EXERCISE 1.

WHAT'S THE CARDINALITY OF THE SET S^n OF STRINGS OF LENGTH n OVER THE ALPHABET S? PROVE YOUR CLAIM.

→ $$|S^n| = |S|^n$$

→ WE CAN PROVE THE ABOVE EQUALITY BY WAY OF INDUCTION ON THE NATURAL NUMBERS.

<u>BASE</u> $n=0$

$$|S^0| = |\{\varepsilon\}| = 1 = |S|^0$$

<u>INDUCTION</u> $n>0$

$$|S^n| = |S| \cdot \underbrace{|S^{n-1}|}_{\text{BY IH THIS IS } |S|^{n-1}} = |S| \cdot |S|^{n-1} = |S|^n$$

ANY STRING x OF LENGTH n OVER S CAN BE SEEN AS $x = s.y$ WHERE $s \in S$ AND $y = S^{n-1}$

EXERCISE 2.

RELATE THE FOLLOWING PAIRS OF FUNCTIONS BY WAY OF ASYMPTOTIC NOTATION

$$f_1(n) = n \lg n \qquad g_1(n) = 10\, n \lg(\lg n)$$

$$f_2(n) = 1000\, n \qquad g_2(n) = \frac{1}{100}\, n \lg n$$

→ LET'S CONSIDER THE FIRST PAIR OF FUNCTIONS

$$\lim_{n\to\infty} \frac{f_2(n)}{g_2(n)} = \lim_{n\to\infty} \frac{\cancel{x}\lg n}{20 \times \lg(\lg(n))} = \lim_{n\to\infty} \frac{\lg n}{20 \lg(\lg(n))} = +\infty$$

THIS GROWS → ASYMPTOT. SLOWER THAN $\lg n$

AS A CONSEQUENCE $f_2(n) = \Omega(g_2(n))$
AND DUALLY $g_2(n) = O(f_2(n))$

EXERCISE 3

WE WOULD LIKE TO FIND APPROPRIATE ENCODINGS FOR THE FOLLOWING DISCRETE SETS:

- THE SET $\mathbb{Q}$ OF RATIONAL NUMBERS
- DISJOINT SUM OF $\mathbb{N}$ AND $\{0,1\}^*$, WHERE THE DISJOINT SUM OF TWO SETS A AND B IS $A+B = \{in_1(a) \mid a \in A\} \cup \{in_2(b) \mid b \in B\}$
- GRAPHS, NAMELY PAIRS IN THE FORM (V, E) SUCH THAT V IS A FINITE SET OF NODES AND $E \subseteq V \times V$ IS A FINITE SET OF EDGES

→ THE RATIONAL NUMBERS CAN BE SEEN AS THOSE REAL NUMBERS IN THE FORM $\frac{a}{b}$ WHERE $a \in \mathbb{Z}$ AND b IS A POSITIVE NATURAL NUMBER.

WE CAN THEN PROCEED BY FIRST ENCODING $\mathbb{Z}$ AS STRINGS AND THEN ENCODING $\mathbb{Q}$ AS A SET OF PAIRS (d, b) WHERE d IS THE ENCODING OF AN INTEGER IN $\mathbb{Z}$ AND b IS THE ENCODING OF A POSITIVE NATURAL NUMBER.

→ ABOUT DISJOINT SUMS, IT'S EASY BECAUSE WE CAN ENCODE $\mathbb{N} + \{0,1\}^*$ AS FOLLOWS

$$\mathrm{inl}(n) \longmapsto 0 \cdot \lfloor n \rfloor$$
$$\mathrm{inr}(x) \longmapsto 1 \cdot x$$