12.05

LANGUAGES FOR AI
JANUARY 28 2020

Time: 2 hours.

In the following $A, B, \ldots$ are propositional variables, $a, b, \ldots$ constant symbols, $f, g, \ldots$ function symbols, $X, Y \ldots$ variables, $p, q, \ldots$ predicate symbols and $\phi, \psi, \ldots$ formulas (unless differently specified).

1. (4 points) Consider the language of propositional logic. Use natural deduction to prove that the following holds, or find a counter-example to show that it does not hold
   - $\vdash \phi \rightarrow (\psi \rightarrow (\phi \wedge \psi))$
   - $\vdash \neg\phi \rightarrow ((\phi \wedge \neg\psi) \rightarrow (\phi \wedge \neg\psi))$
2. (4 points) Transform the following propositional logic formula into an equivalent formula in Conjunctive Normal Form
   - $A \vee (\neg B \wedge (C \rightarrow \neg A))$
3. (4 points) Prove that that $\phi \models \psi$ (ψ is a logical consequence of ϕ) or that $\phi \not\models \psi$ for the following formulas:
   - $\phi : \neg A$ and $\psi : (A \wedge \neg B) \rightarrow (A \wedge B)$
   - $\phi : A \rightarrow B \rightarrow C$ and $\psi : \neg A \vee B \vee C$
4. (5 points) Alice finds two trunks AT and BT in a cave. She knows that each of them either contains a treasure or a fatal trap. The following two inscriptions are found on the trunks:
   (a) On trunk AT is written: "At least one of these two trunks contains a treasure."
   (b) On trunk BT is written: "In AT there's a fatal trap."

   We also know that the following is true:
   - Either both the inscriptions are true, or they are both false.

   Can Alice choose a trunk being sure that she will find a treasure? If this is the case, which trunk should she open?

   Hint: Consider a propositional language where A = "Trunk AT contains the treasure" and B = "Trunk BT contains the treasure". Use this language to formalize the above statements by using propositional logic and then consider the interpretations which satisfy the formula that we know to be true.

5. (5 points) Consider the following sentences:

   (a) All actors and journalists invited to the party are late.

   (b) There is at least a person who is on time.

   (c) There is at least an invited person who is neither a journalist nor an actor.

   Formalize the sentences by using the First Order Language containing the unary predicates *actor, journalist, invited, late* with the obvious meaning (*actor(x)* means that x is an actor etc.) FOL. Is (c) a logical consequence of (a) and (b)? Motivate briefly the answer.

6. (5 points) A binary tree is either empty or it is composed of a root element and two successors, which are binary trees themselves. In Prolog we represent the empty tree by the atom `nil` and the non-empty tree by the term `t(X,L,R)` where X denotes the root node and L and R denote the left and right subtree, respectively. A leaf is a node with no successors. For example, the tree in the Figure is represented by the term `t(a,t(b,nil,nil),t(c,nil,nil))` and b and c are leaves.

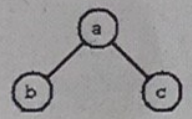

Figure 1: A tree

   Write a Prolog program which defines a predicate `count_leaves/2` to count the leaves of a tree:

   `% count_leaves(T,N) :- the binary tree T has N leaves.`

7. (4 points) Given the Prolog program

```
p(a,X,Z):-p(X,Y,Z).
p(a,b,Z):-p(a,b,Z).
p(b,c,Z):- q(Z),r(Z).
p(f(W)).
r(a).
```

   what is the result of the evaluation of the goal `p(X1, b, Z1)`? Provide a short motivation for the answer.

8. (3 points) Provide a short definition of most general unifier for two terms and explain where it is used.