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Iniziato	venerdì, 17 luglio 2020, 12:19
Stato	Completato
Terminato	venerdì, 17 luglio 2020, 13:19
Tempo impiegato	1 ora
Valutazione	Non ancora valutato

Domanda **1**
Completo
Punteggio max.: 6,00

Construct a deterministic TM of the kind you prefer, which decides the following language:

$$\mathcal{L} = \{w \in \{0,1\}^* \mid \text{between any two occurrences of } 1 \text{ in } w\text{, there is an odd number of } 0\text{s}$$

Study the complexity of TM you have defined.

3-Tape TM

R means RIGHT
S means STOP

Alphabet A = {0, 1, START, BLANK}
States Q = {Q_INIT, Q_0, Q_1, Q_HALT}

Transition function delta:
(Q_INIT, START) -> (Q_0, START, R)
(Q_0, 0) -> (Q_0, 0, R)
(Q_0, BLANK) -> (Q_HALT, BLANK, S) # success (write 1 into the output state)
(Q_1, 1) -> (Q_1, 1, R)
(Q_1, 0) -> (Q_1, 0, R) # here I am counting the zero's and I use the function Increment(n) that increment in the second tape the number of zeros.
(Q_1, 1) -> (Q_0, 1, R) # here if I encounter a 1, then I check in the tape where I was storing the counter for the zero's whether the number is odd (this can be done just by look at the last number. If it is 1, it means that it is odd). Also, in this case I reset the tape setting all the numbers to zeros.
(Q_1, 1) -> (Q_HALT, 1, R) # fail (write 0 into the output state). In this case I do the same process as before, but in case the zero's are even, then the TM halts and it fails.
(Q_1, BLANK) -> (Q_HALT, BLANK, R) # fail (write 0 into the output state)

TM = {A, Q, delta}

The heads of the TMs do not return at the start.
The time complexity of my TM is linear, thus is in P.
The operations that I do is to look at all the elements of the input string. This costs O(n) with n the size of the input.
Moreover, I use the function Increment(n) that we have studied to cost O(2n).
Also, the operations I do is to check whether the string for the counter of zero's, this costs O(n).
To reset the counting zeros - tape the cost is O(n).

Domanda **2**

Completo

Punteggio max.: 7,00

You are required to prove that the following function f is in $\mathbf{FEXP}$. To do that, you can give a TM or define some pseudocode. The function is the is the one that, given a natural number n, returns the prime factors of n.

To compute all prime factors I reason like the follow:

1-while n is divisible by 2, output 2 and divide n by 2.
2- after step 1, n must be odd. Now start a loop from i = 3 to square root of n. While i divides n, output i and divide n by i. After i fails to divide n, increment i by 2 and continue.
3- if n is a prime number and is greater than 2, then n will not become 1 by above two steps. So output n if it is greater than 2.

```
# outputs the number of two's that divide n
while(n % 2 == 0):
      output 2,
      n = n / 2

# n must be odd at this point. So a skip of 2 ( i = i + 2) can be used
for i in range(3, int(squareRoot(n))+1 ,2):
      # while i divides n , print i ad divide n
      while ((n % i) == 0):
         output i,
         n = n / i

# condition if n is a prime number greater than 2
f n > 2:
      output n
```

The complexity of the algorithm is bound to an exponential bound.
To proof that, let k denote the length of the input value n.
Since k=log_2(n), a complexity of O(squareRoot(n)) is equivalent to O(squareRoot(2^k)). Since squareRoot(2^k) = 2^(k/2), this complexity is obviously exponential in terms of input length.

Domanda **3**

Completo

Punteggio max.: 7,00

Consider the following problem:

$$\mathtt{VARSAT} = \{(F, k) \mid F \text{ is a 3-CNF for which there is an assignment satisfying at least } k \text{ c}$$

To which complexity class does $\mathtt{VARSAT}$ belong? Prove your claim.

VARSAT can be reduced to 3SAT (which is the language of all satisfiable 3CNF formulae) because, even though we know that is an assignment satisfying at least k clauses of F, this does not tell us anything about the other clauses. So this is reducible to solve the 3SAT problem which by Cook-Levin Theorem we know that 3SAT is NP-Complete. Thus, VARSAT is NP-Complete too.

Commento:
Everything is OK, except that it's 3SAT which can be reduced to VARSAT.

◄ LAAI - 17072020 - Questions

Vai a...

Exam June 3, 2020 ►