(1) i)

$$\dfrac{\dfrac{\dfrac{[\phi]_{3} \quad \dfrac{[\psi]_{2}}{\psi \vee \neg\phi}\ \vee I\ (1)}{\psi \to \psi \vee \neg\phi}\ \to I\ (2)}{\phi \to (\psi \to (\psi \vee \neg\phi))}\ \to I\ (3)}{}$$

ii) If $\phi = F$ and $\psi = F$ the formula is false, hence it cannot be proved by natural induction

(2) $A \wedge (\neg B \wedge (C \to \neg A)) \equiv A \wedge (\neg B \wedge (\neg C \vee \neg A)) \equiv$
$\equiv A \wedge ((\neg B \wedge \neg C) \vee (\neg B \wedge \neg A)) \equiv$
$(A \wedge \neg B \wedge \neg C) \vee (A \wedge \neg B \wedge \neg A) \equiv A \wedge \neg B \wedge \neg C$

(3) $\psi \equiv \neg(A \wedge \neg C) \vee (B \vee C) \equiv \neg A \vee C \vee B \vee C \equiv$ true

and $\phi \models$ true for any ϕ OR

USING THE TRUTH TABLE TO SHOW THAT WHEN ϕ IS TRUE (THAT IS, WHEN A IS FALSE OR B IS TRUE) ψ IS TRUE (WHICH IS ALWAYS THE CASE).

(4) $G \equiv$ GOLD ROAD TAKES TO THE EXIT
$M \equiv$ MARBLE " " " " "
$S \equiv$ STONE " " " " "

GUARDIAN OF GOLD ROAD : $G \wedge (S \to M) \equiv \phi_1$
" " MARBLE " $\neg G \wedge \neg S \equiv \phi_2$
" " STONE " $G \wedge \neg M \equiv \phi_3$

ALL LIARS: $\neg\phi_1 \wedge \neg\phi_2 \wedge \neg\phi_3 \equiv \psi$. USING TRUTH TABLE THE ONLY INTERPRETATION WHICH MAKES ψ = TRUE IS $S = T$, $M = F$, $G = F$

OR M=T

5

Language

- *A unary function* color*, where* $\mathsf{color}(x)$ *is the color associated to the node* x
- *A unary predicate* node*, where* $\mathsf{node}(x)$ *means that* x *is a node*
- *A binary predicate* edge*, where* $\mathsf{edge}(x, y)$ *means that* x *is connected to* y

Axioms

1. $\forall x \forall y.(\mathsf{edge}(x, y) \rightarrow (\mathsf{color}(x) \neq \mathsf{color}(y))$
   "Two connected node are not equally colored."
2. $\forall x \forall x_1 \ldots \forall x_{k+1}. \left(\bigwedge_{h=1}^{k+1} \mathsf{edge}(x, x_h) \rightarrow \bigvee_{i,j=1,j\neq i}^{k+1} x_i = x_j\right)$
   "A node does not have more than k connected nodes."

6

```
tree_count(nil,0).
tree_count(t(_,nil, nil),0).
tree_count(t(_,L,R),N):-
      tree_count(L,N1), tree_count(R,N2), N is N1+N2+1.
```

Please note that the first clause is needed, othewise the program does not compute the correct number. However, I have considered correct also the solutions without such a first clause.

7 q(W,b,Y) —> (using rule 1 and unifier X/b)
q(b,b,Y) —> (using rule 3 and unifier Z'/Y)
p(Y),r(Y) —> (using rule 4 and unifier Y/f(V))
r(f(V)) —> (using rule 5 and unifier V/a)
[] (empty clause)

The answer is W/b and Y/f(a)

8. Generate and test means that thew program first generate the values for the variables and then check whether the constraint are satifisfied (if not, backtraking occurs).
Constraint and generate means that the program first impose the constraints, which are used to limit the search space, and afterwards the values for instantiating the variablkes are generated.