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1) $\vdash \phi \to (\psi \to (\psi \vee \neg\phi))$

$\vdash \phi \to (\psi \to (\neg\psi \to \neg\phi))$

$[\phi]_1 \quad \underline{[\psi]_2 \; [\neg\psi]_3 \equiv [\psi \to \bot]}$

$\bot$

$\overline{}$ RAA

$\neg Q$

$\overline{} \to I_3$

$\neg\psi \to \neg\phi$

$\overline{} \to I_2$

$\psi \to (\neg\psi \to \neg\phi)$

$\overline{} \to I_1$

$\phi \to (\psi \to (\neg\psi \to \neg\phi))$

$\vdash \neg\phi \to ((\neg\phi \wedge \neg\psi) \to (\phi \wedge \neg\psi))$

Counter example:

ϕ	ψ	$\neg\phi$	$\neg\psi$	$\overbrace{\neg\phi \wedge \neg\psi}^{a}$	$\overbrace{\phi \wedge \neg\psi}^{b}$	$\overbrace{a \to b}^{K}$	$\neg\phi \to K$
T	T	F	F	F	F	T	T
T	F	F	T	F	T	T	T
F	T	T	F	F	F	T	T
F	F	T	T	T	F	F	F

COUNTER EXAMPLE

Since this is false that means that it cannot be proved, namely it is not true in all the interpretations.

2) $A \land (\neg B \land (C \to \neg A))$

$A \land (\neg B \land (\neg C \lor \neg A))$ Distributivity

$A \land ((\neg B \land \neg C) \lor (\neg B \land \neg A))$ Distributivity

$(A \land (\neg B \land \neg C)) \lor (A \land (\neg B \land \neg A))$

$(A \land \neg B \land \neg C) \lor \underbrace{(A \land \neg B \land \neg A)}$ DNF

→ NOTE: this will be always false since $A \land \neg A$

3) $\emptyset : \neg A \lor B \quad \psi : (A \land \neg C) \to (B \lor C)$

I use the Deduction Theorem, so $\models \emptyset \to \psi$

A	B	C	$\neg A$	$\neg C$	$\neg A \lor B$	$\overbrace{A \land \neg C}^{a}$	$\overbrace{B \lor C}^{b}$	$\overbrace{a \to b}^{K}$	$(\neg A \lor B) \to K$
T	T	T	F	F	T	F	T	T	T
T	T	F	F	T	T	T	T	T	T
T	F	T	F	F	F	F	T	T	T
T	F	F	F	T	F	T	F	F	T
F	T	T	T	F	T	F	T	T	T
F	T	F	T	T	T	F	T	T	T
F	F	T	T	F	T	F	T	T	T
F	F	F	T	T	T	F	F	T	T

Since they are all True, that means (from the Deduction Theorem) that ψ is a logical consequence of $\emptyset$.

$\emptyset : (A \to B) \to C \quad \psi : A \to C$

I use the Deduction Theorem, so $\models \emptyset \to \psi$

A	B	C	$A \to B$	$(A \to B) \to C$	$A \to C$	$\emptyset \to \psi$
T	T	T	T	T	T	T
T	T	F	T	F	F	T
T	F	T	F	T	T	T
T	F	F	F	T	F	F
F	T	T	T	T	T	T
F	T	F	T	F	T	T
F	F	T	T	T	T	T
F	F	F	T	F	T	T

Since this is False thanks to the Deduction Theorem, I can conclude that ψ is not a logical consequence of $\emptyset$.

4) I consider the propositions: POTRIMBA PETRU 928850

g, m and s as "the gold road brings you to the exit" (g)

"the marble road brings you to the exit" (m)

"the stone road brings you to the exit" (s)

Then, I express the three statements made by the guardians as compound propositions using g, m and s.

$\neg G1 \equiv (\neg g) \vee (s \wedge \neg m)$

$\neg G2 \equiv g \vee s$

$\neg G3 \equiv (\neg g) \vee m$

g	m	s	$(\neg G1) \wedge (\neg G2) \wedge (\neg G3)$
T	T	T	F
T	T	F	F
T	F	T	F
T	F	F	F
F	T	T	F
F	T	F	F
F	F	T	T
F	F	F	F

$\rightarrow$ this means that the stone road will lead me to the exit.

POTRÎMBA PETRU 928850

5) The language I am going to use is the following:

- A unary function color, where COLOR(X) is the color associated to the node X.
- A unary predicate node, where NODE(X) means that X is a node.
- A binary predicate edge, where EDGE(X,Y) means that X is connected to Y.

The set of axioms are:

- Two connected nodes are not equally colored:

$\forall x \forall y (edge(x,y) \rightarrow (color(x) \neq color(y))$

- A node does not have more than K connected nodes:

$\forall x \forall x_1 \dots \forall x_{K+1} (\bigwedge_{n=1}^{K+1} edge(x, x_n) \rightarrow \bigvee_{i,j=1, j\neq i}^{K+1} x_i = x_j)$

6)

COUNT_NOT_LEAVES(nil, 0).

COUNT_NOT_LEAVES(t(_, nil, nil), 0).

COUNT_NOT_LEAVES(t(_, L, nil), 1).

COUNT_NOT_LEAVES(t(_, nil, R), 1).

COUNT_NOT_LEAVES(t(_, L, R), Res) :- COUNT_NOT_LEAVES(L, ResL), COUNT_NOT_LEAVES(R, ResR), Res is ResL + ResR + 1.

7)

q(W, b, Y) , So the result of the evaluation is: W=a

Y = f(a)

$\mu = \{W/a, X/b, Y/Z\}$

q(a, X, Z) so, the New Goal is: q(b, b, Z)

q(b, b, Z), $\mu = \{Z/Z\}$

so the new goal becomes: p(Z), r(Z)

q(a, X, Z)

this fails because it is not unifiable, so backtrack

q(b, c, Z)

this fails because it is not unifiable, so backtrack

p(f(V)) $\mu = \{Z/f(V)\}$

$\mu = \{Z/f(a)\}$

r(f(a))

So here, to be unified, V is bind with "a". So

8) "Generate and test" idea is to describe the entire problem by means of a generator, that enumerates candidates for the solution and a test that verifies whether a generate candidate is in fact a proper solution.

On the other hand "Constraint on generate" idea is to apply the constraint to the problem in order to eliminate all the candidates that will not be part of the solution since they do not respect the constraints of the problem. After done that, the solutions are generated.