

Iniziato	venerdì, 26 giugno 2020, 12:20
Stato	Completato
Terminato	venerdì, 26 giugno 2020, 13:20
Tempo impiegato	59 min. 49 secondi
Valutazione	Non ancora valutato

Domanda **1**

Completo

Punteggio  
max.: 7,00

Construct a deterministic TM of the kind you prefer, which decides the following language:

$$\mathcal{L} = \{w \in \{0, 1\}^* \mid \text{if } 0^k \text{ is a subsequence of } w, \text{ then } k \leq 2\}.$$

Study the complexity of TM you have defined.

1-tape TM.

states  $Q = \{q_{init}, q_0, q_1, q_2, q_{res}, q_{halt}\}$   
alphabet  $A=\{0,1, start, blank\}$

Transition function:  
( $q_{init}, start$ ) -> ( $q_0, start, R$ )  
( $q_0, 1$ ) -> ( $q_0, 1, R$ ) # If we meet a 1, we keep scanning the string for 0s  
( $q_0, 0$ ) -> ( $q_1, 0, R$ ) # We found a zero  
( $q_0, blank$ ) -> ( $q_{res}, blank, S$ )  
( $q_1, 0$ ) -> ( $q_2, 0, R$ ) # Found a second zero  
( $q_1, 1$ ) -> ( $q_0, 1, R$ ) # found a 1, get back to search 0s  
( $q_2, 1$ ) -> ( $q_0, 1, R$ ) # after 2 zeros, we can only accept if we find a 1 or blank  
( $q_1, blank$ ) -> ( $q_{res}, blank, S$ )  
( $q_2, blank$ ) -> ( $q_{res}, blank, S$ )  
( $q_{res}, blank$ ) -> ( $q_{halt}, blank, S$ )

TM = (Q, A, delta)

Time complexity:  
The complexity is linear w.r.t. input string size because we only pass the string once.

Domanda **2**

Completo

Punteggio  
max.: 7,00

You are required to prove that the following problem  $\mathcal{L}$  is in **NP**. To do that, you can give a NTM or define some pseudocode. The problem is one that, given a natural number  $n$ , checks whether  $n$  is a sum of powers of 3.

We have to check that any solution for the problem can be verified in polynomial time. We do it by defining a pseudocode.

Intuition: Powers of 3 are all dividable by 3. The sum of multiple powers of 3 is also dividable by 3.

So we can just check whether a number "n" is dividable by 3, since we can always express this number "n" as a multiple sum of 3s (3 multiplied by the result of  $n/3$ ).

This is correct because 3 is  $3^1$ , i.e. a power of 3.

Algorithm:

INPUT: n

if (n % 3 == 0) then:

    return True

else:

    return False

Time complexity:

The division can be easily computed in polynomial time by a TM w.r.t. to the length of the encoding of n.

So the algorithm runs in polytime.

Since the algorithm checks whether a solution belongs to the language L, we have proved that L is in NP.

Domanda **3**

Completo

Punteggio

max.: 6,00

The most popular textbook on AI is the one by Russell And Norvig, called "Artificial Intelligence: a Modern Approach". The algorithms presented in it includes many algorithms used in various branches of AI. Let  $RN$  be the set of algorithms explained in the book, appropriately encoded in a succinct way as finite binary strings. To which complexity class does  $RN$  belong? Prove your claim.

We can say that  $RN$  belongs to NP class.  
In fact we can easily verify that an algorithm belongs to the book in polynomial time, by looping all the algorithms one-by-one in the book and say if it's equal to the one taken into consideration.

We could now ask if it's also in P.  
We can express a define rule to generate any string of  $RN$  set. This rule is used to generate an encoding of an algorithm. (after defining for example a mapping from set of algorithms in the book to natural numbers, and from natural numbers to their binary encoding)  
So after defining a way to generate a binary encoding which maps to a certain algorithm of  $RN$  set, we can easily generate a string belonging to the set of encoding of the  $RN$  set using the same rule.  
Thus we can say that  $RN$  is also in P.

Commento:  
Just overly complicated.