

Languages and Algorithms for AI 1st Module

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Plan (1st module)

- Introduction to logic
 - Propositional logic
 - FOL
 - Resolution
 - Unification
- Logic programming
 - Procedural interpretation
 - Declarative interpretation (?)
 - Prolog (part of)
- Constraint programming
 - CLP
 - MiniZinc



Credits

- Part of the following slides are from:
 - Russel Norvig (material for the AI book)
 - Claudio Sacerdoti Cohen (material for the logic course at UniBo, in Italian)



Logic

Good, too, Logic, of course; in itself, but not in fine weather.
Arthur Hugh Clough, 1819-1861

Die Logik muß für sich selber sorgen
Ludwig Wittgenstein, 1889-1951

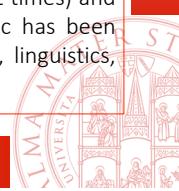


Logic in Wikipedia

Logic (from the ancient Greek Ancient Greek λογική) is the systematic study of the form of valid inference, and the most general laws of truth. A valid inference is one where there is a specific relation of logical support between the assumptions of the inference and its conclusion. In ordinary discourse, inferences may be signified by words such as therefore, thus, hence, ergo, and so on.

There is no universal agreement as to the exact scope and subject matter of logic but it has traditionally included the classification of arguments, the systematic exposition of the 'logical form' common to all valid arguments, the study of proof and inference, including paradoxes and fallacies, and the study of syntax and semantics.

Historically, logic has been studied in philosophy (since ancient times) and mathematics (since the mid-19th century), and recently logic has been studied in cognitive science (encompasses computer science, linguistics, philosophy and psychology).



Many different Logics ...

- Classical logic (propositional, FOL ...): considers truth and inference
If X and Y are true, can I deduce that also W is true?
- Intuitionistic logic: considers constructive proofs
If I can construct a proof for X and for Y, can I construct a proof also for W ?
- Linear logic: considers resources
If I have X and Y, can I exchange them for W?
- Epistemic logic: considers knowledge and belief
If I believe that you know X and Y, can I conclude that I believe W?
- Temporal logic: considers evolution in time
If X and Y will happen, can I conclude that W could happen?



... with many different uses in Computer Science and AI

Classical logic:

AI, Logic Languages, specification and verification of software

Intuitionistic logic:

construction of software correct by design

Linear logic:

resources control

Epistemic logic:

security

Temporal logic:

specification and verification of software



Logics

- Many different logics, but similar problems and techniques
- An interdisciplinary subject:
Philosophy
Mathematics
computer science
AI
- Mathematical Logic =
Application of mathematics methodology to the study of logic
but also
logic as the foundation of mathematics
(different logics => different mathematics)



Why studying logic in Computer Science

- 1) Computer Science derives from Logic
- 2) Foundation of computation and of declarative programming languages
- 3) Foundation of theoretical computer science (computability)
- 4) Proving correctness of software
- 5) Logic as a language paradox free



2) Foundation of computation and of declarative programming languages

Two main classes of programming languages: Declarative (functional and logical) and imperative

Functional languages (LISP, ML, Haskell, OCAML etc.)

Based on (composition of) functions

Strong relations with logic: programming corresponds to proving (in a suitable logic); typing correspond to enunciate theorems.

Logic Languages (Prolog, CLP, CHR, CCP ...)

Based on the notion of relation

Even stronger relations with logic: computation = deduction

Algorithm = Logic + Control



1) Computer Science derives from Logic

- Ancient mathematics (apart from geometry) had no strict formal basis
These are searched in logic and arithmetic
- Mathematical methods are applied to the study of logic
The logical derivation is identified with a computation process (calculemus!)
- Computation processes are studied with surprising results
There exist non computable functions (actually, the majority of functions)
There exists a class of computable functions that can be defined in many, completely different ways (Lambda calculus, Turing machine, logic programs, RAM)
- Computer science is born (about 1930)
- Computer science exists before and independently of computer (as a study of computability)

Computer Science is no more about computers than Astronomy is about telescopes. -- E.W. Dijkstra (1930 – 2002); Turing award 1972.



3) Foundation of theoretical computer science (computability)

Study of the limits of human knowledge

Not everything can be proved to be true or false

Not everything can be calculated

Limits unavoidable to computer science and artificial intelligence and



4) Proving correctness of software

A fundamental result:

Most properties of programs **are not decidable**

It cannot exist an automatic tool that, given a generic program (in a Turing complete language), decide whether the program is correct (**e.g. terminates**)

A possible approach: Given a program and a specification, use logic to prove that the program satisfies the specification.

Automatic proofs: only in some specific cases or approximation of properties.

Assisted proofs: the human does the proof (using logic), the computer verifies it.



5) Logic as a language paradox free

- Paradox: apparently correct reasoning (w.r.t. given rules and axioms) from apparently correct premises which allow to derive an apparently contradictory conclusion
- Natural language allows to express paradoxes, hence not appropriate as a basis for mathematics or CS
- Logic provides languages paradox free



Paradoxes

- False Paradox (find the error):
$$\begin{aligned} x=1 \Rightarrow x^2=x &\Rightarrow x^2-1=x-1 \Rightarrow (x-1)(x+1)=x-1 \Rightarrow \\ &\Rightarrow x+1=1 \Rightarrow x=0 \end{aligned}$$

- Natural language allows paradoxes
(1)

I am a liar

I am liar iff what I am saying is not true

I am liar iff "I am a liar" is not true



Paradoxes

- Natural language allows paradoxes (2)

Definition. Let x be the smallest number which cannot be defined with less than 1000 words

x is defined iff x is the smallest number which cannot be defined with less than 1000 words

x is defined (with less than 1000 words) iff x is not defined (with less than 1000 words)



Why paradoxes

Why these paradoxes?

1. Metalinguistic use of natural language
2. Self-application of a meta linguistic concept

Meta-linguistic: of or relating to a metalanguage

Metalanguage: a language used to talk about language, something about the language



How to avoid paradoxes

How to solve these problem

1. Abandon natural language
2. Use suitable language from mathematical logic



Paradoxes in mathematics

Also mathematic language allows paradoxes

Russel Paradox: Let $X = \{Y \mid Y \notin Y\}$

Then $X \in X$ iff $X \notin X$



Solving the Russel paradox

How to solve Russel paradox:

$\{Y \mid Y \notin Y\}$ is not a set

- Not allowed to use the notion of set in a meta-linguistic way, i.e., the collection of all sets is not a set.
- Sets can be constructed only selecting elements from sets (so $\{Y \in \text{class of all sets} \mid Y \notin Y\}$ is not a set, but a class, hence it does not contain itself).



Paradoxes in computer science

Many programming languages allow higher order functions (A function that takes as input another function)

Meta-linguistic features of programming languages
Auto-application possible

Paradoxes not avoidable



Paradoxes in computer science

Many programming languages allow higher order functions (A function that takes as input another function)

Meta-linguistic features of programming languages
Auto-application possible

Paradoxes not avoidable



Paradoxes in CS 1

Define a function

$$f(g) = \text{not } (g(g))$$

Then $f(f) = \text{not } (f(f))$

Functions computed by programs are not only total !



Paradoxes in CS 2

Assume that a program f exists such that, given as input a program g and an argument x :

$$f(g,x) = \text{true iff } g(x) \text{ terminate}$$

$$f(g,x) = \text{false iff } g(x) \text{ does not terminate}$$

Define

$h(g) = \begin{cases} f(g,g) = \text{true then} \\ \quad \text{while true do skip} \\ \quad \text{else return 0} \end{cases}$

Then

$h(h)$ terminates iff $f(h,h) = \text{false iff } h(h) \text{ does not terminate}$

So f cannot exist! Termination is not decidable



Paradoxes in CS 3

Consider the set T of all possible values of a programming language.

T contains at least booleans and functions.

Assume also that a function in our programming language is a mathematical function. Then

$$T = \{0,1\} \cup T^T$$

Absurd: $|T| < 2 + |T^T|$

Consequence: any programming language cannot express all mathematical functions!



Cantor diagonalization

Theorem. Assume that T contains at least two elements.

$$\text{Then } |T| < |T^T|.$$

Proof.

Obviously it cannot be $|T| > |T^T|$ hence it suffices to prove that $|T| \neq |T^T|$.

Reduction ad absurdum. Assume that $|T| = |T^T|$, then there exists a bijection $g: T \rightarrow T^T$.

Define $f(t) = t'$ such that $t' \neq g(t)(t)$, with $t \in T$ and $f \in T^T$.

Then for each $t \in T$ we have $f \neq g(t)$, since $f \in T^T$ this means that g is not a bijection.



Logic to avoid paradoxes

Curry's paradox

If this sentence is true, then Santa Claus exists *

Call the above sentence A and "then Santa Claus exists" B

To "prove" that A is true:

Assume that the premise of A is true, that is "this sentence is true" is true. We must prove that B is true.

Assuming that the premise of A is true, means to assume that $A \rightarrow B$ is true and then, by modus ponens, B is true. Hence A is true.

Since A is true, and A is the premise in the implication *



Curry's paradox in a logic setting

1. $X := (X \rightarrow Y)$ assumption, the starting point, equivalent to "If this sentence is true, then Y"

2. $X \rightarrow X$ (law of identity)

3. $X \rightarrow (X \rightarrow Y)$ substitute right side of 2, since X is equivalent to $X \rightarrow Y$ by 1

4. $X \rightarrow Y$ from 3 by contraction

5. X substitute 4, by definition in 1

6. Y from 5 and 4 by modus ponens

So a paradox again ...?

Depends on 1, in many logics if Y is false 1 is not possible!



Why studying logic in AI

- a) A tool for expressing human reasoning
- b) Foundation of a classic approach to AI (see later, and see also the Foundations course)
- c) Foundation of (declarative) languages used in AI (see this course)
- d) Direct application in some programming languages used in AI (e.g. PROLOG and CLP, see this course)

What is AI?

Systems that think like humans	Systems that think rationally
Systems that act like humans	Systems that act rationally

Some definitions of AI (Russel and Norvig)

"The exciting new effort to make computers think ... <i>machines with minds</i> , in the full and literal sense." (Haugeland, 1985)	"The study of mental faculties through the use of computational models." (Charniak and McDermott, 1985)
"[The automation of] activities that we associate with human thinking, activities such as decision-making, problem solving, learning ..." (Bellman, 1978)	"The study of the computations that make it possible to perceive, reason, and act." (Winston, 1992)
Acting Humanly "The art of creating machines that perform functions that require intelligence when performed by people." (Kurzweil, 1990) "The study of how to make computers do things at which, at the moment, people are better." (Rich and Knight, 1991)	Acting Rationally "Computational Intelligence is the study of the design of intelligent agents." (Poole <i>et al.</i> , 1998) "AI ... is concerned with intelligent behavior in artifacts." (Nilsson, 1998)

But before the current views some history ...

- Analogously to Computer Science, AI exists before Computers!

René Descartes 1596-1650

Influenced by the automats on display throughout the city of Paris, began to investigate the connection between the mind and body. Dualism:

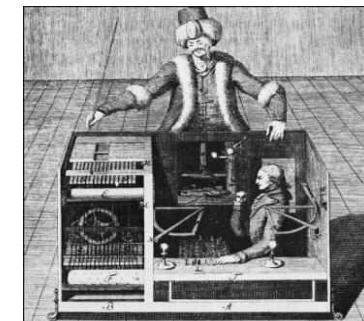
- body works like a machine, that it has material properties.
- mind is not material and does not follow the laws of nature.
- mind interacts with the body at the pineal gland

Cartesio, can be considered a precursor of AI because he considered the constructions of automaton an important cognitive process.¹

1) Bruno G. Bara, Scienza cognitiva. Un approccio evolutivo alla simulazione della mente, Bollati Boringhieri, Torino 1982



AI ante litteram ... fake !



An image from
18th century



AI (and CS) ante litteram in the XVIII century

- G.W. Leibniz (1646-1716) Calculus raziocinator.
- Jacques de Vaucanson(1709-1782) built an artificial duck that could swim and "eat" grain (400 parts in each wing)
- Pierre Jacquet-Droz (1721–1790) swiss-born watchmaker who designed and built animated dolls and automata including a writer and a musician
 - Movements controlled by an input coded on a metallic disc !

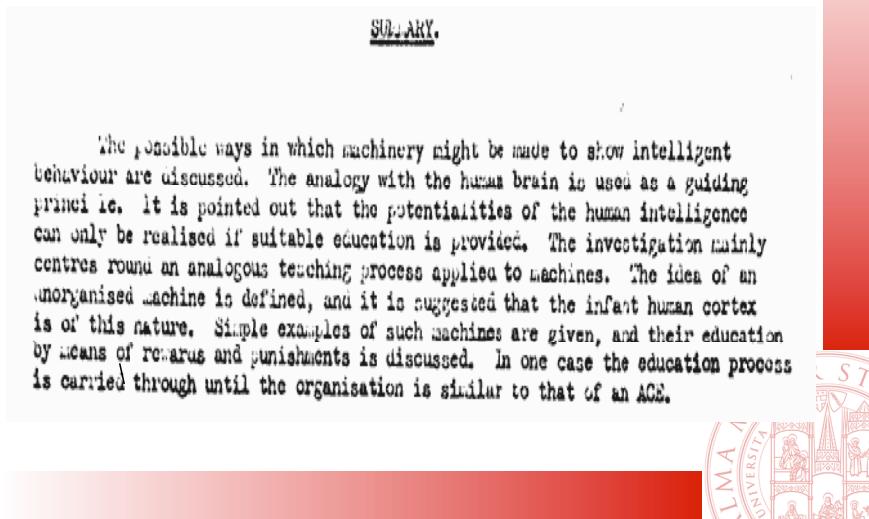


AI before AI term was coined, i.e. Turing

- 1935 Universal Turing Machine
- 1947 Talk by A. Turing in London: "**What we want is a machine that can learn from experience ... the possibility of letting the machine alter its own instructions provides the mechanism for this**".
- 1948. *Intelligent Machinery*. Never published during his life, **a real AI manifesto** contain several ideas later re-invented (including symbolic systems and neural networks).
- 1950. *Computing Machinery and Intelligence*. Including the imitation game, i.e. Turing test (we will see this later)



Intelligent Machinery (1948) http://www.alanturing.net/turing_archive/



Neurons and learning

- W. S. McCulloch and W. H. Pitts (1943). First computational model of the neuron (threshold logic)

At the basis of the research on neural networks

- D.O. Hebb (1949). Hebbian learning.

A theory in neuroscience that proposes an explanation for the adaptation of neurons in the brain during the learning process, describing a basic mechanism for synaptic plasticity neuronal basis of unsupervised learning

- F Rosenblatt (1958). Perceptron.

An algorithm for pattern recognition based on a two-layer computer learning network using simple addition and subtraction

The official start of AI: Dartmouth conference 1956

John McCarthy coined the term AI: "The study is to proceed on the basis of the conjecture that every aspect of learning or any other feature of intelligence can in principle be so precisely described that a machine can be made to simulate it".

McCarthy also invented LISP!



Trenchard More, John McCarthy, Marvin Minsky,
Oliver Selfridge, Ray Solomonoff

The four main current view of AI ...



Thinking humanly: cognitive science

Thinking humanly: imitate the brain

Why not imitate the human brain ? We do not know enough:

- Broca (1861) relation between language and specific areas in the brain
 - Golgi (1873) first visualization of neurons
 - Berger (1929) EEG
 - Ogawa et al (1990) fMRI: measurement of cognitive processes.

Today

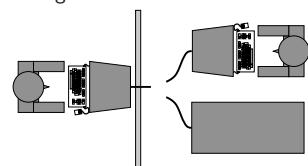
- We know some relations between areas in the brain and specific functions
 - do not know how a cognitive process works
 - do not know how memory works



Acting humanly: The Turing Test

Turing (1950) "Computing machinery and intelligence":

- ◊ "Can machines think?" → "Can machines behave intelligently?"
- ◊ Operational test for intelligent behavior: the [Imitation Game](#)



- ◊ Predicted that by 2000, a machine might have a 30% chance of fooling a lay person for 5 minutes
- ◊ Anticipated all major arguments against AI in following 50 years
- ◊ Suggested major components of AI: knowledge, reasoning, language understanding, learning

Problem: Turing test is not **reproducible**, **constructive**, or amenable to **mathematical analysis**



1960s "cognitive revolution": information-processing psychology replaced prevailing orthodoxy of **behaviorism** (this was concerned with "measures" of stimuli/answers only)

Requires scientific theories of internal activities of the brain

- What level of abstraction? "[Knowledge](#)" or "[circuits](#)"?
- How to validate? Requires
 - 1) Predicting and testing behavior of human subjects (top-down)
 - 2) Direct identification from neurological data (bottom-up)

Both approaches (roughly, [Cognitive Science](#) and [Cognitive Neuroscience](#)) are now distinct from AI

Both share with AI the following characteristic:

the available theories do not explain (or engender) anything resembling human-level general intelligence

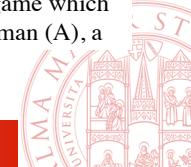


Turing, A.M. (1950). Computing machinery and intelligence. *Mind*, 59, 433-460.

1. The Imitation Game

I propose to consider the question, "Can machines think?" This should begin with definitions of the meaning of the terms "machine" and "think." The definitions might be framed so as to reflect so far as possible the normal use of the words, but this attitude is dangerous. If the meaning of the words "machine" and "think" are to be found by examining how they are commonly used it is difficult to escape the conclusion that the meaning and the answer to the question, "Can machines think?" is to be sought in a statistical survey such as a Gallup poll. But this is absurd. Instead of attempting such a definition I shall replace the question by another, which is closely related to it and is expressed in relatively unambiguous words.

The new form of the problem can be described in terms of a game which we call the 'imitation game.' It is played with three people, a man (A), a



Thinking rationally: Laws of thought, that is Logic

Normative (or prescriptive) rather than descriptive

Aristotle: what are correct arguments/thought processes?

Several Greek schools developed various forms of logic:

notation and rules of derivation for thoughts;
may or may not have proceeded to the idea of mechanization

Direct line through mathematics and philosophy to modern AI

Problems:

- 1) Not all intelligent behavior is mediated by logical deliberation
- 2) What is the purpose of thinking? What thoughts **should** I have out of all the thoughts (logical or otherwise) that I **could** have?



Acting rationally

Rational behavior: doing the right thing

The right thing: that which is expected to maximize goal achievement, given the available information

Doesn't necessarily involve thinking—e.g., blinking reflex—but thinking should be in the service of rational action

Aristotle (Nicomachean Ethics):

Every art and every inquiry, and similarly every action and pursuit, is thought to aim at some good

