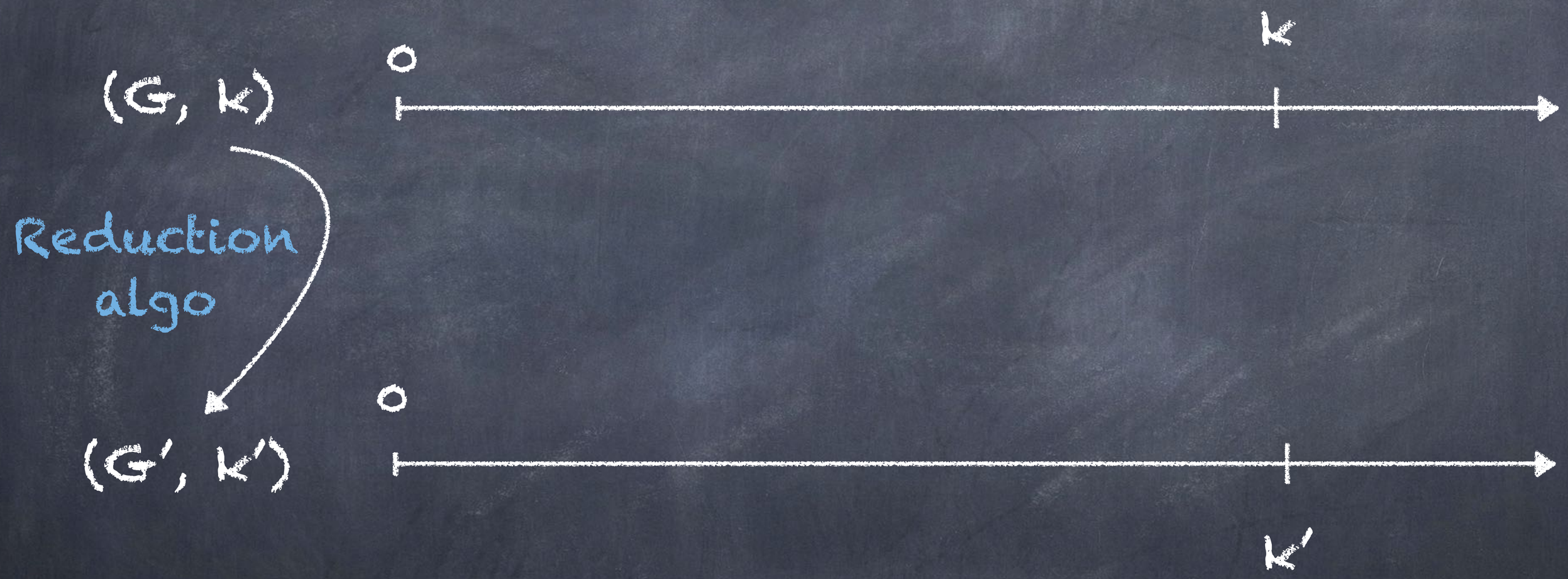

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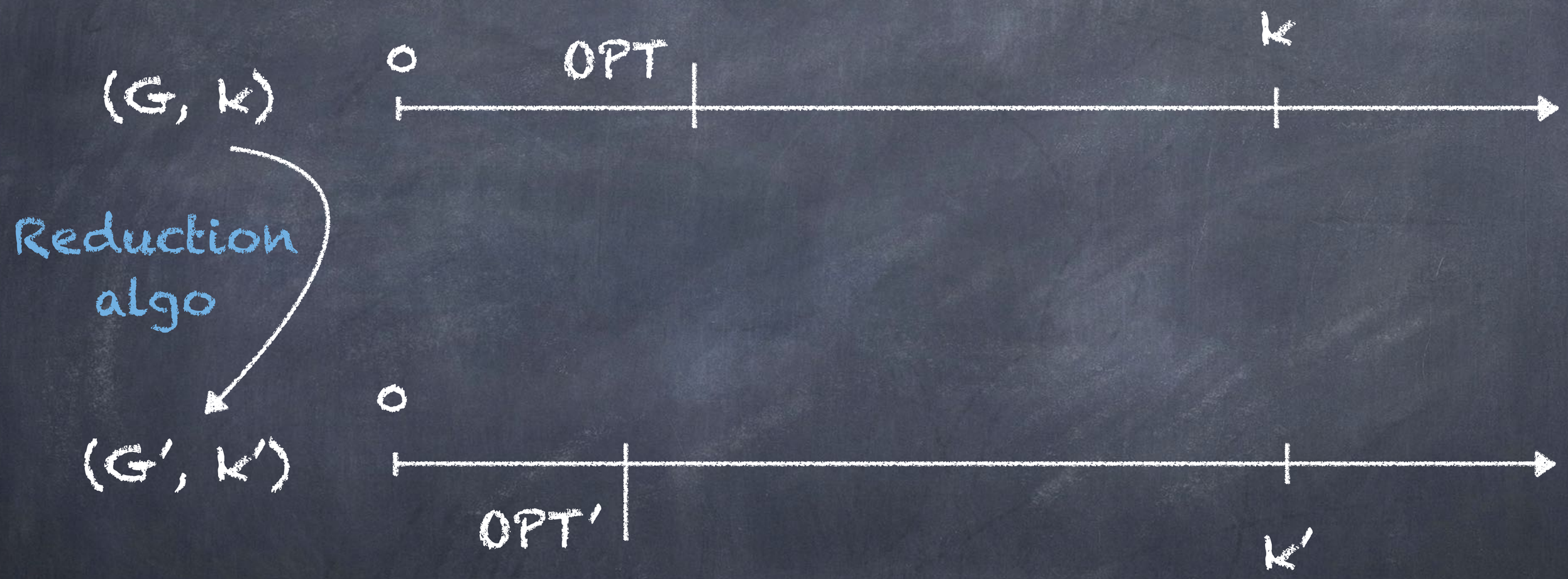

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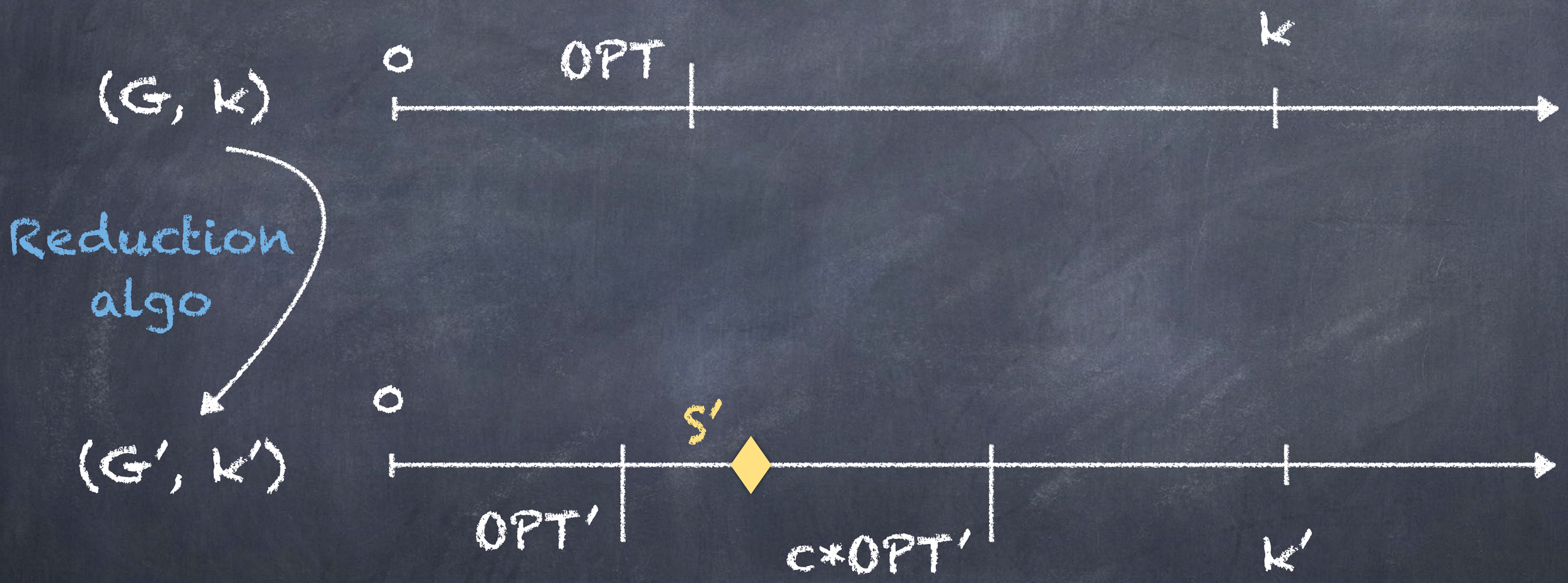

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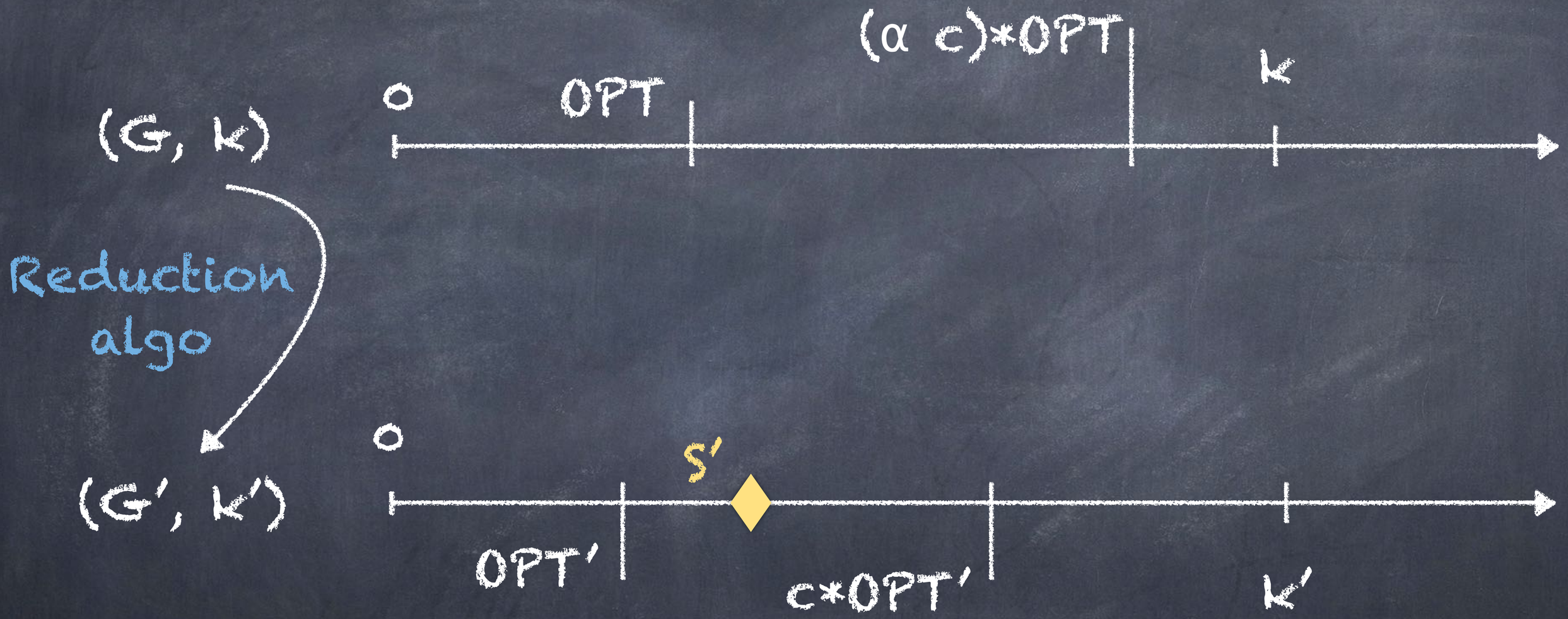

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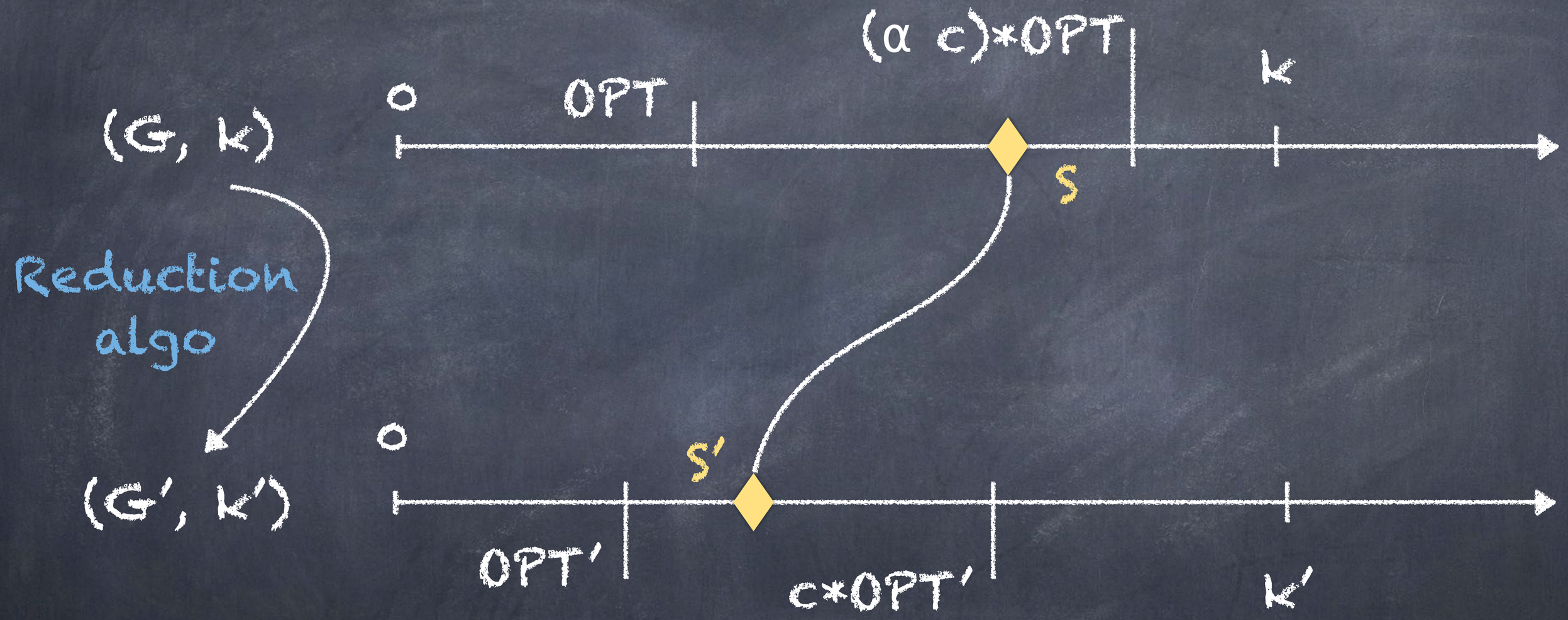

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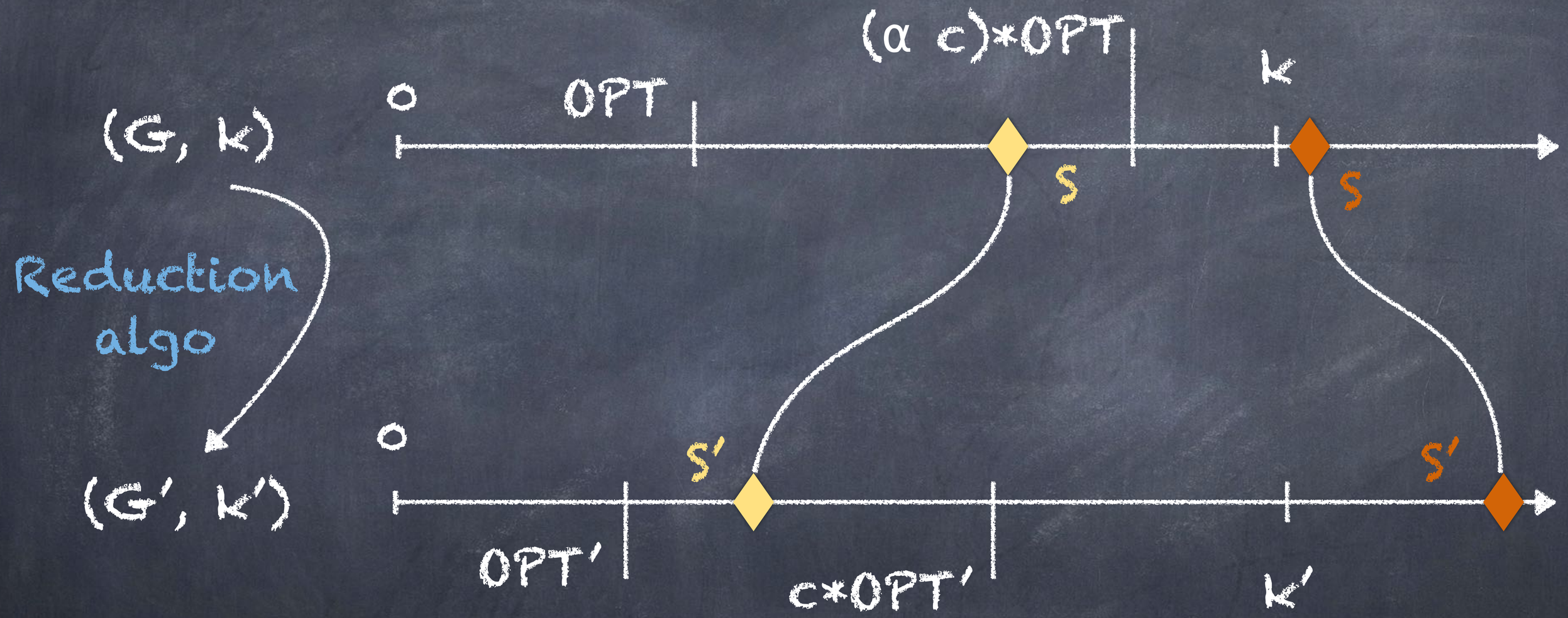

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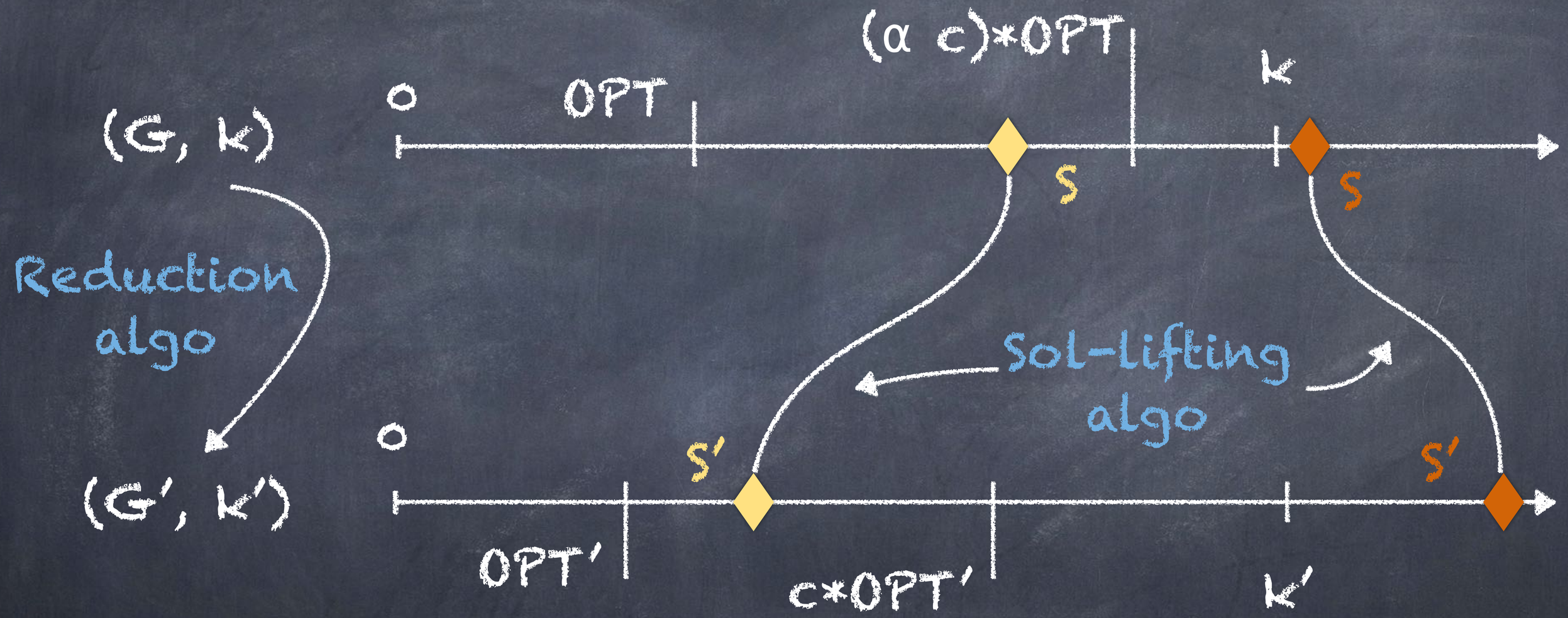

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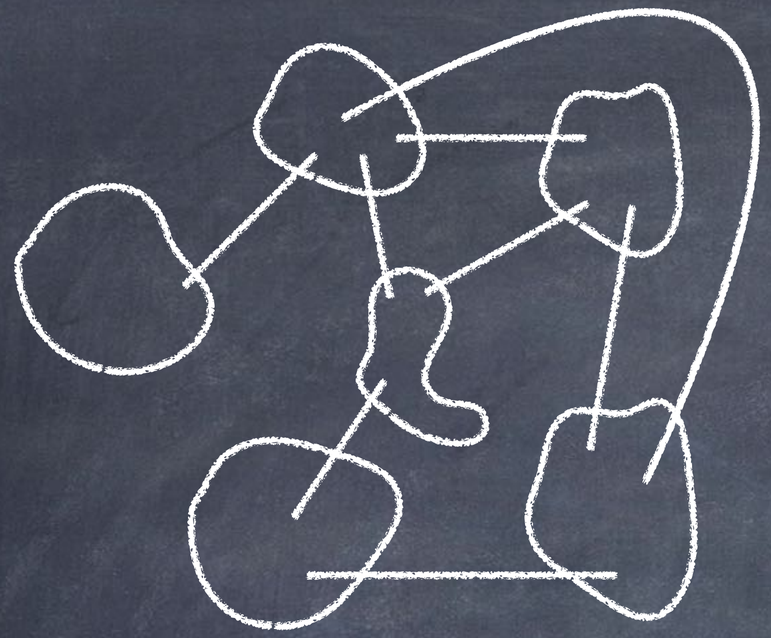
b. Each part is mapped to some vertex is H s.t. V1, V2 adjacent iff h1, h2 are adjacent.



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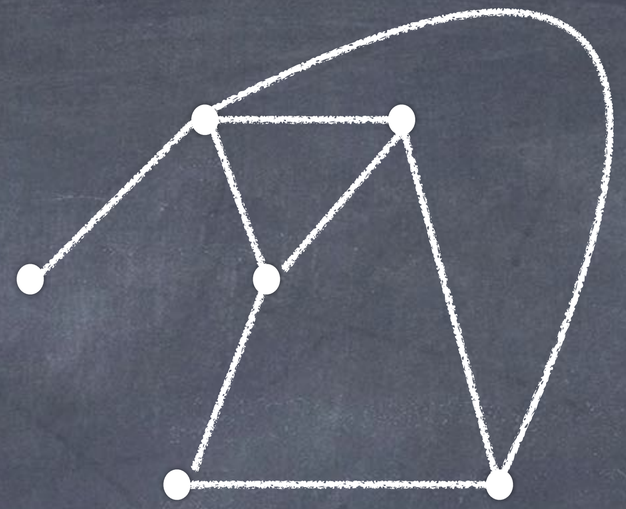

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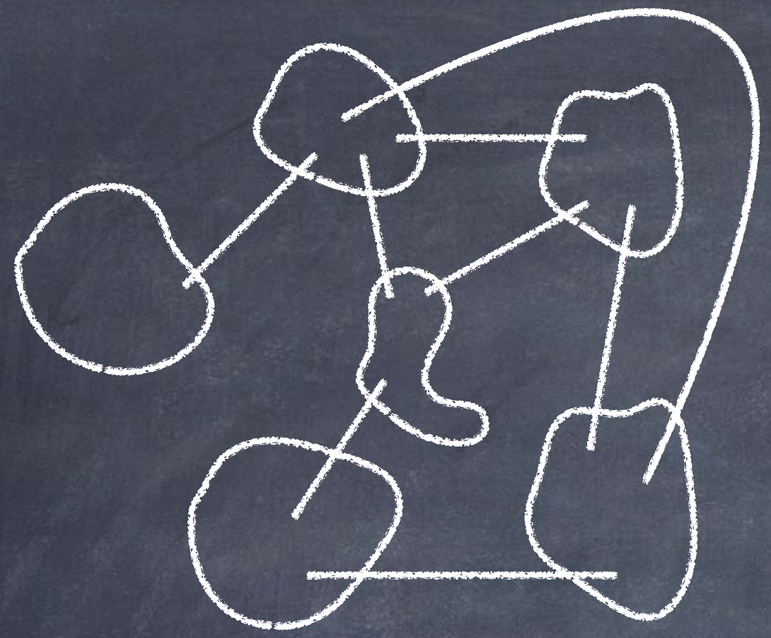

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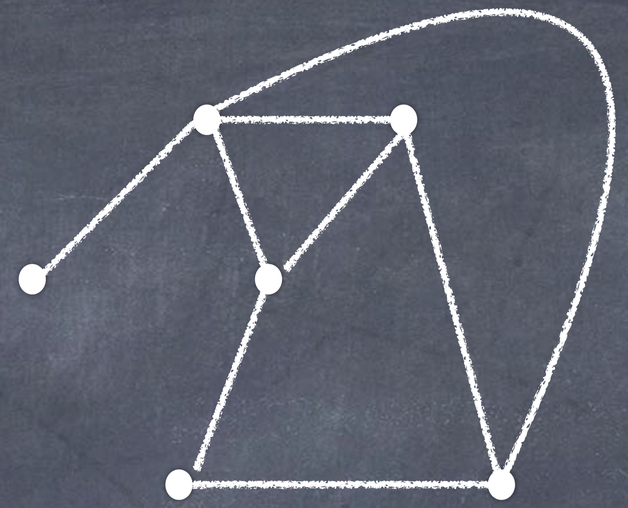

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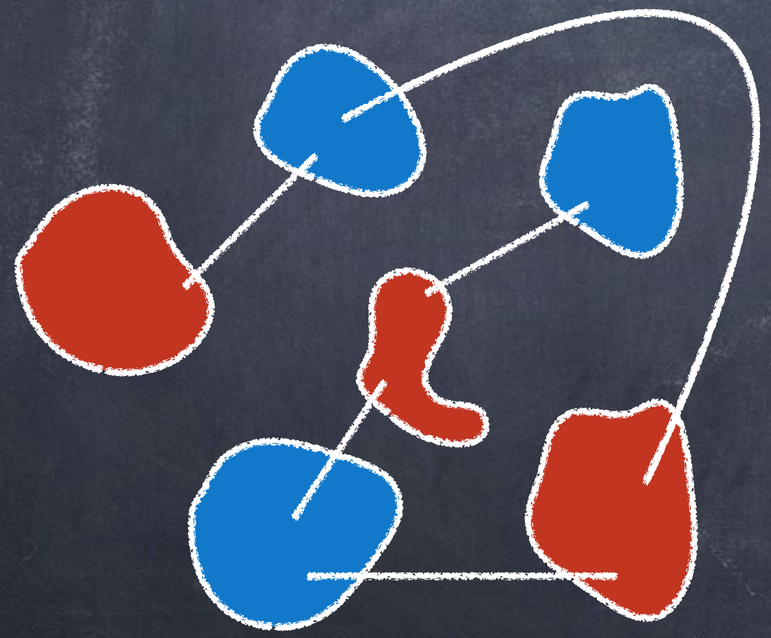

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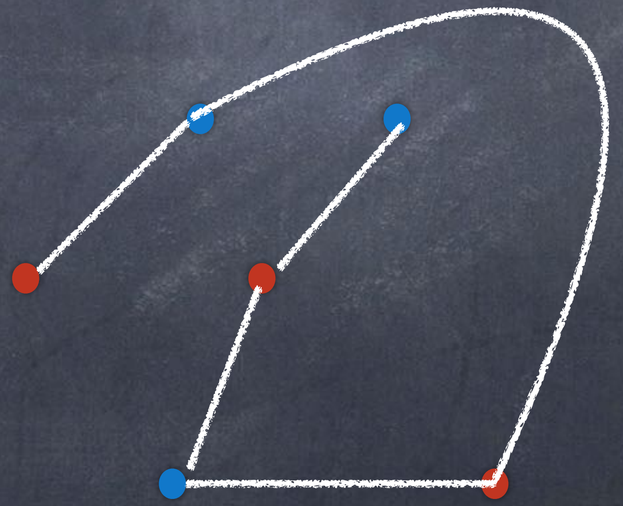

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- 2-factor approx algo for vertex cover in compliment graph

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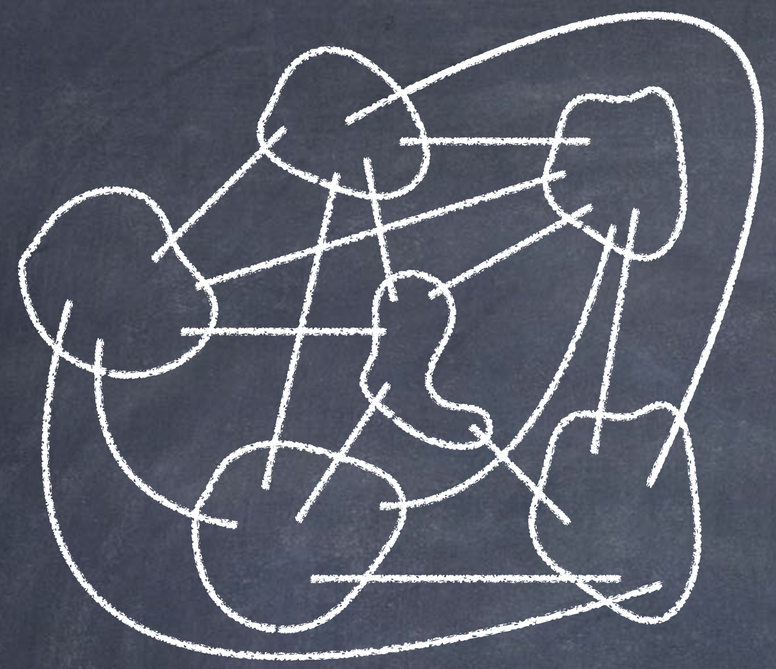

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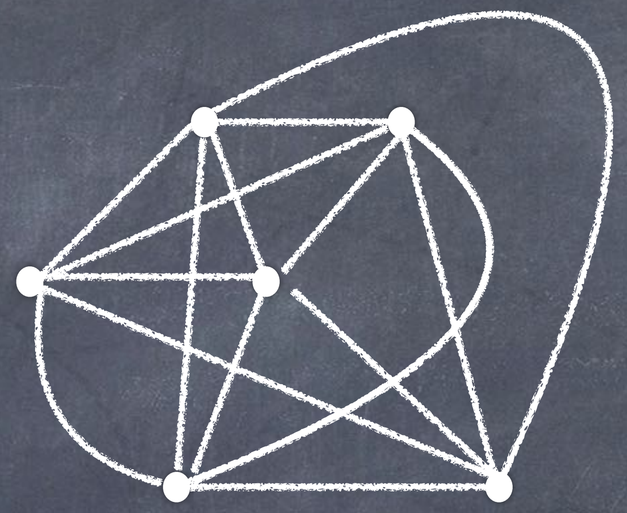
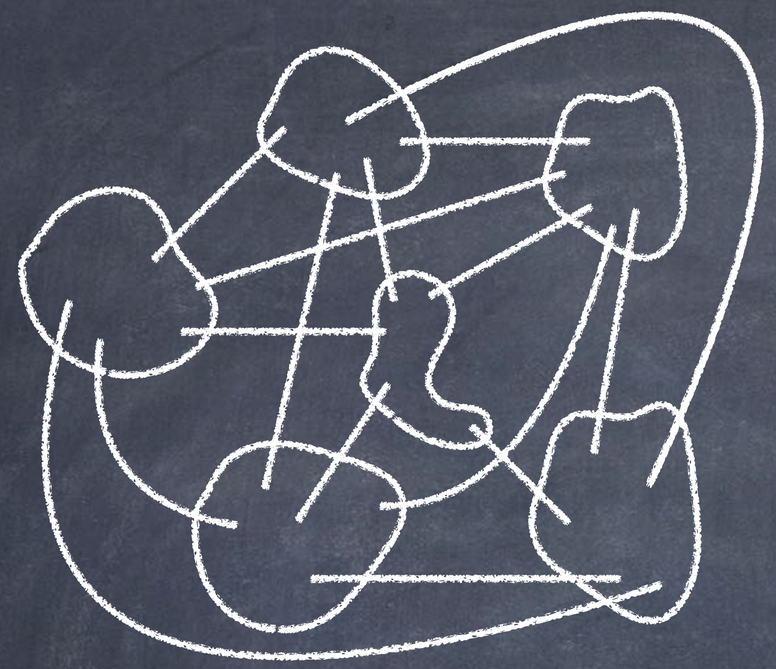


H

CLIQUE CONTRACTION :: Partition of V(G) s. t.

a) Each part (called witness set) is connected
b) any two witness sets are adjacent with each other.



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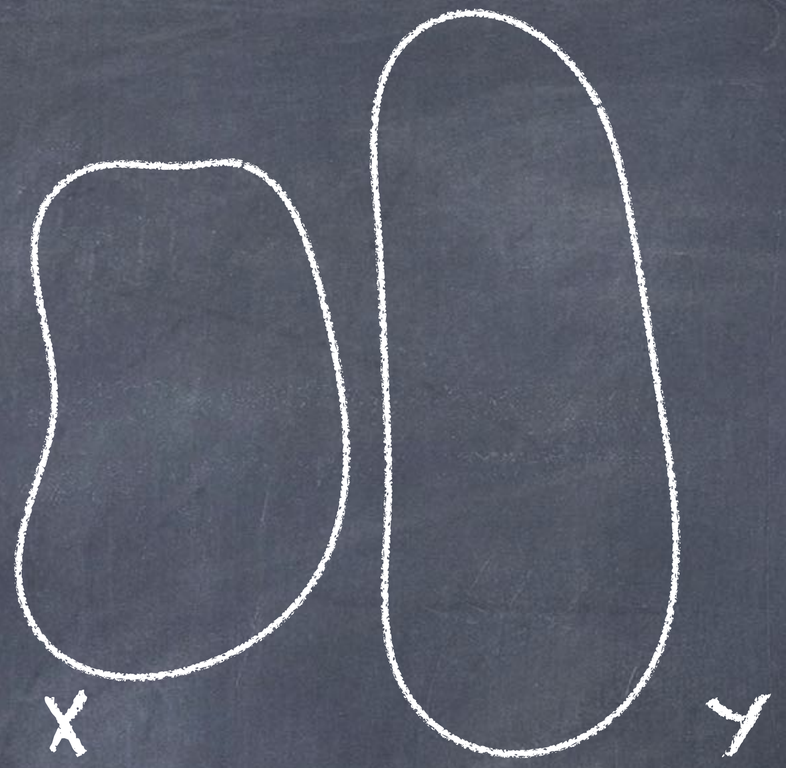

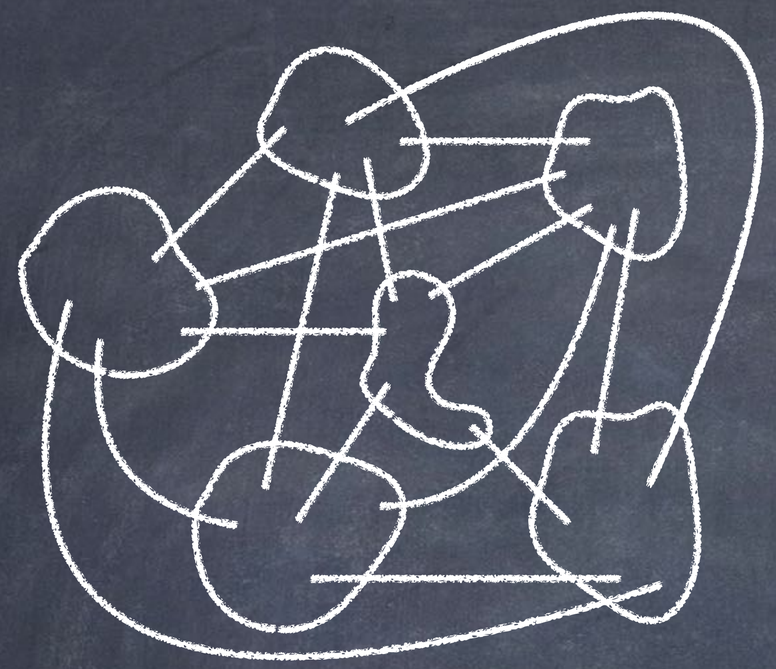

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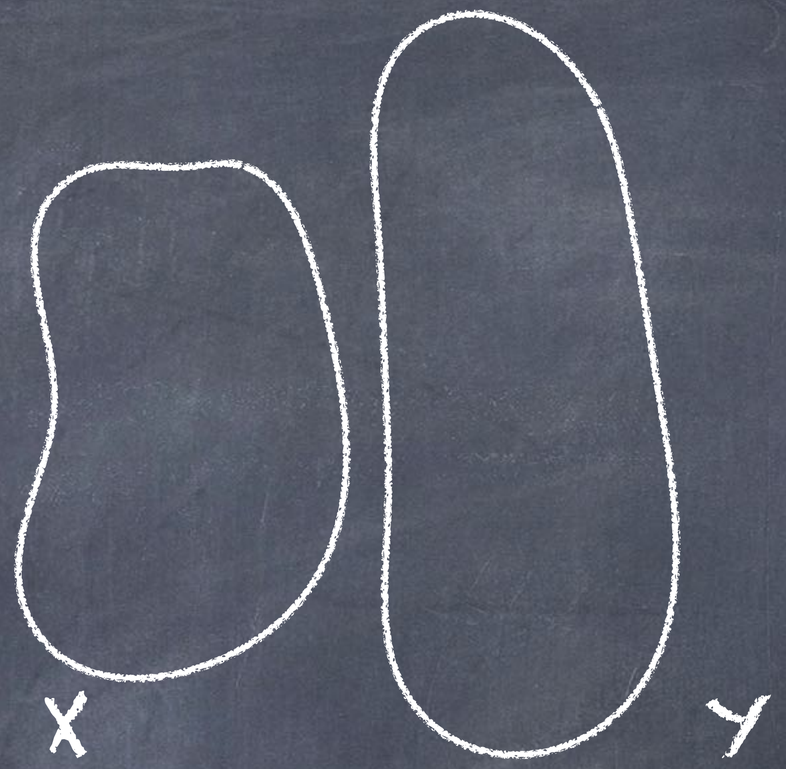

Partition (X, Y) of V(G)

- X is small
- Y induces a clique

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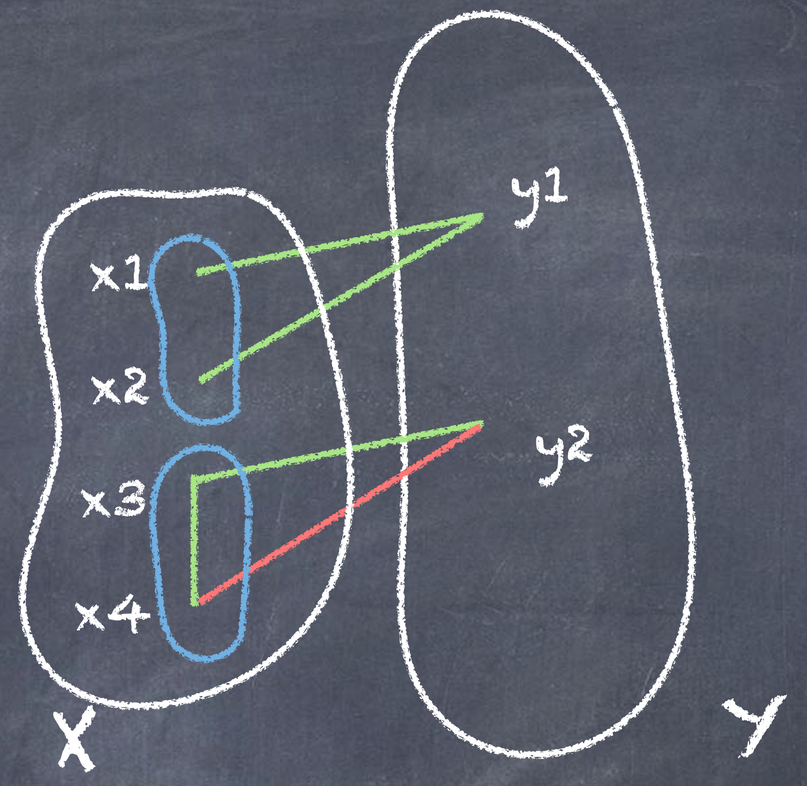

Partition (X, Y) of V(G)

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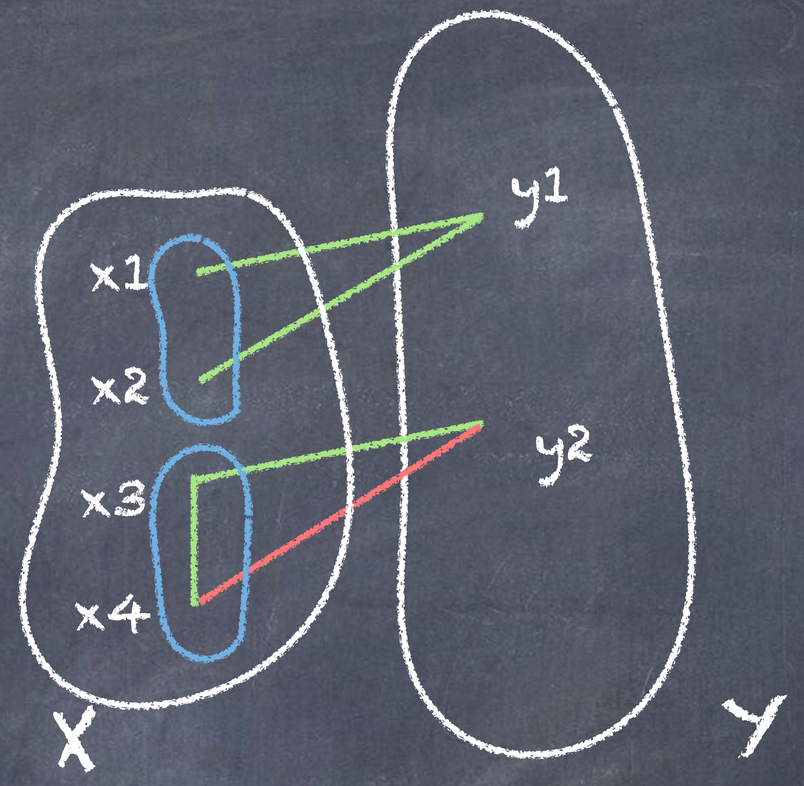

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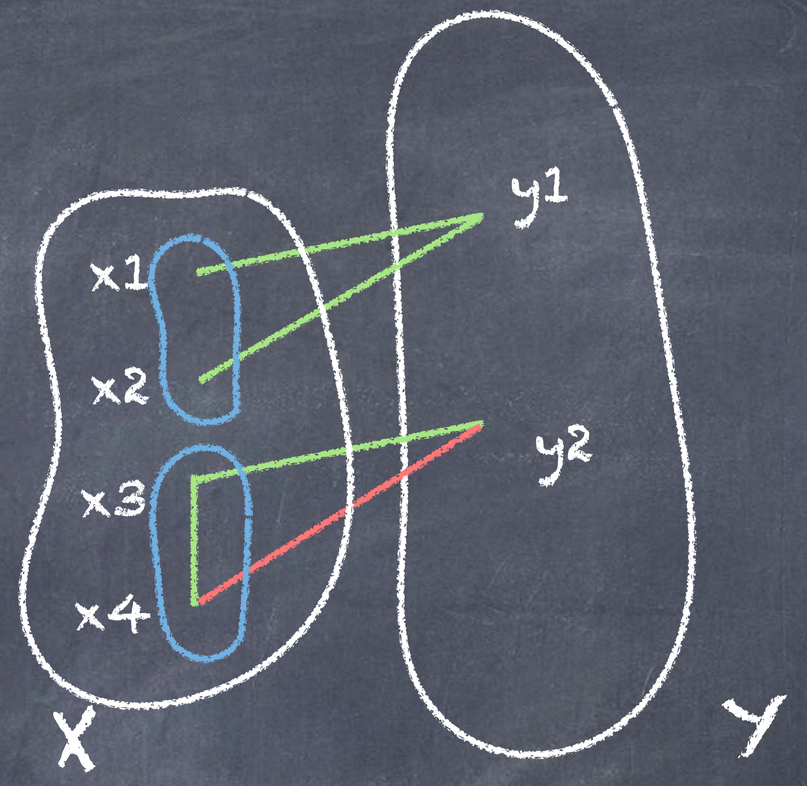

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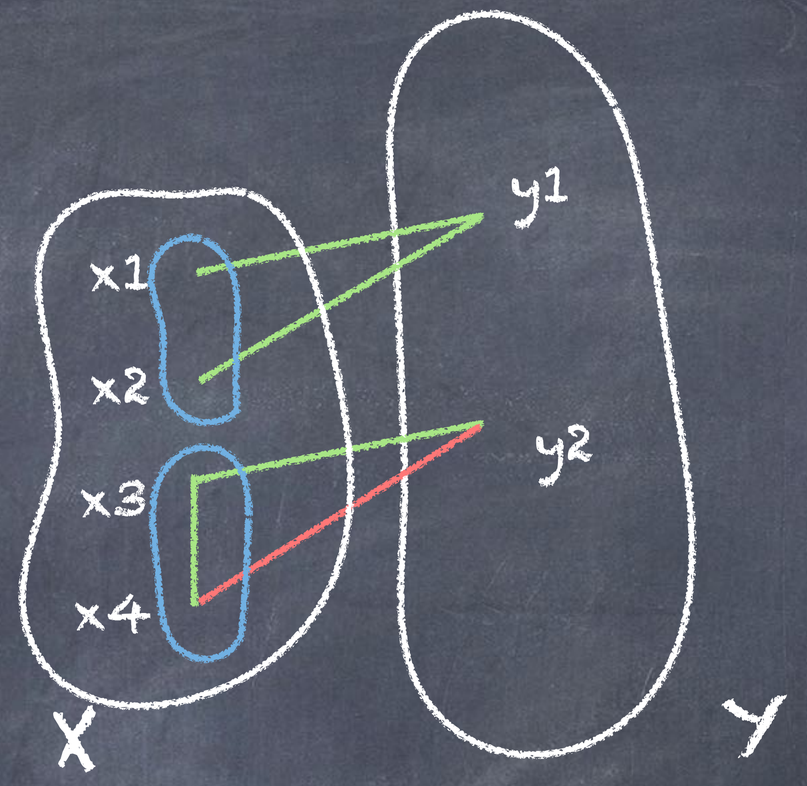

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Role of vertices in Y

$(d + 1)/d <= \alpha$

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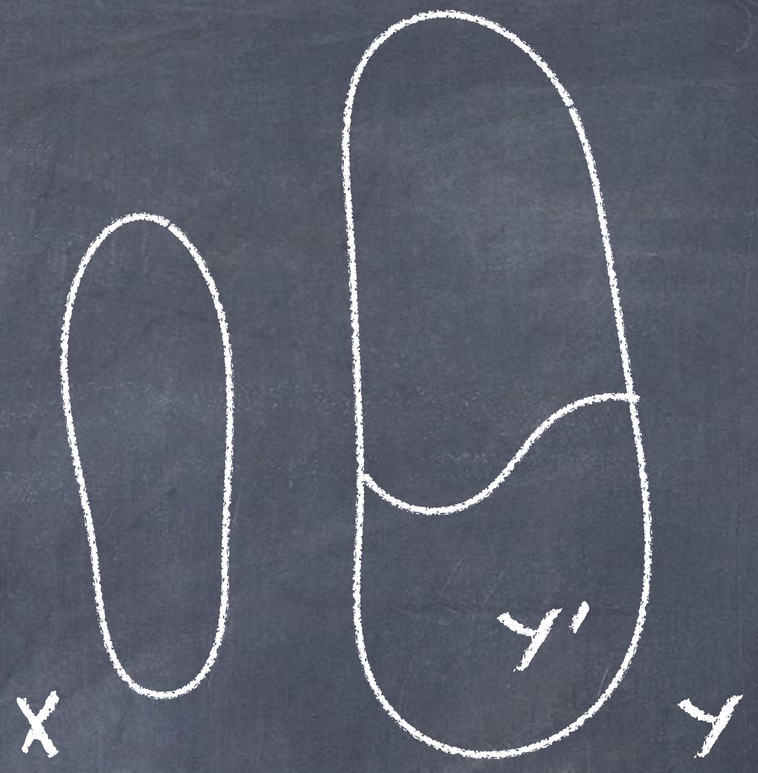
Reduction Rule 2: Mark vertices in Y and delete unmarked vertices.

RR-2 runs in poly-time on large instances.

Y' – deleted vertices



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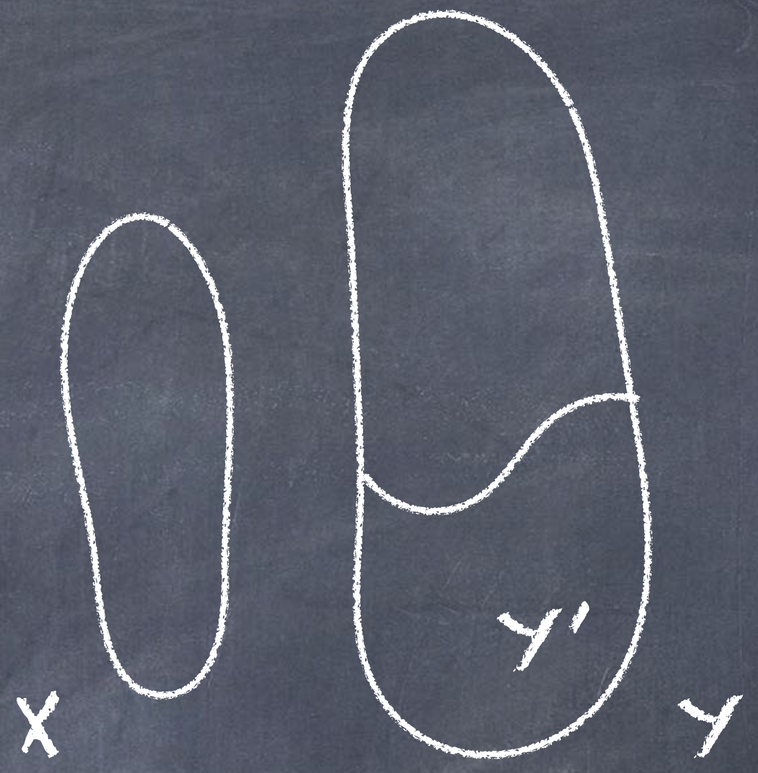
$(G \setminus Y', k)$ reduced instance.



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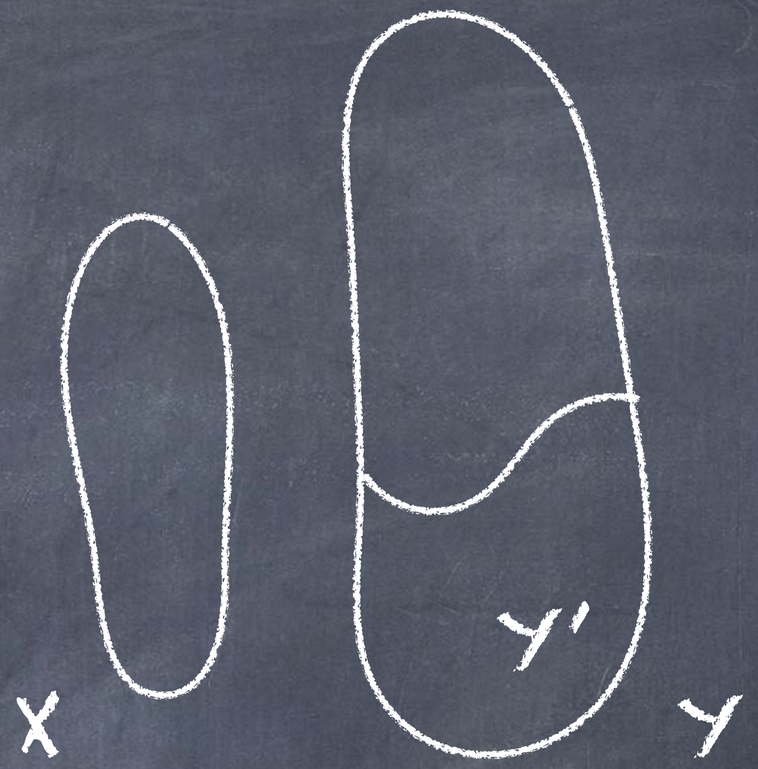

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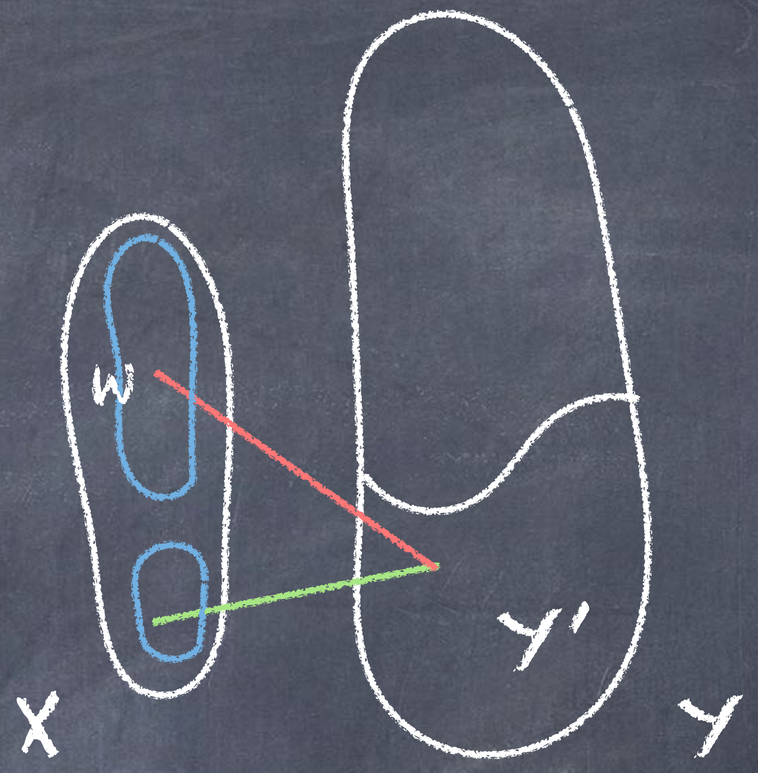

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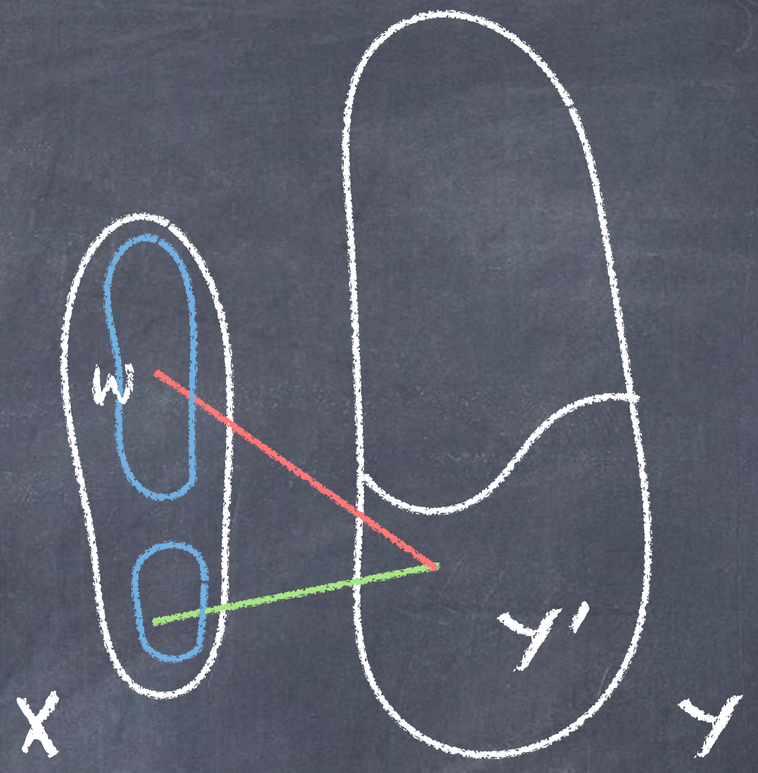

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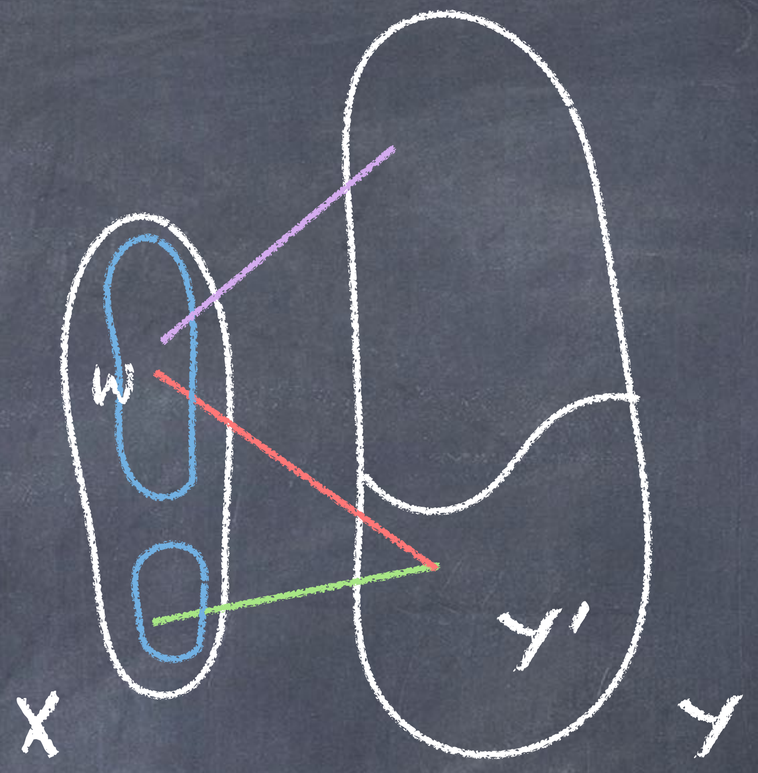

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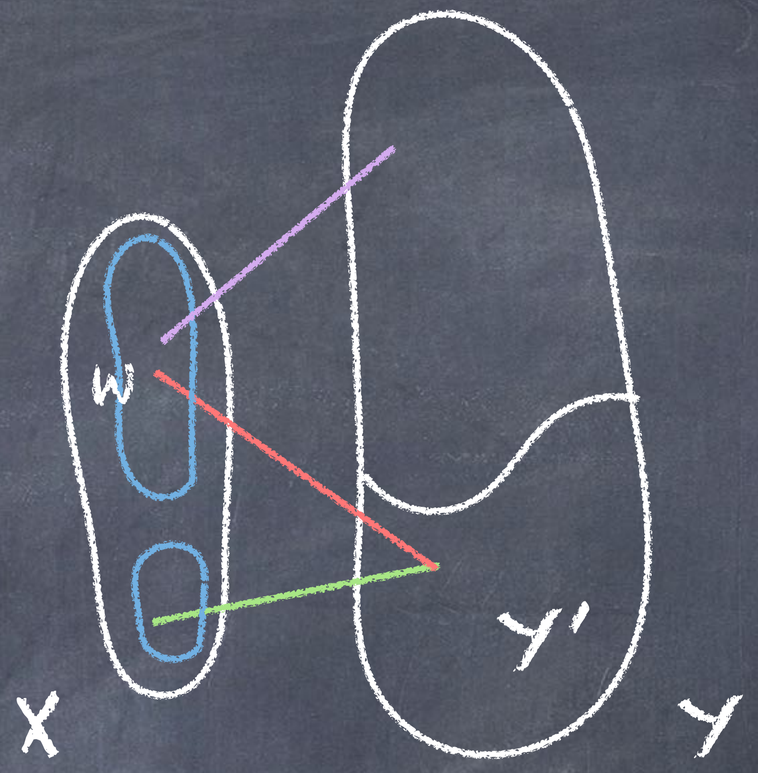

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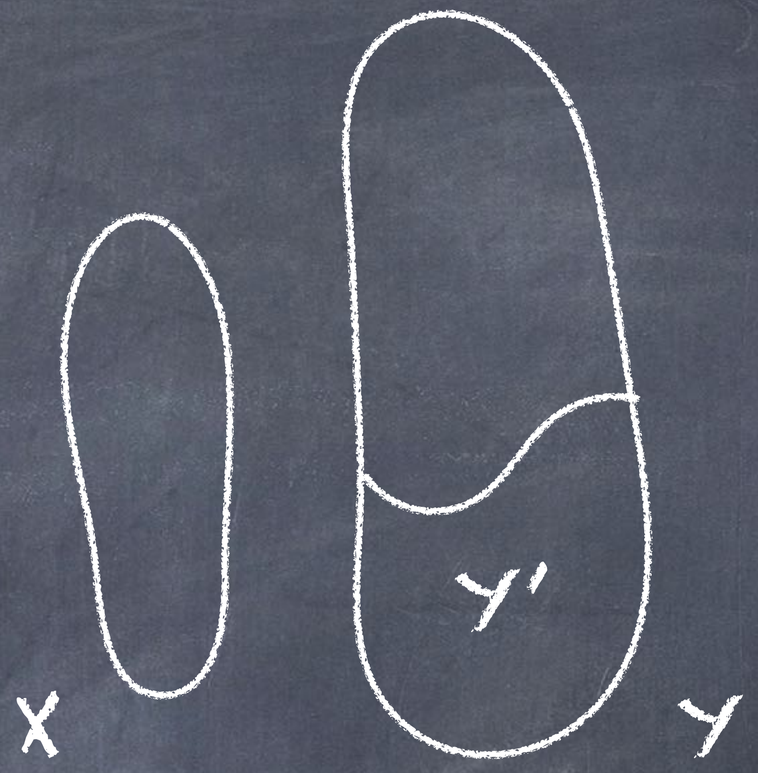
$$\frac{|F|}{OPT(G, k)} <= \alpha \frac{|F'|}{OPT(G', k)} = \alpha\, c$$

$|X| <= 4k$

$|Y \setminus Y'| <= O(k^{d+1})$

(marked vertices)



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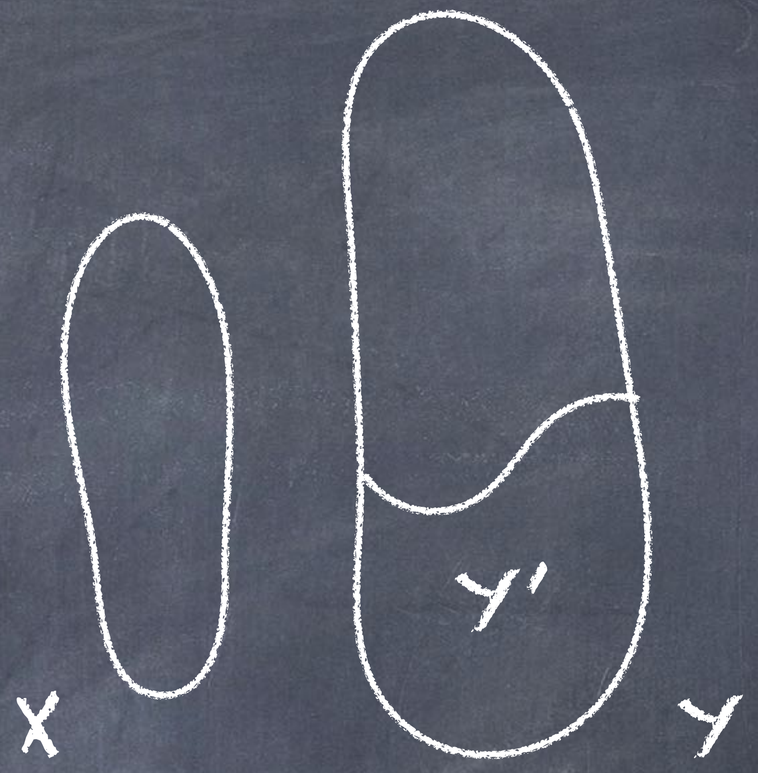

Theorem : For any $\alpha > 1$, CLIQUE CONTRACTION parameterised by the size of solution k admits an α-lossy kernel with $O(k^{d+1})$ vertices where $d = 1/\alpha$.

Parameterized Complexity and Known Results

Overview Of Our Contributions

Brief Description: Lossy Kernel for Clique Cont.

Thank you.