

Languages and Algorithms for AI

Francesca Del Bonifro¹

¹Department of Computer Science and Engineering - UniBO

25/11/2019

Outline

1 Logic Programming

2 Constraints

Lists

Def. *append*/3

append([], L, L).

append([X|T1], L, [X|T2]) :- *append*(T1,L,T2).

Lists: reverse

Write a predicate *rev/2* s.t *rev(L1,L2)* is true if L2 is the reverse of L1.

Lists: reverse

Write a predicate *rev/2* s.t *rev(L1,L2)* is true if L2 is the reverse of L1.

```
rev([], []).
```

```
rev([H|T1], L) :- rev(T1, T), append(T, [H], L).
```

Queries:

```
?- rev([a,b,c],[c,b,a]).
```

```
?- rev([a,b,c],[c,a,b]).
```

```
?- rev([[a,b],c,d],X).
```

Lists: palindrome

Write a predicate *pal/1* s.t *pal(L)* is true if L is palindrome.

Lists: palindrome

Write a predicate *pal/1* s.t *pal(L)* is true if L is palindrome.

pal([]).

pal(R):-*rev*(R,R).

Queries:

?- *pal*([a,b,c,d]).

?- *pal*([a,b,c,c,b,a]).

?- *pal*([a,b,c,b,a]).

?- *pal*([[a,b],c,d,c,[b,a]]).

Lists: first element vs prefix

Write a predicate *first/2* s.t. *first(L,X)* is true if X is the first element of the list L.

Lists: first element vs prefix

- Write a predicate *first/2* s.t. *first(L,X)* is true if X is the first element of the list L.
- Write a predicate *prefix/2* s.t. *prefix(L,X)* is true if X is a prefix of the list L.

Lists: first element vs prefix

Write a predicate *first/2* s.t. *first(L,X)* is true if X is the first element of the list L.

first(L,X) :- append([X],_,L).

Write a predicate *prefix/2* s.t. *prefix(L,X)* is true if X is a prefix of the list L.

prefix(L, X) :- append(X, _, L).

Queries:

?- *first([a,b,c],X).*

?- *prefix([a,b,c],X).*

?- *first([[a,b],b,c],a).*

?- *prefix([[a,b],b,c],a).*

Lists: last element vs suffix

- Write a predicate *last/2* s.t. *last(L,X)* is true if X is the last element of the list L.
- Write a predicate *suffix/2* s.t. *suffix(L,X)* is true if X is a suffix of the list L.

Lists: last element vs suffix

Write a predicate *last/2* s.t. *last(L,X)* is true if X is the last element of the list L.

```
last(L, X) :- append(_, [X], L).
```

Write a predicate *suffix/2* s.t. *suffix(L,X)* is true if X is a suffix of the list L.

```
suffix(L, X) :- append(_, X,L).
```

Queries:

```
?- last([a,b,c],X).
```

```
?- suffix([a,b,c],X).
```

```
?- last([a,[b,c]],c).
```

```
?- suffix([a,[b,c]],c).
```

Lists: same list

Write a predicate *same/2* s.t. *same*(L1,L2) is true if L1 and L2 represent the same list.

Lists: same list

Write a predicate *same/2* s.t. *same(L1,L2)* is true if L1 and L2 represent the same list.

```
same([],[]).
```

```
same([X|T],R):-append([X],T,R).
```

Queries:

```
?- same([a,b,c],[a,f,c]).
```

```
?- same(X,[a,b,c]).
```

```
?- same([],X).
```

```
?- same([],[a]).
```

Train Ticket

How to represent these situations?

2x1 Special Offer: With the special offer 2x1 every Saturday you can travel in 2 paying only one Base ticket:

- The offer is valid for travel on all domestic trains at Business, Premium and Standard service levels, and in 1st and 2nd class;
- 2x1 does not apply to regional trains, Executive service level and couchette, VL and Excelsior services.

Train Ticket

Family Offer: for journeys by family groups made up of from 2 to 5 persons of whom at least 1 is an adult and 1 is a child of less than 12:

- 50% (30% for couchettes and VL) for children under 15;
- 20% for the other persons;
- The Family offer does not apply to journeys on the Excelsior service level.

Train Ticket

Eligibility rules:

■ 2x1

eligibleFor(twoForOne, *ticketFor*(TrainCode, Day, ServiceLevel, Class, ListOfPassengersAndAge)) :-

train(TrainCode, Geo, SupportedServiceLevels, SupportedClasses),

member(ServiceLevel, SupportedServiceLevels),

member(Class, SupportedClasses) ,

Day = saturday ,

Geo \= regional ,

length(ListOfPassengersAndAge,2) ,

member(ServiceLevel, [business, premium, standard]) ,

\+ member(ServiceLevel, [executive, couchette, vl, excelsior]) ,

member(Class, [first, second]) .

Train Ticket

■ Family

eligibleFor(family, *ticketFor*(TrainCode, Day, ServiceLevel, Class, ListOfPassengersAndAge)) :- train(TrainCode, Geo, SupportedServiceLevels, SupportedClasses) ,
member(ServiceLevel, SupportedServiceLevels) ,
member(Class, SupportedClasses) ,
length(ListOfPassengersAndAge,N) ,
N>=2 ,
N<=5 ,
member((_, Age1), ListOfPassengersAndAge) ,
Age1 >= 18 ,
member((_, Age2), ListOfPassengersAndAge) ,
Age2 < 12 ,
ServiceLevel \= excelsior .

Train Ticket

Ex.

`train(676, ic, [business, premium, standard, couchette], [first, second]).`

is a fact added to our knowledge base.

Queries:

?- *eligibleFor*(X, *ticketFor*(676, saturday, business, second, [(_,25), (_,10)])).

?- *eligibleFor*(X, *ticketFor*(676, saturday, standard, first, [(_,18), (_,20)])).

?- *eligibleFor*(X, *ticketFor*(676, monday, premium, second, [(_,18), (_,10)])).

train Ticket

Price rules:

- 2x1

computePrice(twoForOne, *ticketFor*(TrainCode, Day, ServiceLevel, Class, [_, Pass2]), Total) :-

computePrice(twoForOne, *ticketFor*(TrainCode, Day, ServiceLevel, Class, [Pass2]), Total) .

computePrice(twoForOne, *ticketFor*(TrainCode, _, ServiceLevel, Class, [_]), Total) :- price(TrainCode, ServiceLevel, Class, Total) .

train Ticket

■ Family

```
computePrice(family, ticketFor(____, __, __, []), 0).  
computePrice(family, ticketFor(TrainCode, Day, ServiceLevel,  
Class, [(__,Age)|T]), Total) :-  
price(TrainCode, ServiceLevel, Class, Price) ,  
Age < 15 ,  
\+ member(ServiceLevel, [couchette, vl]) ,  
computePrice(family, ticketFor(TrainCode, Day, ServiceLevel,  
Class, T), PartialPrice) ,  
Discounted is Price*0.5 ,  
Total is Discounted + PartialPrice .
```

Train Ticket

■ Family

computePrice(family, *ticketFor*(TrainCode, Day, ServiceLevel, Class, [(_,Age)|T]), Total) :-

price(TrainCode, ServiceLevel, Class, Price) ,

Age < 15 ,

member(ServiceLevel, [couchette, vl]) ,

computePrice(family, *ticketFor*(TrainCode, Day, ServiceLevel, Class, T), PartialPrice) ,

Discounted is Price*0.7 ,

Total is Discounted + PartialPrice .

Train Ticket

■ Family

computePrice(family, *ticketFor*(TrainCode, Day, ServiceLevel, Class, [(,Age)|T]), Total) :-

price(TrainCode, ServiceLevel, Class, Price) ,

Age >= 15 ,

computePrice(family, *ticketFor*(TrainCode, Day, ServiceLevel, Class, T), PartialPrice) ,

Discounted is Price*0.8 ,

Total is Discounted + PartialPrice .

Train Ticket

Generally:

```
finalPrice(Offer, ticketFor(TrainCode, Day, ServiceLevel, Class,  
  ListOfPassengersAndAges), Price):-  
  eligibleFor(Offer, ticketFor(TrainCode, Day, ServiceLevel, Class,  
    ListOfPassengersAndAges)),  
  computePrice(Offer, ticketFor(TrainCode, Day, ServiceLevel,  
    Class, ListOfPassengersAndAges), Price).
```


Train Ticket

For example, add to the knowledge base the price information for the train:

```
price(676, business, first, 100).  
price(676, business, second, 80).  
price(676, premium, first, 90).  
price(676, premium, second, 60).  
price(676, standard, first, 70).  
price(676, standard, second, 50).  
price(676, couchette, first, 130).  
price(676, couchette, second, 110).
```

train Ticket

Queries:

?- finalPrice(family, *ticketFor*(676, monday, premium, second, [(_ ,80),(_ ,10),(_ ,19)]), X).

?- finalPrice(family, *ticketFor*(676, friday, couchette, first, [(_ ,80),(_ ,10),(_ ,19)]), X).

?- finalPrice(twoForOne, *ticketFor*(676, monday, business, first, [(_ ,30),(_ ,50)]), X).

?- finalPrice(twoForOne, *ticketFor*(676, saturday, business, first, [(_ ,30),(_ ,50)]), X).

?- finalPrice(Y, *ticketFor*(676, saturday, premium, second, [(_ ,10),(_ ,19)]), X).

N-queens

Place N Queens on a $N \times N$ chess board s.t. no Queen can attack the others (in one move), i.e. no two queens are on the same row, column, or diagonal.

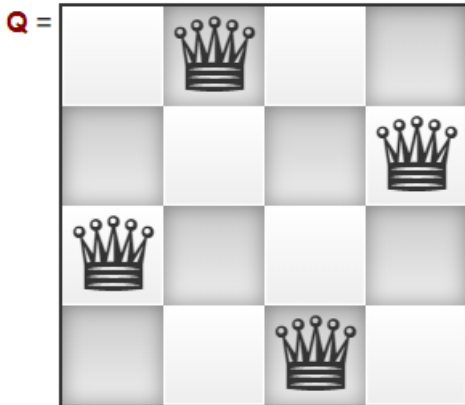
The solution Q is a list of length N . The i -th element of Q is the column number of the queen in the i -th row (the constraint s.t. no queen share the row number with the others is already satisfied).

N-queens

Es. $N=4$

$Q=[2,4,1,3]$

The plugin 'chess' give us a visual representation of lists in this sense.



N-queens

```
:- use_rendering(chess).  
queens(N,Queens) :- queens_list(N,Q), queens(Q,[],Queens).  
queens_list(0,[]).  
queens_list(N,[N|Ns]):- N>0, N1 is N-1, queens_list(N1,Ns).  
queens([],Qs,Qs).  
queens(U,P,Qs) :- select(Q, U,U1), no_attack(P,Q,1),  
queens(U1,[Q|P],Qs).  
no_attack([],_Q, _Nb).  
no_attack([Y|Ys],Queen,Nb) :- Queen =\=Y+Nb,  
Queen=\=Y-Nb, Nb1 is Nb+1, no_attack(Ys,Queen,Nb1).
```

N-queens

Queries:

?- queens(8, Q).

?- queens(20, Q).

?- queens(40, Q).

?- queens(100, Q).

Outline

1 Logic Programming

2 Constraints

Boolean constraints

We are in a village in which every inhabitant (variables) is either a knight or a knave.

knight always say the truth (true value for the correspondent variables)

knave always lie (false value for the correspondent variables)

Boolean constraints

We meet A and B.

A says: "Either I am a knave or B is a knight."

`:- use_module(library(clpb)).`

Boolean constraints

We meet A and B.

A says: "Either I am a knave or B is a knight."

```
:- use_module(library(clpb)).
```

```
knight([A,B]) :- sat(A=:(~ A + B)).
```

Query:

```
?- knight(X).
```

The only solution is: $X=[1,1]$, which means $A=1$, $B=1$.

Boolean constraints

We meet A and B.

A says: "I am a knave, but B isn't."

```
:- use_module(library(clpb)).
```

Boolean constraints

We meet A and B.

A says: "I am a knave, but B isn't."

```
:- use_module(library(clpb)).
```

```
knights([A,B]) :- sat(A:=((~ A * B))).
```

Query:

```
?- knights(X).
```

The only solution is: A=0, B=0.

Boolean constraints

We meet A and B.

A says: "At least one of us is a knave."

```
:- use_module(library(clpb)).
```

Boolean constraints

We meet A and B.

A says: "At least one of us is a knave."

`:- use_module(library(clpb)).`

`knights([A,B]) :- sat(A==card([1,2],[\sim A, \sim B])).`

Query:

`?- knights(X).`

The only solution is: A=1, B=0.

Boolean constraints

We meet A, B and C.

A says: "All of us are knaves.". B says: "Exactly one of us is a knight."

```
:- use_module(library(clpb)).
```


Boolean constraints

We meet A, B and C.

A says: "All of us are knaves.". B says: "Exactly one of us is a knight."

```
:- use_module(library(clpb)).
```

```
knight([A,B,C]) :- sat(A:=((~ A * ~ B * ~ C)),
```

```
sat(B:=card([1],[A,B,C]))).
```

Query:

```
?- knight(X).
```

The only solution is: A=0, B=1, C=0.

Boolean constraints

We meet A, B and C.

A says: "B is a knave.". B says: "A and C are of the same kind."

`:- use_module(library(clpb)).`

Boolean constraints

We meet A, B and C.

A says: "B is a knave.". B says: "A and C are of the same kind."

`:- use_module(library(clpb)).`

`knight([A,B,C]) :- sat(A \equiv \sim B), sat(B \equiv (A \equiv C)).`

Query:

`?- knight(X).`

There are two solutions:

A=0,B=1, C=0

or

A=1, B=0, C=0

N-queens

Place N Queens on a $N \times N$ chess board s.t. no Queen can attack the others (in one move), i.e. no two queens are on the same row, column, or diagonal.

The solution Q is a list of length N . The i -th element of Q is the column number of the queen in the i -th row (the constraint s.t. no queen share the row number with the others is already satisfied).

N-queens

```
:- use_rendering(chess).  
:- use_module(library(clpfd)).  
queens(N, Q) :- length(Q, N), Q ins 1..N, safe_queens(Q).  
safe_queens([]).  
safe_queens([Q1|Q]) :- safe_queens(Q, Q1, 1),  
    safe_queens(Q).  
safe_queens([],_, _).  
safe_queens([Q1|Q], Q0, D0) :-  
    Q0 # \= Q1,  
    abs(Q0 - Q1) # \= D0,  
    D1 #= D0 + 1,  
    safe_queens(Q, Q0, D1).
```

N-queens

Queries:

?- queens(8, Q), labeling([ff], Q).

?- queens(20, Q), labeling([ff], Q).

?- queens(40, Q), labeling([ff], Q).

?- queens(100, Q), labeling([ff], Q).